# Challenge 9

Tuesday, December 1, 2020 1:54 AM

1. Problem 1:-

(a) Given the two-link planar manipulator,

Let,

$\tau$ = vector of joint torques

$\delta x$ = virtual end to end disp.

$F$ = Force.

We know that,

$$\delta x = J(q)\,\delta q. \quad — (1)$$

The virtual work $\delta w \Rightarrow$

$$\delta w = F^T \delta x - \tau^T \delta q \quad — (2)$$

Substituting (1) in (2) :-

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$$\delta\omega = (F^T J - \tau^T)\,\delta q$$

$\therefore$ with generalized co-ordinates of $q$,

$$\tau = J(q)^T F$$

For the given two-link RR the Jacobian matrix is given by :-

$$J = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$$

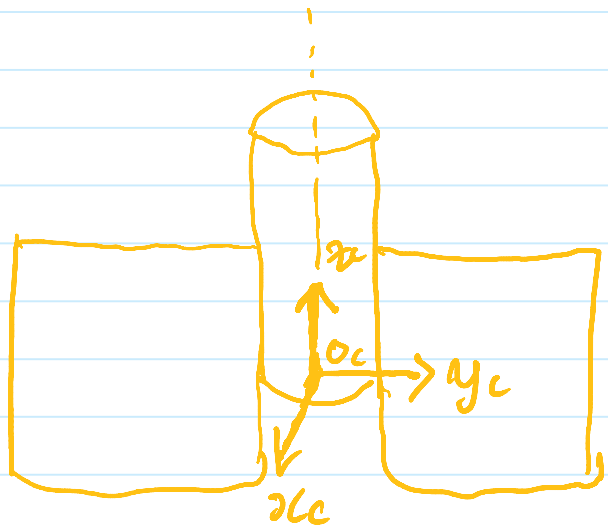
The joint torque necessary to balance an end to end - effector $F = (-1, 1)^T$ is given by $\Rightarrow$

$$\tau = J^T F$$

$$= \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & a_1 c_1 + a_2 c_{12} \\ -a_2 s_{12} & a_2 c_{12} \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\tau = \begin{bmatrix} a_1 (s_1 - c_1) + a_2 (s_{12} - c_{12}) \\ a_2 (s_{12} - c_{12}) \end{bmatrix}$$

6. Round peg into round hole:-




From the compliance frame $O_c x_c y_c z_c$,

$$V^T F = V_x f_x + V_y f_y + V_z f_z + \omega_x n_x + \omega_y n_y + \omega_z m_z . \quad \text{— (1)}$$

Assuming the walls of the hole and the peg are rigid,

$\therefore$ No friction

Hence,

$$V_x = 0 \quad V_y = 0 \quad f_z = 0$$

$\omega_x = 0 \quad \omega_y = 0 \quad n_z = 0$

These are the Natural Constraints.

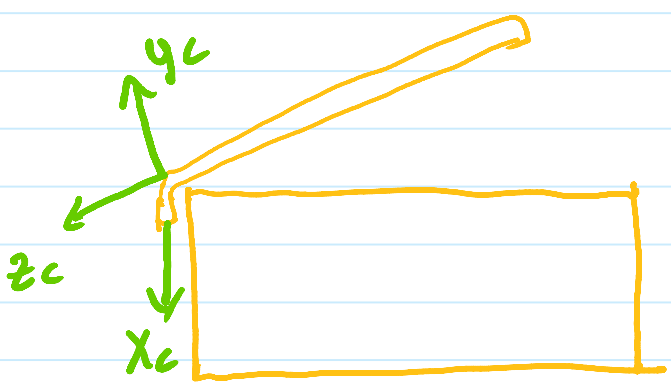
Let us take $V^d$ as the desired speed of insertion into the hole in z-direction.

Hence, the Artificial constraints are :-

$f_x = 0 \quad f_y = 0 \quad V_z = V^d$

$n_x = 0 \quad n_y = 0 \quad \omega_z = 0.$

(c) Opening a box with hinged lid :-




Natural Constraints ;-

$V_x = 0 \quad V_y = 0 \quad V_z = 0$

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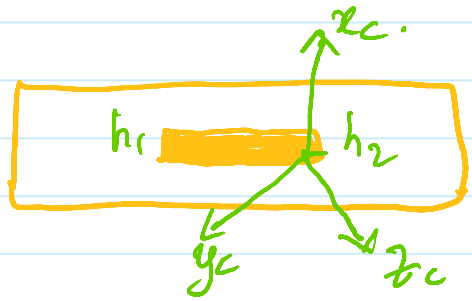
$\omega_y = 0 \quad \omega_z = 0 \quad f_y = 0$

Artificial Constraints:-

$\omega_z = \omega_d \quad \tau_x = I\dot{\omega}_d$

$f_x = 0 \quad f_z = 0 \quad \tau_y = 0 \quad \tau_z = 0$

d) Long two-handled drawer.




Both ends of the handle $h_1$ and $h_2$ have similar compliance frames.

Both $h_1$ and $h_2$ should possess forces in such a way that they produce a straight line motion and avoid side

straight line motion and avoid side ways or rotating motions. Here, it is assumed that the drawer is fixed to rigid horizontal frames, and has less or negligible frictional forces during the motion.

Lets' say that the desired velocity of drawer's movements in and out to be $v^d$.

Then, Natural Constraints ⇒

$v_x = 0 \quad v_y = 0 \quad f_z = 0$

$\omega_x = 0 \quad \omega_y = 0 \quad \omega_z = 0$

Artificial Constraints are as follows:-

$f_x = 0 \quad f_y = 0 \quad v_z = v^d$

$\tau_x = 0 \quad \tau_y = 0 \quad \tau_z = 0$

(e)

(i) Turning a crank is inertial to the

(1) Turning a crank is inertial to the tangent of the circular rotation made by the crank and capacitative along the crank's direction.

(2) Inserting a peg in the hole is inertial in the direction of the hole and capacitative in the directions with rigid constraints of movement (the walls).

(3) Polishing the hood of the car is inertial to the tangent of the hood and is capacitative to the orthogonal region of the hood.

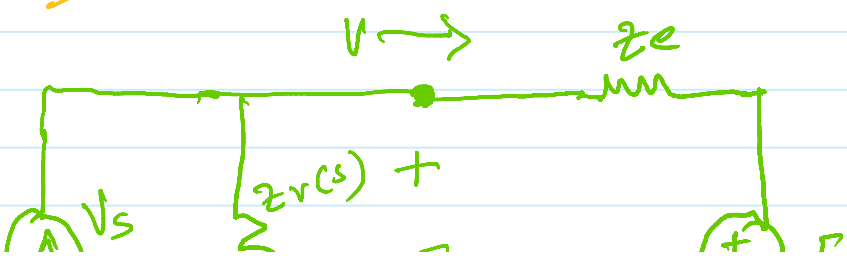
(4) Cutting cloth is a clear example for resistive force in the direction of the cutting action.

(5) Shearing a sheep is resistive to the shearing direction and capacitative to the orthogonal side of shearing action.

(6) Placing stamps is a clear example for capacitative normal to the plane where envelope is placed.

(7). Cutting meat similar to the cloth, is resistive towards the direction of the cutting action.

Problem 2 :-



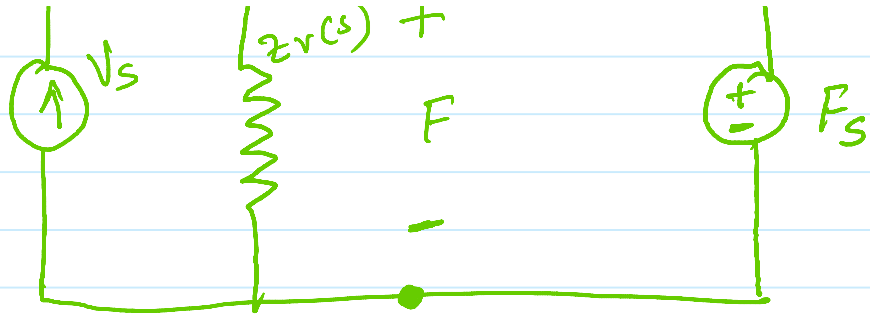



Fig :- Inertial Environment.

Given, the environment one-port is a Thevenin network and the robot on-port is a Norton network.

∴ From the diagram, we can derive that ⇒

$$\frac{V}{V_s} = \frac{Z_r(s)}{Z_e(s) + Z_r(s)} \quad \text{— (1)}$$

Given,

step response velocity command,

$$V_s = \frac{V^d}{s} \quad \text{— (2)}$$

Using (2) in the Final Value Theorem:-

we get the steady state force error ⇒

$$e_{ss} = \frac{-z_e(0)}{z_r(0) + z_e(0)} = 0$$

We also know that, it is an inertial Environment.,

where $z_e(0) = 0$

And,

For non-inertial robots $z_r \neq 0$