EECE 5552

Assistive Robotics

Assignment – 4

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Date: 10/23/2020

ASSIGNMENT - 4

Problem (a):-

Given, $K = \frac{1}{2} J_m \dot{\theta}_m^2 + \frac{1}{2} J_l \dot{\theta}_l^2$ — ①

($\because$ where J_m & J_l are +ve scalar values)

Also, we know that,

$$\theta_m = r\theta_l$$

differentiate both side,

$$\frac{d}{dt}(\theta_m) = r\frac{d}{dt}(\theta_l)$$

$$\dot{\theta}_m = r\dot{\theta}_l \quad — ②$$

Plug ② in ①

$$\therefore K = \frac{1}{2} J_m r^2 \dot{\theta}_l^2 + \frac{1}{2} J_l^2 \dot{\theta}_l^2$$

$$P = Mgl - Mgl\cos\theta_l$$

$$L = K - P$$

$$= \frac{1}{2} J_m r^2 \dot{\theta}_l^2 + \frac{1}{2} J_l \dot{\theta}_l^2 - Mgl + Mgl\cos\theta_l \quad — ③$$

Eulers - Lagranges Equation:-

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \tau$$

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partially differentiate ③ by θ_l

$$\frac{\partial L}{\partial \theta_L} = -Mgl \sin\theta_l \quad — ④$$

$$\frac{\partial L}{\partial \dot{\theta}_L} = J_m . r^2 . \dot{\theta}_l + J_l \dot{\theta}_l \quad — ⑤$$

Differentiating ⑤ w.r.t time

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_L}\right) = J_m r^2 \ddot{\theta}_l + J_l \ddot{\theta}_l \quad — ⑥$$

$$\tau = u \quad — ⑦$$

where τ is the generalized force on system.

Now,

Substituting ④, ⑥ and ⑦ in Euler-Lagranges Eqn

$$J_m r^2 \ddot{\theta}_l + J_l \ddot{\theta}_l + Mgl \sin\theta_l = u$$

→ Eqn of motion for θ_l

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(b) Damping forces can be incorporated in Euler-Lagrange equation by:-

$$\tau = \tau_k + \tau_m + \tau_l$$

$$= U - B_m \dot{\theta}_m - B_l \dot{\theta}_l$$

New equation is

$$(J_m r^2 + J_l)\ddot{\theta}_l + Mgl \sin\theta_l = U - B_m \dot{\theta}_m - B_l \dot{\theta}_l$$

$$= U - (r B_m + B_l)\dot{\theta}_l$$

(c)

$x_1 = \theta_l$ $\qquad$ $\dot{x}_1 = x_2$

$x_2 = \dot{\theta}_l$ $\qquad$ $\dot{x}_2 = \ddot{\theta}_l$

$$\ddot{\theta}_l = \frac{U - (r B_m + B_l)\dot{\theta}_l - Mgd \sin\theta_l}{J_m r^2 + J_l}$$

$$\dot{x}_2 = \frac{U - (r B_m + B_l) x_2 - Mgd \sin(x_1)}{J_m r^2 + J_l}$$

Here, we can observe that as the damping co-efficients increase, change in θ w.r.t time is less even when same force U is applied.
This helps us intuitively increase the effect of damping forces on the system.

Assignment 4:

MATLAB CODE:

```
function simulate_finger()
 close all;
 clc;
 clear;
 % time vector
 t = 0:0.01:2;
 % initial states
 x_10 = 0;
 x_20 = 0;
 % given parameters
 r = 30;
 Jm = 0.001;
 Jl = 0.001;
 M = 0.1;
 g = 0.981;
 l = 0.1;
 u = 0.1;
 Bm_mat = [0.001 0.01 0.1 1];
 Bl_mat = [0.001 0.01 0.1 1];
 % line styles
 lstyl = {'k-','k--','r-','r--'};

 fh1 = figure('Name','A Finger Simulation (thetal)');
 ah1 = axes('parent',fh1);
 hold(ah1,'on');
 xlabel(ah1,'time');
 ylabel(ah1,'theta');
 fh2 = figure('Name','A Finger Simulation (dthetal/dt)');
 ah2 = axes('parent',fh2);
 hold(ah2,'on');
 xlabel(ah2,'time');
 ylabel(ah2,'dtheta');

 for i=1:4
   Bl = Bl_mat(i);
   Bm = Bm_mat(i);
   % solve ODE
```

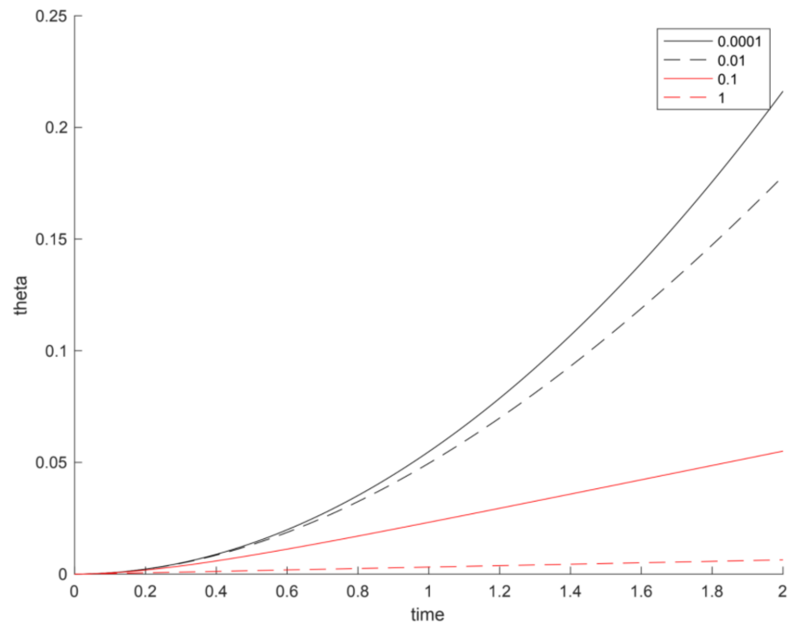
```
   [t,x]=ode45(@func,t,[x_10,x_20]);
   % plot solutions
   plot(ah1,t,x(:,1),lstyl{i});
   plot(ah2,t,x(:,2),lstyl{i});
 end

 legend(ah1,'0.0001','0.01','0.1','1');
 legend(ah2,'0.0001','0.01','0.1','1');
 saveas(ah1,'theta.pdf');
 saveas(ah2,'dtheta.pdf');

 % a simple function
 function dxdt=func(t,x)
   dx1 = x(2);
   dx2 = (u - (r*Bm + Bl)*x(2) - M*g*l*sin(x(1)))/(r*r*Jm + Jl);
   dxdt = [dx1;dx2];
  end
end
```

Results:

Theta w.r.t time:



Dtheta w.r.t time: