NCERT-12.9.7.16

EE24BTECH11034 - K TEJA VARDHAN

Question: Find the solution of the differential equation:

$$\frac{y\,dx - x\,dy}{y} = 0 \tag{1}$$

Solution: Rewriting the equation:

$$y\,dx - x\,dy = 0 \tag{2}$$

Rearranging:

$$\frac{dx}{x} = \frac{dy}{y} \tag{3}$$

Integrate both sides:

$$\int \frac{dx}{x} = \int \frac{dy}{y} \tag{4}$$

This gives:

$$\ln|x| = \ln|y| + C \tag{5}$$

Simplify using properties of logarithms:

$$\ln\left|\frac{x}{y}\right| = C \tag{6}$$

Exponentiate both sides:

$$\frac{x}{y} = e^{C} \tag{7}$$

Let $e^{C} = k$ (where k is an arbitrary positive constant):

$$x = ky \tag{8}$$

The general solution is:

$$\boxed{x = ky} \tag{9}$$

where k is an arbitrary constant which is assumed to be 1.

Numerical Approach:
I used a for loop for finding the y values as the loop proceeds with iterative formula given below. I took some initial value of x and as loop proceeds I assigned it the value as $x+h$. where h is the step size, representing the rate of change.
2. Assigned the values of y for different x-values using a for loop.

The iterative formula for updating y-values is:

$$\frac{dy}{dx} = \frac{y}{x}.h \tag{10}$$

$$y_{n+1} = y_n + \left(\frac{y}{x}\right).h \tag{11}$$

The iterative formula for updating x-values is:

$$x_n = x_{n-1} + h \tag{12}$$

Initial Conditions:

- $x = 1$
- $y = 1$
- $h = 0.01$

Using Matplotlib, I plotted the computed points and the graph of the exact solution to verify that they approximately match.

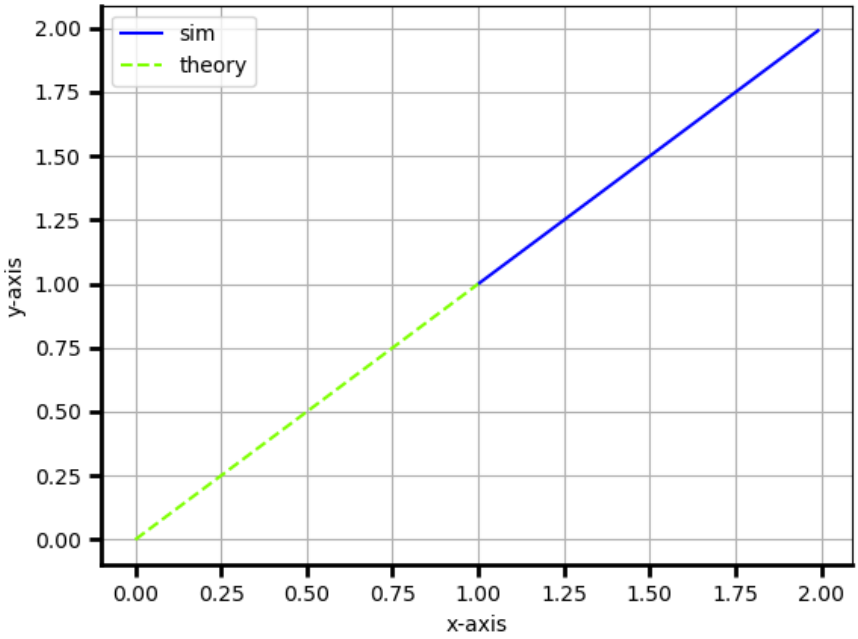

Fig. 0.1: verifying through graph of sim and theory values