ASSIGNMENT 9

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I. JEE PYQ 2022 JULY 26, SHIFT 1

1) If $z \neq 0$ be a complex number such that $\left|z - \frac{1}{z}\right| = 2$, then the maximum value of $|z|$ is:

a) $\sqrt{2}$ b) 1 c) $\sqrt{2}-1$ d) $\sqrt{2}+1$

2) Which of the following matrices can NOT be obtained from the matrix $\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ by a single elementary row operation?

a) $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ c) $\begin{bmatrix} -1 & 2 \\ -2 & 7 \end{bmatrix}$ d) $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$

3) The system of equations
$x + y + z = 6$
$2x + 5y + \alpha z = \beta$
$x + 2y + 3z = 14$
has infinitely many solutions. Then $\alpha + \beta$ is equal to:

a) 8 b) 36 c) 44 d) 48

4) Let the function $f(x) = \begin{cases} \frac{\log_e(1+5x) - \log_e(1+ax)}{x}, & \text{if } x \neq 0 \\ 10, & \text{if } x = 0 \end{cases}$ be continuous at $x = 0$. The α is equal to:

a) 10 b) -10 c) 5 d) -5

5) If $[t]$ denotes the greatest integer $\leq t$, then the value of $\int_0^1 \left[2x - 13x^2 - 5x + 21 + 1\right] dx$ is:

a) $\frac{\sqrt{37}+\sqrt{13}-4}{6}$ b) $\frac{\sqrt{37}-\sqrt{13}-4}{6}$ c) $\frac{-\sqrt{37}-\sqrt{13}+4}{6}$ d) $\frac{-\sqrt{37}+\sqrt{13}+4}{6}$

6) Let $[a_n]_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1$, $\forall n \geq 0$. Then $a_{25} - 2a_{23} - 2a_{22} + 4a_{24}$ is equal to:

a) 483 b) 528 c) 575 d) 624

7) $\sum_{r=1}^{20} \left(r^2 + 1\right)(r!)$ is equal to:

a) $22! - 21!$ b) $22! - 2(21!)$ c) $21! - 2(20!)$ d) $21! - 20!$

8) For $I(x) = \int \frac{\sec^2 x - 2022}{\sin^{2022} x} dx$, if $I\left(\frac{\pi}{4}\right) = 2^{1011}$, then
a) $3^{1010} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$
b) $3^{1010} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$
c) $3^{1011} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$
d) $3^{1011} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

9) If the solution curve of the differential equation $\frac{dy}{dx} = \frac{x+y-2}{x-y}$ passes through the point $(2, 1)$ and $(k+1, 2), k > 0$, then
a) $2\tan^{-1}\left(\frac{1}{k}\right) = \log_e\left(k^2+1\right)$
b) $\tan^{-1}\left(\frac{1}{k}\right) = \log_e\left(k^2+1\right)$
c) $2\tan^{-1}\left(\frac{1}{k+1}\right) = \log_e\left(k^2+2k+2\right)$
d) $2\tan^{-1}\left(\frac{1}{k}\right) = \log_e\left(\frac{k^2+1}{k^2}\right)$

10) Let $y = y(x)$ be the solution curve of the differential equation $\frac{dy}{dx} + \frac{2x^2+11x+13}{x^3+6x^2+11x+6} y = \frac{x+3}{x+1}$, $x > -1$, which passes through the point $(0, 1)$. Then $y(1)$ is equal to:

a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$

11) Let m_1, m_2 be the slopes of two adjacent sides of a square of side a such that $a^2 + 11a + 3\left(m_1^2 + m_2^2\right) = 220$. If one vertex of the square is $(10(\cos\alpha - \sin\alpha), 10(\sin\alpha + \cos\alpha))$, where $\alpha \in \left(0, \frac{\pi}{2}\right)$, and the equation of one diagonal is $(\cos\alpha - \sin\alpha)x + (\sin\alpha + \cos\alpha)y = 10$, then $72\left(\sin^4\alpha + \cos^4\alpha\right) + a^2 - 3a + 13$ is equal to:

a) 119 b) 128 c) 145 d) 155

12) The number of elements in the set $S = \left[x \in \mathbb{R} : 2\cos\left(\frac{x^2+x}{6}\right) = 4^x + 4^{-x}\right]$ is:

a) 1 b) 3 c) 0 d) infinite

13) Let $A(\alpha, -2)$, $B(\alpha, 6)$, and $C\left(\frac{\alpha}{4}, -2\right)$ be vertices of a $\triangle ABC$. If $\left(5, \frac{\alpha}{4}\right)$ is the circumcentre of $\triangle ABC$, then which of the following is NOT correct about $\triangle ABC$:

a) area is 24
b) perimeter is 25
c) circumradius is 5
d) inradius is 2

14) Let Q be the foot of perpendicular drawn from the point $P(1, 2, 3)$ to the plane $x + 2y + z = 14$. If R is a point on the plane such that $\angle PRQ = 60°$, then the area of $\triangle PQR$ is equal to:

a) $\frac{\sqrt{3}}{2}$ b) $\sqrt{3}$ c) $2\sqrt{3}$ d) 3

15) If $(2,3,9)$, $(5,2,1)$, $(1,\lambda,8)$, and $(\lambda,2,3)$ are coplanar, then the product of all possible values of λ is:

a) $\frac{21}{2}$ b) $\frac{59}{8}$ c) $\frac{57}{8}$ d) $\frac{95}{8}$