ASSIGNMENT 12

EE24BTECH11034 - K Teja Vardhan

I. JEE PYQ 2024 JANUARY 30, SHIFT 2

1) Consider the system of linear equations $x+y+z=5$, $x+2y+\lambda^2 z=9$, $x+3y+\lambda z=\mu$ where $\lambda, \mu \in \mathbb{R}$. Then, which of the following statement is NOT correct?
   a) System has infinite number of solutions if $\lambda = 1$ and $\mu = 13$
   b) System is inconsistent if $\lambda = 1$ and $\mu \neq 13$
   c) System is consistent if $\lambda \neq 1$ and $\mu = 13$
   d) System has unique solution if $\lambda \neq 1$ and $\mu \neq 13$

2) For $\alpha, \beta \in \left[0, \frac{\pi}{2}\right]$, let $3\sin(\alpha+\beta) = 2\sin(\alpha-\beta)$ and a real number k be such that $\tan\alpha = k\tan\beta$. Then the value of k is equal to:

   a) $-\frac{2}{3}$ b) -5 c) $\frac{2}{3}$ d) 5

3) Let $A(\alpha, 0)$ and $B(0, \beta)$ be the points on the line $5x + 7y = 50$. Let the point P divide the line segment AB internally in the ratio $7:3$. Let $3x - 25 = 0$ be a directrix of the ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the corresponding focus be S. If from S, the perpendicular on the x-axis passes through P, then the length of the latus rectum of E is equal to:

   a) $\frac{25}{3}$ b) $\frac{32}{9}$ c) $\frac{25}{9}$ d) $\frac{32}{5}$

4) Let $\vec{a} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$, $\alpha, \beta \in \mathbb{R}$. Let a vector $\vec{b}$ be such that the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $\left|\vec{b}\right| = 6$. If $\vec{a}\cdot\vec{b} = 3\sqrt{2}$, then the value of $\left(\alpha^2+\beta^2\right)\left|\vec{a}\times\vec{b}\right|$ is equal to:

   a) 90 b) 75 c) 95 d) 85

5) Let $f(x) = (x+3)(x-2)^3$, $x \in [-4, 4]$. If M and m are the maximum and minimum values of f, respectively in $[-4, 4]$, then the value of $M - m$ is:

   a) 600 b) 392 c) 608 d) 108

6) Let a and b be two distinct positive real numbers. Let 11^{th} term of a GP, whose first term is a and third term is b, is equal to p^{th} term of another GP, whose first term is a and fifth term is b. Then p is equal to:

   a) 20 b) 25 c) 21 d) 24

7) If $x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$ is the locus of a point, which moves such that it is always equidistant from the lines $x + 2y + 7 = 0$ and $2x - y + 8 = 0$, then the value of $g + c + h - f$ equals:

a) 14 b) 6 c) 8 d) 29

8) Let $\vec{a}$ and $\vec{b}$ be two vectors such that $\left|\vec{b}\right| = 1$ and $\left|\vec{b} \times \vec{a}\right| = 2$. Then $\left|\left(\vec{b} \times \vec{a}\right) - \vec{b}\right|^2$ is equal to:

a) 3 b) 5 c) 1 d) 4

9) Let $y = f(x)$ be a thrice differentiable function in $(-5, 5)$. Let the tangents to the curve $y = f(x)$ at $(1, f(1))$ and $(3, f(3))$ make angles $\frac{\pi}{6}$ and $\frac{\pi}{4}$, respectively with positive x-axis. If $27\int_1^3 \left(\left(f'(t)\right)^2 + 1\right) f''(t)\, dt = \alpha + \beta\sqrt{3}$, where α, β are integers, then the value of $\alpha + \beta$ equals

a) -14 b) 26 c) -16 d) 36

10) Let P be a point on the hyperbola $H : \frac{x^2}{9} - \frac{y^2}{4} = 1$, in the first quadrant such that the area of triangle formed by P and the two foci of H is $2\sqrt{13}$. Then, the square of the distance of P from the origin is

a) 18 b) 26 c) 22 d) 20

11) Bag A contains 3 white, 7 red balls and bag B contains 3 white, 2 red balls. One bag is selected at random and a ball is drawn from it. The probability of drawing the ball from the bag A, if the ball drawn in white, is:

a) $\frac{1}{4}$ b) $\frac{1}{9}$ c) $\frac{1}{3}$ d) $\frac{3}{10}$

12) Let $f : \mathbb{R} \to \mathbb{R}$ be defined $f(x) = ae^{2x} + be^x + cx$. If $f(0) = -1$, $f'(0) = 1$, and $f''(0) = 0$, then the value of $\frac{b}{a}$ is equal to:

a) -3 b) 2 c) -2 d) 3