Assignment3

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Problem: A line intersects the Y-axis and the X-axis at the points $P(0, b)$ and $Q(c, 0)$ respectively. If $(2, -5)$ is the midpoint of PQ, then find the coordinates of P and Q.

Solution:

Let the coordinates of points P and Q be represented by the vectors:

$$\mathbf{P} = \begin{pmatrix} 0 \\ b \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} c \\ 0 \end{pmatrix} \tag{0.1}$$

The midpoint M is given as:

$$\mathbf{M} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \tag{0.2}$$

The midpoint formula in vector form is:

$$\mathbf{M} = \frac{1}{2}(\mathbf{P} + \mathbf{Q}) \tag{0.3}$$

Substituting the given values:

$$\frac{1}{2}\left(\begin{pmatrix} 0 \\ b \end{pmatrix} + \begin{pmatrix} c \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \tag{0.4}$$

This simplifies to:

$$\frac{1}{2}\begin{pmatrix} c \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \tag{0.5}$$

Multiplying both sides by 2:

$$\begin{pmatrix} c \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ -10 \end{pmatrix} \tag{0.6}$$

Thus, the coordinates of P and Q are:

$$\mathbf{P} = \begin{pmatrix} 0 \\ -10 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{0.7}$$

$$\mathbf{P} = \begin{pmatrix} 0 \\ b \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} c \\ 0 \end{pmatrix} \tag{0.8}$$

$$\mathbf{M} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \tag{0.9}$$

$$\mathbf{M} = \frac{1}{2}(\mathbf{P} + \mathbf{Q}) \tag{0.10}$$

$$\frac{1}{2}\left(\begin{pmatrix} 0 \\ b \end{pmatrix} + \begin{pmatrix} c \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \tag{0.11}$$

$$\frac{1}{2}\begin{pmatrix} c \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \tag{0.12}$$

$$\begin{pmatrix} c \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ -10 \end{pmatrix} \tag{0.13}$$

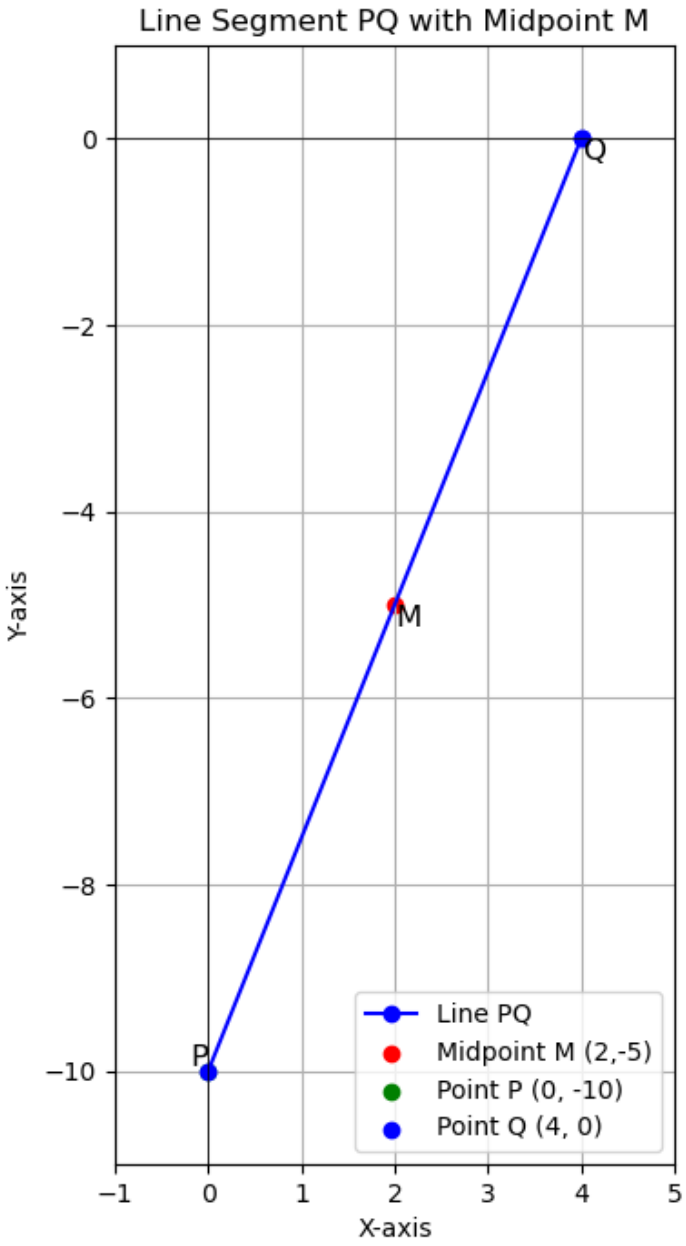

Fig. 0.1: The plot of the points