ASSIGNMENT 13

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I. JEE PYQ 2024 FEB 1, SHIFT 2 16 - 30

1) Let $f(x) = \begin{cases} x-1, & x \text{ is even} \\ 2x, & x \text{ is odd} \end{cases}$, $x \in \mathbb{N}$. If for some $a \in \mathbb{N}$, $f(f(f(a))) = 21$, then $\lim_{x\to a}\left[\frac{|x|^3}{a} - \left[\frac{x}{a}\right]\right]$ is equal to:

a) 121 b) 144 c) 169 d) 225

2) Let the system of equations $x + 2y + 3z = 5$, $2x + 3y + z = 9$, $4x + 3y + \lambda z = \mu$ have infinite number of solutions. Then $\lambda + 2\mu$ is equal to:

a) 28 b) 17 c) 22 d) 15

3) Consider 10 observations $x_1, x_2, \ldots, x_{10}$ such that $\sum_{i=1}^{10}(x_i - \alpha) = 2$ and $\sum_{i=1}^{10}(x_i - \beta)^2 = 40$, where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$, respectively. The $\frac{\beta}{\alpha}$ is equal to:

a) 2 b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) 1

4) Let Ajay will not appear in JEE exam with probability $p = \frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $q = \frac{1}{5}$. Then the probability that Ajay will appear in the exam and Vijay will not appear is:

a) $\frac{9}{35}$ b) $\frac{18}{35}$ c) $\frac{24}{35}$ d) $\frac{3}{35}$

5) Let the locus of the mid points of the chords of the circle $x^2 + (y-1)^2 = 1$ drawn from the origin intersect the line $x + y = 1$ at P and Q. Then, the length of PQ is:

a) $\frac{1}{\sqrt{2}}$ b) $\sqrt{2}$ c) $\frac{1}{2}$ d) 1

6) If three successive terms of a G.P. with common ratio $r\,(r > 1)$ are the lengths of the sides of a triangle and $[r]$ denotes the greatest integer less than or equal to r, then $3[r] + [-r]$ is equal to:

a) 1 b) 7 c) 8 d) 9

7) Let $A = I_2 - 2MM^T$, where M is a real matrix of order 2×1 such that the relation $M^TM = I_1$ holds. If λ is a real number such that the relation $AX = \lambda X$ holds for some non-zero real matrix X of order 2×1, then the sum of squares of all possible values of λ is equal to:

a) 1 b) 2 c) 3 d) 4

8) Let $f((0,\infty)) \to \mathbb{R}$ and $F(x) = \int_0^x tf(t)dt$. If $F\left(x^2\right) = x^4 + x^5$, then $\sum_{r=1}^{12} f\left(r^2\right)$ is equal to:

a) 219 b) 144 c) 156 d) 168

9) If $y = \frac{(\sqrt{x}+1)\left(x^2-\sqrt{x}\right)}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15}\left(3\cos^2 x - 5\right)\cos^3 x$, then $96y\left(\frac{\pi}{6}\right)$ is equal to:

a) 105 b) 13 c) 15 d) 17

10) Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = -\hat{i} - 8\hat{j} + 2\hat{k}$, and $\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three vectors such that $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$. If the angle between the vector $\vec{c}$ and the vector $3\hat{i} + 4\hat{j} + \hat{k}$ is 0, then the greatest integer less than or equal to $\tan^2\theta$ is:

a) 38 b) 2 c) 3 d) 4

11) The lines $L_1, L_2, \ldots, L_{20}$ are distinct. For $n = 1, 2, 3, \ldots, 10$ all the lines L_{2n-1} are parallel to each other and all the lines L_{2n} pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2, ..., L_{20}\}$ is equal to:

a) 101 b) 191 c) 192 d) 193

12) Three points $O(0,0), P(a,a^2), Q(-b,b^2), a > 0, b > 0$ are on the parabola $y = x^2$. Let S_1 be the area of the region bounded by the line PQ and the parabola, and S_2 be the area of the triangle OPQ. If $S_1 = \frac{1}{2}S_2$, then $a + b$ is equal to:

a) 4 b) 5 c) 6 d) 7