EECE 5552

Assistive Robotics

Assignment – 6 and 7

Name: Tejas Krishna Reddy

NUID: 001423166

Date: 11/12/2020

Problem 1 (b):

```
%% Clear Eveything
clc
clear all
close all


%% Constructing a state-space model of DC motor with two inputs (Va, Td) and output
(w).
% Problem 1(b)
% Given Parameters
R = 2.0;                % Ohms
L = 0.5;                % Henrys
Km = 0.1;               % torque constant
Kb = 0.1;               % back emf constant
Kf = 0.2;               % Nms
J = 0.02;               % kg.m^2/s^2


%From figure 2 in the assignment
% Now let us create transfer functions tf
% transfer function h = tf(Numerator, Denominator)
% Armature
h1 = tf(Km,[L R]);
% Equation
h2 = tf(1,[J Kf]);


% w = h2 * (h1*Va + Td)
dcm = ss(h2) * [h1 , 1];
% Back emf
dcm = feedback(dcm,Kb,1,1);


%% Plotting Angular Velocity Response to Step Change in Voltage Va.
stepplot(dcm(1));
```

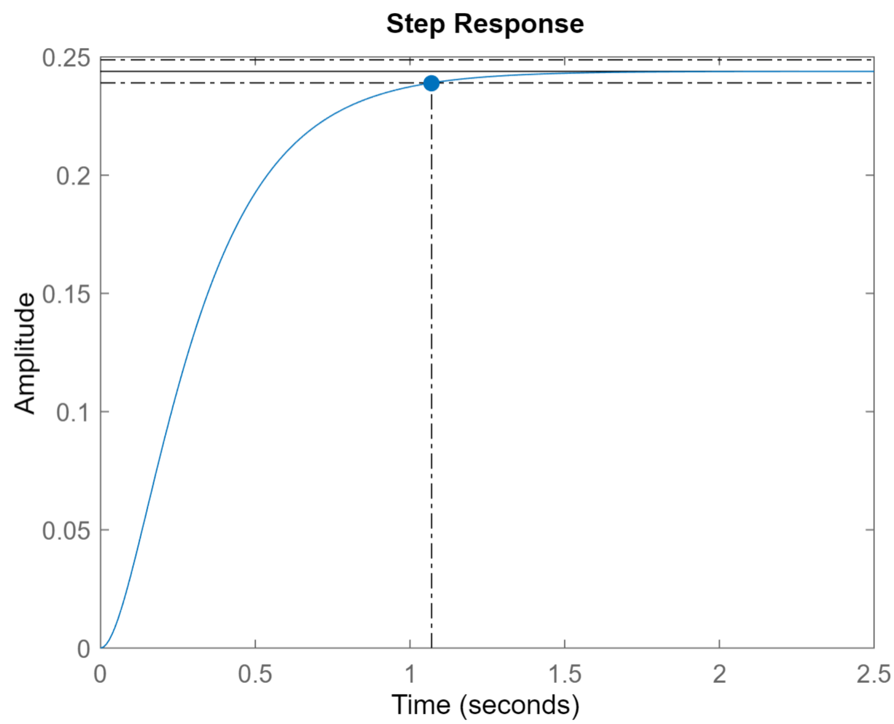


Fig 1: State Space Model Step Response

Problem 1 (c):

```
%% Clear Eveything
clc
clear all
close all


%% Constructing a state-space model of DC motor with two inputs (Va, Td) and output
(w).
% Problem 1(b)
% Given Parameters
R = 2.0; % Ohms
L = 0.5; % Henrys
Km = 0.1; % torque constant
Kb = 0.1; % back emf constant
Kf = 0.2; % Nms
J = 0.02; % kg.m^2/s^2


%From figure 2 in the assignment
% Now let us create transfer functions tf
% transfer function h = tf(Numerator, Denominator)
% Armature
h1 = tf(Km,[L R]);
% Equation
```

```
h2 = tf(1,[J Kf]);


% w = h2 * (h1*Va + Td)
dcm = ss(h2) * [h1 , 1];
% Back emf
dcm = feedback(dcm,Kb,1,1);


%% Plotting Angular Velocity Response to Step Change in Voltage Va.
stepplot(dcm(1));


%% Problem 1(c)
% feedforward gain Kff should be set to the reciprocal of the DC gain from Va to w.
Kff = 1/dcgain(dcm(1))
Kff


%% To evaluate Feed Forward design wrt load disturbances:
t = 0:0.1:15;
Td = -0.1 * (t>5 & t<10);       % load disturbance
u = [ones(size(t)) ; Td];       % w_ref=1 and Td


cl_ff = dcm * diag([Kff,1]);    % add feedforward gain
cl_ff.InputName = {'w_ref','Td'};
cl_ff.OutputName = 'w';


h = lsimplot(cl_ff,u,t);
title('Setpoint tracking and disturbance rejection')
legend('cl\_ff')


% Annotate plot
line([5,5],[.2,.3]);
line([10,10],[.2,.3]);
text(7.5,.25,{'disturbance','T_d = -0.1Nm'},...
            'vertic','middle','horiz','center','color','r');


%% The resultant graph indicates that feedforward control handles disturbances very
poorly.
```

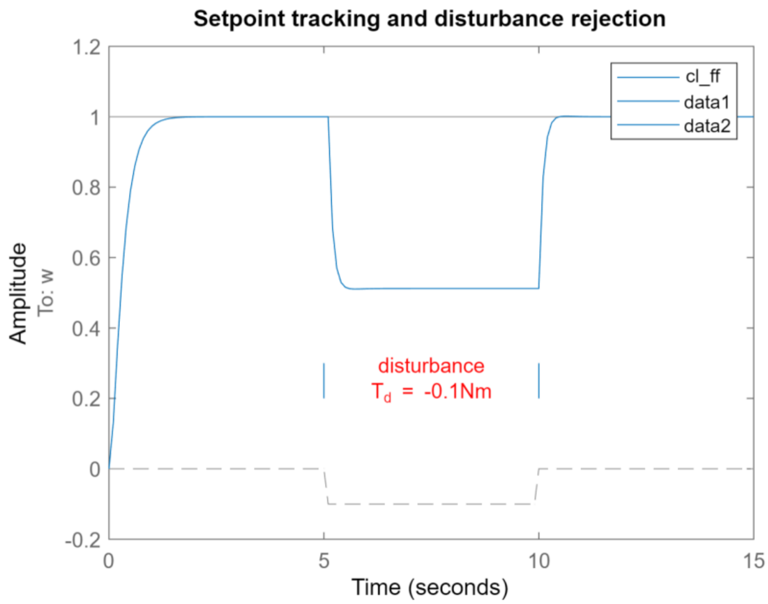


Fig 2: Response to load distribuance.

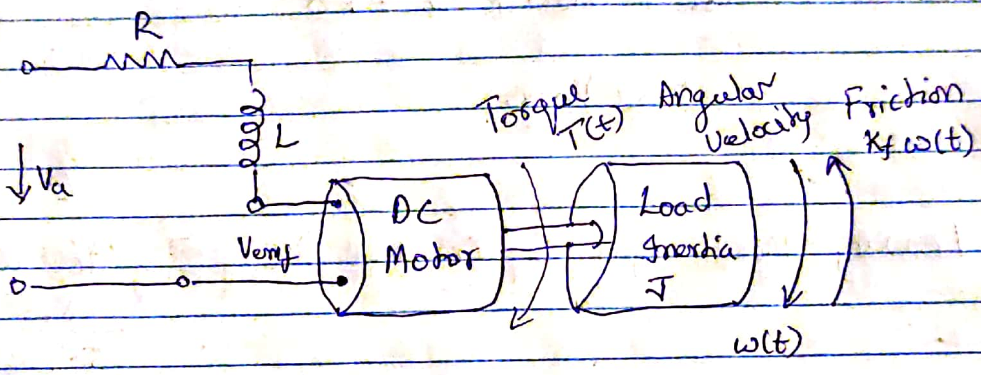
%% The resultant graph indicates that feedforward control handles disturbances very poorly.

Assignment 6 & 7:-

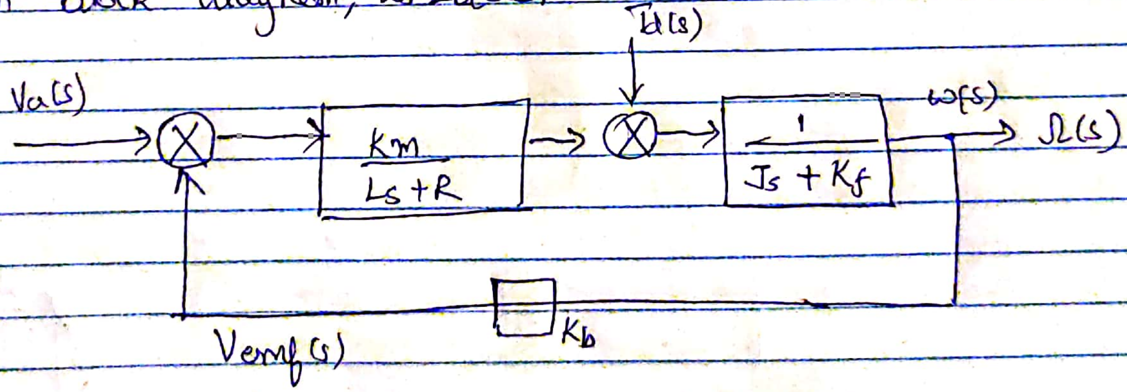
Problom 1 (a):-

The mobility device posses 2 DC Motors, which generate desired angular velocity w_i, $i \in \{R, L\}$ depending on the voltage $V_i(t)$, $i \in \{R, L\}$.

General DC Motor is given by:-



Given block diagram, is below:-



Transfer function:-

$$G_M(s) = \frac{\theta(s)}{V_a(s)}$$

$$G_M(s) = \frac{\theta(s)}{V_a(s)} = \frac{K_m}{[(L_s+R)(J_s+K_f)+K_bK_m]}$$

In general,
DC motors have time constant in armatures which is neglible.
Therefore, equation can further be simplified to:-

$$G_m(s) = \frac{K_m}{R(J_s+K_f)+K_bK_m}$$

$$= \frac{[K_m/(RK_f+K_bK_m]}{T_s+1}$$

$$= \frac{K_m}{T_s+1}$$ //

where time constant $T = RJ/(RK_f+K_bK_m)$

and $K_m = K_m/(RK_f+K_bK_m)$

Also, in an armature controlled DC Motor,
$K_m = K_b$//

Problem 2(a):-

Given, $\begin{bmatrix} V_x(t) \\ V_y(t) \\ \omega \end{bmatrix} = \begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix}$ angular velocities of wheels $[\omega_r, \omega_L]$

$$\dot{\xi} = \begin{bmatrix} V_x(t) \\ V_y(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} C_\theta & -S_\theta & 0 \\ S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ \ell/2 & -\ell/2 & 0 \end{bmatrix} \begin{bmatrix} r_R \omega_R(t) \\ r_L \omega_L(t) \\ 0 \end{bmatrix}$$

There are two passive and two active wheels in the bot. Thus, the motion and velocity of the wheelchair are dependent on the fixed two wheels.

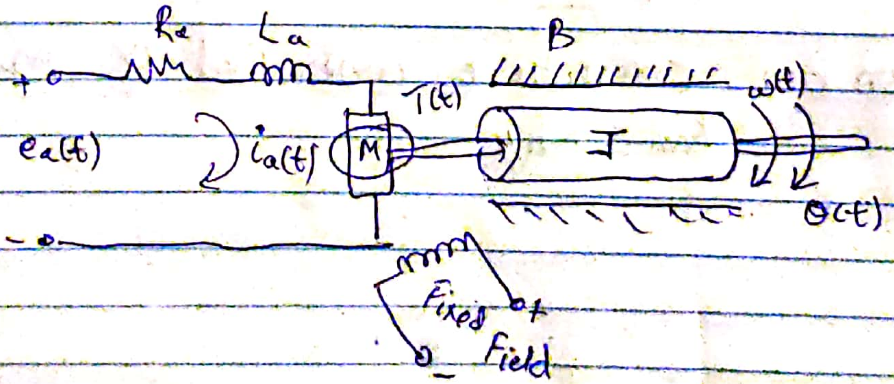
The average linear and angular speed of wheelchair can be calculated as:-

$$\omega_r = \frac{V_{right\,wheel} + V_{left\,wheel}}{2}$$

$$\omega_L = \frac{V_{right\,wheel} + V_{left\,wheel}}{D}$$

Where D = distance between the wheels.

Adding to DC Motor



Transfer function can be given as:-

$$G(s) = \frac{K_t}{[(L_a J)s^2 + (R_a J + bL_a)s + (R_a b_{eq} + K_t K_b)]}$$

$$\Rightarrow \frac{V_m(s)}{V_a(s)} = \frac{K_t}{(R + Ls)(J_m s + B_m) + K_t K_b}$$

If $L \simeq 0$

then,

$$\frac{V_m(s)}{V_s(s)} = \frac{K_t}{RJ_m s + B_m R + K_t K_b}$$

Now let, $T_m = \frac{RJ_m}{B_m R + K_t K_b}$ (time constant)

$$K_m = \frac{K_t}{B_m R + K_t K_b}$$

$$\therefore \frac{V_m(s)}{V_a(s)} = \frac{K_m}{1 + sT_m}$$

Voltage equation of DC Motor system $\Rightarrow$

$$V_a(t) = R i_a(t) + L\frac{di_a(t)}{dt} + e_b(t) \quad -(1)$$

Motor Torque $\Rightarrow$ $T_m(t) = K_t i_a(t) \quad -(2)$

Back emf equation $\Rightarrow$

$$e_b(t) = K_b \omega_m(t) \quad -(3)$$

Substituting eqn ① and ② in ③,

$$V_a(t) = \frac{R\,T_m(t)}{K_t} + K_b\,\omega_m(t) + L\frac{di_a(t)}{dt} \quad -④$$

From ④ we also get:-

$$\omega_m(t) = \frac{V_a}{K_b} - \frac{R}{K_t K_b}\,T_m(t) + \frac{L}{K_b}\,\frac{di_a(t)}{dt}$$

In steady state condition, time t is not involved.
$(L=0)$

$$\omega_m = \frac{V_a}{K_b} - \frac{R}{K_t K_m}\,T_m$$

Assume $T_m = T_L$, $\frac{1}{K_b} = K_m$, $\frac{R}{K_t K_m} = K_L$, $\omega_L = \omega_0$

$\therefore$ $\omega_0 = K_m V_a - K_L T_L$

$\omega_0 = K_s K_m V_i - K_L T_L$

Now, for a robotic wheel chair:-

$$\sin\theta x - \cos\theta y = 0$$

$$\sin(\theta+\phi)x - \cos(\theta+\phi)y - d\cos\phi\theta = 0$$

$$g_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad g_2 = \begin{bmatrix} \sin\theta \\ \sin\theta \\ 1/d\tan\theta \\ 0 \end{bmatrix}$$

when $\theta = \pi/4$

$$g_2 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 1/d \\ 0 \end{bmatrix}$$

Scanned with CamScanner