

ASSIGNMENT 5

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- 1) If $A = [x \in \mathbb{R} : |x| < 2]$ and $B = [x \in \mathbb{R} : |x - 2| \geq 3]$, then:
 - a) $A - B = [-1, 2]$
 - b) $B - A = \mathbb{R} - [-2, 5]$
 - c) $A \cup B = \mathbb{R} - [2, 5]$
 - d) $A \cap B = [-2, -1]$
- 2) If 10 different balls have to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is:
 - a) $\frac{965}{2^{10}}$
 - b) $\frac{945}{2^{10}}$
 - c) $\frac{945}{2^{11}}$
 - d) $\frac{965}{2^{11}}$
- 3) If $x = 2 \sin \theta - \sin 2\theta$ and $y = 2 \cos \theta - \cos 2\theta$, $\theta \in [0, 2\pi]$, then $\frac{d^2y}{dx^2}$ at $\theta = \pi$ is:
 - a) $-\frac{3}{8}$
 - b) $\frac{3}{4}$
 - c) $\frac{3}{2}$
 - d) $-\frac{3}{4}$
- 4) Let f and g be differentiable functions on \mathbb{R} , such that $f \circ g$ is the identity function. If for some $a, b \in \mathbb{R}$, $g'(a) = 5$ and $g(a) = b$, then $f'(b)$ is equal to:
 - a) $\frac{2}{5}$
 - b) 5
 - c) 1
 - d) $\frac{1}{5}$
- 5) In the expansion of $\left(\frac{x}{\cos \theta} + \frac{1}{x \sin \theta}\right)^{16}$ if I_1 is the least value of the term independent of x when $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$ and I_2 is the least value of the term independent of x when $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$, then the ratio $I_2 : I_1$ is equal to:
 - a) 16 : 1
 - b) 8 : 1
 - c) 1 : 8
 - d) 1 : 16
- 6) Let $a, b \in \mathbb{R}$, $a \neq 0$, such that the equation $ax^2 - 2bx + 5 = 0$ has a repeated root α , which is also a root of the equation $x^2 - 2bx - 10 = 0$. If β is the root of this equation, then $a^2 + b^2$ is equal to:
 - a) 24
 - b) 25
 - c) 26
 - d) 28
- 7) Let a function $f : [0, 5] \rightarrow \mathbb{R}$ be continuous, $f(1) = 3$ and F be defined as: $F(x) = \int_1^x t^2 g(t) dt$ where $g(t) = \int_1^t f(u) du$. Then for the function F , the point $x = 1$ is:
 - a) a point of inflection.
 - b) a point of local maxima.

- c) a point of local minima.
d) not a critical point.
- 8) Let $[t]$ denote the greatest integer $\leq t$ and $\lim_{x \rightarrow 0} x \left[\frac{4}{x} \right] = A$. Then the function, $f(x) = [x^2] \sin \pi x$ is discontinuous when x is equal to:
- $\sqrt{(A+1)}$
 - \sqrt{A}
 - $\sqrt{(A+5)}$
 - $\sqrt{(A+21)}$
- 9) Let $a - 2b + c = 1$. If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then
- $f(-50) = 501$
 - $f(-50) = -1$
 - $f(50) = 1$
 - $f(-50) = -501$
- 10) Given: $f(x) = \begin{cases} x, & 0 \leq x < \frac{1}{2} \\ \frac{1}{2}, & x = \frac{1}{2} \\ 1-x, & \frac{1}{2} < x \leq 1 \end{cases}$ and $g(x) = \left(x - \frac{1}{2}\right)^2, x \in \mathbb{R}$. Then the area (in sq. units) of the region bounded by the curves $y = f(x)$ and $y = g(x)$ between the lines $2x = 1$ to $2x = \sqrt{3}$ is:
- $\frac{\sqrt{3}}{4} - \frac{1}{3}$
 - $\frac{1}{3} + \frac{\sqrt{3}}{4}$
 - $\frac{1}{2} + \frac{\sqrt{3}}{4}$
 - $\frac{1}{2} - \frac{\sqrt{3}}{4}$
- 11) The length of the minor axis (along y-axis) of an ellipse of the standard form is $\frac{1}{\sqrt{3}}$. If this ellipse touches the line $x + 6y = 8$, then its eccentricity is:
- $\frac{1}{2} \left(\frac{\sqrt{5}}{3} \right)$
 - $\frac{1}{2} \sqrt{\frac{11}{3}}$
 - $\sqrt{\frac{5}{6}}$
 - $\frac{1}{3} \sqrt{\frac{11}{3}}$
- 12) If z is a complex number satisfying $|\operatorname{Re}(z)| + |\operatorname{Im}(z)| = 4$, then $|z|$ cannot be:
- $\sqrt{7}$
 - $\sqrt{\frac{17}{2}}$
 - $\sqrt{10}$
 - $\sqrt{8}$
- 13) If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n} \theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n} \theta$ where $0 < \theta < \frac{\pi}{4}$, then:
- $y(1+x) = 1$
 - $x(1-y) = 1$
 - $y(1-x) = 1$

d) $x(1+y) = 1$