

NCERT-9.4.5

EE24BTECH11035 - K TEJA VARDHAN

Question: Find the solution of the differential equation:

$$\frac{y dx - x dy}{y} = 0 \quad (1)$$

Solution: Rewriting the equation:

$$y dx - x dy = 0 \quad (2)$$

Rearranging:

$$\frac{dx}{x} = \frac{dy}{y} \quad (3)$$

Integrate both sides:

$$\int \frac{dx}{x} = \int \frac{dy}{y} \quad (4)$$

This gives:

$$\ln |x| = \ln |y| + C \quad (5)$$

Simplify using properties of logarithms:

$$\ln \left| \frac{x}{y} \right| = C \quad (6)$$

Exponentiate both sides:

$$\frac{x}{y} = e^C \quad (7)$$

Let $e^C = k$ (where k is an arbitrary positive constant):

$$x = ky \quad (8)$$

The general solution is:

$$\boxed{x = ky} \quad (9)$$

where k is an arbitrary constant which is assumed to be 1.

Numerical Approach:

I used a for loop for finding the y values as the loop proceeds with iterative formula given below. I took some initial value of x and as loop proceeds I assigned it the value as $x + h$, where h is the step size, representing the rate of change.

2. Assigned the values of y for different x -values using a for loop.

The iterative formula for updating y -values is:

$$\frac{dy}{dx} = \frac{y}{x} \cdot h \quad (10)$$

$$y_{n+1} = y_n + \left(\frac{y}{x}\right) \cdot h \quad (11)$$

The iterative formula for updating x -values is:

$$x_n = x_{n-1} + h \quad (12)$$

Initial Conditions:

- $x = 1$
- $y = 1$
- $h = 0.01$

Using Matplotlib, I plotted the computed points and the graph of the exact solution to verify that they approximately match.

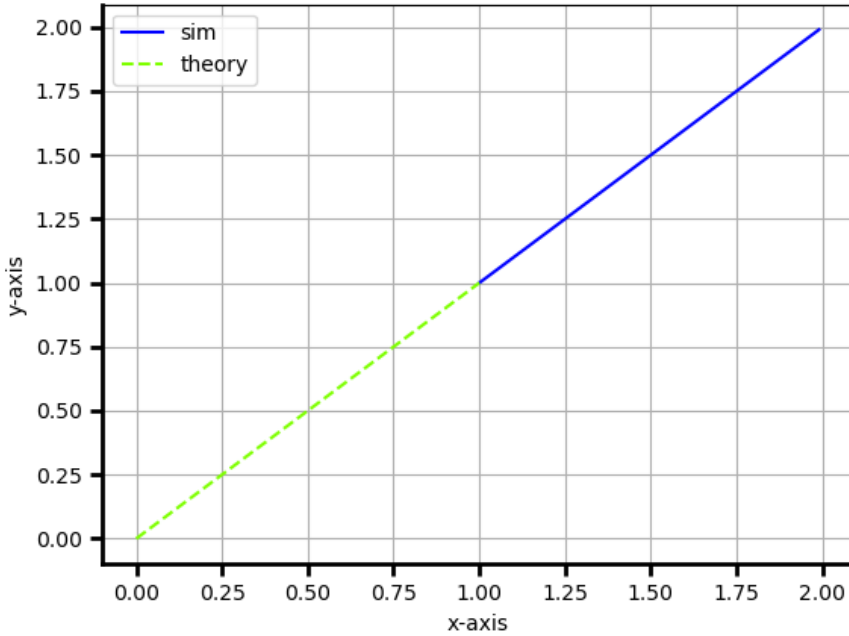


Fig. 0.1: verifying through graph of sim and theory values