Assignment4

Teja Vardhan Shannu

QUESTION

Given the vertices of a triangle PQR as $P(2, 2)$, $Q(-4, -4)$, and $R(5, -8)$, find the length of the median through R.

SOLUTION

We are given the vertices of triangle PQR as follows:

$$P(2, 2), \quad Q(-4, -4), \quad R(5, -8) \tag{0.1}$$

We are asked to find the length of the median through vertex R, using the matrix approach.

STEP 1: FIND THE MIDPOINT OF PQ

The midpoint M of the line segment PQ is calculated as:

$$M = \left(\frac{P_x + Q_x}{2}, \frac{P_y + Q_y}{2} \right) \tag{0.2}$$

Substituting the coordinates of $P(2, 2)$ and $Q(-4, -4)$:

$$M = \left(\frac{2 + (-4)}{2}, \frac{2 + (-4)}{2} \right) = (-1, -1) \tag{0.3}$$

STEP 2: REPRESENT POINTS AS COLUMN VECTORS

We now represent the points R and M as column vectors:

$$\mathbf{R} = \begin{pmatrix} 5 \\ -8 \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \tag{0.4}$$

STEP 3: FIND THE VECTOR FROM R TO M

To find the vector from R to M, we subtract the vector $\mathbf{M}$ from $\mathbf{R}$:

$$\mathbf{RM} = \mathbf{R} - \mathbf{M} = \begin{pmatrix} 5 \\ -8 \end{pmatrix} - \begin{pmatrix} -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 - (-1) \\ -8 - (-1) \end{pmatrix} = \begin{pmatrix} 6 \\ -7 \end{pmatrix} \tag{0.5}$$

STEP 4: FIND THE LENGTH OF THE MEDIAN

The length of the median through R is the magnitude of the vector $\mathbf{RM}$, which is calculated as:

$$|\mathbf{RM}| = \sqrt{6^2 + (-7)^2} = \sqrt{36 + 49} = \sqrt{85} \tag{0.6}$$
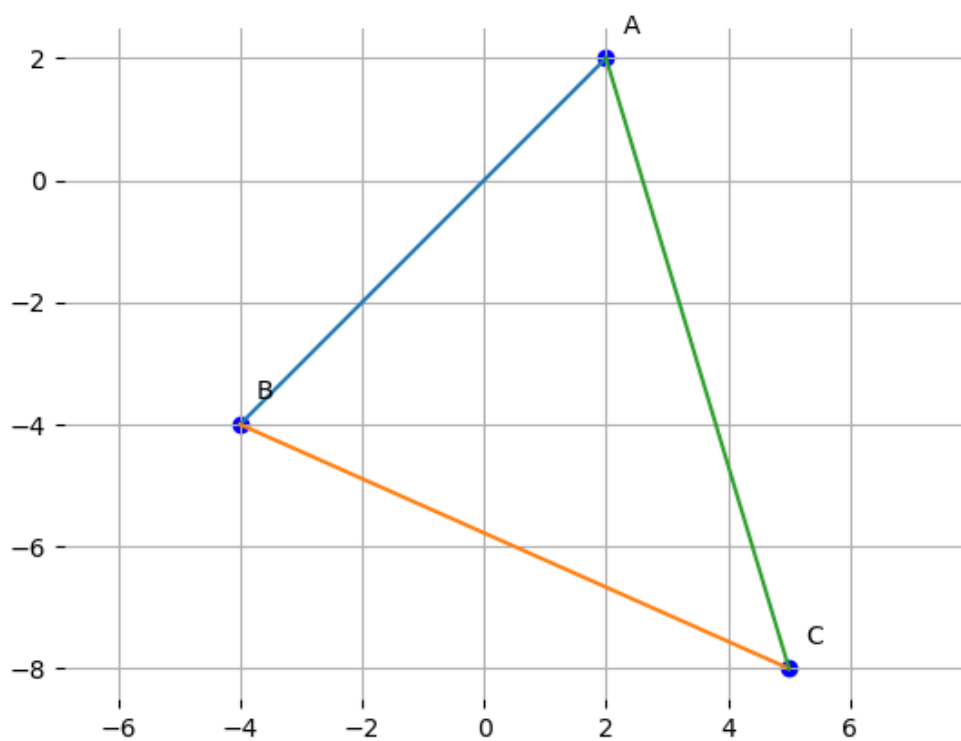
Thus, the length of the median through R is $\sqrt{85}$ units.

Fig. 0.1: The plot of the points