EECE 5552

Assistive Robotics

Assignment – 5

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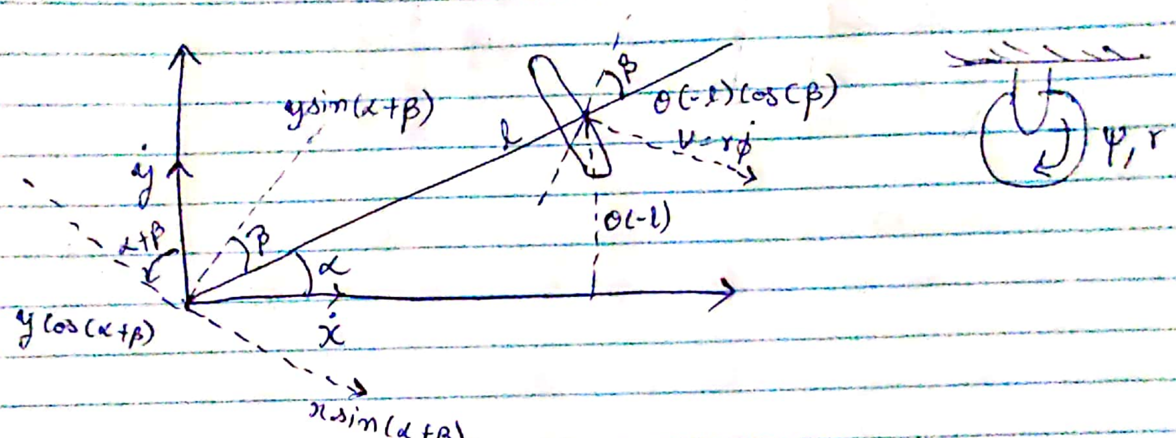
ASSIGNMENT - 5

PROBLEM 1 :- (a)

Redrawing the figure :-



Given,

state vector = $\xi_R = [\,x \quad y \quad \theta\,]^T$

Motion along the direction of the plane must be accompanied by appropriate amount of wheel spin, so there is a pure rolling at the contact point. — (1)

$$[\sin(\alpha+\beta) \; -\cos(\alpha+\beta) \; (-l)\cos(\beta)]\,R(\theta)\,\xi_1 - r\dot\varphi = 0$$

$$[\cos(\alpha+\beta) \; \sin(\alpha+\beta) \; (l)\sin(\beta)]\,R(\theta)\,\xi_1 = 0 \quad -(2)$$

[∵ The sliding constraint for the wheel resulting wheel's motion is orthogonal to the wheel plane = 0].

(b)

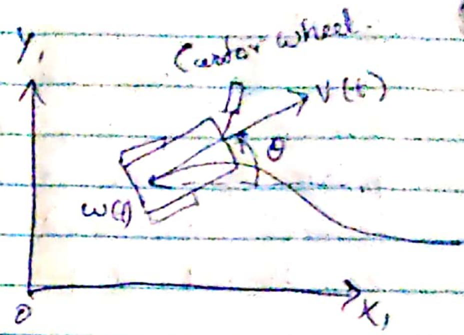
Suppose the wheel's position is such that $\{\alpha = 0, \beta = 0\}$, then this would place the contact point of the wheel on X_I with the plane of the wheel oriented parallel to Y_I. If $\theta = 0$ then sliding constraint (2) reduces to:-

$$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = 0 \quad \text{--- (3)}$$

From (3) we can observe that the component of motion along X_I to be zero and since X_I and X_R are parallel, the wheel is constrained from sliding sideways.

Problem 2 (a):-



Robots state vector =>

$$\varepsilon_1 = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \quad - (1)$$

Mapping the motion of the robot along the axes of global reference frame along with robots local reference frame:-

Hence, orthogonal rotation matrix is represented by:-

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad - (2)$$

Global Reference $\{X_1, Y_1\}$ Motion in terms of local Reference frame $\{X_R, Y_R\}$.

This can be represented by:-

$$\xi_R = R(\theta)\,\varepsilon_1 \quad - (3)$$

Substituting (2), (1) in (3)

$$\xi_R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \quad - (4)$$

If the robot has a $90°$ rotation, then, equation ~~can~~ (4) can be reduced to:-

$$\xi_R = R(90°)\,\xi_I = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} y \\ -x \\ \theta \end{bmatrix}$$

But for a caster wheel:-

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = f(l, r, \theta, \varphi_1, \varphi_2)$$

also

$$\xi_I = R(\theta^{-1})\,\xi_R$$

If one wheel remains stationary and another spins, the robot instantaneously moves with speed - half.

$$x_{R_1} = \frac{1}{2} r\varphi_1 \quad \text{and} \quad x_{R_2} = \frac{1}{2} r\varphi_2 .$$

The rotation velocity ω_1 of wheel of radius $2l$ =

$$\omega_1 = \frac{r\varphi_1}{2l} ;$$

Similarly

$$\omega_2 = -\frac{r\varphi_2}{2l}$$

$$\xi_I = R\theta^{-1} \begin{bmatrix} \frac{r\varphi_1}{2} + \frac{r\varphi_2}{2} \\ 0 \\ \frac{r\varphi_1}{2l} + \frac{(-r\varphi_2)}{2l} \end{bmatrix} \quad \text{--- (5)}$$

$$R\theta^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem(b) :-

Given, 90° wheels $\gamma = 0$

$$\xi_1 = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} V(t) \\ \omega(t) \end{bmatrix} \qquad \dot\varphi = [\dot\varphi_1\ \dot\varphi_2\ \dot\varphi_3]^T$$

Assuming rolling constraints $J_1(\beta)_s$ and wheel spin $\dot\varphi$.

$$\dot\xi_1 = R(\theta)^{-1} J_1(\beta_s)^{-1} J_2 \dot\varphi$$

Suppose robot moves forward along $+X_R$. The right wheel $\alpha = -\pi/2$ $\beta = \pi$, and for left wheel $\alpha = \pi/2$, $\beta = 0$

Now $$J_{1F} = \begin{bmatrix} 1 & 0 & l \\ 1 & 0 & -l \end{bmatrix}$$

Hence, drive robot $\Rightarrow$

$$\dot\xi_1 = R(\theta)^{-1} \begin{bmatrix} 1 & 0 & l \\ 1 & 0 & -l \end{bmatrix}^{-1} J_2 \dot\varphi$$

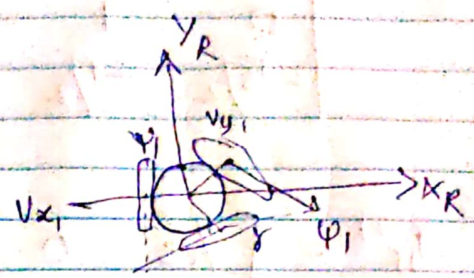
Also, $J_{1F}^{-1} \cdot J_1 = J_1$

Hence equation (5) can be re-written as:-

$$\dot\xi_1 = R(\theta)^{-1} \begin{bmatrix} 1/2 & 1/2 \\ 0 & 0 \\ 1/2l & -1/2l \end{bmatrix} J_2 \dot\varphi \quad \text{--- (6)}$$

Assuming the robot has 3 swedish 90° wheels $\Rightarrow$

All 3 wheels are equidistant,
$P = l$, same radius 'r'.



$$\dot{\xi}_1 = R(\theta)^{-1} J_{1f} J_2 \dot{\varphi}$$

$\gamma = 0$ for swedish 90 wheel.
with respect to local reference frame

$\alpha_1 = \pi/3$, $\alpha_2 = \pi$ and $\alpha_3 = -\pi/3$

$\beta_1 = 0$ $\beta_2 = 0$ $\beta_3 = 0$ $\rightarrow$ since all wheels are tangent to robots circular body.

Hence,

$$J_{1F} = \begin{bmatrix} \sin(\pi/3) & -\cos \pi/3 & -l \\ 0 & -\cos \pi & -l \\ \sin(-\pi/3) & -\cos(-\pi/3) & -l \end{bmatrix}$$

$$J_{1F} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & -l \\ 0 & 1 & -l \\ -\sqrt{3}/2 & -1/2 & -l \end{bmatrix}$$

$$J_{1F}^{-1} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & -l \\ 0 & 1 & -l \\ -\sqrt{3}/2 & -1/2 & -l \end{bmatrix}^{-1}$$

$$J_{1F}^{-1} = \begin{bmatrix} 1/\sqrt{3} & 0 & -1/\sqrt{3} \\ -1/3 & 2/3 & -1/3 \\ -1/3l & -1/3l & -1/3l \end{bmatrix} \quad - (7)$$

substituting (7) in $\xi_1 = R^{-1}(\theta)\, J_{1F}^{-1}\, J_2 \dot{\varphi}$.

$$\xi_1 = R(\theta)^{-1} \begin{bmatrix} 1/\sqrt{3} & 0 & -1/\sqrt{3} \\ -1/3 & 2/3 & -1/3 \\ -1/3l & -1/3l & -1/3l \end{bmatrix} J_2 \dot{\varphi}$$