ASSIGNMENT 9

EE24BTECH11034 - K Teja Vardhan

I. JEE PYQ 2022 JULY 26, SHIFT 1

1) The minimum value of the sum of the squares of the roots of $x^2+(3-a)\,x+1=2a$ is:

a) 4 b) 5 c) 6 d) 8

2) If $z=x+iy$ satisfies $|z|-2=0$ and $|z-i|-|z+5i|=0$, then

a) $x+2y-4=0$ b) $x^2+y-4=0$ c) $x+2y+4=0$ d) $x^2-y+3=0$

3) Let $A=\begin{bmatrix}1\\1\\1\end{bmatrix}$ and $B=\begin{bmatrix}9^2 & -10^2 & 11^2\\ 12^2 & 13^2-14^2 & \\ -15^2 & 16^2 & 17^2\end{bmatrix}$, then the value of A^TBA is:

a) 1224 b) 1042 c) 540 d) 539

4) $\sum_{i,j=0}^{n} nC_i nC_j$ is equal to
a) $2^{2n}-2nC_n$
b) $2^{2n-1}-2n-1C_{n-1}$
c) $2^{2n}-\frac{1}{2}2nC_n$
d) $2^{n-1}+2n-1C_n$

5) Let P and Q be any points on the curves $(x-1)^2+(y+1)^2=1$ and $y=x^2$, respectively. The distance between P and Q is minimum for some value of the abscissa of P in the interval

a) $\left(0,\frac{1}{4}\right)$ b) $\left(\frac{1}{2},\frac{3}{4}\right)$ c) $\left(\frac{1}{4},\frac{1}{2}\right)$ d) $\left(\frac{3}{4},1\right)$

6) If the maximum value of a, for which the function $f_a(x)=\tan^{-1}2x-3ax+7$ is non-decreasing in $\left(-\frac{\pi}{6},\frac{\pi}{6}\right)$, is $\overline{a}$, then $f_{\overline{a}}\left(\frac{\pi}{8}\right)$ is equal to:

a) $8-\frac{9\pi}{4(9+\pi^2)}$ b) $8-\frac{4\pi}{9(4+\pi^2)}$ c) $8\left(\frac{1+\pi^2}{9+\pi^2}\right)$ d) $8-\frac{\pi}{4}$

7) Let $\beta=\lim_{x\to 0}\frac{\alpha x\left(e^{-3x}-1\right)}{\alpha x(e^{3x}-1)}$ for some $\alpha\in\mathbb{R}$. Then the value of $\alpha+\beta$ is:

a) $\frac{14}{5}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$

8) The value of $\log_e 2\frac{d}{dx}\left(\log_{\cos x}\csc x\right)$ at $x=\frac{\pi}{4}$ is

a) $-2\sqrt{2}$ b) $2\sqrt{2}$ c) -4 d) 4

9) $\int_0^{20\pi} \left(|\sin x| + |\cos x|\right)^2 dx$ is equal to:

a) $10\,(\pi + 4)$ b) $10\,(\pi + 2)$ c) $20\,(\pi - 2)$ d) $20\,(\pi + 2)$

10) Let the solution curve $y = f\,(x)$ of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2-1} = \frac{x^4+2x}{\sqrt{1-x^2}}, x \in (-1, 1)$ pass through the origin. Then $\int_{\frac{\sqrt{5}}{2}}^{1} f\,(x)\,dx$ is equal to:

a) $\frac{\pi}{3} - \frac{1}{4}$ b) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$ c) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$ d) $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

11) The acute angle between the pair of tangents drawn to the ellipse $2x^2 + 3y^2 = 5$ from the point $(1, 3)$ is

a) $\tan^{-1}\left(\frac{16}{7\sqrt{5}}\right)$ b) $\tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$ c) $\tan^{-1}\left(\frac{32}{7\sqrt{5}}\right)$ d) $\tan^{-1}\left(\frac{3+8\sqrt{5}}{35}\right)$

12) The equation of a common tangent to the parabolas $y = x^2$ and $y = -\left(x - 2\right)^2$ is

a) $y = 4\,(x - 2)$ b) $y = 4\,(x - 1)$ c) $y = 4\,(x + 1)$ d) $y = 4\,(x + 2)$

13) Let the abscissae of the two points P and Q on a circle be the roots of $x^2 - 4x - 6 = 0$ and the ordinates of P and Q be the roots of $y^2 + 2y - 7 = 0$. If PQ is a diameter of the circle $x^2 + y^2 + 2ax + 2by + c = 0$, then the value of $a + b - c$ is

a) 12 b) 13 c) 14 d) 16

14) If the line $x - 1 = 0$ is a directrix of the hyperbola $kx^2 - y^2 = 6$, then the hyperbola passes through the point

a) $\left(-2\sqrt{5}, 6\right)$ b) $\left(-\sqrt{5}, 3\right)$ c) $\left(\sqrt{5}, -2\right)$ d) $\left(2\sqrt{5}, 3\sqrt{6}\right)$

15) A vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i} + \hat{j}$ and the plane determined by the vectors $\hat{i} - \hat{j}, \hat{i} + \hat{k}$. The obtuse angle between $\vec{a}$ and the vector $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$ is

a) $\frac{3\pi}{4}$ b) $\frac{2\pi}{3}$ c) $\frac{4\pi}{5}$ d) $\frac{5\pi}{6}$