EECE 5552-Assistive Robotics
Assignment 6 & 7

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***Due by 11:59 PM Eastern Time, Friday, November 13**

Problem 1

Consider the smart wheelchair shown below. This mobility device possesses two DC Motors (DCMs) with permanent magnets and they generate desired angular velocities $\omega_i(t),\ i \in \{R, L\}$ depending on the commanded voltage $v_i(t),\ i \in \{R, L\}$.

(a) What is the characteristic polynomial for the simplified DC motors shown below?

Figure 1: Mobility device with two DC motors with permanent magnets.

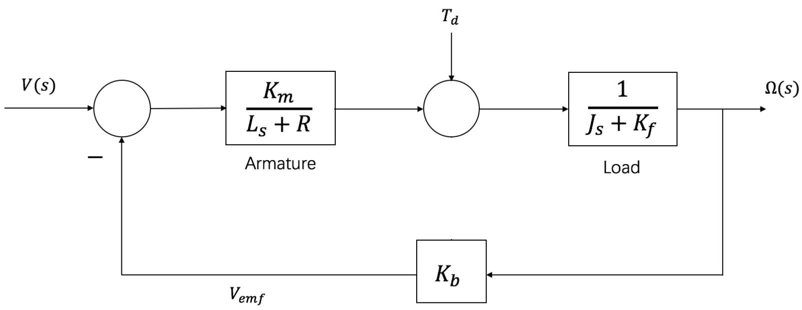


Figure 2: A simplified model of each DC motors with permanent magnets.

(b) The external torque T_d models load disturbances. The physical constants are:

- resistance: $R = 2.0$,
- inductance: $L = 0.5$,
- torque constant: $K_m = 0.1$,
- back-emf constant: $K_b = 0.1$,
- motor inertia: $J = 0.02$,
- damping: $K_f = 0.2$,
- external torque: $T_d = 1$,

Construct the transfer function of the DC motor with two inputs: voltage $V(s)$ and disturbance $T_d(s)$ and one output angular velocity $\Omega(s)$. Use MATLAB to plot the angular velocity response $\Omega(s)$ to a step change in voltage $V(s) = 1$.

Hint1: You can do this with the following commands in MATLAB:

```
h1 = tf(Numerator,Denominator) ;                    % create transfer function
```

Check "stepplot" and "feedback" commands in MATLAB.

(c) You must minimize the speed variations induced by T_d disturbances. What are the solutions you propose (refer to the class notes on closed-loop feedback)?
Hint: Below we will look into a tutorial that applies a feedforward control structure to cancel the unwanted effects of the disturbance term T_d.
Tutorial: It is possible to utilize the feedforward term K_{ff} (shown in Fig. 3) to command the angular velocity Ω to a given value Ω_{ref}.

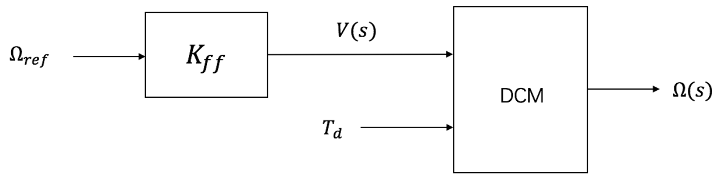


Figure 3: A simple feedforwrd control structure to cancel T_d effects on the tracking performance of Ω.

The feedforward gain K_{ff} should be set to the inverse of the DC gain from $V(s)$ to $\Omega(s)$, i.e., $K_{ff} = \frac{1}{\|\frac{V(s)}{\Omega(s)}\|}$. You can use the following MATLAB command to obtain this value:

```
%% DC motor model
R = 2.0;                    % Ohms
L = 0.5;                    % Henrys
Km = 0.1;                   % torque constant
Kb = 0.1;                   % back emf constant
Kf = 0.2;                   % Nms
J = 0.02;                   % kg.m^2/s^2

h1 = tf(Km,[L R]);                 % armature
h2 = tf(1,[J Kf]);                 % eqn of motion

dcm = ss(h2) * [h1 , 1];           % w = h2 * (h1*Va + Td)
dcm = feedback(dcm,Kb,1,1);        % close back emf loop

Kff = 1/dcgain(dcm(1)) % kff = 4.1
```

In order to evaluate the feedforward design performance in the face of the load disturbance T_d, we simulate the response to a step command $\Omega_{ref} = 1$ with a disturbance $T_d = -0.1$ between $t = 5$ and $t = 10$ seconds:

```
t = 0:0.1:15;
Td = -0.1 * (t>5 & t<10);          % load disturbance
u = [ones(size(t)) ; Td];          % w_ref=1 and Td

cl_ff = dcm * diag([Kff,1]);       % add feedforward gain
cl_ff.InputName = {'w_ref','Td'};
cl_ff.OutputName = 'w';

h = lsimplot(cl_ff,u,t);
title('Setpoint tracking and disturbance rejection')
legend('cl\_ff')

% Annotate plot
line([5,5],[.2,.3]);
line([10,10],[.2,.3]);
text(7.5,.25,{'disturbance','T_d = -0.1Nm'},...
            'vertic','middle','horiz','center','color','r');
```

How do you evaluate the performance of the feedforward solution?

Problem 2

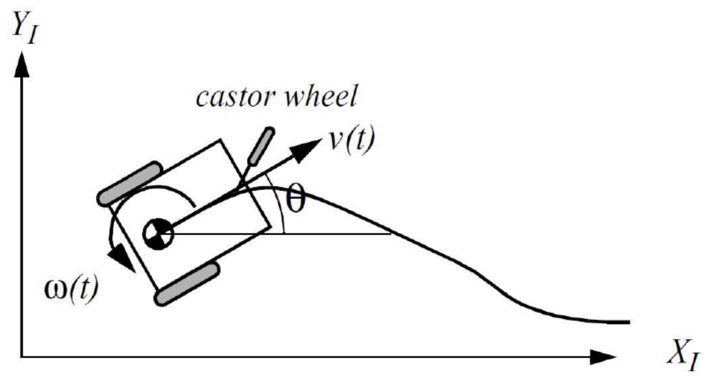


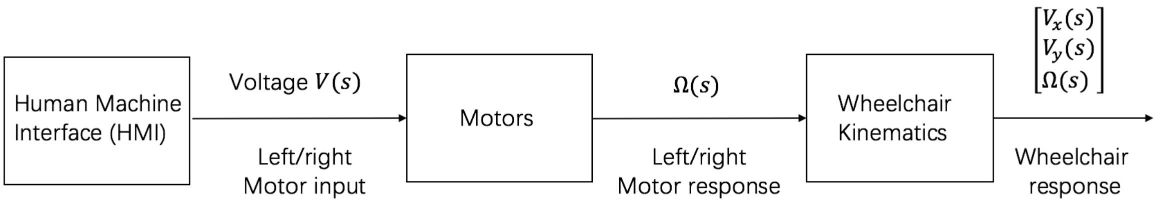
Figure 4: Differential-drive mobile robot revisited (HW. 5, P. 2).

From previous homework (HW. 5, P. 2), the states of a differential-drive robot $\begin{bmatrix} v_x(t) \\ v_y(t) \\ \omega \end{bmatrix} = \begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix}$ are related to the angular velocity of the wheels $[\omega_R \ \omega_L]$ through the following system of equations:

$$\dot{\xi}_I = \begin{bmatrix} v_x(t) \\ v_y(t) \\ \omega(t) \end{bmatrix}_I = \begin{bmatrix} c_\theta & -s_\theta & 0 \\ s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ \frac{l}{2} & \frac{-l}{2} & 0 \end{bmatrix} \begin{bmatrix} r_R\omega_R(t) \\ r_L\omega_L(t) \\ 0 \end{bmatrix}$$

(a) The passive wheels in the smart wheelchair do not contribute to the overall mobility of the device. By ignoring these passive wheels and only considering the two actuated standard wheels the kinematics of the smart wheelchair will resemble that of the simplified differential-drive model shown above. How would you model the motion of the wheelchair based on the voltage $v_i(t) \in \{R, L\}$ commands to the DC motors when $\theta = \frac{\pi}{4}$. (It is not required to simulate this problem with MATLAB.)

Hint: It is not required to simulate this problem with MATLAB. Obtain the transfer functions that relate the DC motor voltage and the wheelchair velocity, e.g., $G_R(s) = \frac{V_x(s)}{V_R(s)}$.


Human Machine Interface (HMI)
Voltage $V(s)$
Left/right Motor input
Motors
$\Omega(s)$
Left/right Motor response
Wheelchair Kinematics
$\begin{bmatrix} V_x(s) \\ V_y(s) \\ \Omega(s) \end{bmatrix}$
Wheelchair response