

Assignment-2

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Question 1-(xi)[ICSE 2017]:

Problem 1. if $a+ib = \frac{x+iy}{x-iy}$
prove that $a^2 + b^2 = 1$ and $\frac{b}{a} = \frac{2xy}{x^2 - y^2}$

Solution:

$$a + ib = \frac{x + iy}{x - iy} \quad (1)$$

let r be the magnitude of the given complex number $a+ib$

so

$$r = \sqrt{a^2 + b^2} \quad (2)$$

$$\tan \theta = \frac{b}{a} \quad (3)$$

$$x + iy = r(\cos \alpha + i \sin \alpha) \quad (4)$$

$$\tan \alpha = \frac{y}{x} \quad (5)$$

$$r(\cos \theta + i \sin \theta) = \frac{\cos \alpha + i \sin \alpha}{\cos \alpha - i \sin \alpha} \quad (6)$$

multiplying denominator on both numerator and denominator we get

$$\cos^2 \alpha - \sin^2 \alpha + 2i \sin \alpha \cdot \cos \alpha \quad (7)$$

on RHS

$$r(\cos \theta + i \sin \theta) = \cos 2\alpha + i \sin 2\alpha \quad (8)$$

$$\Rightarrow r = 1 \quad (9)$$

magnitude of complex number on RHS is 1 so
definitely magnitude on LHS is 1

so comparing both LHS and RHS

$$\theta = 2\alpha \quad (10)$$

applying tan on both sides, we get

$$\tan \theta = \tan 2\alpha \quad (11)$$

$$\frac{b}{a} = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \quad (12)$$

$$\frac{b}{a} = \frac{2y/x}{1 - y^2/x^2} \quad (13)$$

$$\Rightarrow \frac{b}{a} = \frac{2xy}{x^2 - y^2} \quad (14)$$