10.3.3.2

EE24BTECH11034 - K Teja Vardhan

1 Problem Statement

The difference of two supplementary angles by 18, find the angles.

2 equations

since they are supplementary and their difference is 18 hence the equations are,

$$x + y = 180,$$
$$x - y = 18.$$

Our goal is to solve for x and y using multiple methods.

3 Substitution Method

Step 1: Solve one equation for one variable

From the first equation:

$$x + y = 180, \tag{1}$$
$$y = 180 - x. \tag{2}$$

Step 2: Substitute into the second equation

Substitute $y = 180 - x$ into the second equation:

$$x - (180 - x) = 18, \tag{3}$$
$$x - 180 + x = 18, \tag{4}$$
$$2x - 180 = 18. \tag{5}$$

Step 3: Solve for x

$$2x = 198, \tag{6}$$
$$x = 99. \tag{7}$$

Step 4: Substitute back to find y

Substitute $x = 99$ into $y = 180 - x$:

$$y = 180 - 99, \tag{8}$$
$$y = 81. \tag{9}$$

Final Solution

$$\boxed{x = 99, \quad y = 81.}$$

4 Elimination Method

Step 1: Add the two equations

Add the two equations to eliminate y:

$$(x + y) + (x - y) = 180 + 18, \tag{10}$$
$$2x = 198, \tag{11}$$
$$x = 99. \tag{12}$$

Step 2: Substitute x into one equation

Substitute $x = 99$ into $x + y = 180$:

$$99 + y = 180, \tag{13}$$
$$y = 180 - 99, \tag{14}$$
$$y = 81. \tag{15}$$

Final Solution

$$\boxed{x = 99, \quad y = 81.}$$

5 Matrix Method

Step 1: Represent the system in matrix form

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 180 \\ 18 \end{bmatrix}. \tag{16}$$

Let $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, and $B = \begin{bmatrix} 180 \\ 18 \end{bmatrix}$.

Step 2: Compute the determinant of A

$$\det(A) = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}, \tag{17}$$
$$= (1 \times -1) - (1 \times 1), \tag{18}$$
$$= -1 - 1 = -2. \tag{19}$$

Step 3: Find A^{-1}

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}, \tag{20}$$

$$= \frac{1}{-2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}, \tag{21}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}. \tag{22}$$

Step 4: Multiply A^{-1} by B

$$X = A^{-1}B, \tag{23}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 180 \\ 18 \end{bmatrix}, \tag{24}$$

$$= \begin{bmatrix} 99 \\ 81 \end{bmatrix}. \tag{25}$$

Final Solution

$$\boxed{x = 99, \quad y = 81.}$$

6 Cramer's Rule

Step 1: Compute $\det(A)$

$$\det(A) = -2. \tag{26}$$

Step 2: Compute $\det(A_x)$ *and* $\det(A_y)$

Replace the first and second columns of A with B:

$$\det(A_x) = \begin{vmatrix} 180 & 1 \\ 18 & -1 \end{vmatrix}, \tag{27}$$

$$= (180 \times -1) - (1 \times 18) = -198, \tag{28}$$

$$\det(A_y) = \begin{vmatrix} 1 & 180 \\ 1 & 18 \end{vmatrix}, \tag{29}$$

$$= (1 \times 18) - (180 \times 1) = -162. \tag{30}$$

Step 3: Solve for x and y

$$x = \frac{\det(A_x)}{\det(A)} = \frac{-198}{-2} = 99, \tag{31}$$

$$y = \frac{\det(A_y)}{\det(A)} = \frac{-162}{-2} = 81. \tag{32}$$

Final Solution

$$\boxed{x = 99, \quad y = 81.}$$

7 Gaussian Elimination

Step 1: Augmented Matrix Representation

$$\begin{bmatrix} 1 & 1 & | & 180 \\ 1 & -1 & | & 18 \end{bmatrix} \tag{33}$$

Step 2: Forward Elimination

Eliminate x from the second row:

$$R_2 \to R_2 - R_1, \tag{34}$$

$$\begin{bmatrix} 1 & 1 & | & 180 \\ 0 & -2 & | & -162 \end{bmatrix}. \tag{35}$$

Step 3: Back Substitution

Solve for y from the second row:

$$-2y = -162, \tag{36}$$

$$y = 81. \tag{37}$$

Substitute $y = 81$ into the first row:

$$x + 81 = 180, \tag{38}$$

$$x = 99. \tag{39}$$

Final Solution

$$\boxed{x = 99, \quad y = 81.}$$

We are solving the system of equations:

$$x + y = 180, \tag{40}$$
$$x - y = 18. \tag{41}$$

Rewriting the equations:

$$x = 180 - y, \tag{42}$$
$$y = 18 + x. \tag{43}$$

First, we rewrite the question as a system of linear equations.

$$x_1 \implies x, \tag{44}$$
$$x_2 \implies y \tag{45}$$

Converting into matrix form, we get:

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 180 \\ 18 \end{pmatrix} \tag{46}$$
$$\mathbf{A}x = \mathbf{b} \tag{47}$$

To solve the above equation, we apply LU decomposition to matrix $\mathbf{A}$.

Step 2: LU Factorization Using Update Equations

Given a matrix $\mathbf{A}$ of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:

Step-by-Step Procedure:

1. **Initialization:** - Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.

2. **Iterative Update:** - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of $\mathbf{U}$ using the first update equation. - Compute the entries of $\mathbf{L}$ using the second update equation.

3. **Result:** - After completing the iterations, the matrix $\mathbf{A}$ is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where $\mathbf{L}$ is a lower triangular matrix with ones on the diagonal, and $\mathbf{U}$ is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of $\mathbf{U}$)

For each column $j \geq k$, the entries of $\mathbf{U}$ in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix $\mathbf{U}$ by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of $\mathbf{L}$)

For each row $i > k$, the entries of $\mathbf{L}$ in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

LU Factorizing $\mathbf{A}$, we get:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix}, \tag{48}$$

$$\mathbf{L} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \tag{49}$$

$$\mathbf{U} = \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} \tag{50}$$

The solution can now be obtained as:

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 180 \\ 18 \end{pmatrix} \tag{51}$$

Solving for y, we get:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 180 \\ -162 \end{pmatrix} \tag{52}$$

Now, solving for x via back substitution:

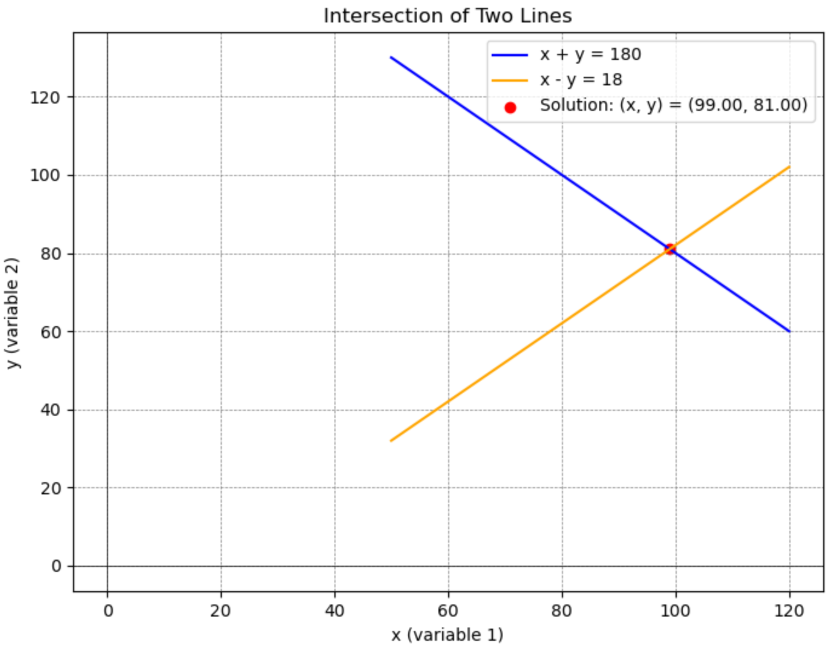
$$\begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 180 \\ -162 \end{pmatrix} \tag{53}$$

$$x_2 = 81, \tag{54}$$

$$x_1 + x_2 = 180 \implies x_1 = 99 \tag{55}$$

Thus, the solution is:

$$x = 99, \ y = 81 \tag{56}$$


Intersection of Two Lines
x + y = 180
x - y = 18
Solution: (x, y) = (99.00, 81.00)
y (variable 2)
x (variable 1)
0
20
40
60
80
100
120