

## Today's agenda

↳ No. of iteration

↳ Big O Notations

## Quiz

↳ How many numbers are in range  $[3, 10]$  {corners included}  
 $\{3, 4, 5, 6, 7, 8, 9, 10\} \rightarrow 8$

$$\begin{array}{c} \text{inc} \quad \text{inc} \\ [a, b] \rightarrow b - a + 1 \end{array} \quad \bigg| \quad \begin{array}{c} \text{inc} \quad \text{exc} \\ [a, b) = b - a \end{array} \quad \bigg| \quad \begin{array}{c} \text{exc} \quad \text{exc} \\ (a, b) = b - a - 1 \end{array}$$

## Quiz

↳ No. of steps for  $N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \dots \rightarrow 1$

$$n * 2 = 10 \quad \Rightarrow \quad n = 10/2$$

$$n + 2 = 10 \quad \Rightarrow \quad n = 10 - 2$$

$$n^2 = 10 \quad \Rightarrow \quad n = \sqrt{10}$$

$$n^2 = 10 \quad \Rightarrow \quad 2 = \log_2(10)$$

$$\log_b a$$

↑ represent  
 ↑ base  
 ↑ number

$$\Rightarrow \log_2 10 = \text{ans} \rightarrow 3.33 \dots$$

$$\Downarrow$$

$$10 : 2^{\text{ans}}$$

$$\text{ans} = 2^{3.33 \dots}$$

$$\log_b a = \text{ans} \Rightarrow a = b^{\text{ans}}$$

$$\rightarrow \log_2 64 = \text{ans} \rightarrow 64 = 2^{\text{ans}} \rightarrow 6$$

$$\rightarrow \log_2 32 = \text{ans} \rightarrow 2^{\text{ans}} = 32 \rightarrow 5$$

$$\rightarrow \log_2 33 = \text{ans} \rightarrow 2^{\text{ans}} = 33 \rightarrow 5. \dots$$

### Properties

$$\textcircled{i} \log_a a^n = n$$

$$\text{ex: } \log_5 5^{10} = 10$$

$$\textcircled{ii} \log_c (a*b) = \log_c a + \log_c b$$

$$\text{ex: } \log_2 10 = \log_2 2 + \log_2 5$$

### Quiz

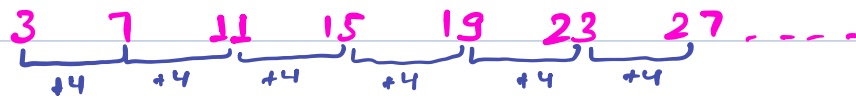
↳ No. of steps for  $N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \dots \rightarrow 1$

$$\left( \left( N * \frac{1}{2} \right) * \frac{1}{2} \right) * \frac{1}{2} \dots = 1$$

$$\frac{N}{2^{\text{Count}}} = 1 \Rightarrow N = 2^{\text{Count}}$$

$\downarrow$   
 $\vdots$   
 $\downarrow$   
 $\text{Count} = \log_2 N$

## A.P. $\rightarrow$ Arithmetic Progression



first term =  $a$  = Starting number

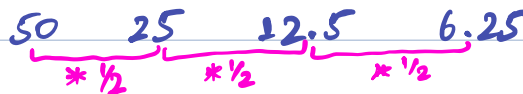
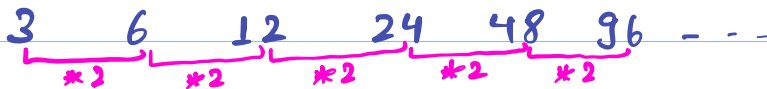
Common diff =  $d$  = difference between consecutive nos

$$\overset{1}{(a+0*d)} \quad \overset{2}{(a+d)} \quad \overset{3}{(a+2*d)} \quad \overset{4}{(a+3*d)} \quad \dots \quad \overset{n\text{th term}}{a+(n-1)*d}$$

$$\hookrightarrow \text{Sum of first } n \text{ terms} = \frac{n}{2} [2*a + (n-1)*d]$$

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## G.P. $\rightarrow$ Geometric Progression



first term =  $a$  = Starting number.

Common ratio:  $r$  = multiple to get next no.

$$\overset{1}{a} \quad \overset{2}{a*r} \quad \overset{3}{a*r^2} \quad \overset{4}{a*r^3} \quad \dots \quad \overset{n\text{th term}}{a*r^{n-1}}$$

$$\hookrightarrow \text{Sum of } n \text{ terms in G.P.} = a * \frac{r^n - 1}{r - 1}$$

## Quiz

```
int sum = 0;
```

```
for (int i = 1; i <= N; i++) {  
    sum = sum + 1;  
}
```

$[1, N] \rightarrow N - 1 + 1$

$N^{\text{th}}$  iterations

$\Downarrow$   
 $O(N)$

## Quiz

```
void func (int n, int m) {
```

```
    for (int i = 1; i <= N; i++) {  
        Print(i);  
    }
```

$\rightarrow [1, N] = N - 1 + 1 = N$  iterations

```
    for (int i = 1; i <= M; i++) {  
        Print(i);  
    }
```

$\rightarrow [1, M] = M - 1 + 1 = M$  iterations

Total =  $N + M$  iterations

$O(N + M)$

$\Downarrow$   
 $N > M \rightarrow O(N)$

$M > N \rightarrow O(M)$

## Quiz

```
int fun (int N) {  
    int S = 0;  
    for (int i = 0; i <= 100; i++) {  
        S = S + i2;  
    }  
    return S;  
}
```

$\rightarrow [0, 100] \rightarrow 100 - 0 + 1 = 101$  iterations  
 $\downarrow$   
 $O(1)$

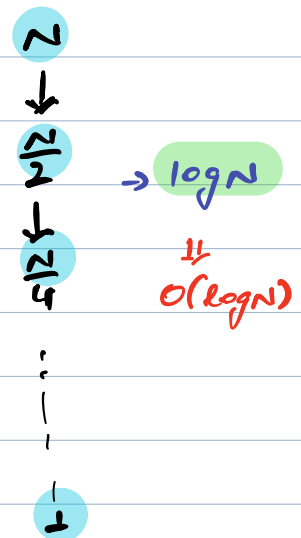
## Quiz

```
void fun (int N) {  
    int S = 0;  
    for (int i = 1; i * i <= N; i++) {  
        S = S + i2;  
    }  
    return S;  
}
```

$\rightarrow [1, \sqrt{N}] \Rightarrow \sqrt{N} - 1 + 1 = \sqrt{N}$  iterations  
 $\downarrow$   
 $O(\sqrt{N})$

## Quiz

```
void fun (int N) {  
    int i = N;  
    while (i >= 1) {  
        i = i/2;  
    }  
}
```



## Quiz

```
void fun (int N) {  
    int S = 0;  
    for (int i = 0; i <= N; i = i*2) {  
        S = S + i;  
    }  
}
```

$[0, 0, 0, 0, \dots]$

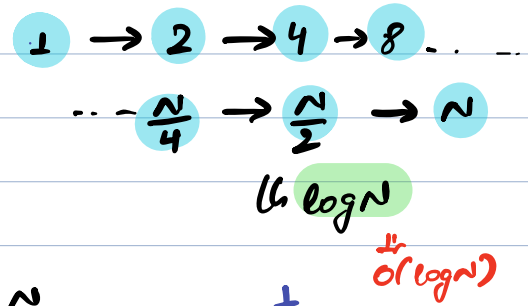
$\uparrow$   
infinite

## Quiz

```

void fun (int N) {
    int S=0;
    for (int i=1; i<=N; i=i*2) {
        S = S+i;
    }
}

```



$\log N$  ←

1 + 2 + 4 + 8 + ... + N

1 + 2 + 4 + 8 + ... + N

1 2 3 4 5 ⇒ 5

5 4 3 2 1 ⇒ 5

Back till 9:30pm



## Nested loops

### Quiz

```

void fun (int n) {
    int S = 0;
    for (int i = 1; i <= 10; i++) {
        for (int j = 1; j <= n; j++) {
            S = S + 10;
        }
    }
}
    
```

3

3

3

i	j	Count
1	[1, N]	N
2	[1, N]	N
3	[1, N]	N
⋮	⋮	⋮
10	[1, N]	N

10 \* N

iterations

↓  
 $O(N)$

### Quiz:

```

void fun (int n) {
    int S = 0;
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= n; j++) {
            S = S + 10;
        }
    }
}
    
```

3

3

3

i	j	Count
1	[1, N]	N
2	[1, N]	N
3	[1, N]	N
⋮	⋮	⋮
N	[1, N]	N

(N \* N)

iterations

↓  
 $O(N^2)$

## Quiz:

```

void fun (int n) {
    int s = 0;
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= i; j++) {
            s = s + 10;
        }
    }
}

```

i	j	Count
1	[1,1]	1
2	[1,2]	2
3	[1,3]	3
⋮	⋮	⋮
N	[1,N]	N

$$\frac{n(n+1)}{2} = \frac{n^2+n}{2} = \frac{n^2}{2} + \frac{n}{2}$$

$$\frac{n(n+1)}{2} \text{ iterations}$$

$O(n^2)$

## Quiz

```

void fun (int n) {
    for (int i = 1; i <= 2^n; i++) {
        Print(i);
    }
}

```

$$[1, 2^n] \rightarrow 2^n \text{ iterations}$$

$$\Downarrow$$

$$O(2^n)$$

## Quiz:

```

void fun (int N) {
    int S = 0;
    for (int i = 1; i <= N; i++) {
        for (int j = 1; j <= 2i; j++) {
            S = S + 10;
        }
    }
}

```

i	j	Count
1	[1, 2 <sup>1</sup> ]	2 <sup>1</sup>
2	[1, 2 <sup>2</sup> ]	2 <sup>2</sup>
3	[1, 2 <sup>3</sup> ]	2 <sup>3</sup>
⋮		+
⋮		
⋮		
⋮		
⋮		
⋮		
N	[1, 2 <sup>N</sup> ]	2 <sup>N</sup>

$$2 + 2^2 + 2^3 + 2^4 + \dots + 2^N = 2 * \frac{2^N - 1}{2 - 1} = 2 * (2^N - 1) \text{ iterations}$$

$$a * \frac{2^N - 1}{2 - 1}$$

$$2 * 2^N - 2 = O(2^N)$$

## Comparison of iteration

$$1 < \log_2 N < \sqrt{N} < N < N \cdot \log N < N\sqrt{N} < N^2 < 2^N \dots$$

## Time Complexity

↳ Big O Notation → Approximation

- i Calculate iteration
- ii Neglect lower order terms
- iii Neglect Constants

$$\text{ex: } (\cancel{20}N^2 + \cancel{20}N + \cancel{30}) \text{ iteration}$$
$$\Downarrow$$
$$O(N^2)$$

ex:

$$\cancel{8}N^2 + \cancel{15}N + \cancel{10}$$
$$\hookrightarrow O(N^2)$$

$$\text{ex: } \cancel{4}N + \cancel{8}N \log N + \cancel{10}$$
$$\hookrightarrow O(N \log N)$$

en:

$$\cancel{16} N \log N + \cancel{15} \sqrt{N} + \cancel{10} 0$$

$$\hookrightarrow O(N \log N)$$