ASSIGNMENT 6

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1) Let $y = mx + c$, $m > 0$ be the focal chord of $y^2 = -64x$ which is tangent to $(x+10)^2 + y^2 = 4$. Then the value of $4\sqrt{2}\,(m+c)$ is equal to

a) 16 b) 24 c) 34 d) 40

2) A continuous differentiable function $f(x)$ is increasing in $\left(-\infty, \frac{3}{2}\right)$ and decreasing in $\left(\frac{3}{2}, \infty\right)$. Then $x = \frac{3}{2}$ is
   a) a point of local maxima
   b) a point of local minima
   c) a point of inflection
   d) None of these

3) If z and w are complex numbers such that $|z\omega| = 1$, $\arg(z) - \arg(w) = \frac{3\pi}{2}$, then find $\arg\left(\frac{1-2z\omega}{1+3z\omega}\right)$

a) $\frac{\pi}{4}$ b) $-\frac{\pi}{4}$ c) $\frac{3\pi}{4}$ d) $-\frac{3\pi}{4}$

4) If an invertible function $f(x)$ is defined as $f(x) = 3x - 2$, and $g(x)$ is also an invertible function such that $f^{-1}\left(g^{-1}(x)\right) = x - 2$, then $g(x)$ is

a) $\frac{x-8}{3}$ b) $\frac{x+8}{3}$ c) $\frac{x-3}{8}$ d) $\frac{x+3}{8}$

5) The probability of selecting integers $a \in [-5, 30]$, such that $x^2 + 2(a+4)x - 5a + 64 > 0$ for all $x \in \mathbb{R}$ is:

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{1}{5}$

6) If $\int_0^a e^{x-[x]}dx = 10e - 9$, then the value of a is $(where\,[.]\,isthegreatestintegerfunction)$

a) $9 + \ln 2$ b) $10 + \ln 2$ c) 10 d) 9

7) If the shortest distance between the lines
   $\mathbf{r_1} = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$
   , $\lambda \in \mathbb{R}$, $\alpha > 0$ and
   $\mathbf{r_2} = -4\hat{i} - \hat{k} + \mu\left(3\hat{i} - 2\hat{j} - 2\hat{k}\right)$
   , $\mu \in \mathbb{R}$ is 9, then the value of α is:

a) 2 b) 4 c) 6 d) $\sqrt{6}$

8) Let $a_{ij} = \begin{cases} 1, & i = j \\ -x, & |i-j| = 1 \\ 2x+1, & \text{otherwise} \end{cases}$
, $A = [a_{ij}]_{3\times 3} = \det(A)$. Then find the sum of local maximum and minimum values of $f(x)$.

a) $\frac{20}{27}$ b) $-\frac{20}{27}$ c) $\frac{88}{27}$ d) $-\frac{88}{27}$

9) Find the coefficient of $a^3b^4c^5$ in $(ab+bc+ca)^6$.

a) 60 b) 45 c) 40 d) 90

10) $x\left(\frac{dy}{dx}\right)\tan\left(\frac{y}{x}\right) = y\tan\left(\frac{y}{x}\right) + x$, $y\left(\frac{1}{2}\right) = \frac{\pi}{6}$. The area bounded by $x = 0$, $x = \frac{1}{\sqrt{2}}$, and $y = y(x)$ is:

a) $\frac{\pi-1}{8}$ b) $\frac{\pi-2}{16}$ c) $\frac{\pi-3}{32}$ d) $\frac{\pi-4}{64}$