

NCERT-9.4.5

EE24BTECH11035 - K TEJA VARDHAN

Question: Find the solution of the differential equation:

$$(e^x + e^{-x}) dy = (e^x - e^{-x}) dx \quad (1)$$

Solution: Rewriting the equation:

$$(e^x + e^{-x}) dy = (e^x - e^{-x}) dx \quad (2)$$

Let:

$$v = e^x + e^{-x} \quad (3)$$

Differentiating v with respect to x :

$$\frac{dv}{dx} = e^x - e^{-x} \quad (4)$$

Thus:

$$e^x - e^{-x} = \sqrt{v^2 - 4} \quad (5)$$

The given equation becomes:

$$v dy = \sqrt{v^2 - 4} dx \quad (6)$$

Substitute:

$$dx = \frac{dv}{\sqrt{v^2 - 4}} \quad (7)$$

This leads to:

$$v dy = dv \quad (8)$$

Rewrite:

$$dy = \frac{1}{v}, dv \quad (9)$$

Integrate both sides:

$$\int dy = \int \frac{1}{v} dv \quad (10)$$

$$y = \ln |v| + C \quad (11)$$

Substitute $v = e^x + e^{-x}$ back:

$$y = \ln(e^x + e^{-x}) + C \quad (12)$$

The solution to the differential equation is:

$$\boxed{y = \ln(e^x + e^{-x}) + C} \quad (13)$$

where C is the constant of integration(Here, it is assumed as 0).

Numerical Approach:

I used a for loop for finding the y values as the loop proceeds with iterative formula given below. I took some initial value of x and as loop proceeds I assigned it the value as $x + h$. where h is the step size, representing the rate of change.

2. Assigned the values of y for different x -values using a for loop.

The iterative formula for updating y -values is:

$$\frac{dy}{dx} = \frac{y_n - x_n}{y_n + x_n} \cdot h \quad (14)$$

$$y_{n+1} = y_n + \left(\frac{y_n - x_n}{y_n + x_n} \right) \cdot h \quad (15)$$

The iterative formula for updating x -values is:

$$x_n = x_{n-1} + h \quad (16)$$

Initial Conditions:

- $x = 0$
- $y = \ln 2$
- $h = 0.01$

Using Matplotlib, I plotted the computed points and the graph of the exact solution to verify that they approximately match.

