12.8.1. ex 2

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Question:
Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
Solution:
Integral to calculate,

$$J = \int_{-a}^{a} 2\frac{b}{a}\sqrt{a^2 - x^2}\,dx \tag{0.1}$$

$$\tag{0.2}$$

Using the trapezoidal rule,

$$J = \int_{a}^{b} f(x)\,dx \approx h\left(\frac{1}{2}f(x) + f(x_1) + f(x_2)\cdots + f(x_{n-1}) + \frac{1}{2}f(b)\right) \tag{0.3}$$

$$h = \frac{b-a}{n} \tag{0.4}$$

$$J = j_n, \text{ where, } j_{i+1} = j_i + k\frac{f(x_{n+1}) + f(x_n)}{2} \tag{0.5}$$

$$\rightarrow j_{i+1} = j_i + \frac{bk}{a}\left(\sqrt{a^2 - x_{n+1}^2} + \sqrt{a^2 - x_n^2}\right) \tag{0.6}$$

$$x_{n+1} = x_n + k \tag{0.7}$$

Theoretical Solution:

$$J = \int_{-2}^{2} 2\frac{b}{a}\sqrt{a^2 - x^2}\,dx \tag{0.8}$$

$$\tag{0.9}$$

Using the following result,

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} \tag{0.10}$$

$$J = 2\frac{b}{a}\left(\frac{\pi a^2}{2}\right) = \pi ab \tag{0.11}$$

assuming $a = 2, b = 3$, the area bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

$$J = \pi(2 * 3) \tag{0.12}$$

$$J = 8\pi \approx 18.84955592153876 \tag{0.13}$$

Computational solution : 18.849477036874855