Discrete Mathematics

UNIT-1:

Foundation: Basics, sets and operations of sets, fundamentals of logic, logical interferences, first order logic and other methods of proofs, Rule of interference for Quantified proposition.

Text books:

1. Joe L. Mott, Abraham kandel, theodare P. Baker, "Discrete Mathematics for example computer scientists and Mathematicians", second Edition, PH, 2019

Set: A set is a well defined, unordered collection of distinct elements of same type.

→ Sets are denoted by capital letters & Elements are written in curly braces & they are seperated by commas (,).

→ Elements are denoted by small letters.

Ex: 1) $A = \{1, 2, 3, 4, 5\}$

2) $B = \{a, e, i, o, u\}$

Ex: $A = \{1, 2, 3, 4, 5\}$ = $\{x \in A / x \in N \text{ \& } x \leq 5\}$.

Roster form — Tabulation form

Cardinality: number of elements in a given set.

Ex: 1) $A = \{1, 2, 3, 4, 5, 8, 9\}$

cardinality of set A is $|A| = 7$

Example 1:

$6 \notin A$ doesn't belongs to A

$5 \in A$ belongs to A

Representation of set:

1) Builder form (Rule method)

2) Tabulation Method (Roster method)

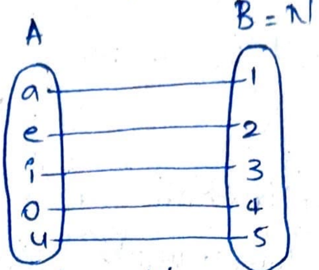
Ex: Description	Set-Builder form	Tabulation form
1) The set of all vowels in English alphabets	$A = \{x \in A / x \in$ vowels in english Alphabets$\}$	$A = \{a, e, i, o, u\}$

2) The set of all prime no's less then 10 — $A = \{x \in A / x \in$ prime no & $x \leq 10\}$ — $A = \{2, 3, 5, 7\}$

Countable set:

If there is a one to one mapping b/w the elements of a set and natural no's.

$A = \{a, e, i, o, u\}$



uncountable set:

If there is no one to one mapping b/w the elements of a set and natural no's.

Empty set or null set:

A set which is containing zero elements.

> denoted by ϕ or $\{\ \}$

* Cardinality of Empty set is 'zero'.

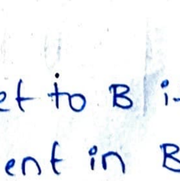
Subset:

Let A & B be two sets then A is said to be subset of B if every element of set A is a element of B.

Ex: $A = \{1, 2, 3, 4, 5\}$

$B = \{1, 2, 3, 4, 5, 6\}$

$(A \subseteq B)$ A is subset of B



Venn diagram.

Proper set:

A set is said to be a proper set to B if A is subset of B & their is atleast one element in B which is not in A.

Ex: $A = \{1, 2, 3, 4, 5\}$

$B = \{1, 2, 3, 4, 5, 6\}$

A is subset of B $(A \subset B)$

Equal set:

Two set A & B are equal

If $A \subseteq B$ & $B \subseteq A$ then we write $A = B$

Power set: Given the set A suppose we construct the consisting of all subset of A. The set so obtained (constructed) is called the power set A. And it is denoted by P(A)

ex: 1) $P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

2) $A = \{1,2,3\}$

$P(A) = \{0, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.

Cardinality of a power set $= 2^n$ [where n is no. of elements in set (A)]

Operations of sets:

1) Union $(A \cup B)$
2) intersect $(A \cap B)$
3) compliment $(A' = U - A)$
4) Difference $(A - B)$
5) Symmetric difference $((A \cancel{\Delta} B) \cup (B - A))$
$(A \Delta B = (A-B) \cup (B-A))$

Universal set:

Suppose in a discussion of all sets that we consider are subset of a certain set 'U'. The set of 'U' is said to be universal set.

(OR)

A set which contains all objects under consideration.

Example:

1) $U = \{1,2,3,4,5,6,7,8,9,10\}$ find i) $A \cup B$
ii) $A \cap B$
$A = \{1,3,4,5,7,9\}$ iii) A' (Compliment of A)
$B = \{2,6,8,9,10\}$. iv) $A-B$ & $B-A$ v) $A \Delta B$

i) $A \cup B = \{1,2,3,4,5,6,7,8,9,10\}$

ii) $A \cap B = \{9\}$

iii) $A' = U - A = \{2,6,8,10\}$

iv) $A - B = \{1,3,4,5,7\}$

$B - A = \{2,6,8,10\}$

v) $A \Delta B = (A-B) \cup (B-A)$

$= \{1,2,3,4,5,6,7,8,10\}$

Ex: 2) $U = \{1,2,3,4,5,6,7,9\}$

$A = \{1,3,5,7,9\}$

$B = \{2,3,4,5\}$

1) $A \cup B = \{1,2,3,4,5,7,9\}$

2) $A \cap B = \{3,5\}$

3) $A' = U - A = \{2,4,6\}$, $U - B = \{1,6,7,9\}$

4) $A - B = \{1,7,9\}$

5) $B - A = \{2,4\}$

6) $A \Delta B = (A-B) \cup (B-A)$

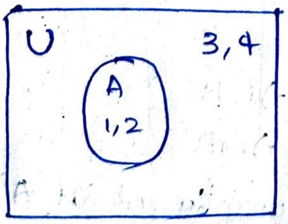
$\{1,2,4,6,7,9\}$

Set Identities:

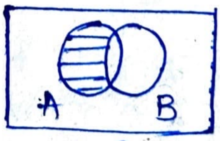
	Union	Intersection
1) Commutative	$A \cup B = B \cup A$	$A \cap B = B \cap A$
2) Identity	$A \cup \emptyset = A$	$A \cap A = A$
3) Idempotent	$A \cup A = A$	$A \cap A = A$
4) Associative	$A \cup (B \cup C) = (A \cup B) \cup C$	$A \cap (B \cap C) = (A \cap B) \cap C$
5) Distributive	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap$
6) Demorgan's	$(A \cup B)' = A' \cap B'$	$(A \cap B)' = A' \cup B'$

$U = \{1,2,3,4\}$ $A = \{1,2\}$ $A' = U - A = \{3,4\}$

Venn diagram

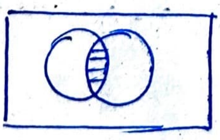


3)

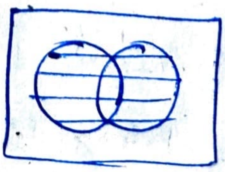


$A - B$

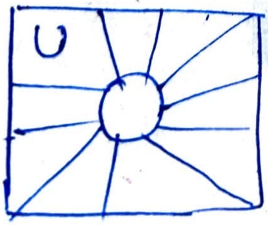
1) $A \cap B$



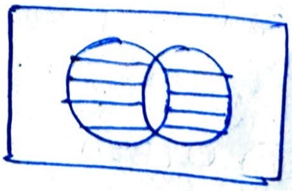
4) $A \cup B$



2) $A' = U - A$



5) $A \Delta B$



Fundamentals of logic:

A sentence which is either true or false but not both, that centance should be declaration sentence. It is also known as 'preposition'.

* Statements are denoted by small letters.

Purpose of logic: It is to construct valid arguments (Truth).

Ex: 1) Hyderabad is in Haryana (False)

2) Every rectange is a square (False)

3) 3 is a prime number (True)

→ A proposition (statement) is a declaration sentence to which it is meaning to assign one and only one of the truth value "True or false".

→ Proposition are combined by means of connectives such as "and, or, if.... then, not, if and only if".

→ these five are the main types of connectives can be defined in terms of (and, or, not) we are going to construct.

Connections:

NOT:- The negation of statement P is denoted by $\sim P$.

Truthtable:

P	$\sim P$
T	F
F	T

AND: the conjection of P & Q a is denoted by $P \wedge q$

P	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

only T & T = T all are F.

Any two proportions can be combined to form compound proposition is called compound proposition.

→ The conjection of the original proposition symbolically $P \wedge q$.

Ex:

1) p: 2 is a prime number

2) q: 3 is a prime number

$p \wedge q$ = True.

OR

The disjunction of p and q is denoted by $p \vee q$ read as p or q.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive OR:

The disjunction of P and q is denoted by $p \bar{\vee} q$

p	q	$p \bar{\vee} q$
T	T	F
T	F	T
F	T	T
F	F	F

If then: (p implies q) $p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

If and only:

The biconditional of p, q is denoted by $p \leftrightarrow q$ or $p \rightarrow q$
$p \leftarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

construct the truth tables for the following compound propositions:

i) $p \wedge (\sim q)$ ii) $\sim p \vee q$ iii) $p \rightarrow \sim q$ iv) $\sim p \vee \sim q$

p	q	$\sim q$	$p \wedge (\sim q)$	$\sim p$	$\sim p \vee q$	$p \rightarrow \sim q$	$\sim p \vee \sim q$
T	T	F	F	F	T	F	F
T	F	T	T	F	F	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T

* 1) $(p \vee q) \wedge r$ 2) $p \vee (q \wedge r)$

p	q	r	$p \vee q$	$(p \vee q) \wedge r$	$q \wedge r$	$p \vee (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	F	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	T	F	T	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

* 1) $(p \wedge q) \rightarrow \sim r$ 2) $q \wedge (\sim r \rightarrow p)$

p	q	r	$p \wedge q$	$\sim r$	$(p \wedge q) \rightarrow \sim r$	$\sim r \rightarrow p$	$q \wedge (\sim r \rightarrow p)$
T	T	T	T	F	F	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	T	T	F
T	F	F	F	T	T	T	F
F	T	T	F	F	T	T	T
F	T	F	F	T	T	F	F
F	F	T	F	F	T	T	F
F	F	F	F	T	T	F	F

Express the following compound propositions in words:-

* p: A circle is a cone

* q: $\sqrt{5}$ is a real number.

r: Exponential series is convergent.

i) $p \wedge \sim q$ ii) $\sim p \vee q$ iii) $p \rightarrow \sim q$ iv) $\sim p \vee \sim q$

Sol:

i) A circle is a cone and $\sqrt{5}$ is not a real numbers.

ii) A circle is not a cone or $\sqrt{5}$ is a real number.

iii) If a circle is a cone then $\sqrt{5}$ is not a real numbers.

iv) A circle is not a cone or $\sqrt{5}$ is not a real numbers.

p: A circle is a cone

q: $\sqrt{5}$ is a real number

r: Exponential series is convergent.

v) $p \rightarrow (q \vee r)$

If A circle is a cone then $\sqrt{5}$ is a real number or exponential series is convergent.

vi) $\vee p \leftrightarrow q$

If a circle is not a cone then $\sqrt{5}$ is a real number &

If $\sqrt{5}$ is a real number then a circle is not a cone.

1) If ravi doesnot visit a friend in this evening then he studies in this evening.

2) If there is cricket telicast in this evening then ravi doesnot study & doesnot visit a friend this evening.

3) If there is no cricket telicast this evening (or) ravi doesnot study & doesnot visit a friend this evening.

p: Ravi does not study this evening

q: Ravi visit a friend this evening $\sim q \rightarrow \sim p$

r: There is a cricket telicast the evening.

Sol: 1) $\sim p \rightarrow \sim q$

2) $r \rightarrow (p \wedge \sim q)$

3) $\sim r \rightarrow (p \wedge q)$

4) $\sim r \vee (p \wedge \sim q)$

Tautology:

Some propositions contains only 'T' in the last column of their truth table are called tautology (or) tantologies

Ex:

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Contraduction:

Some propositions contains only 'F' in the last column of their truth table are called contraduction.

Ex:

p	$\sim p$	$p \vee (\sim p)$
T	F	F
F	T	F

Contingency: A statement which is neither Tautology is nor contraduction is called contingency.

Show that for any propositions p & q

ii) $(p \bar{\vee} q) \vee (p \leftrightarrow q)$ is a tautology.

p	q	$p \bar{\vee} q$	$p \leftrightarrow q$	$(p \bar{\vee} q) \vee (p \leftrightarrow q)$
T	T	F	T	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	T

∴ Given expression represents Tantology.

iii) $p \wedge (\sim p \wedge q)$ is a contraduction:-

p	q	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

∴ Given exp. represente contraduction

Show that for any propositions p and q

iii) $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is a tautology.

p	q	r	$q \to r$	$p \to (q \to r)$	$p \to q$	$p \to r$	$[(p \to q) \to (p \to r)]$
T	T	T	T	T	T	T	
T	T	F	F	F	T	F	
T	F	T	T	T	F	T	
T	F	F	T	T	F	F	
F	T	T	T	T	T	T	
F	T	F	F	T	T	T	
F	F	T	T	T	T	T	
F	F	F	F	T	T	T	

Logical Equivalence: ($\Leftrightarrow$)

Two propositions p & q are said to be logical equivalence whenever p & q have identical Truth values or equivalently whenever the biconditional $p \Leftrightarrow q$ is tautology.

Ex: Prove that for any propositions p & q $(p \to q) \leftrightarrow (\sim p \vee q)$ is logically equivalent.

p	q	$p \to q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\therefore$ LHS $\Leftrightarrow$ RHS

Given proposition are logically equivalent.

Q) PT for any p, q & r $(p \vee q) \to r \Leftrightarrow (p \to r) \wedge (p \to r)$ is logically equivalent.

p	q	r	$p \vee q$	$p \to r$	$(p \vee q) \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

∴ logically equivalent LHS $\Leftrightarrow$ RHS

3) $p \wedge (\sim q \vee r)$ and $p \vee (q \wedge \sim r)$

p	q	r	$\sim q$	$\sim q \vee r$	$\sim r$	$q \wedge \sim r$	$p \vee (q \wedge \sim r)$	$p \wedge (\sim q \vee r)$
T	T	T	F	T	F	F	T	T
T	T	F	F	F	T	T	T	F
T	F	T	T	T	F	F	T	T
T	F	F	T	T	T	F	T	T
F	T	T	F	T	F	F	F	F
F	T	F	F	F	T	T	T	F
F	F	T	T	T	F	F	F	F
F	F	F	T	T	T	F	F	F

LHS $\not\Leftrightarrow$ RHS

Converse, inverse, contrapositive:

1) The converse of $p \to q$ is the proposition of $q \to p$

2) Inverse: The inverse of $p \to q$ is the propositional $\sim p \to \sim q$

3) contrapositive: The contrapositive of $p \to q$ is $\sim q \to \sim q$

Ex: p: a is an integer

q: q is a multiple of 3

$p \to q$: If 2 is an integer then a is a multiple of 3

$q \to p$: If q a is a multiple of 3 then 2 is an Integer.

$\sim p \to \sim q$: If 2 is not an integer then a is not a multiple of 3.

$\sim q \to \sim p$: If a is not a multiple of 3 then 2 is not an Integer

Ex: 2 :-

→ state the converse, inverse, contrapositive of the following

1) If ΔABC is a rightangle triangle then $|AB|^2 + |BC|^2 = |AC|^2$

P: ΔABC is a rightangle triangle.

q: $(AB)^2 + (BC)^2 = (AC)^2$

converse: $(q \to p)$: If $(AB)^2 + (BC)^2 = (AC)^2$ then ΔABC is a rightangle triangle.

Inverse: $(\sim p \to \sim q)$: If ΔABC is not a rightangle triangle then $|AB|^2 + |BC|^2 \neq |AC|^2$.

Contrapositive: $(\sim q \to \sim p)$

If $|AB|^2 + |BC|^2 \neq AC^2$ then ΔABC is not a rightangle triangle.

Inferences:-

Rules of Inferences: 1) Are the templates for constructing valid arguments (or) Deriving conclusions from premises (statements).

Argument: sequence of statements that end with a conclusion (or) set of 1 or more premises and a conclusion

Ex1:- If I love dog (p) then I love cat (q)

I love dog

Therefore I love cat

$p \to q$

$\therefore q$

2) Consider set of propositions $(p_1, p_2, \ldots p_n)$ and q then a compound proposition of the form $(p_1 \wedge p_2 \cdots \wedge p_n) \to q$ is called an argument.

$$[(p \to q) \wedge p] \to q$$

8 rules for Inferences:

Rules		Tautology
1) Modus Ponens	p $p \to q$ $\therefore q$	$(p) \wedge (p \to q) \to q$

2. Modus Tollen's
$$\begin{array}{l} p \to q \\ \sim q \\ \hline \therefore \sim p \end{array}$$
$(p \to q) \wedge (\sim q) \to \sim p$

3. Transitive * rule of syllogism (Hypothetical)
$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$
$(p \to q) \wedge (q \to r) \to (p \to r)$

4. disjunction * syllogism
$$\begin{array}{l} p \vee q \\ \sim p \\ \hline \therefore q \end{array}$$
$(p \vee q) \wedge (\sim p) \to q$

5. Addition
$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$
$p \to (p \vee q)$

6. simplification
$$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array} \text{ (or) } \begin{array}{l} p \wedge q \\ \hline \therefore q \end{array}$$
$(p \vee q) \to p$ (or) $(p \wedge q) \to q$

7. Conjunction
$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$
$(p \wedge q) \to (p \wedge q)$

8. Resolution
$$\begin{array}{l} p \vee q \\ \sim p \vee r \\ \hline \therefore p \vee r \end{array}$$
$(p \vee q) \wedge (\sim p \vee r) \to (q \vee r)$

9. Demorgan's law: $\sim(S \wedge D)$
$(\sim S) \vee (\sim D)$

$$\begin{array}{c} \sim(p \wedge q) \\ \hline \sim p \vee \sim q \end{array}$$

Demorgan's laws

Ex:- 1.

Premises:- "Charan works hard"

"If charan works hard, then he is a dull boy". and "If charan is a dull boy, then he will not get the job".

Conclusion: "Charan will not get the job".

Sol: Let, p: Charan works hard

q: Charan is a dull boy

r: Charan will get the job.

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \text{ (Using M.P)} \end{array}$$

$$\begin{array}{l} q \to \sim r \\ \therefore \sim r \ [\because \text{MP}] \text{ given} \end{array}$$

$\therefore$ Given statements are valid.

Rule P: A premise may be introduced at any point in the derination.

Rule T:- A formula S may be introduced in a derivation If S is a tautology implied by any one or more of the preceding formulas in the derivation.

Ex 1) Demonstrate R is a valid inference from the premises.

$P \to Q,\ Q \to R,\ P$

step	statement & formula	Rule	Justification	Premises.
1)	$P \to Q$	P		{1} {2}
2)	$Q \to R$	P		{1,2}
3)	$P \to R$	T	(1),(2) Transitive	{3}
4)	P	P		
5)	R	T	(3), (4) modus ponens	{1,2,3}

(or)

1)	$P \to Q$	P
2)	P	P
3)	Q	T
4)	$Q \to R$	P
5)	R	T

Ex:- 2) Show that $(R \vee S)$ follows logically from the premises $C \vee D$, $(C \vee D) \to \sim H$, $\sim H \to (A \wedge \sim B)$ and $(A \wedge (\sim B)) \to R \vee S$

Premises	s.no	Statement & Formula	Rule	Justification
{1}	(1)	$C \vee D$	P	
{2}	(2)	$(C \vee D) \to \sim H$	P	
{1,2}	(3)	$\sim H$	T	(1),(2) Modus Ponens
{3}	(4)	$\sim H \to (A \wedge \sim B)$	P	
{1,2,3}	(5)	$(A \wedge (\sim B))$	T	(3),(4) Modus Ponens
{4}	(6)	$(A \wedge \sim B) \to R \vee S$	P	
{1,2,3,4}	(7)	$R \vee S$	T	(5,6) Modus ponens

∴ Given arguments are logically valid.

3) Show that $\sim Q,\ P \to Q \Rightarrow \sim P$ using rules of Inferences.

premises	s.no	Statement & Formula	Rule	Justification.
{1}	(1)	$P \to Q$	P	
{2}	(2)	$\sim Q$	P	
{1,2}	(3)	$\sim P$	T	(1),(2) modus Tollen's

4) show that $(p \to \sim q) \wedge (\sim r \to p) \wedge q \Rightarrow r$

Premises	S.no	statement & Formula	Rule	Justification
{1}	(1)	$(\sim r \to p)$	P	
{2}	(2)	$(p \to \sim q)$	P	
{1,2}	(3)	$\sim r \to \sim q$	T	(1) (2) Transitivity
{3}	(4)	q	P	
{4}	(5)	r	T	(3), (4) Modus Tollens.

***
5) show that SVR is Tautologically implied by PVQ, $P \to R$, $Q \to S$.

Premises	S.No	Statement & Formula	rule	Justification.
{1}	(1)	$P \vee Q$	P	
{1}	(2)	$\sim P \to Q$	T	$(P \vee Q) \Leftrightarrow \sim P \to Q$ (1)
{2}	(3)	$Q \to S$	P	
{1,2}	(4)	$\sim P \to S$	T	(2), (3) Transtivity
{1,2}	(5)	$\sim S \to P$	T	(4) contrapositive
{3}	(6)	$P \to R$	P	
{1,2,3}	(7)	$\sim S \to R$	T	(5),(6) transtivity
{1,2,3}	(8)	$S \vee R$	T	(7) $(\sim S \to R) \Leftrightarrow (S \vee R)$

6) Test whether the following arguments valid

$p \to q$, $r \to s$, $p \vee r$, $\therefore q \vee s$

premises	s.no	statement & Formula	rule	Justification
{1}	(1)	$p \vee r$	P	
{1}	(2)	$\sim p \to r$	T	$(p \vee r) \Leftrightarrow \sim P \to r$ (1)
{2}	(3)	$r \to s$	P	

{1,2}	(4)	$\sim p \to s$	T	(2)(3) Transitivity
{1,2}	(5)	$\sim s \to p$	T	contrapositive
{3}	(6)	$p \to q$	P	
{1,2,3}	(7)	$\sim s \to q$	T	(5), (6) Transitivity
	(8)	$s \vee q$	T	$(\sim p \to r) \Leftrightarrow (p \vee r)$.

→ Check the validity of the argument.

Problem: Show that the following argument is valid.

If today is Wednesday. Then I have a test in Mathematics (or) English. If my English professor is sick, then I will not have a test in English.

Today is Wednesday and my English professor is sick.

Therefore I have a test in Mathematics.

Sol: Let,

W = Today is Wednesday

M: I have a Test in Maths.

E: I have a Test in English.

S: My English professor is sick.

∴ Given argument in symbolic form:

$$\begin{array}{ll} W \to (M \vee E) & T \\ S \to \sim E & T \\ W \wedge S & T \\ \hline \therefore M & \end{array}$$

Consider all premises are True.

1) $W \wedge S$ is true ⇒ W is True & S is True.

2) $S \to \sim E$ is true ⇒ $\sim E$ is true i.e E is false.

3) $W \to (M \vee E)$ W is already true, E is false ∴ M is true.

→ First order logic (or) Predicate logic.

Ex (i): Everyone controlled in the Anurag university has lived in a Hostel. Nithin has never lived in a Hostel

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∴ Nithin is not enrolled in the Anurag University.

Proposition logic is not enough to handle this type of arguments.

Our target is to determine whether the above mentioned argument is valid or invalid.

For this we should understand the predicate logic and first order logic.

In order to understand predicate logic we must understand the following.

1. Predicates
2. Quantifiers.

1. Predicates: It is the statements involving variables which are i.e, true (or) false. until (or) unless the values of variables are specified.

Ex! x is an Animal.

In predicate logic a statement is divided into 2 parts:

1. subject:
2. predicate.

$\exists$ - there exists

$\forall$ for all

<u>Quantifiers</u>: This are the words that refers to quantities such as "some" or "all". It tells for how many elements a given predicate is True.

→ In English Quantifiers are used to express the qualities without giving the exact number.

Ex: all, some, many, none, few etc.

Ex: "Can I have some water?"

Ex: "Nithin has many friends".

Ex: x is an Animal

x is > 3
(or)
x is < 4
(or)
$x = 10$

Quantifiers.

Types of Quantifiers:

1) Universal Quantifiers.

2) Existential Quantifiers

Ex: Let $P(x)$ be a statement "$x+1>x$"

What is the truth value of $P(1) \Rightarrow 1+1>1$

$2>1 \Rightarrow$ True

$\therefore P(x)$ is true for all Integers $\forall x P(x)$

The following list shows the 8-quantifiers and their abbrevated meaning.

1) $\forall x\ P(x)$ – all True

2) $\exists x\ p(x)$ – atleast one true.

3) $\neg(\exists x P(x))$ – None True

4) $\exists x \neg P(x)$ – Atleast one false

5) $\forall x \neg p(x)$ = All false

6) $\sim[\exists x \sim p(x)]$ - None False

7) $\sim[\forall x\ p(x)]$ - Not all True.

8) $\sim[\forall x \sim P(x)]$ - Not all False

Ex: 1) All birds can fly.

Sol: Let $B(x)$: x is a bird

$F(x)$: x can fly

$\therefore \forall x [B(x) \rightarrow F(x)]$

2) Not every graph is planar.

Sol: Let, $G(x)$: x is a graph

$P(x)$: x is planar

$\sim[\forall x G(x) \rightarrow P(x)]$

3) There is a statement who likes Mathematics but not programming.

Sol: $S(x)$: x is a student

$M(x)$: x is a Mathematics

$P(x)$: x does not likes programming

$\exists[x (S(x) \wedge M(x) \wedge P(x))]$

4) Let x and y the real numbers and $P(x,y)$ denotes "$x+y=0$". Find the truth value of.

a) $\forall x \forall y\ P(x,y)$ Sol: False $\quad x=1, y=2$ $\quad 1+2 \neq 0$

b) $\forall x \exists y\ P(x,y)$ Sol: True

c) $\exists y \forall x\ P(x,y)$ Sol: True

d) $\exists x \exists y\ P(x,y)$ Sol: True

* Domain:- Domain specifies the possible values of the variable under consideration.

Ex: Let x and y be the real numbers and $Q(x,y)$ denotes "$xy=0$" Find the valuee of following.

a) $\forall x \forall y\ Q(x,y)$ - False (Let $x=1$, & $y=2$ $x.y=1.2 \neq 0$)

b) $\forall x \exists y\ Q(x,y)$ - True ($\forall x\ \exists y=0 \Rightarrow x.y=0$)

c) $\exists x \forall y\ Q(x,y)$ - True ($\forall y\ \exists x=0 \Rightarrow x.y=0$)

d) $\exists x \exists y\ Q(x,y)$ - True

⇒ Existential Quantifiers:- The Existential quantifier of $p(x)$ is the proposition.

"There exists an element x in the domain such that $p(x)$."

Different ways to say $\exists x\ p(x)$

1) "There is an x such that $p(x)$"

2) "There is atleast one x such that $p(x)$".

3) "For some x $p(x)$".

⇒ Translating English sentences to logical expressions.

Problem:- Consider these statements of which first three are premisses and the fourth one is a valid conclusion.

"All humming birds are richly colored".

"No large birds live on honey".

"Birds that do not live on honey are dull in color".

"Humming birds are small".

Sol: Let $P(x)$: "x is a humming bird".

$Q(x)$: x is large.

$R(x)$: x lives on honey

$S(x)$: x is richly colored.

Assuming that the domain consisting all the birds.

S1: $\forall x\,[P(x) \rightarrow S(x)]$

S2: $\forall(x)\,[Q(x) \rightarrow \sim R(x)]$

S3: $\forall(x)\,[\sim R(x) \rightarrow \sim S(x)]$

Conclusion: $\forall x\,[P(x) \rightarrow \sim Q(x)]$

* Rules of Inferences for Quantified statements:-

1) Universal specification: $\dfrac{\forall x\, p(x)}{\therefore\ p(c) \text{ for some elements } c}$

2) Universal generalization: $\dfrac{p(c) \text{ for an arbitary 'c'}}{\therefore\ \forall x\, p(x)}$

3) Existentialization:-

4) Existential specification:- $\dfrac{\exists x\, p(x)}{\therefore\ p(c) \text{ for some element } c}$

4)

→ Consider the argument:

Ex: All men are fallible.

All kings are men.

~~The~~ ∴ All kings are fallible

Prove the validity of a given argument.

Sol: Let $m(x)$: x is a man

$F(x)$: x is fallible

$K(x)$: x is king

Given Arguments 1) $\forall x\,[M(x) \rightarrow F(x)]$

2) $\forall x\,[K(x) \rightarrow M(x)]$

3) $\forall x\,[K(x) \rightarrow F(x)]$

S.no	Statement & Formula	Rule	Justification.
1)	$\forall x\,[M(x) \rightarrow F(x)]$	P	
2)	$M(c) \rightarrow F(c)$	T	(1) U.S
3)	$\forall x\,[K(x) \rightarrow M(x)]$	P	
4)	$K(c) \rightarrow M(c)$	T	(3) U.S
5)	$K(c) \rightarrow F(c)$	T	(4) (2) Transitive
6)	$\forall x\,[K(x) \rightarrow F(x)]$	T	(5) Universal Generalization

Ex-2: Lions are dangerous animals
There are lions.

Conclusion: There are dangerous animals.

Sol: Let $L(x)$: x is a lion
$D(x)$: x is a dangerous animal

$\therefore$ Given statements $\rightarrow$ $\forall x[L(x) \rightarrow D(x)]$
$\exists x L[x]$

$\therefore \exists x D(x)$

S.no	Statement	Rule	Justification
1.	$\forall x[L(x) \rightarrow D(x)]$	P	
2.	$L(c) \rightarrow D(c)$	T	(1) Universal specification
3.	$\exists x L(x)$	P	
4.	$L(c)$ P	T	(3) Existantion specification
5.	$D(c)$	T	(2) (4) M.P
6.	$\exists x D(x)$	T	(5) Existential Generalisation.

Ex: 3: Test the validity of the following quantified statements.
Every living thing is plant or animal David's dog is alive and it is not a plant.
All animals have hearts.
Hence, davids dog has a heart.

Sol: Let $L(x)$: x is a living thing.
$P(x)$: x is a plant.
$A(x)$: x is Animal
a: David's dog
$H(x)$: x has a heart.

$\therefore$ given statements.
$\rightarrow$ $\forall x[L(x) \rightarrow (P(x) \vee A(x))]$
$L(a) \wedge \sim p(a)$
$\forall x[A(x) \rightarrow H(x)]$

$\therefore H(a)$

S.No	Statement	rule	Justification
1.	$\forall x[L(x) \to P(x) \vee A(x)]$	P	
2.	$L(c) \to [P(c) \vee A(c)]$	T	(1) U.S
3.	$L(a) \wedge \sim p(a)$	P	
4.	$L(c)$	T	(3) conjuction rule
5.	$P(c) \vee A(c)$	T	(3), (4) M.P
6.	$\sim P(c) \to A(c)$	T	(5) equivalence relation
7.	$\sim P(c)$	T	(3) conjuction rule.
8.	$A(c)$	T	(6) (7) M.P
9.	$\forall(x)[A(x) \to H(x)]$	P	
10.	$A(c) \to H(c)$	T	(9) U.S
11.	$H(c)$	T	(8) (10) M.P

4. ST $\exists x M(x)$ logically from the premises $\forall x[H(x) \to M(x)]$ and $\exists x H(x)$

$$\begin{array}{l} \forall x[H(x) \to M(x)] \\ \exists x\{H(x) \\ \hline \therefore \exists x M(x) \end{array}$$

S.No	statement	Rule	Justification.
1.	$\forall x[H(x) \to M(x)]$	P	
2.	$H(c) \to M(c)$	T	(1) U.S
3.	$\exists x H(x)$	P	
4.	$H(c)$	T	(3) E.S
5.	$M(c)$	T	(2) (4) M P