# 6.5.2.1

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**Question**:
Find the Maximum and Minimum values of the function

$$y(x) = |x + 2| - 1$$

**Solution :**
**Threotical Solution :**

As $y(x)$ is modular function, The vertex of the function is at the point where,

$$|x + 2| = 0 \tag{0.1}$$
$$x = -2 \tag{0.2}$$

Substitute $x = -2$ in the function, gives :

$$y(-2) = -1 \tag{0.3}$$

Hence, the vertex is at $(-2, -1)$, which is the **minimum** point and it can be explained by
,

$$y = \begin{cases} (x + 2) - 1 & \text{if } x \geq -2, \\ -(x + 2) + -1 & \text{if } x < -2 \end{cases}$$

The derivative of $y$ is:

$$\frac{dy}{dx} = \begin{cases} 1 & \text{if } x > -2, \\ -1 & \text{if } x < -2. \end{cases}$$

At $x = -2$, the derivative does not exist because of the abrupt change in slope. However, we can observe the behavior of the function:
- For $x < -2$, the derivative $\frac{dy}{dx} = -1 < 0$, indicating that the function is decreasing.
- For $x > -2$, the derivative $\frac{dy}{dx} = 1 > 0$, indicating that the function is increasing.

Therefore, **minimum** value of $y(x) = -1$ at $x = -2$

Now,
As $x \to \infty$ or $x \to -\infty$, the absolute value $|x + 2| \to \infty$, and the positive part of this term dominates. Thus:

$$y \to \infty \tag{0.4}$$

This means the function increases without bound, so there **is no maximum value.**

**Computational solution :**

Minimum value of the function can be done by **Gradient Descent** method:

- **Choose a starting point** - $x_0$ away from $x = -2$.
- **Update the position iteratively**:

$$x_{n+1} = x_n - \eta \cdot \frac{dy}{dx}$$

  Here, $\eta = 0.01$, $\eta$ is learning rate.
- **Behaviour in each region**:
  **for** $x > -2$: $\frac{dy}{dx} = 1$,

$$x_{n+1} = x_n - \eta \tag{0.5}$$

  This causes $x_n$ to decrease toward $x = -2$.
  **for** $x < -2$: $\frac{dy}{dx} = -1$,

$$x_{n+1} = x_n + \eta \tag{0.6}$$

  This causes $x_n$ to decrease toward $x = -2$.
  **At** $x = -2$:
  the gradient changes direction abruptly, and the iteration stops because the function value is minimized at this point.

Maximum value of the function can be done by **Gradient ascent** method:

Here,

$$x_{n+1} = x_n + \eta \cdot \frac{dy}{dx} \tag{0.7}$$

**Similarly finding behaviour in each region**:
**for** $x > -2$:

$$x_{n+1} = x_n + \eta \tag{0.8}$$

This causes $x_n$ to increase indefinitely, moving away from $x = -2$.
**for** $x < -2$:

$$x_{n+1} = x_n - \eta \tag{0.9}$$

This causes $x_n$ to increase indefinitely, moving away from $x = -2$.

The function increase without bound as $x \to \infty$ or $x \to -\infty$, so **gradient ascent** will not converge to a maximum. The iteration will continue indefinitely. so, **No maximum exists**

**Computational results :**

-Absolute Minimum

$$x \approx -2, \; y(x) \approx -1 \tag{0.10}$$

-No Absolute Maximum