EECE 5552-Assistive Robotics
Assignment 4

Alireza Ramezani

***Due by 11:59 PM Eastern Time, Friday, October 23**

Euler-Lagrange equations for an n-DOF system, where $q = (q_1; ...; q_n)$, are given by:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_\kappa} - \frac{\partial \mathcal{L}}{\partial q_\kappa} = \tau_\kappa, \quad \kappa = 1, ..., n$$

In these equations, τ_k is called generalized force (or moment) and $\mathcal{L}$ is called Lagrangian and is given by:

$$\mathcal{L} = K - P\text{: Kinetic-Potential Energy}$$

Consider a single link of an n-DOF robotic arm (shown in Fig. 1). The link is attached to an actuator at one end. A control input u is applied by the motor to the link. Consequently a rotary motion is generated at the link. The relation between the moter angle (θ_m) and link angle (θ_l) is given by:

$$\theta_m = r\theta_l$$

where r is the gear ration of the actuator.

(a) Apply Euler-Lagrange equations to drive the equations of motion.
Hint: The Kinetic Energy of the link is given by:

$$K = \tfrac{1}{2}J_m\dot{\theta}_m^2 + \tfrac{1}{2}J_l\dot{\theta}_l^2$$

where J_m and J_l are some positive scalar values.
Hint: The Potential Energy is simply given by:

$$P = Mgl(1 - \cos\theta_l)$$

where M, g and l are some positive scalars (Note: l is the distance of the center of mass (CM) from the joint, shown in Fig. 1).

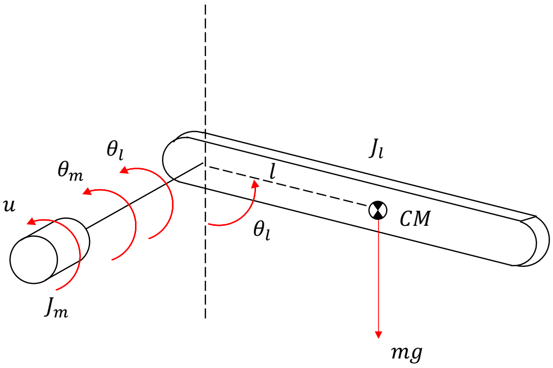

Figure 1: Rotating a link with an actuator.

(b) Damping terms are velocity dependent external dissipative forces that affect the motion of a joint. The motor-related and link-related damping coefficients are denoted by B_m and B_l, respectively. Consequently, these damping forces are given by:

$$\tau_m = -B_m \dot{\theta_m}$$
$$\tau_l = -B_l \dot{\theta}_l$$

How would you incorporate these terms in Euler-Lagrange Equations?

(c) Let's assume that we use a servo drive with the gear ratio $r = 30$ to actuate the finger in the robotic hand shown in Fig. 3. If the parameters $J_m = J_l = 0.001$, $M = 0.1$, $g = 9.81$, $l = 0.1$, $u = 0.1$ and the initial states of the link are $(\theta_l; \dot{\theta}_l) = (0; 0)$, find the solutions of the system $\sum_2$ for the damping values of $B_m = B_l \in \{0.001, 0.01, 0.1, 1\}$. How do you interpret your results?

Please, only plot the numeric solutions to θ_l and $\dot{\theta}_l$ versus time for the time interval 0 to 2. No analytic solutions are required.

Figure 2: Robotic hand.

Hint: You can use MATLAB to find the answers to this problem.
Hint: Consider a change of variable given by

$$x_1 = \theta_l$$
$$x_2 = \dot{\theta}_l$$

After substituting x_1 and x_2 in $\sum_2$ a new system $\sum_2'$, which is a first-order ordinary differential equation (ODE), is obtained:

$$\sum_2' : \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = f(x_1, x_2) + g(x_1, x_2)u$$

Now, use Matlab to integrate this ODE.

Tutorial:
Below, solving a simple ODE using ODE45 in MATLAB is shown. ODE45 is a function for solving nonstiff differential equations.

$$\Sigma : \begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = -\alpha x_2 - \sin(x_1) \end{cases}$$

MATLAB code:

```
function func_a_simple_ODE()
close all;
```

```
% time vector
t=0:0.01:2 ;

% initial states
x_10=1 ;
x_20=10;

fh1 = figure('Name','A simple ODE (x1)');
ah1 = axes('parent',fh1);
hold(ah1,'on');
xlabel(ah1,'time') ;

fh2 = figure('Name','A simple ODE (x2)');
ah2 = axes('parent',fh2);
hold(ah2,'on');
xlabel(ah2,'time') ;

alpha_vec = [1,2,3];
for i=1:3

    alpha = alpha_vec(i);

    % solve ODE
    [t,x]=ode45(@func,t,[x_10,x_20]);

    % plot solutions
    plot(ah1,t,x(:,1));
    plot(ah2,t,x(:,2));
end

% a simple function
    function dxdt=func(t,x)
        dx1  = x(2) ;
        dx2  = -alpha*x(2)+sin(x(1));
        dxdt =[dx1;dx2] ;
    end
end
```

Useful links:
https://www.mathworks.com/help/matlab/ref/ode45.html
https://www.12000.org/my_notes/matlab_ODE/index.htm