EECE 5552

Assistive Robotics

Assignment – 3

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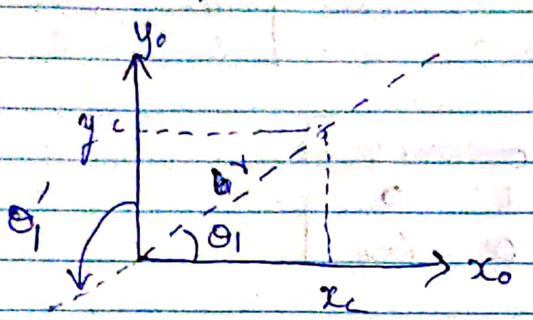
Problem 1 :-

(a) Redrawing the lines near θ_1 in the diagram.



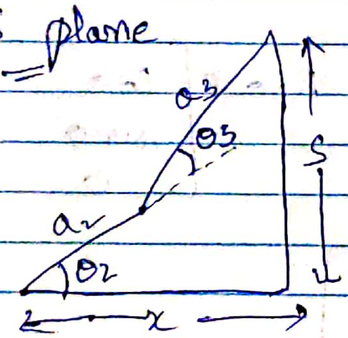
$$\tan\theta_1 = \frac{y_c}{x_c}$$

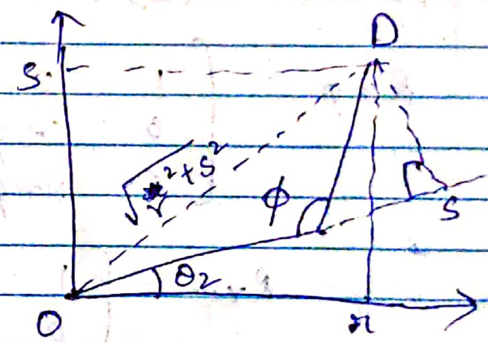
$$\theta_1 = \tan^{-1}\left(\frac{y_c}{x_c}\right)$$

$$\theta_1' = \pi + \tan^{-1}(y_c/x_c)$$

$$r = \sqrt{x_c^2 + y_c^2}$$

R-S plane





Now,
$$\phi = \pi - \theta_3$$

From ΔOCD

$$(\sqrt{r^2+s^2}) = a_2^2 + a_3^2 + 2a_2a_3\cos\phi \quad \rightarrow \text{Cosine Rule}$$

$$r^2+s^2 = a_2^2 + a_3^2 - 2a_2a_3\cos(\pi-\phi_3)$$

$$\cos(\pi - \phi_3) = \frac{a_2^2 + a_3^2 - (r^2+s^2)}{2a_2a_3} \quad — ①$$

∴ We also know that,

$$\cos(\pi - \theta) = -\cos\theta \quad — ②$$

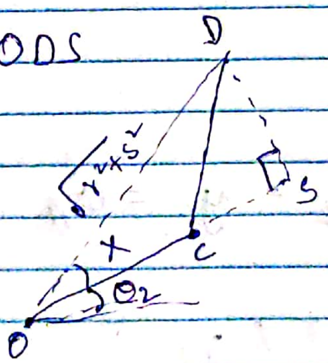
Applying ② in ①

$$-\cos\theta_3 = \frac{a_2^2 + a_3^2 - (r^2+s^2)}{2a_2a_3}$$

$$\theta_3 = \cos^{-1}\left[\frac{-a_2^2 - a_3^2 + (r^2+s^2)}{2a_2a_3}\right]$$

$$\theta_3 = \cos^{-1}\left[\frac{r^2+s^2 - a_2^2 - a_3^2}{2a_2a_3}\right] //$$

Redrawing ΔODS



Now,

θ_2 can be expressed as:-

$$\theta_2 = (\theta_2 + x) - x \quad — ⑤$$

R.H.S

$$\tan(\theta_2 + x) = \frac{s}{r} \quad \text{from figure}$$

$$\theta_2 + x = \tan^{-1}\left(\frac{s}{r}\right) \quad — ③$$

$$\tan x = \frac{DS}{OS} \quad \text{(from figure)}$$

$$= \frac{DS}{OC + CS}$$

$$x = \tan^{-1}\left(\frac{a_3 \sin\theta_3}{a_2 + a_3 \cos\theta_3}\right) \quad — \quad (4)$$

$\theta_2 = \gamma + \tan^{-1}$

Substituting (3) and (4) in (5)

$$\theta_2 = \tan^{-1}\left(\frac{s}{r}\right) + \tan^{-1}\left(\frac{a_3 \sin\theta_3}{a_2 + a_3 \sin\theta_3}\right)$$

1 (b):-

Given $x_c = y_c = 0$

$$\theta_1 = \tan^{-1}\left(\frac{x_c}{y_c}\right)$$

$$\theta_1 = \tan^{-1}(\infty)$$

$$\theta_1 = \pi/2$$

($\because$ $\tan^{-1}$(infinity) = 90 degrees).

$$\theta_3 = \cos^{-1}\left(\frac{r^2+s^2-a_2^2-a_3^2}{2a_2a_3}\right)$$

$$= \cos^{-1}\left(\frac{x_c^2+y_c^2+(z_c-d_1)^2-a_2^2-a_3^2}{2a_2a_3}\right)$$

$$\theta_3 = \cos^{-1}\left(\frac{(z_c-d_1)^2-a_2^2-a_3^2}{2a_2a_3}\right) \quad // \text{ Since } x_c=y_c=0 //$$

$$\theta_2 = \tan^{-1}\left(\frac{s}{r}\right) + \tan^{-1}\left(\frac{a_3 \sin\theta_3}{a_2+a_3\cos\theta_3}\right)$$

$$= \tan^{-1}\left(\frac{z_c-d_1}{\sqrt{x_c^2+y_c^2}}\right) + \tan^{-1}\left(\frac{a_3 \sin\theta_3'}{a_2+a_3\cos\theta_3}\right)$$

$$\theta_2 = \pi/2 + \tan^{-1}\left(\frac{a_3\sin\theta_3}{a_2+a_3\cos\theta_3}\right) \quad //$$

when $x_c=y_c=0$,
$\theta_3' = \theta_3$

Problem 1 (c) :-

if $d \neq 0$

then $\theta_1 = \pm \pi/2$

hence 2 solutions exists

Problem 2 (a) :-

$$A_1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} \cos\theta_4 & 0 & -\sin\theta_4 & 0 \\ \sin\theta_4 & 0 & \cos\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} \cos\theta_5 & 0 & \sin\theta_5 & 0 \\ \sin\theta_5 & 0 & -\cos\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, to find $T_0^6 = A_1 * A_2 * A_3 * A_4 * A_5 * A_6$

First, $T_0^2 = A_1 * A_2$

$$= \begin{bmatrix} \cos\theta_1 \cos\theta_2 & -\sin\theta_1 & \cos\theta_1 \sin\theta_2 & -d_2 \sin\theta_1 \\ \cos\theta_2 \sin\theta_1 & \cos\theta_1 & \sin\theta_1 \sin\theta_2 & d_2 \cos\theta_1 \\ -\sin\theta_2 & 0 & \cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = \begin{bmatrix} \cos\theta_1 \cos\theta_2 & -\sin\theta_1 & \cos\theta_1 \sin\theta_2 & d_3 \cos\theta_1 \sin\theta_2 - d_2 \sin\theta_1 \\ \cos\theta_2 \sin\theta_1 & \cos\theta_1 & \sin\theta_1 \sin\theta_2 & d_2 \cos\theta_1 + d_3 \sin\theta_2 \sin\theta_1 \\ -\sin\theta_2 & 0 & \cos\theta_2 & d_3 \cos\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^4 = \begin{bmatrix} \cos\theta_1 \cos\theta_2 \cos\theta_4 - \sin\theta_1 \sin\theta_4, & -\cos\theta_1 \sin\theta_2 - \cos\theta_4 \sin\theta_1, & -\cos\theta_4 S\theta_1 - C\theta_1 C\theta_2 S\theta_4 & d_3 C\theta_1 S\theta_2 - d_2 S\theta_1 \\ C\theta_1 S\theta_4 + C\theta_2 C\theta_4 S\theta_1, & -S\theta_1 S\theta_2, & C\theta_1 C\theta_4 - C\theta_2 S\theta_1 S\theta_4, & d_2 C\theta_1 + d_3 S\theta_1 S\theta_2 \\ -C\theta_4 S\theta_2 & -C\theta_2 & S\theta_2 S\theta_4 & d_3 \cos\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^5 = \begin{bmatrix} -C\theta_5 S\theta_1 S\theta_4 - C\theta_1 C\theta_2 C\theta_4 - C\theta_1 S\theta_2 S\theta_5, \; -C\theta_4 S\theta_1 - C\theta_1 C\theta_2 S\theta_4, \\ C\theta_1 C\theta_5 S\theta_2 - S\theta_5 S\theta_1 S\theta_4) - C\theta_1 C\theta_2 C\theta_4, \; d_3 C\theta_1 S\theta_2 - d_2 S\theta_1 \\ C\theta_5 C\theta_1 S\theta_4 + C\theta_2 C\theta_4 S\theta_1 - S\theta_1 S\theta_2 S\theta_5, \; C\theta_1 C\theta_4 - C\theta_2 S\theta_1 S\theta_4, \\ S\theta_5 C\theta_1 S\theta_4 + C\theta_2 C\theta_4 S\theta_1, \; C\theta_5 S\theta_1 S\theta_2, \; d_2 C\theta_1 \; d_3 S\theta_1 S\theta_2 \\ -C\theta_2 S\theta_5 C\theta_4 C\theta_5 S\theta_2, \; S\theta_2 S\theta_4, \; C\theta_4 C\theta_5 - C\theta_4 S\theta_2 S\theta_5, \; d_3 \cos\theta_2 \\ 0 \quad 0 \quad 0 \quad 1 \end{bmatrix}$$

$$T_0^6 = \begin{bmatrix} -c\theta_6 c\theta_5 s\theta_1 s\theta_4 - c\theta_1 c\theta_2 c\theta_4 + c\theta_1 s\theta_2 s\theta_5 - s\theta_6 c\theta_4 s\theta_1 + c\theta_1 c\theta_2 s\theta_4 , & \sin\theta_6 c\theta_5 s\theta_1 s\theta_4 - c\theta_1 c\theta_2 c\theta_4 + c\theta_1 s\theta_2 s\theta_5 - c\theta_6 c\theta_4 s\theta_1 + c\theta_1 c\theta_2 s\theta_4 , & c\theta_1 c\theta_2 c\theta_4 , & d_3 c\theta_1 s\theta_2 - d_2 s\theta_1 d_6 s\theta_5 s\theta_1 s\theta_4 - c\theta_1 c\theta_2 c\theta_4 - c\theta_1 c\theta_5 s\theta_2 \\ c\theta_6 c\theta_5 c\theta_1 s\theta_4 + c\theta_2 c\theta_4 s\theta_1 - s\theta_1 s\theta_2 s\theta_5 + s\theta_6 c\theta_1 c\theta_4 - c\theta_2 s\theta_1 s\theta_4 , & c\theta_6 c\theta_1 c\theta_4 - c\theta_2 s\theta_1 s\theta_4 - s\theta_6 c\theta_5 c\theta_1 s\theta_4 + c\theta_2 c\theta_4 s\theta_1 - s\theta_1 s\theta_2 s\theta_5 , & s\theta_5 c\theta_1 s\theta_4 + c\theta_2 c\theta_4 s\theta_1 + c\theta_5 s\theta_1 s\theta_2 , & d_6 s\theta_5 c\theta_1 s\theta_4 + c\theta_2 c\theta_4 s\theta_1 + c\theta_5 s\theta_1 s\theta_2 + d_2 c\theta_1 + d_3 s\theta_1 s\theta_2 \\ s\theta_2 s\theta_4 s\theta_6 - c\theta_6 c\theta_2 s\theta_5 + c\theta_4 c\theta_5 s\theta_2 , & s\theta_6 c\theta_2 s\theta_5 + c\theta_4 c\theta_5 s\theta_2 + c\theta_6 s\theta_2 s\theta_4 , & c\theta_2 c\theta_5 - c\theta_4 s\theta_2 s\theta_5 , & d_3 c\theta_2 + d_6 c\theta_2 c\theta_5 - c\theta_4 s\theta_2 s\theta_5 \\ 0 , & 0 , & 0 , & 1 \end{bmatrix}$$

Problem 2(b):-

$$J = \begin{bmatrix} J_v \\ \hline J_\omega \end{bmatrix} \quad \text{form.}$$

$$J_i = \begin{cases} \begin{bmatrix} z_{i-1} \times (o_6 - o_{i-1}) \\ z_{i-1} \end{bmatrix} & \text{for } i = 1, 2 \\ \begin{bmatrix} z_2 \\ 0 \end{bmatrix} & \text{for } i = 3 \\ \begin{bmatrix} z_{i-1} \times (o_6 - o_{i-1}) \\ \hline z_{i-1} \end{bmatrix} & \text{for } i = 4, 5, 6 \end{cases}$$

Jacobian Mat :-

$[-d6(\sin\theta_5 c\theta_1 s\theta_4 + c\theta_2 c\theta_4 s\theta_1) + c\theta_5 s\theta_1 s\theta_2 - d2\, c\theta_1$
$-d3\, s\theta_1 s\theta_2 ,\ c\theta_1 d3\, c\theta_2 + d6\, c\theta_2 c\theta_5 - c\theta_4 s\theta_2 s\theta_5 ,$
$-d6\, s\theta_5 c\theta_4 s\theta_1 + c\theta_1 c\theta_2 s\theta_4 ,\ -d6(c\theta_1 s\theta_2 s\theta_5 + c\theta_5 s\theta_1$
$s\theta_4 - c\theta_1 c\theta_2 c\theta_4 c\theta_5) ,\ 0\ ]$

$[\ d3 \cos\theta_1 s\theta_2 - d2\, s\theta_1 - d6(s\theta_5 s\theta_4 - c\theta_1 c\theta_2 c\theta_4)$
$- c\theta_1 c\theta_5 s\theta_2 ,\ s\theta_1 d3\, c\theta_2 + d6\, c\theta_2 c\theta_5 - c\theta_4 s\theta_2 s\theta_5 ,$
$s\theta_1 s\theta_2 ,\ d6\, s\theta_5 (c\theta_1 c\theta_4 - c\theta_2 s\theta_2 s\theta_4) ,\ d6\, c\theta_1 c\theta_5 s\theta_4 -$
$s\theta_1 s\theta_2 s\theta_5 + c\theta_2 c\theta_4 c\theta_5 s\theta_1) ,\ 0\ ]$

$[\ 0 ,\ -d3\, s\theta_2 - d6\, c\theta_5 s\theta_2 - d6\, c\theta_2 c\theta_4 s\theta_5 ,\ c\theta_2 ,$
$d6\, s\theta_2 s\theta_4 s\theta_5 ,\ -d6(c\theta_2 s\theta_5 + c\theta_4 c\theta_5 s\theta_2) ,\ 0\ ]$

$[\ 0 ,\ -\sin\theta_1 ,\ c\theta_1 s\theta_2 ,$
$\cos\theta_1 s\theta_2 ;\ c\theta_4 s\theta_1 - c\theta_1 c\theta_2 s\theta_4 ,\ c\theta_1 c\theta_5 s\theta_2 - s\theta_5 s\theta_1 s\theta_4$
$- c\theta_1 c\theta_2 c\theta_4\ ]$

$[\ 0 ,\ c\theta_1 ,\ s\theta_1 s\theta_2 ,$
$s\theta_1 s\theta_2 ,\ c\theta_1 c\theta_4 - c\theta_2 s\theta_1 s\theta_4 ,$
$s\theta_5 (c\theta_1 s\theta_4 + c\theta_2 c\theta_4 s\theta_1) + c\theta_5 s\theta_1 s\theta_2\ ]$

$[\ 1 ,\ \cos\theta_2$
$0 ,\ \cos\theta_2$
$\sin\theta_2 * \sin\theta_4 ,$
$\cos\theta_2 \cos\theta_5 - \cos\theta_4 * \sin\theta_2 * \sin\theta_5\ ]$