Assignment-2

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Question 1-(xi)[ICSE 2017]:

Problem 1. *if a+ib = $\frac{x+iy}{x-iy}$ prove that a^2+b^2 = 1 and $\frac{b}{a} = \frac{2xy}{x^2-y^2}$*

Solution:

$$a+ib = \frac{x+iy}{x-iy} \tag{1}$$

let r be the magnitude of the given complex number a+ib

so r= $\sqrt{a^2+b^2}$

$$\tan\theta = \frac{b}{a} \tag{2}$$

$$x+iy = s(\cos\alpha + i\sin\alpha) \tag{3}$$

$$\tan\alpha = \frac{y}{x} \tag{4}$$

r$(\cos\theta$+i$\sin\theta)$=$\frac{\cos\alpha+i\sin\alpha}{\cos\alpha-i\sin\alpha}$

multiplying denominator on both numerator and denominator we get

r$(\cos\theta + i\sin\theta)$ = $\cos^2\alpha$-$\sin^2\alpha$ + 2i$\sin\alpha\cdot\cos\alpha$

$$r(\cos\theta + i\sin\theta) = \cos 2\alpha + i\sin 2\alpha \tag{5}$$

$$\implies r = 1 \tag{6}$$

magnitude of complex number on RHS is 1 so definitely magnitude on LHS is 1

so comparing both LHS and RHS $\theta = 2\alpha$

applying $\tan$ on both sides, we get $\tan\theta = \tan 2\alpha$

$$\frac{b}{a} = \frac{2\tan\alpha}{1-\tan^2\alpha} \tag{7}$$

$$\frac{b}{a} = \frac{2y/x}{1-y^2/x^2} \tag{8}$$

$$\implies \frac{b}{a} = \frac{2xy}{x^2-y^2} \tag{9}$$