Day 8: Convolutional Neural Networks

Summer STEM: Machine Learning

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Learning Objectives

- How is an image represented in a computer?
- What is convolution?
- How is convolution used in neural networks?
- What is a convolutional layer? How does it aid with feature extraction?
- What is a Tensor, what does its shape represent?
- How do we represent RGB images in a computer?

Outline

Extending Logistic Regression

- Motivation: Feature engineering in the model

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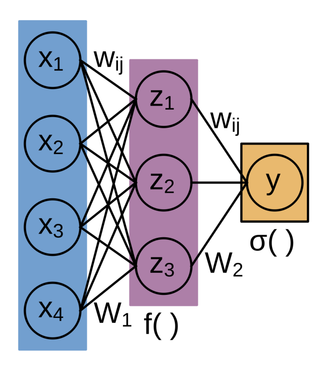

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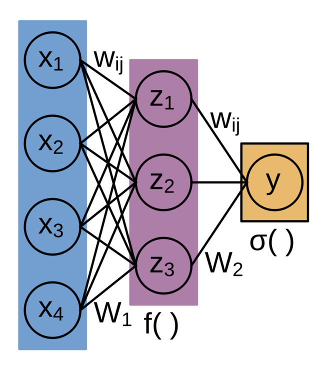

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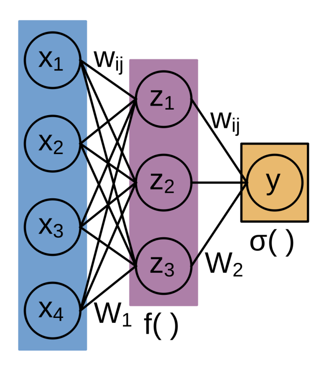

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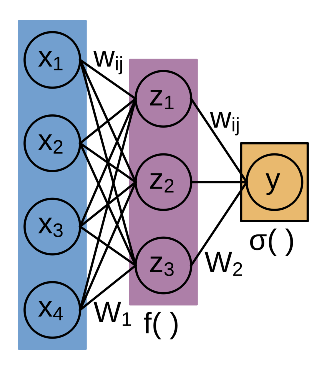

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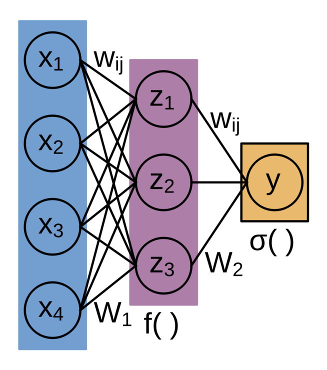
- But why stop here?...



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    - **Multi-Class Classification**: Soft-max Output

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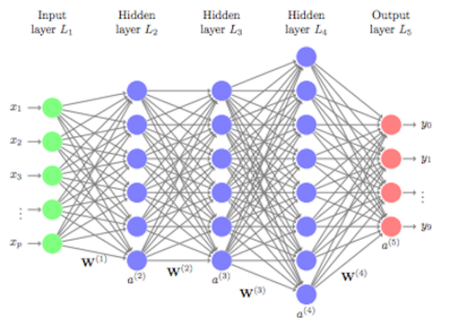

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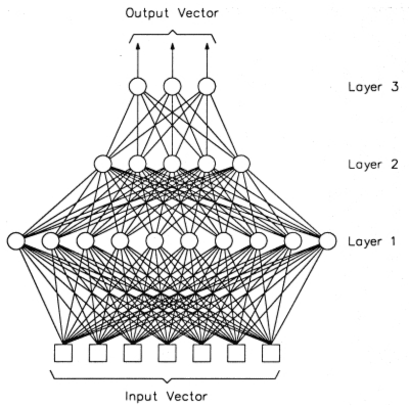

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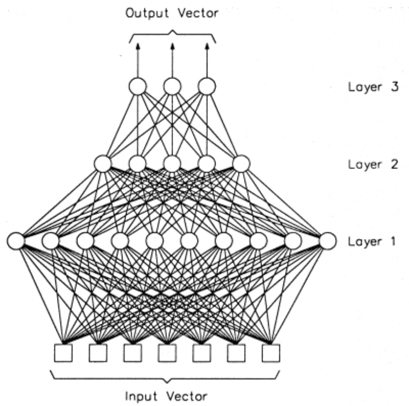

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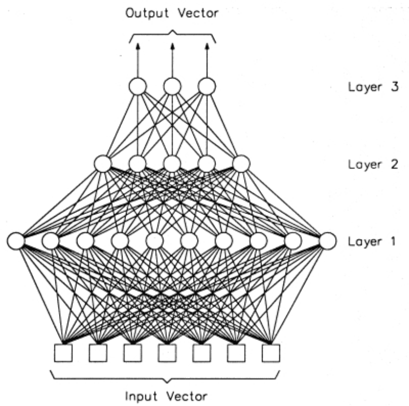

Outline

Better performance with images

- Encoding locality
- How does an MLP see an image?
- Is this how we see images?

Examples: Lena & Mandrill

Outline

Images in Computer

- Images are stored as arrays of quantized numbers in computers
- Gray scale image: 2D matrices with each entry specifying the intensity (brightness) of a pixel
    - Pixel values range from 0 to 255, 0 being the darkest, 255 being the brightest

```
[[255 255 255]
 [255   0 255]
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```
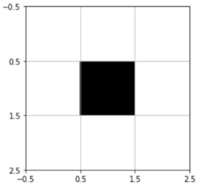
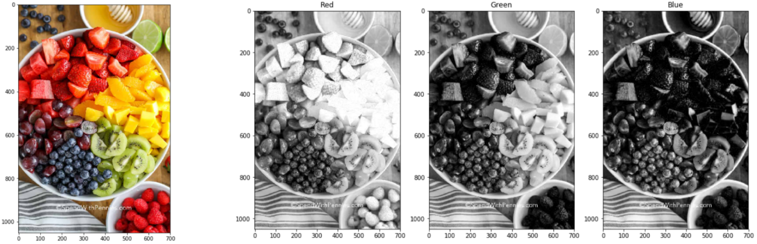



Color Images

- Color image: 3D array, 2 dimensions for space, 1 dimension for color
  - Can be thought of as three 2D matrices stacked together into a cube, each 2D matrix specify the amount of each color: Red ,Green ,Blue value at each pixel



- Shape of this image: (1050,700,3)
- There are 1050x700 pixels, 3 channels: R,G,B

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- Higher definition images often contain millions of pixels $\rightarrow$ It is not practical to use fully connected network
- Fully connected network treat each individual pixel as a feature, it does not utilize the positional relationship between pixels

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Convolution

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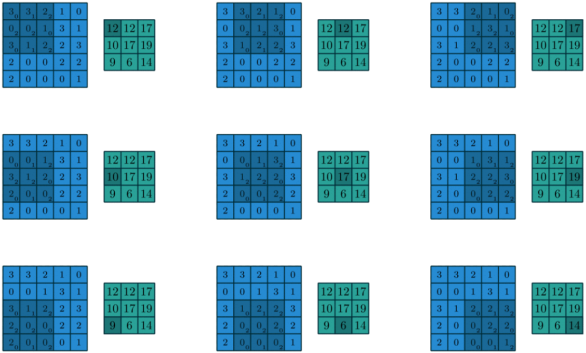
- Equation:

At each offset (j_1, j_2) compute:

$$Z[j_1, j_2] = \sum_{k_1=0}^{K_1-1} \sum_{k_2=0}^{K_2-1} W[k_1, k_2] X[j_1 + k_1, j_2 + k_2]$$

Example of a Convolultion



Kernel

$$W = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

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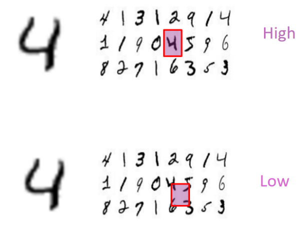
- This allows us to learn the positional relationship between pixels
- Use of different kernels allows us to detect features



Convolution for Multiple Channels

- A kernel for each channel. Could be same kernel, or different
- Perform a convolution for each of the channel, with the respective kernel
- Sum the results

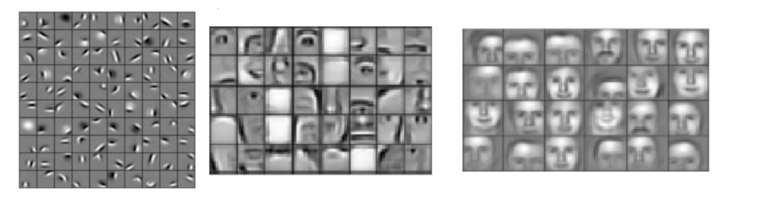
Outline

Averaging Kernels

- Uniform Kernel: $\frac{1}{K_x K_y}\begin{bmatrix}1 & .. & 1\\ 1 & .. & 1\\ 1 & .. & 1\end{bmatrix}$
  $K_x =$ Number of Columns
  $K_y =$ Number of Rows
- Gaussian Kernel is a blurring kernel too.

Edge Detection

- Initial layers in a deep neural networks detect small patterns like lines, curves or edges.
- Subsequent layers combine these local features to create more complex features.



Edge Detection

- Using Sobel filters:
  - Vertical Edge Detection $G_x = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$
  - Horizontal Edge Detection $G_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

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Thank You!

- Next Class: ...