NCERT-9.4.5

EE24BTECH11035 - K TEJA VARDHAN

Question: Find the solution of the differential equation:

$$(e^x + e^{-x})\,dy = (e^x - e^{-x})\,dx \tag{1}$$

Solution: Rewriting the equation:

$$(e^x + e^{-x})\,dy = (e^x - e^{-x})\,dx \tag{2}$$

Let:

$$v = e^x + e^{-x} \tag{3}$$

Differentiating v with respect to x:

$$\frac{dv}{dx} = e^x - e^{-x} \tag{4}$$

Thus:

$$e^x - e^{-x} = \sqrt{v^2 - 4} \tag{5}$$

The given equation becomes:

$$v\,dy = \sqrt{v^2 - 4}\,dx \tag{6}$$

Substitute:

$$dx = \frac{dv}{\sqrt{v^2 - 4}} \tag{7}$$

This leads to:

$$v\,dy = dv \tag{8}$$

Rewrite:

$$dy = \frac{1}{v}, dv \tag{9}$$

Integrate both sides:

$$\int dy = \int \frac{1}{v}\,dv \tag{10}$$

$$y = \ln|v| + C \tag{11}$$

Substitute $v = e^x + e^{-x}$ back:

$$y = \ln(e^x + e^{-x}) + C \tag{12}$$

The solution to the differential equation is:

$$\boxed{y = \ln(e^x + e^{-x}) + C} \tag{13}$$

where C is the constant of integration(Here, it is assumed as 0).

Numerical Approach:

I used a for loop for finding the y values as the loop proceeds with iterative formula given below. I took some initial value of x and as loop proceeds I assigned it the value as $x+h$. where h is the step size, representing the rate of change.

2. Assigned the values of y for different x-values using a for loop.

The iterative formula for updating y-values is:

$$\frac{dy}{dx} = \frac{y_n - x_n}{y_n + x_n}.h \tag{14}$$

$$y_{n+1} = y_n + \left(\frac{y_n - x_n}{y_n + x_n}\right).h \tag{15}$$

The iterative formula for updating x-values is:

$$x_n = x_{n-1} + h \tag{16}$$

Initial Conditions:

- $x = 0$
- $y = \ln 2$
- $h = 0.01$

Using Matplotlib, I plotted the computed points and the graph of the exact solution to verify that they approximately match.

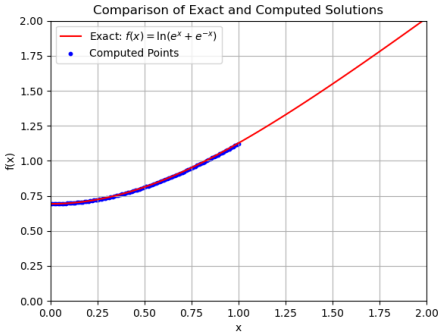

Fig. 0.1: verifying through graph of sim and theory values