Assignment-2

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Question 1-(xi)[ICSE 2017]:

Problem 1. *if* $a+ib = \frac{x+iy}{x-iy}$ *prove that* $a^2+b^2 = 1$ *and* $\frac{b}{a} = \frac{2xy}{x^2-y^2}$

Solution:

$$a+ib = \frac{x+iy}{x-iy} \tag{1}$$

let r be the magnitude of the given complex number a+ib
so

$$r = \sqrt{a^2+b^2} \tag{2}$$

$$\tan\theta = \frac{b}{a} \tag{3}$$
$$x+iy = s(\cos\alpha + i\sin\alpha) \tag{4}$$
$$\tan\alpha = \frac{y}{x} \tag{5}$$

$$r(\cos\theta + i\sin\theta) = \frac{\cos\alpha + i\sin\alpha}{\cos\alpha - i\sin\alpha} \tag{6}$$

multiplying denominator on both numerator and denominator we get

$$\cos^2\alpha - \sin^2\alpha + 2i\sin\alpha\times\cos\alpha \tag{7}$$

on RHS

$$r(\cos\theta + i\sin\theta) = \cos 2\alpha + i\sin 2\alpha \tag{8}$$
$$\implies r = 1 \tag{9}$$

magnitude of complex number on RHS is 1 so definitely magnitude on LHS is 1
so comparing both LHS and RHS

$$\theta = 2\alpha \tag{10}$$

applying $\tan$ on both sides, we get

$$\tan\theta = \tan 2\alpha \tag{11}$$

$$\frac{b}{a} = \frac{2\tan\alpha}{1-\tan^2\alpha} \tag{12}$$
$$\frac{b}{a} = \frac{2y/x}{1-y^2/x^2} \tag{13}$$
$$\implies \frac{b}{a} = \frac{2xy}{x^2-y^2} \tag{14}$$