GATE Revision Series 2023

Aditya Vangala (M.Tech (Res) IISc Bangalore)
Faculty at Ace Engineering Academy

Subjects: Engineering Maths for all branches and Signals and Systems for ECE, EE and IN.

Email Id: adityasrnvs@gmail.com
Telegram Id: adityasrnvs

Aditya Vangala



# Linear Algebra

1. Basics
2. Determinants
3. Rank
4. System of linear eq$^{ns}$
5. eigen values and eigen values

CSE

LU

ECE

vector space.

Aditya Vangala

ACE

$A_{m \times n}$ , $B_{n \times p}$

Total multiplications $= m \cdot n \cdot p$

Total Additions $= m(n-1)p$

Aditya Vangala

ACE

Idempotent matrix : $A^2 = A$

Involutary matrix $A^2 = I$

Nilpotent matrix $N^P = O$

orthogonal matrix $A \cdot A^T = A^T \cdot A = I$ or $A^T = A^{-1}$

sym matrix $A^T = A$ or $a_{ij} = a_{ji}$

skew sym $A^T = -A$

Hermitian $\bar{A}^T = A$

skew Herm $\bar{A}^T = -A$

Aditya Vangala

ACE

Identity matrix :- $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

upper triangular $\begin{pmatrix} u_{11} & u_{12} & u_{12} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$

Diagonal

$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$

Lower $\begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix}$

Aditya Vangala

ACE

## Determinants :-

$$\begin{vmatrix} a & b & c \\ d & e & f \\ x & y & z \end{vmatrix} = a(ez - fy) - b(dz - fx) + c(dy - ex)$$

Aditya Vangala

ACE

# Elementary Row operations :-

Type 1 : Interchange Row $i$ and $j$

$$|B| = -|A|$$

Type 2 : $R_i \to \alpha R_i \quad , \alpha \neq 0$

$$|B| = \alpha |A|$$

$C = \alpha A$

$|C| = \alpha^n |A|$

Type 3 : $R_i \to R_i + \alpha R_j$

$$|B| = |A|$$

$R_1 \to 2R_1 + R_2$ ✗

$R_1 \to R_1 + 2R_2$ ✓

Aditya Vangala

ACE

Since 1995

Operations that do not change Det

$R_1 \to R_1 + 2R_2$

$R_1 \to R_1 + R_2 + R_3 + \cdots + R_n$

Operations that change the Det value

$R_1 \to 2R_1 + R_2$

Aditya Vangala

ACE

1. $|A \cdot B| = |A| \cdot |B|$

2. $|A^2| = |A|^2$

3. $|A^n| = |A|^n$

4. $|A+B| \neq |A| + |B|$

5. $\begin{vmatrix} a & b & c \\ d & e & f \\ a & b & c \end{vmatrix} = 0$ $\quad \begin{vmatrix} a & b & c \\ d & e & f \\ \alpha a & \alpha b & \alpha c \end{vmatrix} = 0$ $\quad \begin{vmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 0 \end{vmatrix} = 0$

Aditya Vangala

ACE

A is skew sym matrix of odd order $|A| = 0$

$$\begin{vmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{vmatrix} = u_{11} \cdot u_{22} \cdot u_{33}$$

Rank :-

1. Convert the given matrix to row echelon form.

2. Rank is n.o of non-zero rows.

Elementary row operations do not change rank of the matrix.

Aditya Vangala

ACE

Linearly independent and dependent vectors:-

$|v_1\ v_2\ v_3 \cdots v_n| \neq 0$ L.I

$|v_1\ v_2\ v_3 \cdots v_n| = 0$ L.D

Aditya Vangala

ACE

Since 1995

System of linear eqns :-

Homogeneous sys

$Ax = 0$

Trivial solution

$R(A) = n$

Non-trivial Solution

$R(A) < n$

$n$ is number of variables.

Aditya Vangala

ACE

Non-Homogeneous sys

$Ax = b$

No solution

$R(A|b) \neq R(A)$

unique solution

$R(A|b) = R(A) = n$

Infinitely many solution

$R(A|b) = R(A) < n$

## Eigen Values and Eigen vectors :-

Aditya Vangala

ACE

$|A - \lambda I| = 0$

$(A - \lambda I)X = 0$

$$\boxed{A \cdot X = \lambda \cdot X}$$

$\lambda \longrightarrow A$

$\lambda^2 \longrightarrow A^2$

$\lambda^m \longrightarrow A^m$

$f(\lambda) \longrightarrow f(A)$

$a_2\lambda^2 + a_1\lambda + a_0 \longrightarrow a_2A^2 + a_1A + a_0I$

$\frac{1}{\lambda} \longrightarrow A^{-1}$

$\frac{|A|}{\lambda} \longrightarrow \text{Adj } A$

Aditya Vangala

ACE

Trace(A) = Sum of eigen values

|A| = Product of eigen values.

Real Sym matrix $\Rightarrow$ All eigen values are real

Skew Sym $\Rightarrow$ Eigen values are either 0 or Purely Imaginary

Diagonal, triangular matrix : eigen values are principal diagonal itself.

Sym matrix :-

$\lambda_1 , \lambda_2$

$e_1 , e_2$

$e_1^T \cdot e_2 = 0$

Cayley Hamilton theorem :-

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$$

$|A - \lambda I| = 0$

$\lambda^2 - 5\lambda + 4 = 0$

$A^2 - 5A + 4I = O$

$A^2$

1. $5A - 4I$
2. $5A + 4I$

Aditya Vangala

ACE

$$\begin{pmatrix} 2 & 3 & 4 \\ 0 & 3 & 5 \\ 0 & 0 & 7 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\lambda = 2, 3, 7$

$A.m(\lambda = 2) = 1$

$A.m(\lambda = 3) = 1$

$A.m(\lambda = 7) = 1$

$\lambda = 1, 1, 1$

$A.m(\lambda = 1) = 3$

Aditya Vangala

ACE

$G.M = n - R(A - \lambda I)$

$=$ Number of L.I eigen vectors of A corresponding eigen value $\lambda$

If $A = \begin{bmatrix} 10 & 2k+5 \\ 3k-3 & k+5 \end{bmatrix}$ is a symmetric matrix, the value of k is .

Aditya Vangala

ACE

(GATE-2022)

(A) 8

(B) 5

(C) -0.4

(D) $\frac{1+\sqrt{1561}}{12}$

$A^T = A$

$$A^T = \begin{pmatrix} 10 & 3K-3 \\ 2K+5 & K+5 \end{pmatrix}$$

$2K+5 = 3K-3$

$5+3 = 3K-2K$

$K = 8$

The two vectors [ 1 1 1 ] and [ 1 $a$ $a^2$ ] where $a = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$ and $j = \sqrt{-1}$ are

$V_1$ $V_2$

$\omega$

(GATE-2011)

Aditya Vangala

ACE

(a) Orthonormal

(b) Orthogonal ✓

(c) Parallel

(d) Collinear

$1, \omega, \omega^2$ are cube roots of unity.

$1 + \omega + \omega^2 = 0$

$$V_1 \cdot V_2 = 1.1 + 1.a + 1.a^2$$
$$= 1 + a + a^2$$
$$= 0$$

Dot product = 0

$$|V_1| = \sqrt{1^2 + 1^2 + 1^2}$$
$$= \sqrt{3}$$

Euclidean norm (length) of the vector $[4 \quad -2 \quad -6]^T$ is

Aditya Vangala

ACE

GATE-2019

(a) $\sqrt{56}$

(b) $\sqrt{24}$

(c) $\sqrt{48}$

(d) $\sqrt{12}$

$$\|V\| = \sqrt{4^2 + (-2)^2 + (-6)^2}$$

$$= \sqrt{16 + 4 + 36}$$

$$= \sqrt{56}$$

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

Aditya Vangala

ACE

$R_1 \to R_1 + R_2 + R_3 + R_4$

$$\begin{vmatrix} 5 & 5 & 5 & 5 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix}$$

$$5\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$R_4 \to R_4 - R_1$

$$5\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$= 5(1 \times 1 \times 1 \times 1)$

$= \underline{\underline{5}}$

Aditya Vangala

ACE

Consider the matrix

$$J_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$I_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

which is obtained by reversing the order of columns of the identity matrix $I_6$. Let $P = I_6 + \alpha J_6$ where $\alpha$ is a non-negative real number. The value of $\alpha$ for which $|P| = 0$ is (GATE-14)

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & \alpha \\ 0 & 1 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 1 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 1 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 1 & 0 \\ \alpha & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

For $\alpha = 1$

$|P| = 0$

The matrix $A = \begin{bmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{bmatrix}$ has $|A| = 100$ and $trace(A) = 14$. The value of $|a - b|$ is

Aditya Vangala

ACE

GATE-2016

$$|A| = 0(\ ) + 0(\ ) + 0(\ ) + b(-1)^{4+4} \begin{vmatrix} a & 0 & 3 \\ 2 & 5 & 1 \\ 0 & 0 & 2 \end{vmatrix} = 100$$

$$b\left(a(10) - 0(\ ) + 3(0)\right) = 100$$

$$10ab = 100$$

$$ab = 10$$

$$\text{Trace}(A) = 14$$

$$a + 5 + 2 + b = 14$$

$$a + b = 7$$

$$ab = 10$$

$$(a-b)^2 = (a+b)^2 - 4ab$$

$$= 7^2 - 4(10)$$

$$= 49 - 40$$

$$(a-b)^2 = 9$$

$$a - b = \pm 3$$

$$\underline{\underline{|a-b| = 3}}$$

Aditya Vangala

ACE

The inverse of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ is

$A^{-1} = \dfrac{\text{Adj } A}{|A|}$

GATE-2019

(a) $\begin{bmatrix} -10 & 4 & 9 \\ 15 & -4 & -14 \\ -5 & 1 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & \frac{-4}{5} & \frac{-9}{5} \\ -3 & \frac{4}{5} & \frac{14}{5} \\ 1 & \frac{-1}{5} & \frac{-6}{5} \end{bmatrix}$

(c) $\begin{bmatrix} -2 & \frac{4}{5} & \frac{9}{5} \\ 3 & \frac{-4}{5} & \frac{-14}{5} \\ -1 & \frac{1}{5} & \frac{6}{5} \end{bmatrix}$

(d) $\begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$

$A \cdot A^{-1} = I$

$A^{-1} \cdot A = I$

Aditya Vangala

ACE

$$\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} -10 & 4 & 9 \\ 15 & -4 & -14 \\ -5 & 1 & 6 \end{bmatrix} =$$

$-20 + 45 - 20 = 5$ ✗

$$\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} -2 & \frac{4}{5} & \frac{9}{5} \\ 3 & \frac{-4}{5} & \frac{-14}{5} \\ -1 & \frac{1}{5} & \frac{6}{5} \end{bmatrix} =$$

$-4 + 9 - 4 = 1$ ✓

The rank of the matrix is $\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$

Aditya Vangala

ACE

Since 1995

2 min

GATE-17

$R_4 \to R_4 + R_1 + R_2 + R_3 + R_5$

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

$R_2 \leftrightarrow R_3$, $R_4 \leftrightarrow R_5$

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$R(A) = 4$

$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$, $B = \begin{bmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{bmatrix}$



If the rank of the matrix $A$ is $N$ then the rank of matrix $B$ is

GATE-2014

(a) $\frac{N}{2}$

(b) $N-1$

(c) $N$ ✓

(d) $2N$

$B = A \cdot A^T$

$B = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \cdot \begin{pmatrix} p & r \\ q & s \end{pmatrix} = \begin{pmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{pmatrix}$

$R(A \cdot A^T) = R(A) = N$

$X = [x_1\ x_2\ \cdots\ x_n]^T$ is an $n$-tuple non-zero vector.
The $n \times n$ matrix $V = XX^T$

Aditya Vangala

ACE

GATE-2007

(a) has rank zero

(b) has rank 1

(c) is orthogonal

(d) has rank n

If the vectors (1,-1,2), (7,3,x) and (2,3,1) in $R^3$ are linearly dependent then $x = ?$ GATE-21

Aditya Vangala

ACE

$$\begin{vmatrix} 1 & -1 & 2 \\ 7 & 3 & x \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$$1(3-3x)+1(7-2x)+2(21-6)=0$$

$$3-3x+7-2x+30=0$$

$$40=5x$$

$$x=8$$

Aditya Vangala

ACE

Given $M = \begin{bmatrix} 2 & 3 & 7 \\ 6 & 4 & 7 \\ 4 & 6 & 14 \end{bmatrix}$, which of the following statement(s) is/are correct?

GATE-2022(MSQ)

(A) The rank of $M$ is 2 True

(B) The rank of $M$ is 3 False

(C) The rows of $M$ are linearly independent False

(D) The determinant of $M$ is 0 True

$R_3 = 2R_1$

$|M| = 0$

$R(M) < 3$

$\begin{vmatrix} 2 & 3 \\ 6 & 4 \end{vmatrix} \neq 0$

$R(M) = 2$

(A, D)

Consider the following system of equation

$$3x + 2y = 1$$
$$4x + 7z = 1$$
$$x + y + z = 3$$
$$x - 2y + 7z = 0$$

Which of the following is true? GATE-14

(a) The system has no solution

(b) The system has infinitely many solutions

(c) The system has unique solution

(d) The rank of the augmented matrix of the system is 2

Aditya Vangala

ACE

x, y and z are the 3 variables

$n = 3$

$$(A|b) = \begin{pmatrix} 1 & 1 & 1 & 3 \\ 4 & 0 & 7 & 1 \\ 3 & 2 & 0 & 1 \\ 1 & -2 & 7 & 0 \end{pmatrix}$$

$R_2 \to R_2 - 4R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - R_1$

$$\begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -4 & 3 & -11 \\ 0 & -1 & -3 & -8 \\ 0 & -3 & 6 & -3 \end{pmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -1 & -3 & -8 \\ 0 & -4 & 3 & -11 \\ 0 & -3 & 6 & -3 \end{pmatrix}$$

$R_3 \to R_3 - 4R_2$

$R_4 \to R_4 - 3R_2$

$$\begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -1 & -3 & -8 \\ 0 & 0 & 15 & 21 \\ 0 & 0 & 15 & 21 \end{pmatrix}$$

$R_4 \to R_4 - R_3$

$$\begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -1 & -3 & -8 \\ 0 & 0 & 15 & 21 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$R(A|b) = 3$

$R(A) = 3$

$n = 3$

$R(A|b) = R(A) = n$

unique solution

The system of linear equations in real $(x, y)$ given by

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & 5-2\alpha \\ \alpha & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

involves a real parameter $\alpha$ and has infinitely many non-trivial solutions for special value(s) of $\alpha$. Which one or more among the following options is/are non-trivial solution(s) of $(x, y)$ for such special value(s) of $\alpha$? GATE-2022(MSQ)

Aditya Vangala



(a) $x = 2, y = -2$ $\Rightarrow x = -y$

(b) $x = -1, y = 4$ $\Rightarrow x = -\frac{1}{4} y$

(c) $x = 1, y = 1$

(d) $x = 4, y = -2$

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & 5-2\alpha \\ \alpha & 1 \end{bmatrix} = (0 \;\; 0)$$

$$2x + \alpha y = 0$$

$$(5-2\alpha)x + y = 0$$

$$\begin{pmatrix} 2 & \alpha \\ 5-2\alpha & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$AX = 0$$

Aditya Vangala

ACE

$$\begin{pmatrix} 2 & \alpha \\ 5-2\alpha & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$AX = 0$$

$$r(A) < n$$

$$r(A) < 2$$

$$|A| = 0$$

$$2 - \alpha(5-2\alpha) = 0$$

$$2 - 5\alpha + 2\alpha^2 = 0$$

Roots $\alpha = \frac{1}{2}, 2$

$$AX = 0$$

1 $\alpha = 1/2$

$$\begin{pmatrix} 2 & 1/2 \\ 4 & 1 \end{pmatrix} \quad R_2 \to R_2 - 2R_1$$

$$\begin{pmatrix} 2 & 1/2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$2x + \frac{1}{2}y = 0$$

$$2x = -\frac{1}{2}y$$

$$x = -\frac{1}{4}y$$

Aditya Vangala

ACE

(2) $\alpha = 2$

$$\begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$$

$R_2 \to R_2 - \frac{1}{2}R_1$ ✓

$$\begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$2x + 2y = 0$

$x = -y$

Consider solving the following system of simultaneous equations using LU decomposition.

$$x_1 + x_2 - 2x_3 = 4$$
$$x_1 + 3x_2 - x_3 = 7$$
$$2x_1 + x_2 - 5x_3 = 7$$

where L and U are denoted as

$$\begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

Which one of the following is the correct combination of values for $L_{32}$, $U_{33}$, and $x_1$ ?

(a) $L_{32} = 2, U_{33} = \frac{-1}{2}, x_1 = -1$

(b) $L_{32} = 2, U_{33} = 2, x_1 = -1$

(c) $L_{32} = \frac{-1}{2}, U_{33} = 2, x_1 = 0$

(d) $L_{32} = \frac{-1}{2}, U_{33} = \frac{-1}{2}, x_1 = 0$

GATE-2022

Which of the following is/are the eigenvector(s) for the matrix given below?

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix}$$

GATE-2022(MSQ)

(a) $\begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} -1 \\ 0 \\ 2 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix}$

Aditya Vangala

ACE

$AX = \lambda X$

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{pmatrix} 9 - 6 - 4 \\ 8 - 6 - 1 \\ -20 + 15 + 5 \\ -32 + 21 + 12 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$AX = 1 \cdot X$

Aditya Vangala

ACE

# Probability

$$P(A) = \frac{\text{Number of elements in } A}{\text{Number of elements in } S}$$

1. $0 \leq P(A) \leq 1$

2. $P(S) = 1$

3. $P(A \cup B) = P(A) + P(B) \quad (A \cap B = \phi)$

Aditya Vangala

ACE

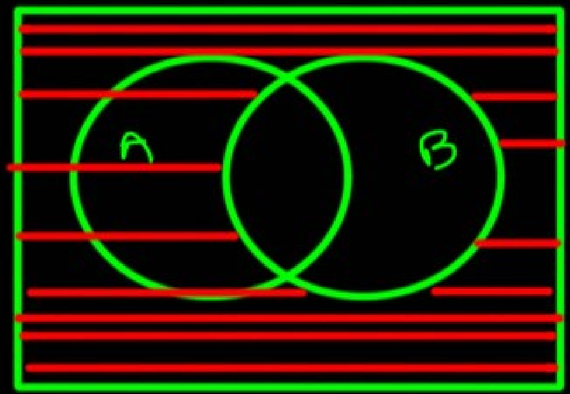
$P(A^c) = 1 - P(A)$ ( At least )

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$
$- P(A \cap B) - P(B \cap C) - P(C \cap A)$
$+ P(A \cap B \cap C)$

Aditya Vangala

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$




$\leftarrow B^c$

Aditya Vangala

ACE

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

If A and B are mutually exclusive

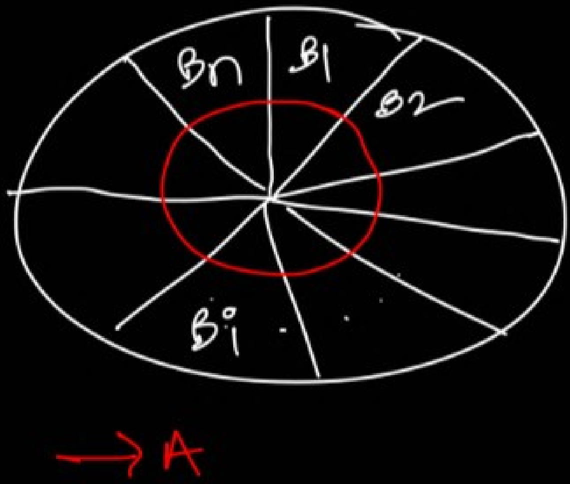
$P(A \cap B) = 0$ $\quad P(A|B) = 0$ $\quad P(B|A) = 0$

If A and B are independent

$P(A|B) = P(A)$ $\quad P(B|A) = P(B)$

Aditya Vangala

ACE




① $B_1, B_2 \ldots B_n$ are mutually exclusive

② $B_1 \cup B_2 \ldots \cup B_n = S$

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \ldots + P(A|B_n)P(B_n)$$

$$P(B_1|A) = \frac{P(A|B_1)\,P(B_1)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \ldots + P(A|B_n)P(B_n)}$$

$$P(B_2|A) = \frac{P(A|B_2)P(B_2)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \ldots + P(A|B_n)P(B_n)}$$

A box contains the following three coins.

I. A fair coin with head on one face and tail on the other face

II. A coin with heads on both faces.

III. A coin with tails on both faces

A coin picked randomly from the box and tossed. Out of the two remaining coins in the box, one coin is then picked randomly and tossed. If the first toss results in a head, the probability of getting a head in the second toss is GATE-2021

A cab was involved in a hit and run accident at night. You are given the following data about the cabs in the city and the accident.

(i) 85% of cabs in the city are green and the remaining cabs are blue.

(ii) a witness identified the cab involved in the accident as blue.

(iii) it is known that a witness can correctly identify the cab colour only 80% of the time.

Which of the following options is closest to the probability that the accident was caused by a blue cab? (GATE 2018)

(a) 12% (b) 15% (c) 41% (d) 80%

A Sender (S) transmits a signal, which can be one of the two kinds: H and L with probabilities 0.1 and 0.9 respectively, to a receiver (R). In the graph below, the weight edge $(u, v)$ is the probability of receiving $v$ when $u$ is transmitted, where $u, v \in \{H, L\}$. For example, the probability that the received signal is L given the transmitted signal was H is 0.7. If the received signal is H, the probability that the transmitted signal was H is

(GATE-2021)

Aditya Vangala

ACE

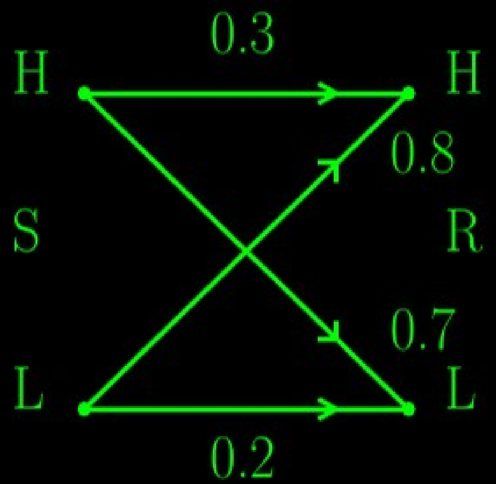
$P(H) = 0.1 \quad P(L) = 0.9$

$P(Rx = L \mid Tx = H) = 0.7$

$P(Rx = H \mid Tx = L) = 0.8$

$P(Rx = H \mid Tx = H) = 0.3$

$P(Rx = L \mid Tx = L) = 0.2$




$$P(Tx = H \mid Rx = H) = \frac{(0.1)(0.3)}{(0.1)(0.3) + 0.9\,(0.8)}$$

$$=$$

An urn contains 5 red and 7 green balls. A ball is drawn at random and its colour is noted. The ball is placed back into the urn along with another ball of the same colour. The probability of getting a red ball in the next draw is GATE-16

Aditya Vangala

ACE

(a) $\frac{5}{12}$

(b) $\frac{5}{7}$

(c) $\frac{6}{12}$

(d) $\frac{8}{12}$

$5R + 7G$

$$P(R) = \frac{5}{5+7} = \frac{5}{12}$$

$$P(G) = \frac{7}{5+7} = \frac{7}{12}$$

A bag has r red balls and b black balls. All balls are identical except for their colours. In a trial, a ball is randomly drawn from the bag, its colour is noted and the ball is placed back into the bag along with another ball of the same colour. Note that the number of balls in the bag will increase by one, after the trial. A sequence of four such trials is conducted. Which one of the following choices gives the probability of drawing a red ball in the fourth trial? GATE-2021

Aditya Vangala

ACE

(a) $\left(\frac{r}{r+b}\right)\left(\frac{r+1}{r+b+1}\right)\left(\frac{r+2}{r+b+2}\right)\left(\frac{r+3}{r+b+3}\right)$

(b) $\frac{r}{r+b}$

(c) $\frac{r}{r+b+3}$

(d) $\frac{r+3}{r+b+3}$

$$P(R) = \frac{r}{r+b}$$

Two players A and B alternately keep rolling a fair die. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is GATE-15

Aditya Vangala



(a) $\frac{5}{11}$

(b) $\frac{1}{2}$

(c) $\frac{7}{13}$

(d) $\frac{6}{11}$

$P(6) = 1/6$ $\quad P(6^c) = 5/6$

Break upto 11.20 am

$6$ $\quad 1/6$

A

$6^c\ 6^c\ 6$ $\quad (5/6)(5/6)(1/6)$

A B A

$6^c\ 6^c\ 6^c\ 6^c\ 6$ $\quad (5/6)^4 \cdot (1/6)$

A B A B A

A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is GATE-12

Aditya Vangala

ACE

(a) $\frac{1}{3}$

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

Aditya Vangala

ACE

Random variables :-

$f(x) = P(X = x)$ PMF

$$F(x) = P(X \leq x) = \sum_{x_i=-\infty}^{x} f(x_i) \quad \text{CDF}$$

$$= \sum_{x_i=-\infty}^{x} P(X = x_i)$$

D.R.V

Aditya Vangala

ACE

## C.R.V

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$f(x) = \frac{d}{dx} F(x)$$

$$P(X > x) = \int_{x}^{\infty} f(x)\,dx$$

$$P(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} f(x)\,dx = P(x_1 < X < x_2)$$

Aditya Vangala

ACE

$$E[X] = \sum_{\forall x} x \, p(X=x)$$

$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$E[X^n] = \int_{-\infty}^{\infty} x^n f(x)\, dx$$

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$$

$$E[aX] = a\,E[X]$$

$$E(b) = b$$

$$E[aX+b] = a\,E(X) + b$$

$$E[X+Y] = E(X) + E(Y)$$

Aditya Vangala

ACE

$$Var(X) = E\left[\left(X - E(X)\right)^2\right]$$

$$= E(X^2) - \left(E(X)\right)^2$$

$$Var(aX) = a^2 \, Var(X)$$

$$Var(b) = 0$$

$$Var(aX+b) = a^2 \, Var(X)$$

$$Var(X+Y) = Var(X) + Var(Y) + 2\,Cov(X,Y)$$

$$Cov(X,Y) = E[XY] - E(X) \cdot E(Y)$$

If $X$ and $Y$ are independent

$$\mathrm{Cov}(X,Y) = 0$$

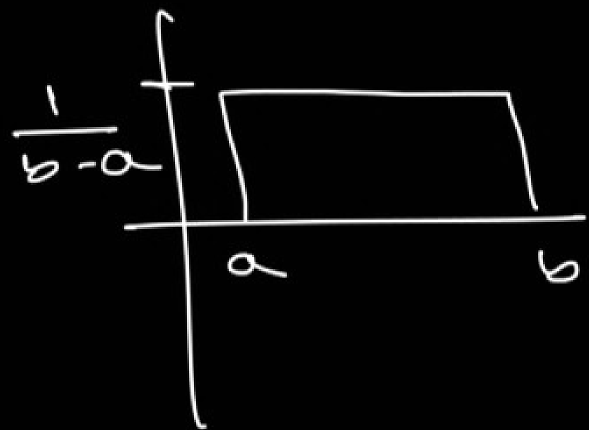
$$E(XY) = E(X)\cdot E(Y)$$

$$\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$$

$$\mathrm{Var}(X-Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$$

# Uniform R.V

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$$




$$E(x) = \frac{b+a}{2}$$

$$E(x^2) = \frac{a^2+ab+b^2}{3}$$

$$Var(x) = \frac{(b-a)^2}{12}$$

Aditya Vangala

ACE

## Exponential R.V

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$P(X > x) = e^{-\lambda x} \longrightarrow P(X > 2) = e^{-2\lambda}$$

$\rightarrow E(X) = \dfrac{1}{\lambda}$ $\qquad E(X^2) = \dfrac{2}{\lambda^2}$

$\rightarrow Var(X) = \dfrac{1}{\lambda^2}$

## Normal or Gaussian R.V

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

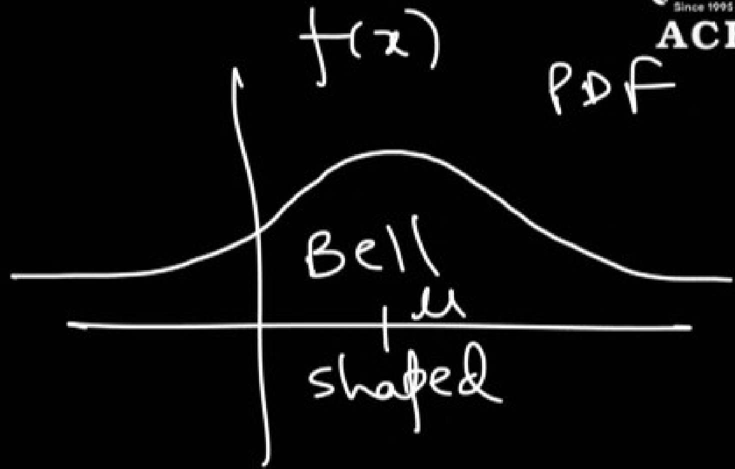
$$E(X) = \mu$$

$$Var(X) = \sigma^2$$

$$P(X > \mu) = 1/2$$

$$P(X < \mu) = 1/2$$

PDF




CDF is S-shaped

Aditya Vangala

ACE

$$Z = \frac{X - \mu}{\sigma}$$ Standard Normal R.V

$$E(Z) = 0$$

$$Var(Z) = 1$$

$$P(-1 \le Z \le 1) = 0.6827$$

$$P(-2 \le Z \le 2) = 0.9545$$

$$P(-3 \le Z \le 3) = 0.9973$$

Aditya Vangala

ACE

Poisson R.V

$$P(X=r) = \frac{e^{-\lambda}\lambda^{r}}{r!}, \quad r = 0, 1, 2, 3 \ldots$$

$$E[X] = \lambda$$

$$Var(X) = \lambda$$

$$E(X^2) = \lambda^2 + \lambda$$

Aditya Vangala

ACE

<u>Binomial</u> <u>R.V</u>

$$P(X=r) = {}^{n}C_{r}\, p^{r} (1-p)^{n-r}, \quad r = 0, 1, 2, \ldots n$$

$$E[X] = np$$

$$Var(X) = np(1-p)$$

Aditya Vangala

ACE

statistics :-

The probability density function of a random variable X is $P_x(x) = e^{-x}$ for $x \geq 0$ and 0 otherwise. The expected value of the function $g_x(x) = e^{\frac{3x}{4}}$ is

GATE-15

Aditya Vangala

ACE

$$E[g(x)] = \int_{-\infty}^{\infty} g(x)\, f(x)\, dx$$

$$= \int_{0}^{\infty} e^{\frac{3x}{4}} \cdot e^{-x}\, dx$$

$$= \underline{\hspace{3cm}}$$

Passengers try repeatedly to get a seat reservation in any train running between two stations until they are successful. If there is 40% chance of getting reservation in any attempt by a passenger, then the average number of attempts that passengers need to make to get a seat reserved is

GATE-2017

Aditya Vangala

ACE

$$p = 0.4$$

| $X$ | 1 | 2 | 3 | 4 | 5 | ... |
|---|---|---|---|---|---|---|
| $P(X)$ | $p$ | $(1-p)p$ | $(1-p)^2 p$ | $(1-p)^3 \cdot p$ | $(1-p)^4 \cdot p$ | ... |

$$E[X] = 1\cdot(p) + 2(1-p)(p) + 3(1-p)^2 p + \cdots$$

$$= \frac{1}{p}$$

$$= \frac{1}{0.4} = 2.5$$

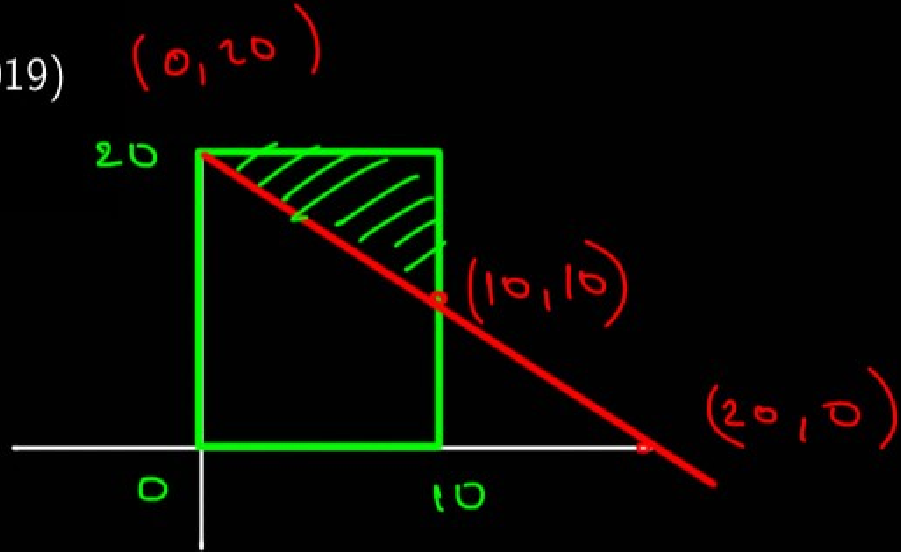
Aditya Vangala

ACE

The variable $X$ takes a value between 0 and 10 with uniform probability distribution. The variable $Y$ takes a value between 0 and 20 with uniform probability distribution. The probability of the sum of variables $X + Y$ being greater than 20 is

(GATE 2019)

(a) 0.5 (b) 0 (c) 0.33 (d) 0.25




$X+Y>20$

$X+Y=20$

$$P(X+Y>20) = \frac{\frac{1}{2}(1\cancel{0})(1\cancel{0})}{1\cancel{0}\times 2\cancel{0}} = \frac{1}{4}$$

Consider two exponentially distributed random variables $X$ and $Y$, both having a mean of 0.50. Let $Z = X + Y$ and $r$ be the correlation coefficient between $X$ and $Y$. If the variance of $Z$ equals 0, then the value of $r$ is (round off to 2 decimal places).

(GATE–2020)

Aditya Vangala

ACE

Since 1995

$X \rightarrow E.R.V$

$\frac{1}{\lambda_1} = 0.5 \Rightarrow \lambda_1 = 2$

$Y \rightarrow E.R.V$

$\frac{1}{\lambda_2} = 0.5$

$\lambda_2 = 2$

$$r = \frac{\text{cov}(X,Y)}{\sigma_X \cdot \sigma_Y}$$

$$\text{Var}(X) = \frac{1}{\lambda_1^2} = \frac{1}{4}$$

$$\sigma_X = \frac{1}{2}$$

$$\sigma_Y = 1/2$$

$$\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y) + 2\,\text{cov}(X,Y) = 0$$

$$\frac{1}{4} + \frac{1}{4} + 2\,\text{cov}(X,Y) = 0$$

$$\text{cov}(X,Y) = -1/4$$

$$r = \frac{-1/4}{1/2 \cdot 1/2} = \underline{\underline{-1}}$$

Aditya Vangala

ACE

The lengths of a large stock of titanium rods follow a normal distribution with a mean of 440 mm and a standard deviation of 1mm. What is the percentage of rods whose lengths lie between 438 mm and 441 mm

GATE-19

(a) 81.85%
(b) 68.4%
(c) 99.75%
(d) 86.64%

$$P(438 < L < 441)$$

$$P\left(\frac{438-\mu}{\sigma} < \frac{L-\mu}{\sigma} < \frac{441-\mu}{\sigma}\right)$$

$$P\left(\frac{438-\mu}{\sigma} < z < \frac{441-\mu}{\sigma}\right)$$

Let $X$ be a normal random variable with mean 1 and variance 4. The probability $P(X < 0)$ is

GATE-13

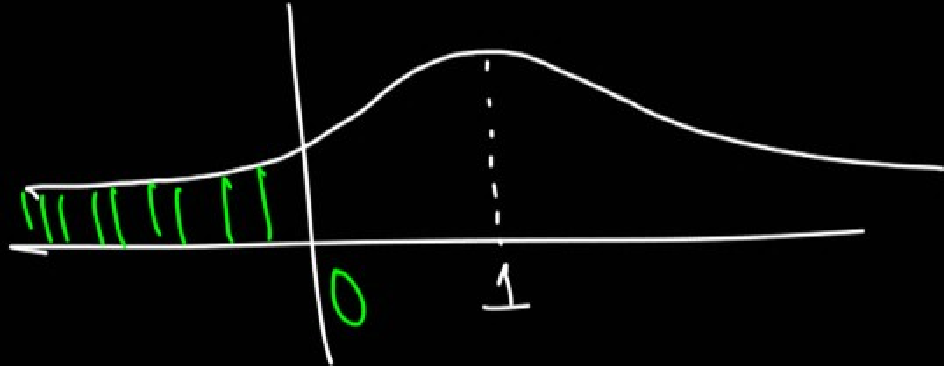
(a) 0.5

(b) greater than zero and less than 0.5

(c) greater than 0.5 and less than 1.0

(d) 1.0

Aditya Vangala




(b)

An Observer counts 240 vehicles per hour at a specific highway location. Assume that the vehicle arrival at the location is Poisson distributed. The Probability of having one vehicle arriving over a 30 second time interval is

GATE-14

Aditya Vangala

ACE

$$P(x=r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$

$$P(x=1) = \frac{e^{-\lambda}\lambda^{1}}{1!}$$

$240V \longleftrightarrow 1 \text{ hour}$

$240 \longrightarrow 60 \times 60 \text{ sec}$

$\frac{240}{60 \times 2} \longleftrightarrow 30 \text{ sec}$

$2 \longleftrightarrow 30 \text{ sec}$

$\lambda = 2$

A fair coin is tossed 20 times. The probability that head will appear exactly 4 times in the first ten tosses, and tail will appear exactly 4 times in the next ten tosses is

GATE-20

Aditya Vangala

ACE

$$P(X=r) = {}^{n}C_{r}\, p^{r} (1-p)^{n-r}$$

$$\left( {}^{10}C_{4} \left(\tfrac{1}{2}\right)^{4} \left(\tfrac{1}{2}\right)^{10-4} \right) \cdot {}^{10}C_{4} \left(\tfrac{1}{2}\right)^{4} \left(\tfrac{1}{2}\right)^{6}$$

An unbiased coin is tossed an infinite number of times. The probability that the fourth head appears at the tenth toss is

Aditya Vangala

ACE

GATE-14

(a) 0.067

(b) 0.073

(c) 0.082

(d) 0.091

$$\left( {}^{9}C_{3} \left(1/2\right)^{3} \left(1/2\right)^{6} \right) \cdot 1/2$$

3 heads in

9 tosses

Let $X_1$, $X_2$ be two independent normal random variables with means $\mu_1$, $\mu_2$ and standard deviations $\sigma_1$, $\sigma_2$, respectively. Consider $Y = X_1 - X_2$, $\mu_1 = \mu_2 = 1$, $\sigma_1 = 1$, $\sigma_2 = 2$. Then

Aditya Vangala

Since 1995 ACE

GATE-18

(a) $Y$ is normally distributed with mean 0 and variance 1

(b) $Y$ is normally distributed with mean 0 and variance 5

(c) $Y$ has mean 0 and variance 5, but is not normally distributed

(d) $Y$ has mean 0 and variance 1, but is not normally distributed

$Y = \alpha_1 X_1 + \alpha_2 X_2$ is also N.R.V

$E(Y) = \alpha_1 \mu_1 + \alpha_2 \mu_2$

$Var(Y) = \alpha_1^2 \sigma_1^2 + \alpha_2^2 \sigma_2^2$

$E(Y) = 1(1) + (-1)(1) = 0$

$Var(Y) = 1^2 (1) + (-1)^2 (2)^2$

$= 5$

Consider a binomial random variable $X$. If $X_1, X_2, X_3, \cdots, X_n$ are independent and identically distributed samples from the distribution of $X$ with sum $Y = \sum_{i=1}^{n} X_i$, then the distribution of $Y$ as $n \to \infty$ can be approximated as

Aditya Vangala

ACE

GATE-2021

A. Exponential

B. Binomial

C. Bernoulli

D. Normal

$$Y = X_1 + X_2 + X_3 + \cdots + X_n$$

$n \to \infty$ $\quad Y \Rightarrow$ Normal R.V

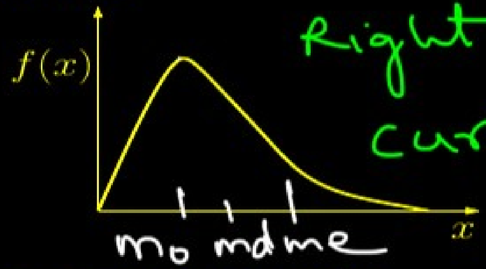
or

Gaussian R.V

Central limit theorem

A probability distribution with right skew is shown in figure. The correct statement for the probability distribution is GATE 2018






Right skewed curved

(a) Mean is equal to mode

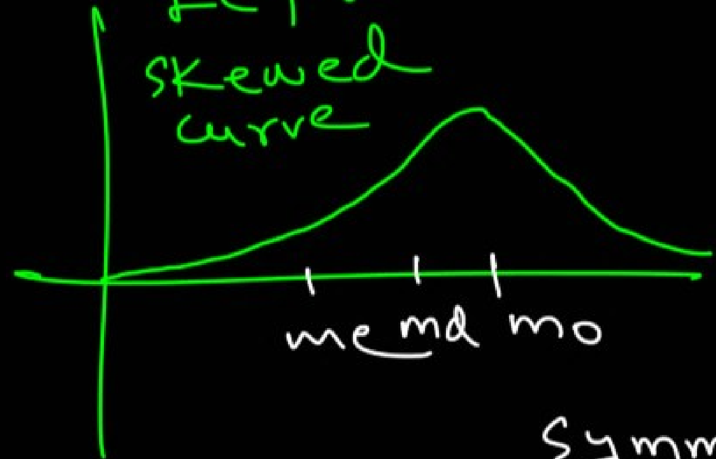
(b) Mean is greater than median but less than mode

(c) Mean is greater than median and mode

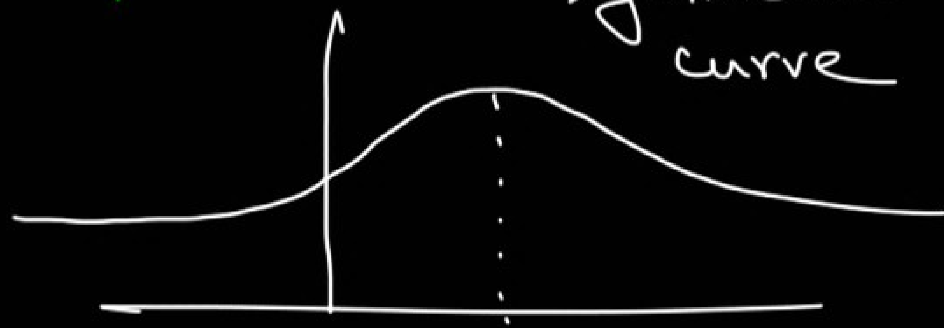
(d) Mode is greater than median

Lack of Sym => Skewness

Left skewed curve




Symmetrical curve



me = md = mo

Marks obtained by 100 students in an examination are given in the table

Aditya Vangala

ACE

| S. No | Marked Obtained | Number of students |
|---|---|---|
| 1 | 25 | 20 |
| 2 | 30 | 20 |
| 3 | 35 | 40 |
| 4 | 40 | 20 |

What would be the mean, median, and mode of the marks obtained by the students? GATE-2014

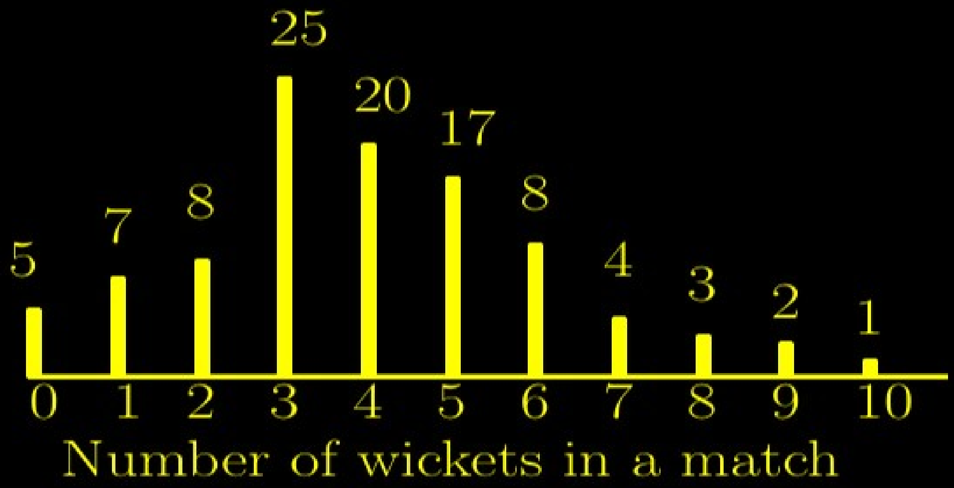
(a) Mean 33, Median 35, Mode 40

(b) Mean 35, Median 32.5, Mode 40

(c) Mean 33, Median 35, Mode 35

(d) Mean 33, Median 32.5, Mode 35

The bar graph shows the frequency of the number of wickets taken in a match by a bowler in her career. For example, in 17 of her matches, the bowler has taken 5 wickets each. The median number of wickets taken by the bowler in a match is (rounded off to one decimal place).GATE-2022




Aditya Vangala

ACE

$\left.\begin{matrix}0\\0\\\vdots\\0\end{matrix}\right\}$ 5times

$\left.\begin{matrix}1\\1\\\vdots\\1\end{matrix}\right\}$ 7 times

50 and 51th obs are 4, 4

$$\text{median} = \frac{4+4}{2} = \underline{\underline{4}}$$

Aditya Vangala

ACE

E.E Correlation and Regression

E.C.E joint PDF and CDF

Aditya Vangala

ACE

Since 1995

## Calculus :-

$$\lim_{x \to a} f(x) = l$$

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

## Continuity :-

$$\lim_{x \to a} f(x) = f(a)$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Aditya Vangala

ACE

Indeterminate forms :-

$\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \times \infty, \infty^0, 0^0, 1^\infty$

If $\lim_{x \to a} f(x) = 0$, $\lim_{x \to a} g(x) = 0$ and

$\lim_{x \to a} \frac{f'(x)}{g'(x)} = L$ then $\lim_{x \to a} \frac{f(x)}{g(x)} = L$

If $\lim_{x \to a} f(x) = \infty$, $\lim_{x \to a} g(x) = \infty$ and

$\lim_{x \to a} \frac{f'(x)}{g'(x)} = L$ then $\lim_{x \to a} \frac{f(x)}{g(x)} = L$

## Mean value theorems:-

Rolles theorem:- $f'(c) = 0 \quad c \in (a,b)$

Lagranges theorem:- $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

Cauchy theorem:- $\dfrac{f'(c)}{g'(c)} = \dfrac{f(b) - f(a)}{g(b) - g(a)}$

Aditya Vangala

ACE

## maximum and minimum

(1) closed Interval

(2) open Interval

Aditya Vangala

ACE

# Integration

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$|x|$, $[x]$, piecewise functions.

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

Aditya Vangala

ACE

$$\int_{-a}^{a} f(x)\,dx = \quad 2\int_{0}^{a} f(x)\,dx \qquad f(-x) = f(x)$$

$$0 \qquad f(-x) = -f(x)$$

Aditya Vangala

ACE

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx$$

$\Gamma(1) = 1$

$\Gamma(2) = 1$

$\Gamma(3) = 2!$

$\Gamma(4) = 3!$

$\Gamma(n+1) = n!$

$\Gamma(n+1) = n\Gamma(n)$

$\Gamma(1/2) = \sqrt{\pi}$

$\Gamma(-1/2) = -2\sqrt{\pi}$

# Beta function

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

$$= 2\int_0^{\pi/2} \sin^{2m-1}x \cos^{2n-1}x \, dx$$

$$= \frac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n)}$$

Aditya Vangala

ACE

A function $f(x)$ is defined as

$$f(x) = \begin{cases} e^x & x < 1 \\ \ln x + ax^2 + bx & x \geq 1 \end{cases}$$

where $x \in R$.

Which one of the following statements is TRUE?

(GATE-17)

(a) $f(x)$ is NOT differentiable at $x = 1$ for any values of $a$ and $b$.

(b) $f(x)$ is differentiable at $x = 1$ for the unique values of $a$ and $b$.

(c) $f(x)$ is differentiable at $x = 1$ for all values of $a$ and $b$ such that $a + b = e$.

(d) $f(x)$ is differentiable at $x = 1$ for all values of $a$ and $b$.

Let $f(x)$ be diff at $x=1$

$\Rightarrow f(x)$ is conti at $x=1$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$e^1 = \ln 1 + a + b$$

$$\Rightarrow a + b = e \quad \text{--- } 1$$

$$f'(x) = \begin{cases} e^x & x < 1 \\ \frac{1}{x} + 2ax + b & x \geq 1 \end{cases}$$

$$f'(1^-) = f'(1^+)$$

$$e^1 = \frac{1}{1} + 2a + b$$

$$2a + b = e - 1 \quad \text{--- } 2$$

Aditya Vangala

ACE

$$a + b = e$$

$$2a + b = e - 1$$

$$- \quad - \quad -$$

---

$$-a = e - (e - 1)$$

$$-a = 1$$

$$a = -1$$

$$\Rightarrow a + b = e$$

$$-1 + b = e$$

$$b = e + 1$$

The value of $\lim_{x\to\infty}\left(1+x^2\right)^{e^{-x}}$ is

(GATE-2015)

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) $\infty$

Aditya Vangala

ACE

Since 1995

$\infty^0 \to$ Convert this either to

$\frac{0}{0}$ or $\frac{\infty}{\infty}$

use Logarithm

The limit

$$p = \lim_{x \to \pi} \left( \frac{x^2 + \alpha x + 2\pi^2}{x - \pi + 2\sin x} \right)$$

has a finite value for a real $\alpha$. The value of $\alpha$ and the corresponding limit $p$ are

(GATE-2022)

(a) $\alpha = -3\pi$ and $p = \pi$

(b) $\alpha = -2\pi$ and $p = 2\pi$

(c) $\alpha = \pi$ and $p = \pi$

(d) $\alpha = 2\pi$ and $p = 3\pi$

$$p = \lim_{x \to \pi} \left( \frac{x^2 + \alpha x + 2\pi^2}{x - \pi + 2\sin x} \right) = \frac{\pi^2 + \alpha\pi + 2\pi^2}{\pi - \pi + 2\sin\pi}$$

$$= \frac{3\pi^2 + \alpha\pi}{0}$$

Aditya Vangala

$$3\pi^2 + \alpha\pi = 0$$

$$\alpha\cancel{\pi} = -3\pi^{\cancel{2}}$$

$$\alpha = -3\pi$$

ACE

Let $f(x) = x^2 - 2x + 2$ be a continuous function defined on $x \in [1, 3]$. The point $x$ at which the tangent of $f(x)$ becomes parallel to the straight line joining $f(1)$ and $f(3)$ is (GATE-2021)

Aditya Vangala

ACE

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Apply LMVT

Suppose that $f : R \to R$ is a continuous function on the interval [-3, 3] and a differentiable function in the interval (-3, 3) such that for every $x$ in the interval $f'(x) \leq 2$. If $f(-3) = 7$ then $f(3)$ is at most (GATE-2021)

Aditya Vangala

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$a = -3 \quad b = 3$

$$f'(c) = \frac{f(3) - f(-3)}{3 - (-3)}$$

$$f'(c) = \frac{f(3) - 7}{6} \leq 2$$

$$f(3) - 7 \leq 12$$

$$f(3) \leq 19$$

Ans $\underline{\underline{19}}$

The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval [1, 6] is (GATE–12)

(a) 21

(b) 25

(c) 41

(d) 46

Aditya Vangala

ACE

Working Rule:

Step 1: Find $p$, $q$, $r$, $s$ and $t$

Step 2: Equate $p$ and $q$ to zero for obtaining stationary points.

Step 3: Find $r$, $s$ and $t$ at each stationary point.

(a) If $rt - s^2 > 0$ and $r > 0$ then $f(x, y)$ has a minimum at that stationary point.

(b) If $rt - s^2 > 0$ and $r < 0$ then $f(x, y)$ has a maximum at that stationary point.

(c) If $rt - s^2 < 0$ then $f(x, y)$ has no extremum at that stationary point and such points are called saddle points.

(d) If $rt - s^2 = 0$ then the case is undecided.