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EE24BTECH11034 - K Teja Vardhan

1) Let $u(x,y) = x^3 + ax^2y + bxy^2 + 2y^3$ be a harmonic function and $v(x,y)$ its harmonic conjugate. If $v(0,0) = 1$, then $|a + b + v(1,1)|$ is equal to ____________
2) Let γ be the triangular path connecting the points $(0,0)$, $(2,2)$ and $(0,2)$ in the counter-clockwise direction in $\mathbb{R}^2$. Then

$$I = \oint_\gamma \sin\left(x^3\right) dx + 6xydy$$

is equal to ____________
3) Let y be the solution of

$$y\prime + y = |x|, \quad x \in \mathbb{R}, \quad y(-1) = 0.$$

Then $y(1)$ is equal to
 a) $\frac{2}{e} - \frac{2}{e^2}$
 b) $\frac{2}{e} - 2e^2$
 c) $2 - \frac{2}{e}$
 d) $2 - 2e$
4) Let X be a random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \le x < \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} \le x < 1 \\ 1 & 2 \le x < 1. \end{cases}$$

Then $P\left(\frac{1}{4} < X < 1\right)$ is equal to ____________
5) Let γ be the curve which passes through $(0,1)$ and intersects each curve of the family $y = cx^2$ orthogonally. Then γ also passes through the point
 a) $\left(\sqrt{2}, 0\right)$
 b) $\left(0, \sqrt{2}\right)$
 c) $(1,1)$
 d) $(-1,1)$
6) Let $S(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(nx) + b_n \sin(nx)\right)$ be the Fourier series of the 2π periodic function defined by $f(x) = x^2 + 4\sin(x)\cos(x), -\pi \le x \le \pi$. Then

$$\left|\sum_{n=0}^{\infty} a_n - \sum_{n=1}^{\infty} b_n\right|$$

is equal to ____________

7) Let $y(t)$ be a continuous function on $[0, \infty]$. If

$$y(t) = t\left(1 - 4\int_0^t y(x)\,dx\right) + 4\int_0^t xy(x)\,dx,$$

then $\int_0^{\pi/2} y(t)\,dt$ is equal to ____________

8) Let $S_n = \sum_{k=1}^{n} \frac{1}{k}$ and $I_n = \int_1^n \frac{x-[x]}{x^2}dx$. Then $S_{10} + I_{10}$ is equal to

a) $\ln 10 + 1$

b) $\ln 10 - 1$

c) $\ln 10 - \dfrac{1}{10}$

d) $\ln 10 + \dfrac{1}{10}$

9) For any $(x, y) \in \mathbb{R}^2 \setminus B(0,1)$, let

$$\begin{aligned} f(x,y) &= \text{distance}\left((x,y), B(0,1)\right) \\ &= \inf\left[\sqrt{(x-x_1)^2 + (y-y_1)^2} : (x_1, y_1) \in B(0,1)\right]. \end{aligned}$$

Then, $|\nabla f(3,4)|$ is equal to ____________

10) Let $f(x) = \int_0^x e^{t^2-t^2}dt$ and $g(x) = \int_0^x \frac{e^{t^2+t^2}}{1+t^2}dt$. Then $f(\sqrt{\pi}) + g(\sqrt{\pi})$ is equal to ____________

11) Let $M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ be a real matrix with eigenvalues 1, 0, and 3. If the eigenvectors corresponding to 1 and 0 are $[1,1,1]^T$ and $[1,-1,0]^T$, respectively, then the value of $3f$ is equal to ____________

12) Let $M = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $e^M = I + M + \frac{1}{2!}M^2 + \frac{1}{3!}M^3 + \cdots$. If $e^M = [b_{ij}]$, then

$$\frac{1}{e}\sum_{i=1}^{3}\sum_{j=1}^{3} b_{ij}$$

is equal to ____________

13) Let the integral $I = \int_0^4 f(x)\,dx$, where $f(x) = \begin{cases} x & 0 \le x \le 2 \\ 4-x & 2 \le x \le 4. \end{cases}$

Consider the following statements P and Q:

P: If I_2 is the value of the integral obtained by the composite trapezoidal rule with two equal sub-intervals, then I_2 is exact.

Q: If I_3 is the value of the integral obtained by the composite trapezoidal rule with three equal sub-intervals, then I_3 is exact.
Which of the above statements hold TRUE?

a) both P and Q
b) only P
c) only Q
d) Neither P nor Q