ASSIGNMENT 8

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I. JEE PYQ 2022 JUNE 26, SHIFT 1

1) If $\vec{a} \cdot \vec{b} = 1$, $\vec{b} \cdot \vec{c} = 2$ and $\vec{c} \cdot \vec{a} = 3$, then the value of $\left[\vec{a} \times \left(\vec{b} \times \vec{c}\right), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times \left(\vec{b} \times \vec{a}\right)\right]$ is:

a) 0 b) $-6\vec{a} \cdot \left(\vec{b} \times \vec{c}\right)$ c) $12\vec{c} \cdot \left(\vec{a} \times \vec{b}\right)$ d) $-12\vec{b} \cdot (\vec{c} \times \vec{a})$

2) Let a biased coin be tossed 5 times. If the probability of getting 4 heads is equal to the probability of getting 5 heads, then the probability of getting at most two heads is:

a) $\frac{275}{6^5}$ b) $\frac{36}{5^4}$ c) $\frac{181}{5^5}$ d) $\frac{46}{6^4}$

3) The mean of the numbers a, b, 8, 5, 10 is 6 and their variance is 6.8. If M is the mean deviation of the numbers about the mean, then $25M$ is equal to:

a) 60 b) 55 c) 50 d) 45

4) Let $f(x) = 2\cos^{-1} x + 4\cot^{-1} x - 3x^2 - 2x + 10$, $x \in [-1, 1]$. If $[a, b]$ is the range of the function, then $4a - b$ is equal to:

a) 11 b) $11 - \pi$ c) $11 + \pi$ d) $15 - \pi$

5) Let $\wedge$, $\vee$, ϵ $[\wedge, \vee]$ be such that $p \vee q \Rightarrow [(p \wedge q) \vee r]$ is a tautology. Then $(p \vee q) \wedge r$ is logically equivalent to:

a) $[(p \wedge r) \vee q]$ b) $[(p \wedge r) \wedge q]$ c) $[(p \wedge r) \wedge q]$ d) $[(p \vee r) \wedge q]$

6) The sum of the cubes of all the roots of the equation $x^4 - 3x^3 - 2x^2 + 3x + 1 = 10$ is:

a) 10 b) 27 c) 36 d) 45

7) There are ten boys $B_1, B_2, \ldots, B_{10}$ and five girls $G_1, G_2, \ldots, G_5$ in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is:

a) 1120 b) 960 c) 1080 d) 1200

8) Let the common tangents to the curves $4\left(x^2+y^2\right)=9$ and $y^2=4x$ intersect at the point Q. Let an ellipse, centered at the origin O, has lengths of semi-minor and semi-major axes equal to OQ and 6, respectively. If e and l respectively denote the eccentricity and the length of the latusrectum of this ellipse, then $\frac{l}{e^2}$ is equal to:

a) 16 b) 32 c) 25 d) 4

9) Let $f(x)=\max\left(|x+1|,|x+2|,\ldots,|x+5|\right)$. Then $\int_{-6}^{0} f(x)\,dx$ is equal to:

a) 11 b) 21 c) 13 d) 23

10) Let the solution curve $y=y(x)$ of the differential equation $\left(4+x^2\right)dy-2x\left(x^2+3y+4\right)dx=0$ pass through the origin. Then $y(2)$ is equal to:

a) 11 b) 12 c) 13 d) 14

11) If $\sin^2(10^\circ)\sin(20^\circ)\sin(40^\circ)\sin(50^\circ)\sin(70^\circ)=\alpha-\frac{1}{16}\sin(10^\circ)$, then $16+\alpha^{-1}$ is equal to:

a) 20 b) 40 c) 80 d) 160

12) Let $A=[n\in\mathbb{N}:\text{H.C.F.}(n,45)=1]$ and $B=[2k:k\in[1,2,\ldots,100]]$. Then the sum of all the elements of $A\cap B$ is:

a) 1000 b) 1020 c) 1040 d) 1060

13) The value of the integral $\frac{48}{\pi^4}\int_0^\pi\left(\frac{3\pi x^2}{2}-x^3\right)\frac{\sin x}{1+\cos^2 x}dx$ is equal to:

a) 9 b) 12 c) 15 d) 18

14) Let $A=\sum_{i=1}^{10}\sum_{j=1}^{10}\min(i,j)$ and $B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max(i,j)$. Then $A+B$ is equal to:

a) 1000 b) 1100 c) 1200 d) 1300

15) Let $S=(0,2\pi)-\left[\frac{\pi}{2},\frac{3\pi}{4},\frac{3\pi}{2},\frac{7\pi}{4}\right]$. Let $y=y(x)$, $x\in S$, be the solution curve of the differential equation $\frac{dy}{dx}=\frac{1}{1+\sin 2x}$, $y\left(\frac{\pi}{4}\right)=\frac{1}{2}$. If the sum of abscissas of all the points of intersection of the curve $y=y(x)$ with the curve $y=\sqrt{2}\sin x$ is $\frac{k\pi}{12}$, then k is equal to:

a) 11 b) 13 c) 15 d) 17