AI1110
Assignment 11

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CS21BTECH11059

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Outline

question 9.33

Find $S(\omega)$ if a) $R(\tau) = e^{-}\alpha\tau^{2}$ b) $R(\tau) = e^{-}\alpha\tau^{2}\cos\omega_{0}$

first Part

a) take -iω = s for simplicity

$$\int_{-\infty}^{\infty} e^{-\alpha\tau^2} e^{-s\tau} d\tau \tag{1}$$

$$= e^{\frac{s^2}{4\alpha}} \int_{-\infty}^{\infty} e^{-\alpha(\tau+\frac{s}{2\alpha})^2} \tau \tag{2}$$

$$= \sqrt{\frac{\pi}{\alpha}} \times e^{\frac{s^2}{4\alpha}} \tag{3}$$

$$= \sqrt{\frac{\pi}{\alpha}} \times e^{\frac{-\omega^2}{4\alpha}} \tag{4}$$

second Part-1

b)

$$= \int_{-\infty}^{\infty} e^{-\alpha\tau^2} e^{-s\tau} \cos\omega_0 d\tau \tag{5}$$

$$= \frac{\int_{-\infty}^{\infty} e^{-\alpha\tau^2 - s\tau}(e^{i\omega_0\tau} + e^{-i\omega_0\tau})}{2} d\tau \tag{6}$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} e^{-\alpha\tau^2 - s\tau + i\omega_0\tau} dx + \frac{1}{2}\int_{-\infty}^{\infty} e^{-\alpha\tau^2 - s\tau - i\omega_0\tau} dx \tag{7}$$

$$\tag{8}$$

first let us solve $\int_{-\infty}^{\infty} e^{-\alpha\tau^2 - s\tau + i\omega_0\tau} dx$

second part-2

$$\int_{-\infty}^{\infty} e^{(\sqrt{-\alpha}x+\frac{i\omega+i\omega_0}{2\sqrt{-\alpha}})^2+\frac{(i\omega+i\omega_0)^2}{4\alpha}} \tag{9}$$

$$\tag{10}$$

let u = $\frac{-2kx+i\omega+i\omega_0}{2\sqrt{k}}$ So $\frac{du}{dx}$ = - $\sqrt{k}$

$$= -\int_{-\infty}^{\infty} \frac{e^{-\frac{\sqrt{-k}u}{\sqrt{k}}+\frac{(i\sqrt{-k}(\omega+i\omega_0))^2}{4k}}}{\sqrt{k}} \tag{11}$$

$$= \frac{\sqrt{\pi}}{2\sqrt{k}} e^{\frac{-(\omega+i\omega_0)^2}{4k}} \int_{-\infty}^{\infty} 2\frac{e^{-u^2}}{\sqrt{k}} du \tag{12}$$

$$= \int_{-\infty}^{\infty} 2\frac{e^{-u^2}}{\sqrt{\pi}} du = 1 \tag{13}$$

second Part-3

Therefore total integral is :

$$= \frac{\sqrt{\pi}}{2\sqrt{k}} e^{\frac{-(\omega + i\omega_0)^2}{4k}} + \frac{\sqrt{\pi}}{2\sqrt{k}} e^{\frac{-(\omega - i\omega_0)^2}{4k}} \tag{14}$$