ASSIGNMENT 5

EE24BTECH11034 - K Teja Vardhan

I. JEE PYQ JAN 9, SHIFT 2

1) If $A = [x \in \mathbb{R} : |x| < 2]$ and $B = [x \in \mathbb{R} : |x-2| \geq 3]$, then:
   a) $A - B = [-1, 2]$
   b) $B - A = \mathbb{R} - [-2, 5]$
   c) $A \cup B = \mathbb{R} - [2, 5]$
   d) $A \cap B = [-2, -1]$
2) If 10 different balls have to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is:

   a) $\frac{965}{2^{10}}$ b) $\frac{945}{2^{10}}$ c) $\frac{945}{2^{11}}$ d) $\frac{965}{2^{11}}$

3) If $x = 2\sin\theta - \sin 2\theta$ and $y = 2\cos\theta - \cos 2\theta$, $\theta \in [0, 2\pi]$, then $\frac{d^2y}{dx^2}$ at $\theta = \pi$ is:

   a) $-\frac{3}{8}$ b) $\frac{3}{4}$ c) $\frac{3}{2}$ d) $-\frac{3}{4}$

4) Let f and g be differentiable functions on $\mathbb{R}$, such that $f \circ g$ is the identity function. If for some $a, b \in \mathbb{R}$, $g'(a) = 5$ and $g(a) = b$, then $f'(b)$ is equal to:

   a) $\frac{2}{5}$ b) 5 c) 1 d) $\frac{1}{5}$

5) In the expansion of $\left(\frac{x}{\cos\theta} + \frac{1}{x\sin\theta}\right)^{16}$ if I_1 is the least value of the term independent of x when $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$ and I_2 is the least value of the term independent of x when $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$, then the ratio $I_2 : I_1$ is equal to:

   a) $16 : 1$ b) $8 : 1$ c) $1 : 8$ d) $1 : 16$

6) Let $a, b \in \mathbb{R}, a \neq 0$, such that the equation $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation $x^2 - 2bx - 10 = 0$. If β is the root of this equation, then $a^2 + b^2$ is equal to:
   a) 24
   b) 25
   c) 26
   d) 28
7) Let a function $f : [0, 5] \to \mathbb{R}$ be continuous, $f(1) = 3$ and F be defined as: $F(x) = \int_1^x t^2 g(t)dt$ where $g(t) = \int_1^t f(u)du$. Then for the function F, the point $x = 1$ is:
   a) a point of inflection.
   b) a point of local maxima.

c) a point of local minima.
d) not a critical point.

8) Let $[t]$ denote the greatest integer $\leq t$ and $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$. Then the function, $f(x) = [x^2]\sin \pi x$ is discontinuous when x is equal to:
a) $\sqrt{(A+1)}$
b) $\sqrt{A}$
c) $\sqrt{(A+5)}$
d) $\sqrt{(A+21)}$

9) Let $a - 2b + c = 1$. If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then
a) $f(-50) = 501$
b) $f(-50) = -1$
c) $f(50) = 1$
d) $f(-50) = -501$

10) Given: $f(x) = \begin{cases} x, & 0 \leq x < \frac{1}{2} \\ \frac{1}{2}, & x = \frac{1}{2} \\ 1-x, & \frac{1}{2} < x \leq 1 \end{cases}$ and $g(x) = \left(x - \frac{1}{2}\right)^2, x \in \mathbb{R}$. Then the area (in sq. units) of the region bounded by the curves $y = f(x)$ and $y = g(x)$ between the lines $2x = 1$ to $2x = \sqrt{3}$ is:
a) $\frac{\sqrt{3}}{4} - \frac{1}{3}$
b) $\frac{1}{3} + \frac{\sqrt{3}}{4}$
c) $\frac{1}{2} + \frac{\sqrt{3}}{4}$
d) $\frac{1}{2} - \frac{\sqrt{3}}{4}$

11) The following system of linear equations $7x + 6y - 2z = 0$ $3x + 4y + 2z = 0$ $x - 2y - 6z = 0$ has
a) infinitely many solutions, $[x, y, z]$ satisfying $y = 2z$
b) infinitely many solutions $[x, y, z]$ satisfying $x = 2z$
c) no solution
d) only the trivial solution

12) If $p \to (p \wedge \neg q)$ is false, then the truth values of p and q are respectively:

a) F, T b) T, F c) F, F d) T, T

13) The length of minor axis (along y-axis) of an ellipse of the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line $x + 6y = 8$, then its eccentricity is:
a) $\frac{1}{2}\left(\frac{\sqrt{5}}{3}\right)$
b) $\frac{1}{2}\sqrt{\frac{11}{3}}$
c) $\sqrt{\frac{5}{6}}$
d) $\frac{1}{3}\sqrt{\frac{11}{3}}$

14) If z is a complex number satisfying $|\mathrm{Re}(z)| + |\mathrm{Im}(z)| = 4$, then $|z|$ cannot be:
   a) $\sqrt{7}$
   b) $\sqrt{\frac{17}{2}}$
   c) $\sqrt{10}$
   d) $\sqrt{8}$

15) If $x = \sum_{n=0}^{\infty}(-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$ where $0 < \theta < \frac{\pi}{4}$, then:
   a) $y(1+x) = 1$
   b) $x(1-y) = 1$
   c) $y(1-x) = 1$
   d) $x(1+y) = 1$