Assignment-2

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Q) if a+ib = $\frac{x+iy}{x-iy}$
prove that a^2+b^2 = 1 and $\frac{b}{a} = \frac{2xy}{x^2-y^2}$

I. Solution

let us solve the equation
The equation is given by

$$a+ib = \frac{x+iy}{x-iy} \tag{1}$$

$$\tag{2}$$

let r be the magnitude of the given complex number a+ib

$$\tag{3}$$

so r= $\sqrt{a^2+b^2}$

$$\tan\theta = \frac{b}{a}, let x+iy = s(\cos\alpha + i\sin\alpha) \tag{4}$$

$$and \tan\alpha = \frac{y}{x} \tag{5}$$

r($\cos\theta$+isin θ)=$\frac{\cos\alpha+i\sin\alpha}{\cos\alpha-i\sin\alpha}$

$$\tag{6}$$

multiplying denominator on both numerator and denominator we get

$$r(\cos\theta+i\sin\theta) = \cos^2\alpha - \sin^2\alpha + 2i\sin\alpha\cdot\cos\alpha \tag{7}$$

$$r(\cos\theta+i\sin\theta) = \cos 2\alpha + i\sin 2\alpha \tag{8}$$

$$\tag{9}$$

magnitude of complex number on RHS is 1 so definitely magnitude on LHS is 1

$$\implies r = 1 \tag{10}$$

so comparing both LHS and RHS $\theta = 2\alpha$
applying $\tan$ on both sides, we get $\tan\theta = \tan 2\alpha$

$$\frac{b}{a} = \frac{2\tan\alpha}{1-\tan^2\alpha} \tag{11}$$

$$\frac{b}{a} = \frac{2y/x}{1-y^2/x^2} \tag{12}$$

$$\implies \frac{b}{a} = \frac{2xy}{x^2-y^2} \tag{13}$$