AI1110 ASSIGNMENT-9

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***Abstract*—This document contains the solution for Assignment 9 (papoulis question-4.13)**

QUESTION 4.13 :

A fair coin is tossed three times and the random variable x equals the total number of heads. Find and sketch $F_x(x)$ and $f_x(x)$.

Solution : let x be a random variable which maps to 1 when coin denotes head and 0 when it denotes tail.

Event	Description
$X = 0$	probability of getting 0 heads
$X = 1$	probability of getting 1 heads
$X = 2$	probability of getting 2 heads
$X = 3$	probability of getting 3 heads

TABLE I
EVENTS AND DESCRIPTION

probability of getting r heads is $\Pr(X = k) = \binom{n}{k} \times p^k \times (1-p)^k$ so

$$\Pr(X = 0) = \binom{3}{0} \times \frac{1}{2}^0 \times (1 - \frac{1}{2})^3 = \frac{1}{8} \tag{1}$$

$$\Pr(X = 1) = \binom{3}{1} \times \frac{1}{2}^1 \times (1 - \frac{1}{2})^2 = \frac{3}{8} \tag{2}$$

$$\Pr(X = 2) = \binom{3}{2} \times \frac{1}{2}^2 \times (1 - \frac{1}{2})^1 = \frac{3}{8} \tag{3}$$

$$\Pr(X = 3) = \binom{3}{3} \times \frac{1}{2}^3 \times (1 - \frac{1}{2})^0 = \frac{1}{8} \tag{4}$$

the $F_x(x)$ i.e PMF is given by :

$$\begin{cases} 0, & k < 0 \\ \frac{1}{8}, & k = 0 or 3 \\ \frac{3}{8}, & k = 1 or 2 \end{cases} \tag{5}$$

the $f_x(x)$ CDF is given by :

$$\begin{cases} 0, & k < 0 \\ \frac{1}{8}, & k = 1 \\ \frac{1}{2}, & k = 2 \\ \frac{7}{8}, & k = 3 \\ 1, & k > 3 \end{cases} \tag{6}$$

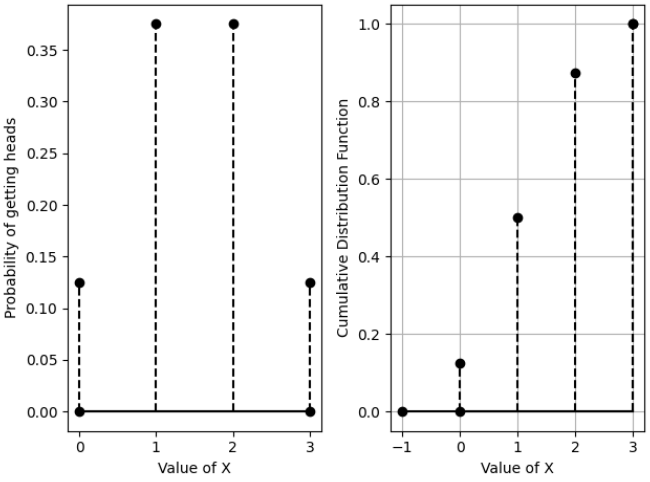

Fig. 0. Plot of the PMF (left) and CDF (right) of an unbiased die.