

ASSIGNMENT 13

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I. JEE PYQ 2024 JANUARY 30, SHIFT 2

- 1) Let $f(x) = \begin{cases} x-1, & x \text{ is even} \\ 2x, & x \text{ is odd} \end{cases}$, $x \in \mathbb{N}$. If for some $a \in \mathbb{N}$, $f(f(f(a))) = 21$, then $\lim_{x \rightarrow a} \left[\frac{|x|^3}{a} - \left[\frac{x}{a} \right] \right]$ is equal to:
 - a) 121
 - b) 144
 - c) 169
 - d) 225
- 2) Let the system of equations $x + 2y + 3z = 5$, $2x + 3y + z = 9$, $4x + 3y + \lambda z = \mu$ have infinite number of solutions. Then $\lambda + 2\mu$ is equal to:
 - a) 28
 - b) 17
 - c) 22
 - d) 15
- 3) Consider 10 observations x_1, x_2, \dots, x_{10} such that $\sum_{i=1}^{10} (x_i - \alpha) = 2$ and $\sum_{i=1}^{10} (x_i - \beta)^2 = 40$, where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$, respectively. The $\frac{\beta}{\alpha}$ is equal to:
 - a) 2
 - b) $\frac{3}{2}$
 - c) $\frac{5}{2}$
 - d) 1
- 4) Let Ajay will not appear in JEE exam with probability $p = \frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $q = \frac{1}{5}$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is:
 - a) $\frac{9}{35}$
 - b) $\frac{18}{35}$
 - c) $\frac{24}{35}$
 - d) $\frac{3}{35}$
- 5) Let the locus of the mid points of the chords of the circle $x^2 + (y-1)^2 = 1$ drawn from the origin intersect the line $x + y = 1$ at P and Q . Then, the length of PQ is:
 - a) $\frac{1}{\sqrt{2}}$
 - b) $\sqrt{2}$
 - c) $\frac{1}{2}$
 - d) 1
- 6) If three successive terms of a G.P. with common ratio r ($r > 1$) are the lengths of the sides of a triangle and $[r]$ denotes the greatest integer less than or equal to r , then $3[r] + [-r]$ is equal to:
 - a) 1
 - b) 7
 - c) 8
 - d) 9
- 7) Let $A = I_2 - 2MM^T$, where M is a real matrix of order 2×1 such that the relation $M^T M = I_1$ holds. If λ is a real number such that the relation $AX = \lambda X$ holds for some non-zero real matrix X of order 2×1 , then the sum of squares of all possible values of λ is equal to:

- a) 1 b) 2 c) 3 d) 4

8) Let $f((0, \infty)) \rightarrow \mathbb{R}$ and $F(x) = \int_0^x t f(t) dt$. If $F(x^2) = x^4 + x^5$, then $\sum_{r=1}^{12} f(r^2)$ is equal to:

- a) 219 b) 144 c) 156 d) 168

9) If $y = \frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15} (3 \cos^2 x - 5) \cos^3 x$, then $96y\left(\frac{\pi}{6}\right)$ is equal to:

- a) 105 b) 13 c) 15 d) 17

10) Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = -\hat{i} - 8\hat{j} + 2\hat{k}$, and $\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three vectors such that $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$. If the angle between the vector \vec{c} and the vector $3\hat{i} + 4\hat{j} + \hat{k}$ is 0, then the greatest integer less than or equal to $\tan^2 \theta$ is:

- a) 38 b) 2 c) 3 d) 4

11) The lines L_1, L_2, \dots, L_{20} are distinct. For $n = 1, 2, 3, \dots, 10$ all the lines L_{2n-1} are parallel to each other and all the lines L_{2n} pass through a given point P . The maximum number of points of intersection of pairs of lines from the set $[L_1, L_2, \dots, L_{20}]$ is equal to:

- a) 101 b) 191 c) 192 d) 193

12) Three points $O(0, 0)$, $P(a, a^2)$, $Q(-b, b^2)$, $a > 0, b > 0$ are on the parabola $y = x^2$. Let S_1 be the area of the region bounded by the line PQ and the parabola, and S_2 be the area of the triangle OPQ . If the minimum value of $\frac{S_1}{S_2}$ is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to:

- a) 7 b) 8 c) 9 d) 10

13) The sum of squares of all possible values of k , for which area of the region bounded by the parabolas $2y^2 = kx$ and $ky^2 = 2(y - x)$ is maximum, is equal to:

- a) 10 b) 8 c) 14 d) 16

14) If $\frac{dx}{dy} = \frac{1+x-y^2}{y}$, $x(1) = 1$, then $5x(2)$ is equal to:

- a) 11 b) 5 c) 15 d) 17

15) Let ABC be an isosceles triangle in which A is at $(-1, 0)$, $\angle A = \frac{2\pi}{3}$, $AB = AC$ and B is on the positive x -axis. If $BC = 4\sqrt{3}$ and the line BC intersects the line $y = x + 3$ at (α, β) , then $\frac{\beta^4}{\alpha^2}$ is:

a) 36

b) 32

c) 64

d) 128