ASSIGNMENT 11

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I. JEE PYQ 2023 JANUARY 31, SHIFT 1

1) If $\sin^{-1}\left(\frac{\alpha}{17}\right) + \cos^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{77}{36}\right) = 0$, $0 < \alpha < 13$, then $\sin^{-1}(\sin\alpha) + \cos^{-1}(\cos\alpha)$ is equal to:

a) π b) 16 c) 0 d) $16 - 5\pi$

2) Let a circle C_1 be obtained on rolling the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ upwards 4 units on the tangent T to it at the point $(3, 2)$. Let C_2 be the image of C_1 in T. Let A and B be the centers of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn from A and B on the x-axis. Then the area of the trapezium $AMNB$ is:

a) $2\left(2 + \sqrt{2}\right)$ b) $4\left(1 + \sqrt{2}\right)$ c) $3 + 2\sqrt{2}$ d) $2\left(1 + \sqrt{2}\right)$

3) S1: $(p \Rightarrow q) \vee (p \wedge (\neg q))$ is a tautology.
S2: $((\neg p) \Rightarrow (\neg q)) \wedge ((\neg p) \vee q)$ is a contradiction. Then

a) only $(S2)$ is correct b) both $(S1)$ and $(S2)$ are correct c) both $(S1)$ and $(S2)$ are wrong d) only $(S1)$ is correct

4) The value of $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{(2+3\sin x)}{\sin x(1+\cos x)} dx$ is equal to:

a) $\frac{7}{2} - \sqrt{3} - \log_e \sqrt{3}$
b) $-2 + 3\sqrt{3} + \log_e \sqrt{3}$
c) $\frac{10}{3} - \sqrt{3} + \log_e \sqrt{3}$
d) $\frac{10}{3} - \sqrt{3} - \log_e \sqrt{3}$

5) A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is:

a) $\frac{5}{7}$ b) $\frac{2}{7}$ c) $\frac{3}{7}$ d) $\frac{5}{6}$

6) Let 5 digit numbers be constructed using the digits $0, 2, 3, 4, 7, 9$ with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is:

a) 2997 b) 2998 c) 2999 d) 2996

7) Let $a_1, a_2, \ldots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then $12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}} + \cdots + \frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$ is equal to:

a) 3 b) 4 c) 8 d) 6

8) Let θ be the angle between the planes $P_1 = \vec{r} \cdot \left(\hat{i} + \hat{j} + 2\hat{k}\right) = 9$ and $P_2 = \vec{r} \cdot \left(2\hat{i} - \hat{j} + \hat{k}\right) = 15$. Let L be the line that meets P_2 at the point $(4, -2, 5)$ and makes an angle θ with the normal of P_2. If α is the angle between L and P_2, then $\left(\tan^2\theta\right)\left(\cot^2\alpha\right)$ is equal to:

a) 9 b) 3 c) $\frac{9}{16}$ d) $\frac{16}{9}$

9) Let $\alpha > 0$ be the smallest number such that the expansion of $\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$ has a term $\beta x^{-\alpha}$, $\beta \in \mathbb{N}$. Then α is equal to:

a) 10 b) 2 c) 14 d) 16

10) Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = \sqrt{14}$, $\left|\vec{b}\right| = \sqrt{6}$, and $\left|\vec{a} \times \vec{b}\right| = \sqrt{48}$. Then $\left(\vec{a} \cdot \vec{b}\right)^2$ is equal to:

a) 16 b) 25 c) 36 d) 49

11) Let the line $L : \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P. Let the point Q be the foot of perpendicular from the point $R(1, -1, -3)$ on the line L. If α is the area of triangle PQR, then α^2 is equal to:

a) $\frac{16}{3}$ b) $\frac{25}{3}$ c) $\frac{36}{3}$ d) $\frac{49}{3}$

12) The remainder on dividing 5^{99} by 11 is:

a) 1 b) 2 c) 3 d) 4

13) If the variance of the frequency distribution

X_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

is 2.5, then α is equal to:

a) 7 b) 8 c) 9 d) 10

14) Let for $x \in \mathbb{R}$ $f(x) = \frac{x+|x|}{2}$ and $g(x) = \begin{cases} x, & x < 0 \\ x^2, & x \geq 0 \end{cases}$. Then the area bounded by the curve $y = (f \circ g)(x)$ and the lines $y = 0$, $2y - x = 15$ is equal to:

a) $\frac{225}{4}$ b) $\frac{425}{4}$ c) $\frac{325}{4}$ d) $\frac{525}{4}$

15) Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11, is equal to:

a) 780 b) 781 c) 782 d) 783