EECE 5552-Assistive Robotics
Assignment 5

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***Due by 11:59 PM Eastern Time, Tuesday, November 3**

Problem 1

(a) Fig. 1 depicts a fixed standard wheel attached to a wheeled assistive robot.

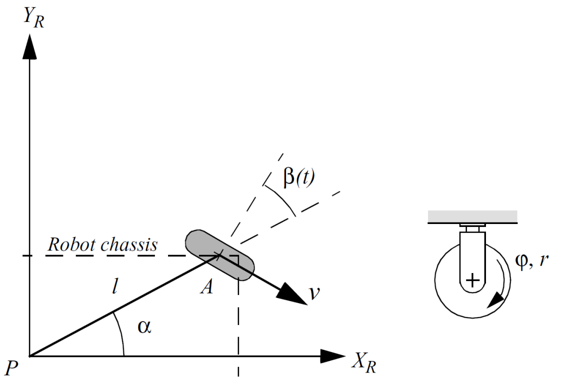

Figure 1: A fixed standard wheel.

$\{X\ Y\}_R$ is the robot local coordinate frame and the position of point A is described by the polar coordinates l and α. The steering angle β is constant and the wheel has time-dependant rolling motion $\varphi(t)$. The wheel radius is denoted by r. The states of the wheeled robot x, y, θ – positions and orientation – and the state vector $\xi = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$ are given with respect to the world coordinate frame $\{X\ Y\}_I$. Obtain the kinematic constraints acting on the system?
Hint :Use the class notes on wheeled assistive technologies and re-write the pure rolling and no side-slip constraints for the fixed standard wheel.

(b) Suppose the wheel is in a position such that $\alpha = 0$ and $\beta = 0$. At this configuration the point A will fall on X_R and the wheel plane will be parallel to Y_R. If $\theta = 0$, then obtain the no side slip constraint. How would you interpret the obtained equation?

Problem 2

(a) For the above differential-drive robot with a passive caster wheel and two fixed standard wheels obtain the kinematic equations that relate the instantaneous velocity of the system in the world frame $\dot{\xi}_I = \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T = \begin{bmatrix} v(t) & w(t) \end{bmatrix}^T$ to the angular velocity of the wheels $\dot{\varphi}(t) = [\dot{\varphi}_R\ \dot{\varphi}_L]^T$.
Hint 1: Simply fuse the pure rolling and no side slip constraints for the system:

$$\begin{bmatrix} J_1(\beta_s) \\ C_1(\beta_s) \end{bmatrix} R(\theta)\dot{\xi}_I = \begin{bmatrix} J_2\dot{\varphi} \\ 0 \end{bmatrix}$$

where $J_1(\beta_s)$ and $C_1(\beta_s)$ are directly from the rolling and no slide slip constraints. $R(\theta)$ maps

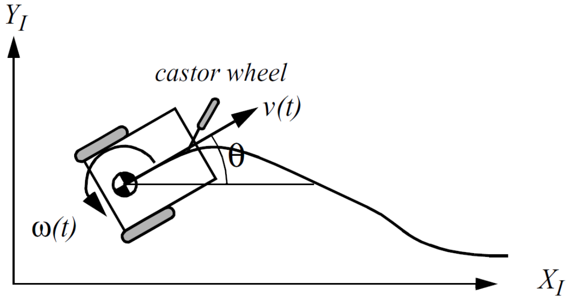

Figure 2: Differential-drive mobility system.

the world instantaneous velocity of the system $\dot{\xi}_I$ to $\dot{\xi}_R$. The heading angle is denoted by θ and J_2 is given by $J_2 = \begin{bmatrix} r_R & 0 \\ 0 & r_L \end{bmatrix}$, where r_R and r_L are the radius of the right and left wheels. The vector of angular velocity $\dot{\varphi}$ is created by stacking the angular velocity of the right and left wheels: $\dot{\varphi} = [\dot{\varphi}_R \; \dot{\varphi}_L]^T$.
Hint 2: Since the caster wheel is passive, therefore, ignore it in your analysis.

(b) Consider the three-wheel omnidrive robot shown below.

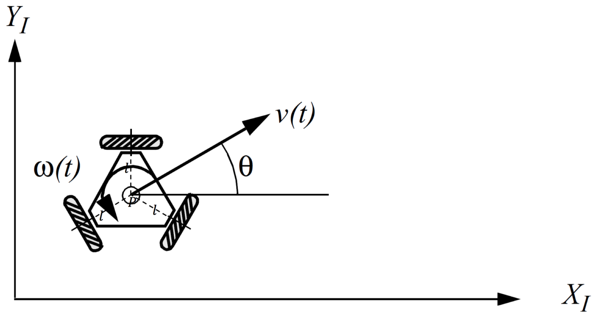

Figure 3: A three-wheel omnidrive robot.

The wheels are 90-degree wheels, i.e., $\gamma = 0$ (Note: refer to the class nodes for the definition of γ.) We assume that the distance of each wheel and P is l and each wheel has the radius r. Obtain the instantaneous velocity of the robot with regard to world frame:

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v(t) \\ w(t) \end{bmatrix}$$

in terms of the angular velocity of each wheel $\dot{\varphi} = [\dot{\varphi}_1 \; \dot{\varphi}_2 \; \dot{\varphi}_3]^T$.
Hint 1: First, impose a suitable robot coordinate frame $\{X \; Y_R\}$. E.g., the local coordinate frame can be located at point P and the X_R axis can be co-linear with the axis of the wheel 2, shown in figure 4.

Hint 2: As with the differential-drive robot, there is no steerable wheels here. Therefore, $J_1(\beta_s)$ simplifies to J_{1f} and

$$\dot{\xi}_I = R^{-1}(\theta) J_{1f}^{-1} J_2 \dot{\varphi}.$$

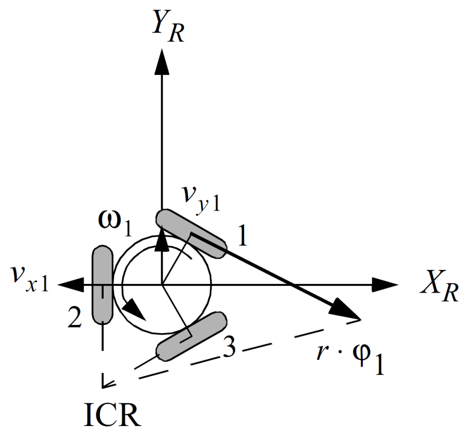

Figure 4: A choice of local reference frame $\{X\ Y\}_R$.