Topical review The Jaynes-Cummings model

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The Jaynes-Cummings model

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Abstract. The Jaynes-Cummings model (JCM), a soluble fully quantum mechanical model of an atom in a field, was first used (in 1963) to examine the classical aspects of spontaneous emission and to reveal the existence of Rabi oscillations in atomic excitation probabilities for fields with sharply defined energy (or photon number). For fields having a statistical distributions of photon numbers the oscillations collapse to an expected steady value. In 1980 it was discovered that with appropriate initial conditions (e.g. a near-classical field), the Rabi oscillations would eventually revive, only to collapse and revive repeatedly in a complicated pattern. The existence of these revivals, present in the analytic solutions of the JCM, provided direct evidence for discreteness of field excitation (photons) and hence for the truly quantum nature of radiation. Subsequent study revealed further non-classical properties of the JCM field, such as a tendency of the photons to antibunch. Within the last two years it has been found that during the quiescent intervals of collapsed Rabi oscillations the atom and field exist in a macroscopic superposition state (a Schrödinger cat). This discovery offers the opportunity to use the JCM to elucidate the basic properties of quantum correlation (entanglement) and to explore still further the relationship between classical and quantum physics. The relative simplicity of the JCM and the ease with which it can be extended through analytic expressions or numerical computation continues to motivate attention, as evidenced by the growing abundance of publications. We here present an overview of the theory of the JCM and some of the many extensions and generalizations that have appeared.

1. Introduction

As originally defined (Jaynes and Cummings 1963), the Jaynes-Cummings model (JCM) comprised a single two-state atom (molecule) interacting with a single near-resonant quantized cavity mode of the electromagnetic field (see figure 1). The model was first used to examine the classical aspects of spontaneous emission. It was subsequently discovered that the JCM atomic population histories presented direct evidence for the discreteness of photons. It was also found that the cavity field, once modified by the JCM interaction with the atom, had statistical properties not found in classical fields. More recently, the JCM has been used to elucidate quantum correlation and the formation of macroscopic quantum states.

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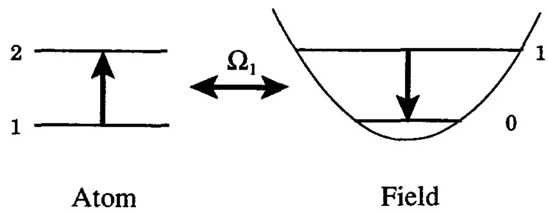

Figure 1. The Jaynes–Cummings model consists of a single two-level atom coupled to a quantized single-mode field, represented as a harmonic oscillator; the coupling between atom and field is characterized by a Rabi frequency Ω_1. Loss of excitation in the atom appears as a gain in excitation of the oscillator.

In the three decades that have passed since Jaynes and Cummings first presented what has become the JCM there has arisen a sizeable scientific industry devoted to exploiting and extending this work (for reviews see Aravind and Hirschfelder (1984), Haroche and Raimond (1985), Yoo and Eberly (1985), Barnett *et al.* (1986), Meystre and Sargent (1990), Le Kien and Shumovsky (1991), Milonni and Singh (1991), Meystre (1992)). In part, the interest has been stimulated by the discovery of a number of remarkable properties of the model, along with the possibility of finding solutions (often exact) to fundamental models of a quantum theory of interacting fields and atoms. Perhaps equally important as a motivating force has been the remarkable advance in cavity quantum electrodynamics (QED) experiments involving single atoms (usually Rydberg atoms, see Filipowicz *et al.* (1985), Knight (1986 *b*)) within single-mode cavities (the micromaser, see Haroche and Raimond (1985), Meschede *et al.* (1985), Rempe *et al.* (1986), Brune *et al.* (1987), Haroche and Kleppner (1989), Hinds (1991)) which have turned the theory from an academic curiosity into a useful and testable enterprise. These various motives have certainly provided incentive for extending and generalizing what is still called the Jaynes–Cummings model. The following Sections comment on a few of these generalizations: a variety of initial conditions; dissipation and damping; multiple atoms; multilevel atoms; more elaborate multimode descriptions of the field; and more elaborate interactions (multiphoton or intensity-dependent) than the original single-photon electric-dipole interaction.

2. The JCM Hamiltonian

The original JCM Hamiltonian can be expressed in the form

$$H=\hbar\omega\hat{a}^\dagger\hat{a}+E_1\hat{S}_{11}+E_2\hat{S}_{22}+\frac{\hbar}{2}\Omega_1(\hat{a}^\dagger\hat{S}_{12}+\hat{S}_{21}\hat{a}), \tag{1}$$

where E_k is the energy of atomic state ψ_k, the atom–field coupling constant Ω_1 is the vacuum (or single-photon) Rabi frequency, and $\hat{S}_{jk}$ is a transition operator acting on atomic states, defined by the expressions

$$\hat{S}_{jk}|\psi_n\rangle=\delta_{kn}|\psi_j\rangle, \qquad \text{or,} \qquad \hat{S}_{jk}\hat{S}_{nm}=\delta_{kn}\hat{S}_{jm}. \tag{2}$$

When restricted to two states, as is the case here, it is common to express these atomic operators in terms of Pauli (spin) matrices,

$$\sigma_x=\hat{S}_{12}+\hat{S}_{21}, \qquad \sigma_y=i(\hat{S}_{12}-\hat{S}_{21}), \qquad \sigma_2=\hat{S}_{22}-\hat{S}_{11}, \tag{3}$$

thereby emphasizing the close association between a two-state atom and a spin-$\frac{1}{2}$ system. (The notation $\sigma_+ = \hat{S}_{21}$ and $\sigma = \hat{S}_{12}$ is often used.) The photon creation and annihilation operators $\hat{a}^\dagger$ and $\hat{a}$, with commutator

$$[\hat{a}, \hat{a}^\dagger] = 1 \tag{4}$$

act on photon number states $|n\rangle$, eigenstates of the photon number operator $\hat{a}^\dagger \hat{a}$,

$$\hat{a}^\dagger \hat{a}|n\rangle = n|n\rangle, \qquad \hat{a}^\dagger|n\rangle = (n+1)^{1/2}|n+1\rangle, \qquad \hat{a}|n\rangle = \sqrt{n}|n-1\rangle. \tag{5}$$

The field frequency ω, the atomic energies E_k, and the (vacuum) Rabi frequency Ω_1, appear as arbitrary parameters in the theory, although in applications they are fixed by physical considerations (e.g. cavity volume V and atomic transition moment d, as in the expression $|\Omega_1|^2 = 4d^2\omega/\hbar V\varepsilon_0$). Missing from the JCM Hamiltonian of equation (1) are such effects as cavity loss, multiple cavity modes, atomic sublevel degeneracy and atomic polarizability (leading to dynamic Stark shifts), all of which have subsequently been examined.

To place this theory into an experimental context one may consider the micromaser (Filipowicz *et al.* 1985, 1986, Meschede *et al.* 1985, Guzman *et al.* 1989): a stream of velocity-selected excited Rydberg atoms pass through a cold high Q single-mode microwave cavity at a rate that allows only a single atom in the cavity at any time. Time $t=0$ marks the entrance of an atom into the cavity.

A few numbers may be instructive. In the micromaser configuration of Rempe *et al.* (1987), the cavity Q value was between 6×10^7 and $2{\cdot}7\times10^8$ at a field frequency of 21 GHz, so that the field decay rates are around $10^2\,\mathrm{s}^{-1}$. (Microwave cavities are now available with much higher Q values; the Garching micromaser of Walther and coworkers (Rempe *et al.* 1989) now operates at $Q>10^{10}$.) The Rydberg transitions used in this microwave experiment have principal quantum number of about 63. The dipole moments for such atoms are so large that the vacuum Rabi frequency is around 10^4 Hz. This exceeds the loss rates and enables the observation of the interaction of a single atom with a single photon. Recent progress in construction of high Q optical cavities now permit this same observation at visible-light frequencies (see Morin *et al.* (1992)).

It should be appreciated that an atom placed suddenly into a cavity requires time to accommodate the single-mode conditions; the needed time is basically the interval required for light to travel to the cavity wall and return (see Mallalieu *et al.* (1988)).

3. The atomic JCM Hamiltonian

The construction of the JCM Hamiltonian is such that each photon creation accompanies an atomic de-excitation, and each photon annihilation accompanies atomic excitation.Therefore in addition to the conservation of atomic probability,

$$\langle \hat{S}_{11}\rangle + \langle \hat{S}_{22}\rangle = 1, \tag{6}$$

there occurs a conservation of excitation

$$\langle \hat{a}^\dagger \hat{a}\rangle + \langle \hat{S}_{22}\rangle = \text{constant}. \tag{7}$$

Thus, as Jaynes and Cummings pointed out, the problem becomes that of an infinite set of uncoupled two-state Schrödinger equations, each pair identified by the number of photons that are present when the atom is in the lowest-energy state. In this sense, the JCM is closely related to the classic Lee model of quantum field theory (Barton 1963). We can write the atom–field state-vector for specified photon number as a combination of two basis states,

$$\Psi(n,t)=\exp(-in\omega t-\mathrm{i}E_1t/\hbar)[C_1(n,t)\phi_1(n)+C_2(n,t)\phi_2(n)], \tag{8}$$

where $\phi_k(n)$ is the atom–field product state

$$\phi_1(n)=|n\rangle\psi_1, \qquad \phi_2(n)=|n-1\rangle\psi_2. \tag{9}$$

The 2×2 Hamiltonian matrix of one such pair has the form

$$\boldsymbol{H}(n)=\frac{\hbar}{2}\begin{bmatrix} 0 & \Omega(n) \\ \Omega(n) & 2\Delta \end{bmatrix}, \qquad \Psi(n,t)=\exp(in\omega t-iE_1t/\hbar)\begin{bmatrix} C_1(n,t) \\ C_2(n,t) \end{bmatrix}, \tag{10}$$

involving as parameters the cavity-atom detuning Δ and the n-photon Rabi frequency $\Omega(n)$

$$\hbar\Delta=E_2-E_1-\hbar\omega, \qquad \Omega(n)=n^{1/2}\Omega_1. \tag{11}$$

4. The JCM solutions for populations; Rabi oscillations

The separate paired equations of the simplified JCM Hamiltonian (10) are just those of a two-state atom in a monochromatic field (Allen and Eberly 1975, Meystre and Sargent 1990); it is this simplification that made possible the exact solutions of Jaynes and Cummings (1963), presented independently by Paul (1963). As with their earlier occurrence in magnetic resonance (Rabi 1937), they predict sinusoidal changes in populations now known as Rabi oscillations. In particular, if the atom is initially unexcited while the field has exactly n photons, then the population inversion at time t is (see Shore (1990), p. 640)

$$P_2(t)-P_1(t)=|C_2(n\,t,)|^2-|C_1(n,t)|^2=-1+\sin^2(2\theta_n)[1-\cos(\lambda_n t)], \tag{12}$$

whereas if an excited atom encounters this field of n photons, then the inversion is

$$\begin{aligned} P_2(t)-P_1(t)&=|C_2(n+1,t)|^2-|C_1(n+1,t)|^2 \\ &=1-\sin^2(2\theta_{n+1})[1-\cos(\lambda_{n+1}t)]. \end{aligned} \tag{13}$$

These expressions use the (dressed-state) mixing angle θ_n (see Shore (1990), p. 640; Cohen-Tannoudji *et al.* (1992)) and the flopping frequency λ_n

$$\tan(2\theta_n)=\Omega(n)/\Delta, \qquad \lambda_n=[\Delta^2+\Omega(n)^2]^{1/2}. \tag{14}$$

The assertion that the JCM is analytically soluble is equivalent to the observation that there are (simple) algebraic expressions for the (two) probability amplitudes $C_k(n,t)$ or for the (two) eigenvalues of the matrix $\boldsymbol{H}(n)$ and the corresponding pair of eigenvectors, the two-component dressed states $\Phi_k(n)$,

$$\boldsymbol{H}(n)\Phi_k(n)=\hbar\Lambda_k(n)\Phi_k(n), \tag{15}$$

for any choice of detunings, Rabi frequencies, and photon number. When the system is resonant ($\Delta=0$) the two dressed states are independent of photon number. The resulting superposition of atomic states is

$$\psi_{\pm}=\frac{1}{\sqrt{2}}[\exp(\mathrm{i}\varphi)\psi_1\pm\Psi_2], \tag{16}$$

where φ is the phase of the field (and hence of a complex-valued Rabi frequency). Given dressed states of $\boldsymbol{H}(n)$ (or those of the more elaborate Hamiltonians discussed below) we can construct the atom–field statevector at any time t as

$$\Psi(n,t)=\exp(-\mathrm{i}n\omega t-\mathrm{i}E_1 t/\hbar)\sum_k c_k(n)\Phi_k(n)\exp[-\mathrm{i}\Lambda_k(n)t], \tag{17}$$

where the numbers $c_k(n)$ are chosen to reproduce specified initial conditions $\Psi(n,0)$. In particular, one can examine limiting cases of very long times (Meystre *et al.* 1975), stationary properties (Savage 1988), long-time statistical properties of the irregularities (Hioe *et al.* 1983) or time averaged behaviour (Cresser and Hillery 1989, Hillery and Schwartz 1991).

5. The Heisenberg picture: spin one-half atom

Although the initial treatments of the JCM used the Schrödinger picture, with attendant statevectors and probability amplitudes, the model also permits simple expressions for solutions to the Heisenberg equations,

$$\hbar\frac{\partial}{\partial t}\hat{M}(t)=\mathrm{i}[H,\hat{M}(t)], \tag{18}$$

with emphasis on time varying operators $\hat{M}(t)$. The first application of Heisenberg equations to the JCM was by Ackerhalt in his thesis (1974*b*). By defining the subsidiary operators (see Ackerhalt and Eberly (1974), Ackerhalt and Rzazewski (1975), Knight and Milonni (1980), Narozhny *et al.* (1981), Meystre and Sargent (1990), Shore (1990), Section 10.7).

$$N=\hat{a}^\dagger\hat{a}+\hat{S}_{22},\qquad \hat{\Lambda}=[(\Omega_1)^2(\hat{N}-1)+\Delta^2]^{1/2}, \tag{19}$$

$$2\hat{C}=\Delta(\hat{S}_{22}-\hat{S}_{11})-\Omega_1(\hat{a}^\dagger\hat{S}_{12}+\hat{S}_{21}\hat{a}), \tag{20}$$

$$2\hat{r}_\pm=2\hat{C}\pm\hat{\Lambda},\quad \hat{R}_\pm(t)=\exp(\mathrm{i}\hat{r}_\pm t)\hat{\Lambda}^{-1}, \tag{21}$$

one can express the Ackerhalt solutions as

$$\hat{a}^\dagger(t)=\exp(\mathrm{i}\omega t)\{[\hat{R}_-(t)\hat{r}_+-\hat{R}_+(t)\hat{r}_-]\hat{a}^\dagger(0)+\tfrac{1}{2}\Omega_1[\hat{R}_+(t)-\hat{R}_-(t)]\hat{S}_{21}(0)\}, \tag{22}$$

$$\hat{S}_{21}(t)=\exp(\mathrm{i}\omega t)\{[\hat{R}_+(t)\hat{r}_+-\hat{R}_-(t)\hat{r}_-]\hat{S}_{21}(0)-\tfrac{1}{2}\Omega_1[\hat{R}_+(t)-\hat{R}_-(t)]\hat{a}^\dagger(0)\}. \tag{23}$$

Any two-state system is mathematically equivalent to a spin one-half particle, and thus the formalism of the JCM can make use of Pauli matrices as the fundamental atomic operators. Three-level generalizations have been considered by Bogolubov *et al.* (1984, 1985 *a,b*, 1986 *a,b*, 1987 and by Buck and Sukumar (1984).

The dressed Ackerhalt operators of equations (22) and (23) demonstrate the mixing of the cavity field oscillator with the nonlinear atomic oscillator. If only the lowest dressed states are excited (i.e. the field is weak), then it is profitable to think of normal modes of the interacting oscillators, separated by their coupling (the vacuum Rabi frequency, see Kaluzny *et al.* (1983), Sanchez-Mondragon *et al.* (1983), Agarwal (1984, 1985, 1991), Carmichael (1991)). Zhu *et al.* (1991) have shown how vacuum Rabi splitting can be derived using linear dispersion theory; in their view one can regard it as no more quantum mechanical than the oscillator strength. This vacuum 'normal mode' a.c. Stark splitting would be revealed in either emission spectra or in the absorption spectra of a weak probe field. The splitting has been observed experimentally in an *optical* cavity of ultra-high finesse (Thompson *et al.* 1992) and in microwave Rydberg experiments by Bernadot *et al.* (1992). (The latter work examined not a single atom but five atoms, with appropriate collective enhancement.)

6. The density matrix

Closely akin to Heisenberg equations is the Liouville equation of motion for the density matrix $\rho(t)$

$$\hbar\frac{\partial}{\partial t}\rho(t) = -\mathrm{i}[H, \rho(t)], \tag{24}$$

whose solution is used to evaluate expectation values as a trace

$$\langle M(t)\rangle = \mathrm{Tr}\,[\hat{M}\rho(t)]. \tag{25}$$

The density matrix offers a simple way of incorporating statistical distributions of initial conditions into quantum mechanics. More importantly, the Liouville equation generalizes in a straightforward manner to permit incorporation of dissipative mechanism and homogeneous relaxation, as noted below.

When the system comprises two parts, the field and atom components of the JCM, then the trace consists of a sum over atom attributes and field attributes. By carrying out the separate sums we define reduced density matrices for the atom and field subsystem,

$$\rho^A(t) = \mathrm{Tr}_F\,\rho(t), \qquad \rho^F(t) = \mathrm{Tr}_A\,\rho(t), \tag{26}$$

with which to evaluate expectation values of atom operators $\hat{A}$ and field operators $\hat{F}$

$$\langle \hat{A}\rangle = \mathrm{Tr}_A\,[\rho^A(t)\hat{A}], \qquad \langle \hat{F}\rangle = \mathrm{Tr}_F\,[\rho^F(t)\hat{F}]. \tag{27}$$

As examples, we evaluate the ground-state population and mean photon number by the formulas

$$P_1(t) = \mathrm{Tr}_A\,[\rho^A(t)\hat{S}_{11}] = \rho^A_{11}(t), \qquad \langle n(t)\rangle = \langle \hat{a}^\dagger\hat{a}\rangle = \mathrm{Tr}_F\,[\rho^F(t)\hat{a}^\dagger\hat{a}]. \tag{28}$$

When applied to the traditional JCM the atom density matrix has dimension two, and so it is quite manageable. The field density matrix in a Fock-state basis is much larger: when the field description is by means of photon numbers the field density matrix has denumerably infinite dimension, although for numerical work one need only consider photon numbers no larger than a few times the initial mean photon number.

7. Bloch variables

Discussions and an intuitive, pictorial understanding of the behaviour of a lossless two-level atom are greatly facilitated by the introduction of a three-dimensional abstract vector space in which the atomic behaviour maps onto the motion of a vector (the Bloch vector, see Feynman, Vernon and Hellwarth (1957), Allen and Eberly (1975), Shore (1990), Section 8.5), whose three real-valued components in the JCM are

$$R_1 = \rho^A_{21} + \rho^A_{12}, \qquad R_2 = \mathrm{i}(\rho^A_{12} - \rho^A_{21}), \qquad R_3 = \rho^A_{22} - \rho^A_{11}. \tag{29}$$

The magnitude of the Bloch vector is given through

$$|\mathbf{R}|^2 \equiv (R_1)^2 + (R_2)^2 + (R_3)^2 = 2\mathrm{Tr}\,(\rho^A)^2 - 1. \tag{30}$$

The usefulness of this vector for interpreting behaviour of the fully quantized JCM has recently been pointed out by Gea-Banacloche (1992). When the atom is in a pure quantum state the Bloch vector has unit length, and it therefore moves on a sphere of unit radius (the Bloch sphere). A mixed state (an incoherent superposition) has a shorter Bloch vector. The completely random state is a Bloch vector of zero length. The component R_3 measures the population inversion $P_2 - P_1$: an initially unexcited atom corresponds to $R_3 = -1$ (the south pole of the Bloch sphere), while an excited atom has $R_3 = +1$ (the north pole). The component in the equatorial 1, 2 plane is proportional to the dipole moment. By convention the 1 and 2 axes are chosen so that R_1 is in phase with the electric field, while R_2 is the out-of-phase component. The position $R_1 = 1$ corresponds to the coherent superposition of energy states that defines the dressed state ψ_+. Figure 2 shows an example.

8. Atom observables

Given an idealized model of radiation–matter interaction, such as the single atom–single mode JCM, the most obvious observables are the instantaneous values of the probabilities $P_k(t)$ of finding the atom in atom state ψ_k at time t. This probability is the expectation value of the atomic operator $\hat{S}_{kk}$

$$P_k(t) = \langle \hat{S}_{kk} \rangle. \tag{31}$$

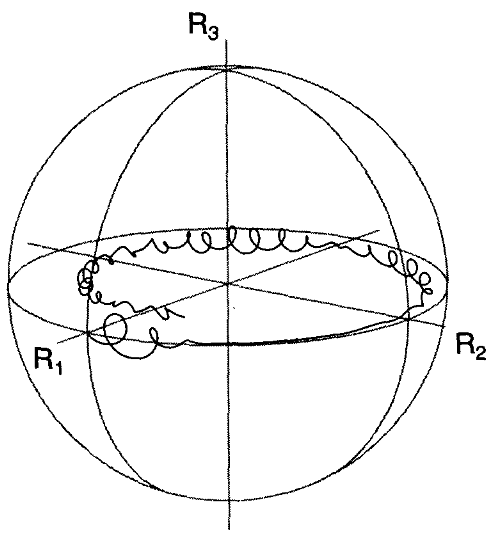

Figure 2. An example of the time evolution of the density matrix for the JCM, as displayed with components of the Bloch vector. Depicted evolution is for atom prepared in the coherent superposition $(1/\sqrt{2})(\psi_1 + \psi_2)$ and field initially in a coherent state with mean photon number $\langle n \rangle = 9$. The total time shown is a little over twice the revival time. Taken from Gea-Banacloche (1992).

When the overall field–atom system is in a pure quantum state $\Psi(t)$ then this probability is

$$P_k(t)=\sum_n |\langle n, \psi_k | \Psi(t)\rangle|^2. \tag{32}$$

More typically these population histories are ensemble averages over initial distributions of atom and field properties (e.g. an initial distribution of photon numbers).

Perhaps the most notable aspect of a two-state atom interacting coherently with a monochromatic classical field is the periodic transfer of population between the ground state and the excited state (e.g. Knight and Milonni 1980). Jaynes and Cummings showed that these same Rabi oscillations will occur for a quantized field, if the field is initially in a photon-number state, and they showed that almost indistinguishable oscillations occur in neo-classical theory. In particular, an excited atom entering a vacuum cavity undergoes spontaneous Rabi oscillations. These were remarkable conclusions. But they were only the first of many fascinating discoveries about the JCM.

The Rabi oscillations of the JCM differ dramatically from the irreversible exponential decay that occurs in free space. This is because an atom within a cavity undergoes reversible spontaneous emission, as it repeatedly emits and then reabsorbs radiation. This simplest example occurs when an excited atom enters a cold dark cavity (the photon vacuum). The JCM predicts exactly periodic interchange of energy between atom and field as the atom repeatedly emits and then reabsorbs the single quantum of energy. When the cavity and Bohr frequencies coincide, the population oscillations occur at the vacuum Rabi frequency (in suitable units this is the product of atom dipole transition moment and the single-photon electric field of the cavity). If the atom leaves in its unexcited state, then we can infer that the cavity contains exactly one photon. More generally, an atom may undergo some integral number of complete Rabi cycles during the time $\tau=L/v$ that it remains within a cavity of length L while moving at speed v. This can occur for special field configurations such that the photon number N forces the condition (see Filipowicz *et al.* (1986), Slosser and Meystre (1990))

$$\Omega(N)\tau=2\pi m, \qquad \text{integer } m. \tag{33}$$

Such a cavity field, the quantum counterpart of a semiclassical 2π pulse, leaves the atom unchanged.

9. Field observables; fluctuations

The basic JCM for an initial condition of sharply defined atomic and field energy (a photon number state) exhibits periodic exchange of energy between field and atom. The Rabi oscillations of population are complemented by oscillations of field energy, as expressed by the mean photon number,

$$\langle \hat{n}\rangle=\langle \hat{a}^\dagger \hat{a}\rangle, \tag{34}$$

and so the JCM offers a soluble case of a quantum field that changes with time.

Advances in both theory and experiment during the last 30 years have brought an interest in the statistical properties of quantum states of the electromagnetic field (see Glauber (1972), Geneux *et al.* (1973), Louisell (1975), Perina (1984)).

The simplest statistical characteristic is the distribution of possible field energies, as expressed by the probability $p_n(t)$ of finding n photons at time t. When the atom-field system is in state $\Psi(t)$, this probability is obtained from the formula

$$p_n(t)=\sum_k |\langle n,\psi_k|\Psi(t)|^2. \tag{35}$$

Whereas 30 years ago one would have shown interest in $p_n(t)$ and $\langle\hat{n}\rangle$, today practitioners of quantum optics have an interest in a variety of other statistical properties of a field, including the photon number variance

$$\langle(\Delta\hat{n})^2\rangle=\langle\hat{a}^\dagger\hat{a}\hat{a}^\dagger\hat{a}\rangle-\langle\hat{a}^\dagger\hat{a}\rangle^2=\langle\hat{n}^2\rangle-\langle\hat{n}\rangle^2, \tag{36}$$

and the Mandel Q parameter (Mandel 1979), a normally ordered variance or normalized second factorial moment which directly measures the deviation of the photon statistics from that for a coherent-field Poisson distribution:

$$Q(t)=\frac{\langle\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}\rangle-\langle\hat{a}^\dagger\hat{a}\rangle^2}{\langle\hat{a}^\dagger\hat{a}\rangle}=\frac{\langle(\Delta\hat{n})^2\rangle-\langle\hat{n}\rangle}{\langle\hat{n}\rangle}. \tag{37}$$

These statistical variables, together with higher moments of the photon number operator, offer quantitative measures of how much a given field differs from a classical field (see Glauber (1965)). In particular, the Mandel Q parameter vanishes for a Poisson distribution, i.e. a classical state. It provides information about the tendency of photons to arrive in bunches: when $Q>0$ the photons are bunched (super-Poisson), while for $Q<0$ the photons are antibunched (sub-Poisson, a purely quantum regime; see Short and Mandel (1983), Diedrich and Walther (1987), Diedrich *et al.* (1988), Wang *et al.* (1991)).

The field generated by the fluorescence of a two-state atom has definite non-classical photon statistics: the antibunching originates in the requirement that before an atom can emit a second photon it must be reexcited. As noted below, it is known that the JCM field exhibits a number of non-classical effects, even when the cavity begins in a near-classical state.

The electric and magnetic field operators of the JCM are proportional to the quadrature operators

$$\hat{X}=\frac{1}{2}(\hat{a}+\hat{a}^\dagger), \qquad \hat{Y}=\frac{1}{2\mathrm{i}}(\hat{a}-\hat{a}^\dagger). \tag{38}$$

Expectation values of these operators provide histories of the field (Bandilla and Ritze 1988, Goldberg and Harrison 1991). These operators have the variances

$$\langle(\Delta X)^2\rangle=\frac{1}{2}\langle\hat{a}^\dagger\hat{a}\rangle+\frac{1}{4}+\frac{1}{4}\langle\hat{a}\hat{a}\rangle+\frac{1}{4}\langle\hat{a}^\dagger\hat{a}^\dagger\rangle-\frac{1}{4}\langle\hat{a}^\dagger+\hat{a}\rangle^2, \tag{39}$$

$$\langle(\Delta Y)^2\rangle=\frac{1}{2}\langle\hat{a}^\dagger\hat{a}\rangle+\frac{1}{4}-\frac{1}{4}\langle\hat{a}\hat{a}\rangle-\frac{1}{4}\langle\hat{a}^\dagger\hat{a}^\dagger\rangle+\frac{1}{4}\langle\hat{a}^\dagger+\hat{a}\rangle^2. \tag{40}$$

When either of these variances is less than 1/4, the field is said to be squeezed (Slusher and Yurke 1986, Loudon and Knight 1987). The cavity field surrounding a two-state atom, initially excited, exhibits a time-varying pattern of squeezing (Meystre and Zubairy 1982, Wodkiewicz *et al.* 1987, Aravind and Hu 1988, Barnett and Knight 1988, Hillery 1989).

10. Initial conditions: mixtures; oscillation collapse

With the introduction of a Hamiltonian one has, in principle, established the time evolution of a quantum system. To complete the definition of a model one must also specify initial conditions. The mathematics is simplest when one proposes a single atom of precisely known energy (i.e. a particular excitation state) brought suddenly into a cavity with precisely known energy (i.e. a definite number of photons). In recent years such initial conditions have indeed become experimentally realizable, but even so the more usual condition is a cavity for which one can only specify statistical properties of the single-mode field. Over the years there have been studies of the JCM interacting with a great variety of initial single-mode fields, each with different statistical properties. These studies have shown that there can be remarkable differences in the behaviour of the system with different initial conditions.

To treat the general situation, we begin by considering the atom–field statevector $\Psi(n, t)$ that evolves in time from an initial state in which there are exactly n photons. When the original field has some unknown photon number, specified only by a probability $\mathscr{P}_m$ for observing m photons, then the atomic excitation probabilities and photon distributions are calculated as the sums

$$P_k(t)=\sum_{m=0}^{\infty}\mathscr{P}_m|\langle m,\psi_k|\Psi(m,t)\rangle|^2=\sum_{m=0}^{\infty}\mathscr{P}_m|C_k(m,t)|^2, \tag{41}$$

$$p_n(t)=\sum_{\substack{m=0\\k}}^{\infty}\mathscr{P}_m|\langle n,\psi_k|\Psi(m,t)\rangle|^2=\sum_k\mathscr{P}_{n+k-1}|C_k(n+k-1,t)|^2. \tag{42}$$

One expects, and finds, that superpositions of periodic solutions will produce damped oscillations (Cummings collapses; Cummings (1965)). An important example occurs when two-state atoms encounter a cavity maintained at a finite temperature T, so that the photon number distribution is that of one mode of black-body radiation, given by the Bose–Einstein probability distribution

$$\mathscr{P}_m(T)=\frac{1}{1+\langle n\rangle}\left(\frac{\langle n\rangle}{1+\langle n\rangle}\right)^m, \tag{43}$$

$$\langle n\rangle=[\exp(\hbar\omega/kT)-1]^{-1}, \qquad \langle\Delta n^2\rangle=\langle n\rangle^2+\langle n\rangle. \tag{44}$$

This distribution (termed thermal or chaotic) maximizes the field entropy for fixed mean photon number (Glauber 1972). Figure 3 illustrates the effect of mixtures by plotting the population inversion $w(t)=P_2(t)-P_1(t)$ for an initially excited two state atom interacting with a single-mode cavity field in which there is a thermal distribution of photon numbers. Here, and in our other plots, times are expressed in units of the inverse of the mean Rabi frequency, so that the time displayed in the figure (T) is related to the true time (t) by expression $T=\Omega_1 t(\langle n\rangle)^{1/2}$. The uppermost frame shows the vacuum Rabi oscillations that occur when the atom enters an initially empty cavity. Subsequent frames show the atomic behaviour as the initial mean photon number increases. The very wide range of photon numbers that are present in such a thermal field (see equation (44)) give, in turn, such a broad distribution of Rabi frequencies that there is almost no trace of population oscillations visible in an ensemble average (Cummings 1965, Knight and Radmore 1982, Reti and Vetri 1982, Haroche and Raimond 1985). By recognizing that the collapse occurs on a time scale fixed by the spread of Rabi frequencies one finds the collapse time for a thermal field with large $\langle n\rangle$ to be (Barnett *et al.* 1986)

$$(t_c)^{-1}=\frac{1}{2}\Omega_1(\langle n\rangle)^{1/2}=\frac{1}{2}\Omega(\langle n\rangle), \qquad \text{for } \langle n\rangle \gg 1. \tag{45}$$

(With our choice of time units, this collapse occurs at the fixed time $T=2$ in each frame.)

11. Initial field as coherent superposition; revivals

When the initial field is expressible as a coherent superposition of photon-number states and the atom is initially unexcited, then the initial statevector is

$$\Psi(0)=\sum_n f_n|n\rangle\psi_1=\sum_n f_n\Psi(n,0), \qquad \mathscr{P}_m=|f_m|^2. \tag{46}$$

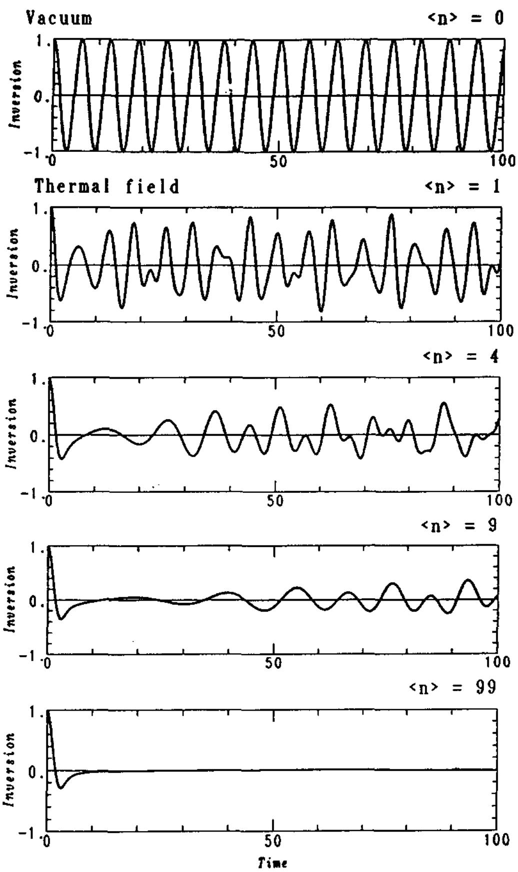

Figure 3. The population inversion $w(t)=P_2(t)-P_1(t)$ for an initially excited two state atom interacting with a single-mode cavity field in which there is a thermal distribution of photon numbers. Mean photon numbers of the initial field are $\langle n\rangle=0$, 1, 4, 9 and 99. Times are in units of the inverse of mean Rabi frequency. Note the lack of Rabi oscillations; instead there occur irregular fluctuations.

The JCM has the important property that each statevector $\Psi(n,t)$ evolves independently, and so we obtain the expression

$$\Psi(t)=\sum_n f_n\Psi(n,t)=\sum_{n,k} f_n C_k(n,t)|n-k+1\rangle\psi_k. \tag{47}$$

The probability for observing the atom in state k at time t is

$$P_k(t)=\langle\hat{S}_{kk}\rangle=\sum_n |f_n C_k(n,t)|^2=\rho^A_{kk}(t), \tag{48}$$

while the probability of observing exactly n photons is

$$p_n(t)=\sum_k |f_{n+k-1}C_k(n+k-1,t)|^2. \tag{49}$$

For real-valued f_m these expressions are identical with those for mixtures, equations (41, 42). The remaining elements of the reduced density matrix for the atom are

$$\langle\hat{S}_{21}\rangle=\rho^A_{12}(t)=\rho^A_{21}(t)^*=\exp(\mathrm{i}\omega t)\sum_n f_{n-1}C_1(n-1,t)C_2(n,t)^*f_n{}^*. \tag{50}$$

The expectation values for photon annihilation and creation operators are

$$\langle\hat{a}\rangle=\langle\hat{a}^\dagger\rangle^*=\exp(-\mathrm{i}\omega t)\sum_{k,n}(n-k+1)^{1/2}f_nC_k(n,t)C_k(n-1,t)^*f_{n-1}{}^*. \tag{51}$$

The reduced density matrix for the field, in a number representation, is

$$\rho^F_{nm}(t)=\exp[\mathrm{i}(m-n)t]\sum_k f_{n+k-1}C_k(n+k-1,t)C_k(m+k-1,t)^*f_{m+k-1}{}^*. \tag{52}$$

From such formulae, and comparable ones for various powers of the photon creation and annihilation operators, one can obtain any required property of the atom or the field from values of the Schrödinger amplitudes $C_k(n,t)$.

Following the work of Glauber (1963 *a,c*) on coherent states, it is known that the most classical of single-mode quantum states is a coherent state,

$$|\alpha\rangle=\exp(\alpha\hat{a}^\dagger-\alpha^*\hat{a})|n=0\rangle\equiv D(\alpha)|0\rangle=\sum_{n=0}^{\infty}f_n(\alpha)|n\rangle, \tag{53}$$

$$f_n(\alpha)\equiv\langle n|\alpha\rangle=\frac{\alpha^n}{(n!)^{1/2}}\exp\left(-\frac{1}{2}|\alpha|^2\right), \tag{54}$$

whose photon probabilities follow a Poisson distribution,

$$\mathscr{P}_m(\alpha)=|f_m(\alpha)|^2=\frac{\langle n\rangle^m}{m!}\exp(-\langle n\rangle), \tag{55}$$

$$\langle n\rangle=|\alpha|^2,\qquad \langle(\Delta n)^2\rangle=\langle n\rangle. \tag{56}$$

Thus it was natural to investigate, as several people did (e.g. Faist *et al.* (1972), Stenholm (1973), Meystre *et al.* (1975), von Foerster (1975)), the behaviour of a two-state atom when it encountered a coherent state of a single-mode quantum field. As with a thermal cavity, it was found that the Rabi oscillations which occur with photon number states, and for a monochromatic classical field, do not persist indefinitely when the field is initially prepared in a coherent state. Instead, the oscillations collapse to yield constant populations.

Figure 4 shows the behaviour of an initially excited two-state atom subsequent to interacting with a cavity in which there is a single-mode coherent state field. As the field becomes more intense, and the mean photon number larger, the Rabi oscillations persist for longer intervals (with our units of time as inverse mean Rabi frequency), but inevitably the incommensurability of Rabi oscillations for different photon numbers acts to wash out the periodicity of population transfers. The envelope surrounding the oscillations is a Gaussian and the collapse time for a coherent-state field, obtained from the inverse of the photon number distribution, is (Eberly *et al.* 1980, Barnett *et al.* 1986).

$$(t_c)^{-1}=\frac{\Omega(\langle n\rangle)}{2(\langle n\rangle)^{1/2}}=\frac{1}{2}\Omega_1, \tag{57}$$

independent of photon number and governed solely by the vacuum Rabi frequency (Knight and Radmore 1982 *b*). As the mean photon number increases there will occur more oscillations under the envelope of collapse, but the collapse time remains fixed at the inverse of the vacuum Rabi frequency.

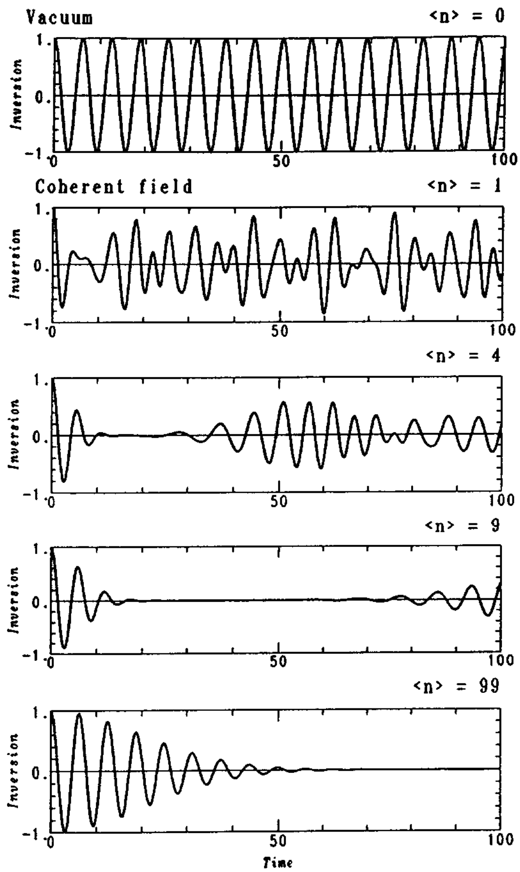

Figure 4. Population inversion, as in figure 3, for a cavity field that is initially in a coherent state. Mean photon numbers of the initial field are $\langle n\rangle=0$, 1, 4, 9 and 99. Times are in units of the inverse of the mean Rabi frequency. Note the Cummings collapse of Rabi oscillations followed by quiescence.

What was much more remarkable than the collapses, and unforeseen, was that the Rabi oscillations should revive after a quiescent interval. Thanks to the work of Eberly and his coworkers (Eberly *et al.* (1980), Narozhny *et al.* (1981), Yoo *et al.* (1981), see also Dung *et al.* (1990)), this phenomenon is now understood both qualitatively and quantitatively (an interesting approach, based on coherent states rather than number states, has been presented by Stenholm (1981)). This revival time is essentially the inverse of the separation between distinct Rabi frequencies (Narozhny *et al.* 1981, Barnett *et al.* 1986),

$$t_R = t_c 2\pi(\langle n \rangle)^{1/2}. \tag{58}$$

Thus the behaviour of a two-state atom interacting via one-photon transitions (in the rotating wave approximation) with a single-mode coherent state, is surprisingly irregular: Rabi oscillations that collapse, remain quiescent, revive, collapse again, and repeatedly undergo a complicated pattern of collapses and revivals. The discovery of quantum revivals, along with the possibility of finding solutions (often exact) to fundamental models of a quantum theory of interacting fields and atoms, have certainly provided incentive for extending and generalizing the Jaynes–Cummings model. (The micromaser revival is very closely related to, but not identical to the JCM revival; for a comparison of experiments with the collapse and revival predicted by micromaser theory, see Wright and Meystre (1989).)

Figure 5 displays an example of the collapse and revival phenomena in population inversion, as well as in other statistical properties of the field: the mean photon number, the Mandel Q parameter, and the normalized electric field variance. These latter quantities show the non-classical behaviour of the field, specifically sub-Poisson statistics (from Q) and squeezing (from the field variance). The squeezing that occurs for the JCM when the initial field is a coherent state, as first shown by Meystre and Zubairy (1982), is slight but perceptible when the mean photon number is fairly small (only 16 in the present example). However, for sufficiently large initial photon numbers n_0, the bound on $\Delta X > (\frac{1}{4})/\sqrt{n_0}$ can become arbitrarily small, and squeezing can become complete periodically (Kuklinski and Madajczyk 1988; Hillery 1989).

For comparison, figure 5 also shows the population inversion for a thermal field (Knight and Radmore 1982 *a*). The reason for the qualitative difference between these two initial fields can be appreciated from a glance at the photon probability distributions shown in figure 6. The great breadth of the thermal distribution, which washes out any traces of Rabi oscillations, is apparent. A thermal field can be represented as a mixture of coherent states with a Gaussian distribution of mean fields; each component collapses and revives, but the Gaussian average smears out the evolution into a complicated sequence of interfering revivals (Arroyo-Correa and Sanchez-Mondragon 1990).

Plots of population or inversion, such as those of the previous figures, give the impression that the system is dormant following the collapse. This is not correct. As Narozhny *et al.* (1981) showed, the atomic dipole moment continues to change during this interval, reaching a broad extremum at the half revival time. The field, as well as the atom, continues to evolve even though the populations remain unchanged.

A variety of other initial fields have been studied (e.g. Vidiella-Barranco *et al.* (1992)), including various squeezed states (Milburn 1984, Puri and Agarwal 1987, Agarwal and Dutta Gupta 1989, Rustamov *et al.* 1989, Savage 1989, Kim *et al.*

1990). As an example, photon numbers distributed according to a binomial distribution (a binomial field state, Miller and Mishkin (1967), Stoler *et al.* (1985), Sharma *et al.* (1989), applied to a three-level atom by Goggin *et al.* (1990)),

$$\mathscr{P}_m(p, N) = \frac{N!}{(N-m)!m!} p^m (1-p)^{N-m}, \tag{59}$$

$$\langle n \rangle = pN, \qquad \langle (\Delta n)^2 \rangle = pN(1-p), \tag{60}$$

interpolate between a coherent state ($p \rightarrow 0$, $N \rightarrow \infty$ but $pN = \alpha$) and a pure number state ($p=1$, $n=N$). The case $p=0$ is the vacuum, and the case $p=\frac{1}{2}$ is the Bernoulli state. Thus for $p=1$, $n=N$, the distribution produces the regular

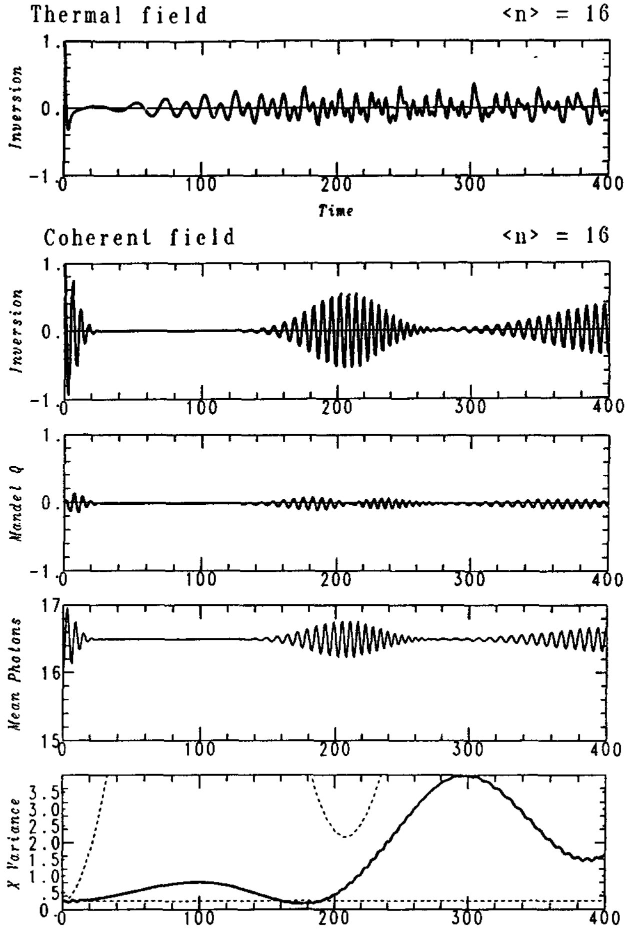

Figure 5. Population inversion over longer time interval, for initial mean photon number of $\langle n \rangle = 16$. Top frame shows thermal field, lower frames show a coherent field. Also shown for the coherent field are the mean photon number, the Mandel Q parameter, and the field variance ΔX (solid line; the dashed line shows ΔY). Times are in units of the inverse of the mean Rabi frequency. Note the Eberly revival of Rabi oscillations, followed by collapse. Note also the non-classical behaviour: sub-Poisson statistics and squeezing.

sinusoidal oscillations characteristic of a field with a precisely determined number of quanta. Another possibility for interpolating between two extremes are the Glauber–Lachs states which combine a coherent state and a thermal mixture (Satyanarayana *et al.* 1992). The logarithmic states (Simon and Satyanarayana 1988) and the negative binomial states (Agarwal 1992) each interpolate between thermal and coherent states (see Agarwal (1992) for a tabular comparison of the statistical properties of the most common distributions, including Q functions).

The preceding examples of photon distributions, devised for analytic convenience, produce field states that will evolve in time when interacting with atoms. The micromaser offers opportunity, by supplying a continuous stream of fresh atoms to the cavity, of achieving a steady state in which the field remains in a definite quantum state. By introducing single-velocity excited atoms, so that each atom remains in the cavity for a time τ, one can produce a photon-number state (Filipowicz *et al.* 1986). More generally, when atoms enter the cavity in a coherent superposition state,

$$\psi = \alpha\psi_1 + \beta\psi_2, \tag{61}$$

the cavity field can evolve toward a cotangent state (Slosser and Meystre 1990, Meystre *et al.* 1991), with probabilities defined through the recursion relations

$$\mathcal{P}_m(\tau) = |\alpha/\beta|^2 \cot^2\left(\frac{1}{4}\Omega_1 \tau \sqrt{m}\right)\mathcal{P}_{m-1}(\tau). \tag{62}$$

Special cases include photon number states.

12. Phase space

With the development of quantum optics has come the recognition that full characterization of a quantum field requires far more than simply a specification of

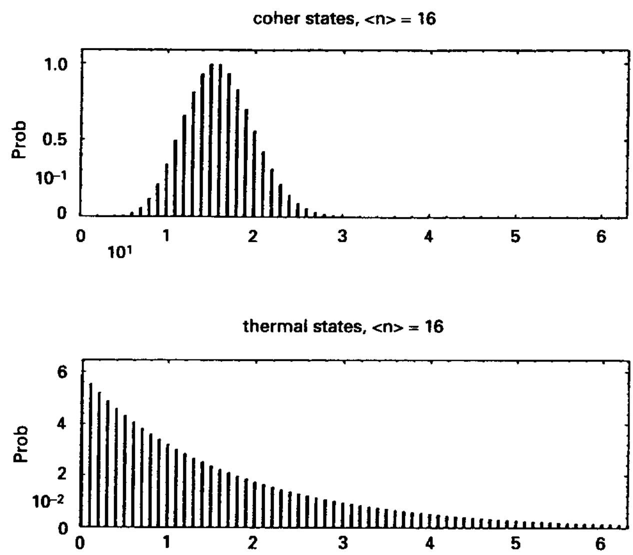

Figure 6. Initial distribution of photon numbers $\mathcal{P}_n$ against n with mean photon number $\langle n\rangle = 16$ for a coherent state and for a thermal state.

mean and variance of photon number. One requires an infinite number of photon moments or correlation functions. Alternatively, one can examine phase-space properties of the harmonic oscillator that embodies the dynamics of the single mode field. For this purpose there exist a variety of real-valued Cahill–Glauber quasi-probability functions (Cahill and Glauber 1968 *a,b*, Louisell 1973)

$$W(\alpha, s) = \frac{1}{\pi} \int d^2\xi \exp(\alpha\xi^* + \alpha^*\xi)\, \chi(\xi, s), \tag{63}$$

defined as complex Fourier transforms of the characteristic function $\chi(\xi, s)$

$$\chi(\xi, s) = \langle D(\xi, s) \rangle, \qquad D(\xi, s) = \exp\left(\xi \hat{a}^\dagger + \xi^* \hat{a} + \frac{s}{2} |\xi|^2 \right), \tag{64}$$

based, in turn upon the s-ordered Glauber displacement operator $D(\xi, s)$. The ordering choice depends on the nature of the relevant measurement. Normal ordering ($s = 1$) is usually associated with *absorptive* measurements. These involve the Glauber–Sudarshan P distribution function. The symmetric ordering choice $s = 0$ defines the Wigner function, whereas the antinormally ordered choice $s = -1$ gives the Q function (Cahill and Glauber 1968 *a,b*):

$$Q(\alpha, t) = W(\alpha, s = -1). \tag{65}$$

This function may be regarded as the projection of the field on to a basis of coherent states, as expressed by the formula

$$Q(\alpha, t) = \langle \alpha | \rho^F | \alpha \rangle = \sum_{n,m} f_n(\alpha)^* \rho^F_{nm} f_m(\alpha). \tag{66}$$

For a system that begins in a definite atomic state one can obtain the Q function from the atomic probability amplitudes, $C_1(n, t)$ and $C_2(n, t)$ for a two-level system, through the construction

$$Q(\alpha,t) = \sum_n \left| \sum_k f_{n-k+1}(\alpha) f_n C_k(n, t) \right|^2. \tag{67}$$

The quasi-probability Q function has useful pictorial properties (see Kim *et al.* (1989)). For a number state, it is a circular doughnut centred at the origin and with mean radius $(\langle n \rangle)^{1/2}$, whereas a coherent state appears as a Gaussian mound centred at some point in the complex alpha plane. Squeezing appears as an elongation of circular contours.

For a two-state atom interacting with a coherent state, the field quasi-probability function $Q(\alpha, t)$ appears initially as a Gaussian distribution in the complex α plane. With the usual choice of real α, the distribution is centred on the horizontal axis, at a distance $(\langle n \rangle)^{1/2}$ from the origin. As shown by Eiselt and Risken (1989, 1991) (see also Miller *et al.* (1992), and Daeubler *et al.* (1992)) this distribution subsequently bifurcates into two peaks that move in opposite directions around a circular path whose radius equals the square root of the initial mean photon number. As the peaks meet at the opposite side of the circle from the start, they produce a revival of Rabi oscillations. Further revivals occur as the peaks repeatedly pass each other along the circular track.

Figure 7 shows the appearance of the Q function of the JCM for a coherent-state initial field having mean photon number of nine. The topmost frame is the initial Gaussian form. The next frame depicts the Q function during the collapse interval. At this time the density matrix is a coherent superposition of two distinct states. The final frame shows the function during the first revival.

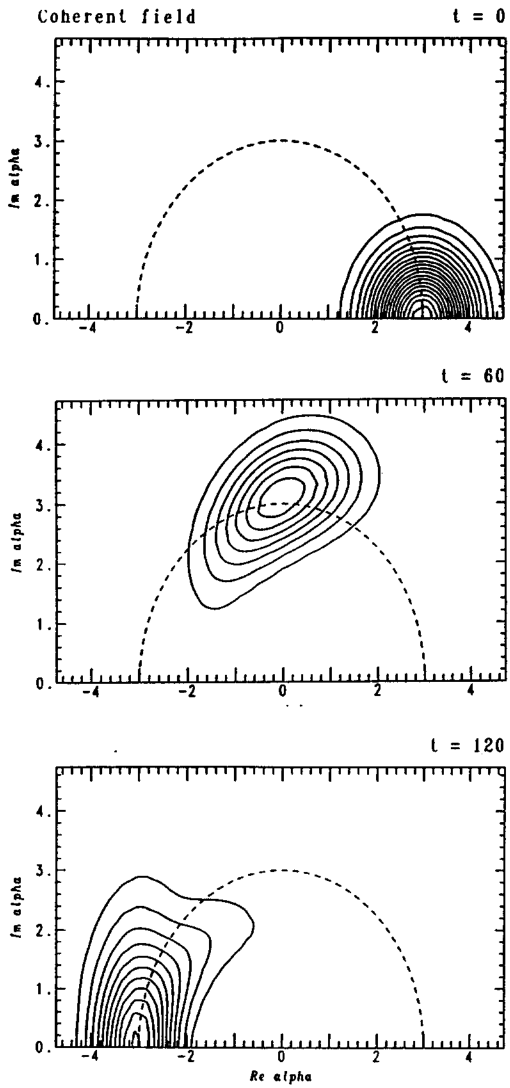

Figure 7. Phase plane plot of $Q(\alpha)$ function at various times, for an initially excited atom interacting with a coherent field, with initial mean photon number $\langle n\rangle=9$. The figure shows only the upper half of the complex plane; the lower half is a mirror image of this. Times (in units of mean Rabi frequency) are $t=0$, $t=60$ (during collapse) and $t=120$ (during revival). Note the formation of two wavepackets, moving in opposite directions along the circular path with constant mean photon number (dashed semicircle).

It is interesting to contrast this behaviour with that which occurs when the initial state is a thermal field. When we remember that the initial thermal field can be represented as a broad Gaussian centred on the origin, then the subsequent complicated evolution is poorly illustrated by the relevant Q function.

13. Initial atomic coherence

Although it is natural to assume that the atoms begin in one of their two excitation states, or in an incoherent average distribution between the two states, one can also consider atoms that are initially in a coherent superposition of the two states (Krause *et al.* 1986, Zaheer and Zubairy 1989). The initial condition, for a field in some precise quantum state (specifiable as a superposition of photon number states) and the atom also in a precisely defined state (a coherent superposition of energy states ψ_k) reads

$$\Psi(0)=\Phi^F\psi^A, \qquad \Phi^F=\sum_{n=0}^{\infty} f_n|n\rangle, \qquad \psi^A=\sum_{k=1}^{N} g_k\psi_k. \tag{68}$$

When the atom arrives at the cavity in such a state then the subsequent behaviour is sensitive to the relative phase between the cavity field and the atom superposition (Puri and Agarwal 1987, Agarwal and Puri 1988, Joshi and Puri 1989, Zaheer and Zubairy 1989). More remarkably, the collapse and revival phenomena that so strikingly characterize the interaction of a coherent field with an energy state are absent if the atom is initially in one of the dressed states ψ_+ or ψ_- defined by equation (16). These are states that, in the semiclassical treatment of a specified field interacting with a two-state atom, remain constant in time (Bai *et al.* 1985, Lu *et al.* 1986). As shown by Gea-Banacloche (1990, 1991), such an atomic dressed state $\psi_\pm$, when combined with a coherent state $|\alpha\rangle$ whose mean photon number is large, will evolve in time very nearly as product states

$$\Psi(0)=\psi_+\Phi\rightarrow\frac{1}{\sqrt{2}}[\exp(\mathrm{i}\varphi)\,\psi_2\exp(-\mathrm{i}\pi t/t_\mathrm{R})+\psi_1]\Phi_+(t), \tag{69}$$

$$\Psi(0)=\psi_-\Phi\rightarrow\frac{1}{\sqrt{2}}[\exp(\mathrm{i}\varphi)\,\psi_2\exp(+\mathrm{i}\pi t/t_\mathrm{R})-\psi_1]\Phi_-(t), \tag{70}$$

where t_R is the recurrence time. Because the statevector is essentially a disentangled product of field and atomic states, the atom never really becomes correlated with the field. Any measurement on the atoms *alone* can therefore impart essentially *no* information about the field. The field states $\Phi_+(t)$ and $\Phi_-(t)$ are slight distortions of coherent states whose phase increases (or decreases) steadily with time at a rate $\pi t/t_\mathrm{R}$,

$$\Phi_\pm(t)=\exp\left[\mp\frac{1}{2}\mathrm{i}t\Omega_1(\hat{a}^\dagger\hat{a})^{1/2}\right]|\alpha\rangle\approx\exp(\mp\mathrm{i}\pi t/t_\mathrm{R})|\alpha_\pm(t)\rangle, \tag{71}$$

$$\alpha_\pm(t)=\alpha\exp(\pm\mathrm{i}\pi t/t_\mathrm{R}). \tag{72}$$

The Q function for one of these states appears as a wavepacket moving steadily along a circular path of radius $|\alpha|$: the packet for Φ_+ moves clockwise while that for Φ_- moves counterclockwise. The meeting of these two packets, at the recurrence time, produces an interference that appears as a revival of Rabi oscillations

(Eiselt and Risken 1989). With either of these initial conditions, or indeed for any other pure atomic state at $t=0$, the atom at the half revival time is in the coherent superposition state

$$\psi_c = \frac{1}{\sqrt{2}}[-\mathrm{i}\exp(\mathrm{i}\varphi)\psi_2 + \psi_1], \tag{73}$$

which acts as an 'attractor state' at this special time. By starting with an atom in an excited (or unexcited) state, one deals with a coherent superposition of these dressed states. These are the two amplitudes that evolve as the bifurcation of the Q function. Subsequent time evolution will entangle the field and atom states. However, at the half-revival time the atom is again nearly in a pure state (Phoenix and Knight 1991 *a,b*), namely the coherent superposition state of equation (73). At those moments the field is a macroscopic superposition state composed of two field states that have the same amplitude but opposite phase. The purity of this macroscopic superposition or 'Schrödinger-cat' field state increases as the mean photon number increases. Interesting bimodal effects, evident in plots of the Q function, occur when the two-state JCM is driven by a strong external field, as discussed by Alsing and Carmichael (1991).

14. Entanglement

The atom and the cavity field of the JCM serve as paradigms of coupled quantum mechanical systems. The strong correlation (entanglement) that develops between atom and field during the course of time offers the means of inferring properties of the field from measurements upon atoms. On the other hand, the disappearance of correlation can accompany the formation of a macroscopic quantum state of the field. Thus the JCM offers opportunities to elucidate and test purely quantum correlations.

One can say that an atomic observable $\hat{A}$ and a field observable $\hat{F}$ are uncorrelated if the expectation value of their product factors into separate expectation values,

$$\langle \hat{A}\hat{F}\rangle = \langle \hat{A}\rangle\langle \hat{F}\rangle. \tag{74}$$

If all possible atom and field operators are uncorrelated then the atom and field subsystems are statistically independent, and the density matrix factors into independent reduced density matrices,

$$\rho = \rho^A \otimes \rho^F. \tag{75}$$

One may say that the field and the atom are entangled (correlated) if a measurement of some attribute of the field (or atom) provides information about the atom (or field). To avoid commitment to a particular operator it is desirable to base quantitative estimators of correlation upon some property of density matrices. Because any density matrix has unit trace (i.e. probabilities sum to unity), and because the density matrix for a pure quantum state has the property $\rho^2=\rho$, one simple measure of entanglement is

$$p^A = \mathrm{Tr}\,(\rho^A)^2, \qquad p^F = \mathrm{Tr}\,(\rho^F)^2. \tag{76}$$

This number will be unity if the atom (or field) is in a pure state, and will otherwise be smaller. One can show tht $p^A = p^F$, so that computations require only the elements of the atomic density matrix. Equation (30) shows that the magnitude of the Bloch vector offers an equivalent measure of purity: it shrinks as the state becomes mixed. (For an example of a two-state atom, see Gea-Banacloche (1990).)

As a note of caution, it should be mentioned that in the micromaser (at present the nearest example of an experimental realization of the JCM), we cannot directly observe the microwave field. Instead, we deduce field properties from the measured state of the atoms as they exit the interaction cavity. If the field and atom dynamically disentangle, then nothing we can do to the emerging atoms will reveal to us that the field left in the cavity is such an interesting macroscopic state.

15. Entropy and entanglement

Entropy offers a quantitative measure of the disorder of a system, and of the purity of a quantum state. We define the dimensionless Von Neumann entropy of a quantum system as (Wehrl 1978)

$$S = -\mathrm{Tr}\,(\rho \ln \rho). \tag{77}$$

For a pure state this entropy vanishes, $S=0$, whereas for a statistical mixture the entropy is non-zero, $S \neq 0$. The entropy of a system remains constant whenever the time-dependent Schrödinger equation governs the entire time evolution. Of more interest, therefore, are the partial entropies of system components, such as the field and atom subsystems of the JCM. From the reduced density matrices of the atom and field we form the partial entropies (Aravind and Hirschfelder 1983, Phoenix and Knight 1988, Barnett and Phoenix 1989)

$$S_A(t) = -\mathrm{Tr}_A\,[\rho^A(t) \ln \rho^A(t)], \qquad S_F(t) = -\mathrm{Tr}_F\,[\rho^F(t) \ln \rho^F(t)]. \tag{78}$$

Unlike the entropy of the complete system, these partial entropies may change with time. According to a theorem of Araki and Lieb (1970), see Phoenix and Knight (1988), if the combined system begins as a pure quantum state, then at all times the component entropies are equal, $S_A(t) = S_F(t)$. Under these circumstances a decrease of partial entropy means that each subsystem evolves toward a pure quantum state, whereas a rise in partial entropy means that the two components tend to lose their individuality and become correlated or entangled. The individual components are in their purest state when their entropies are smallest. The components are most strongly correlated when their individual entropies are large. Thus a plot of $S_A(t)$, like a plot of the purity $p^A(t)$ of equation (76), will reveal a history of atom–field entanglement.

Examples of partial entropy changes for photon-number states, as presented by Aravind and Hirschfelder (1984), show periodicities correlated with population oscillations. The behaviour for a coherent-state field is more complicated. Starting from initially pure states of atom and field, and hence from zero partial entropies, one expects, and finds, that the partial entropies increase (with modulation at the Rabi frequency) during the interval of collapsing Rabi oscillations. Figure 8 shows this atomic (or field) entropy. One might guess that the field and atom return most closely to their initial pure state at the peak of the revivals. Surprisingly, the peak of the revival does not correspond to the recreation of a pure atomic state, as measured by small entropy; instead the system returns most closely to a pure state of atom or field at the half-revival time $t = t_R/2$, during the collapse, when population appears static (Phoenix and Knight 1988, 1991 *a*,*b*, Gea-Banacloche 1990, 1991, 1992). The momentarily created nearly-pure atomic state, the ψ_c of equation (73), is a coherent superposition of the two energy states of the atom. Because it occurs especially with large mean-photon numbers, it provides an example of a macroscopic superposition (Schrödinger cat) state (Yurke *et al.* (1990), Buzek *et al.* (1992), see also Yurke and Stoler (1986)). The occurrence of revivals proves the existence of the superposition states: cavity damping will introduce dephasing during the collapse interval, and thereby eliminate the interferences that appear as revivals (Barnett and Knight 1986*a,b*, Eiselt and Risken 1989, 1991). Savage and coworkers (Savage *et al.* 1990) have investigated how macroscopic quantum superpositions can be created by single-atom dispersion. Slosser *et al.* (1990), Wilkins and Meystre (1991) and Meystre *et al.* (1990) have discussed generation and detection of Schrödinger cats in micromasers. Orszag *et al.* (1992) have shown that even when the initial atom is in a mixed state, evolution into a pure state can occur. Vidiella-Barranco *et al.* (1992) have investigated the interaction of a two-level atom in the JCM with a Schrödinger-cat superposition of coherent field states, and they show how the atomic dynamics can be used to sense the difference between superpositions and mixtures of field states.

16. Extending the JCM

Although the basic JCM continues to provide new insights into some of the most basic properties of quantum mechanics, and occasional unexpected results,

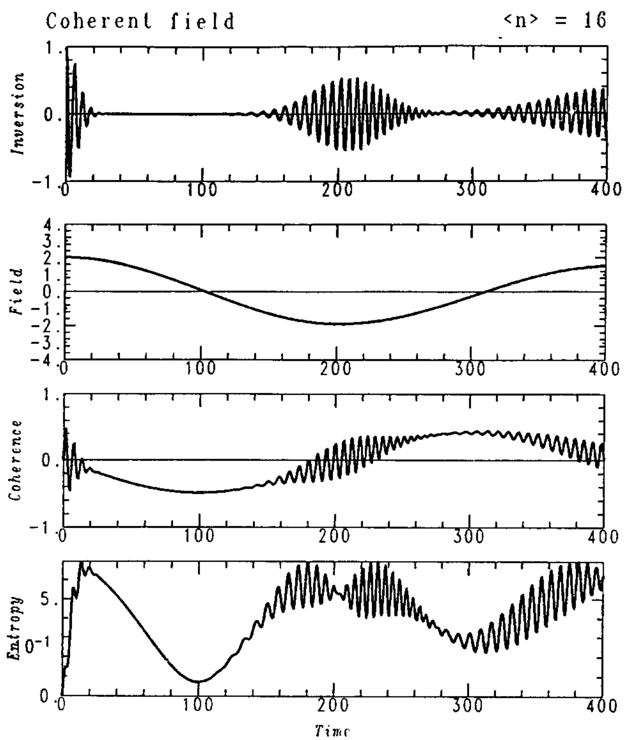

Figure 8. Histories of field $\langle X \rangle$, coherence $\langle \hat{S}_{12} \rangle$ and partial entropy (for atom or field; they are equal), for the coherent-state example of figure 5. Top frame repeats the population inversion of that figure.

much effort has been directed toward extending and generalizing the original model. In part the extensions have aimed at more realistic descriptions of experiments. The inclusion of photon loss (finite cavity Q factor) falls into this category. Other papers ask, more academically, what more general Hamiltonians have analytic solutions. Two-photon transitions fall into this category. Many of these more elaborate Hamiltonians may be experimentally realizable; often they predict remarkable properties. The following Sections discuss illustrative examples.

17. Beyond the RWA

The original JCM assumed that each photon change balanced an atomic change in such a way that the combination of photon energy plus atom energy remained constant. Jaynes and Cummings recognized that, just as with semiclassical theory, a more complete description of the radiation interaction should include additional terms, in the form

$$H=\hbar\omega\hat{a}^{\dagger}\hat{a}+E_1\hat{S}_{11}+E_2\hat{S}_{22}+\frac{\hbar}{2}\Omega_1(\hat{a}^{\dagger}+\hat{a})(\hat{S}_{12}+\hat{S}_{21}). \tag{79}$$

The original JCM is a quantized version of the rotating wave approximation (RWA); the additional terms are counter-rotating terms (Rabi *et al.* 1954, Shirley 1965). Typically these are treated, if at all, as sources of slight frequency shifts (Bloch–Siegert shifts: Bloch and Siegert (1940), Shirley (1965), Aravind and Hirschfelder (1984)). Although these terms are small, they can profoundly affect the long-time behaviour of the system. In particular, the semiclassical version of the JCM is perfectly periodic, whereas the inclusion of counter-rotating terms in the semiclassical equations leads to chaotic behaviour (Milonni *et al.* 1983, 1987, Graham and Höhnerbach 1984, Kujawski 1988). Zaheer and Zubairy (1988) have presented a solution of the JCM with the counter-rotating terms in the weak coupling regime. Phoenix (1989) has made a perturbative study of the role of the counter-rotating terms in the JCM and finds a small *phase* dependence of the atomic inversion, arising from interferences between rotating and counter-rotating contributions. Armstrong and Feneuille (1973) find there are phase-dependent corrections of a related kind in the Bloch–Siegert shifts. Recent discussions of the JCM without the RWA include Crisp (1991) and Seke (1991).

18. Dissipation and damping

The Rabi oscillations that persist indefinitely when a (hypothetical) two state atom exchanges energy with a single mode of an idealized lossless cavity will, in any experimental test, damp out. Apart from the collapse produced by the destructive interference of different Rabi frequencies associated with the distribution of photon numbers, two dissipative mechanisms affect atoms in real cavities: spontaneous emission into any continuum of field modes other than the JCM cavity mode, say at rate γ, and the loss of energy from the cavity (through walls or mirrors) at a rate ω/Q, parametrized by the quality factor Q (Sachdev 1984, Haroche and Raimond 1985, Barnett and Knight 1986*b*, Puri and Agarwal 1986, 1987, Kuklinski and Madajczyk 1988, Nayak *et al.* 1988, Eiselt and Risken 1989, 1991, Castro Neto and Caldeira 1990, Cirac *et al.* 1991).

Spontaneous emission at rate γ can be incorporated into the JCM after the fashion of Weisskopf and Wigner, by introducing a complex-valued atom energy, whose imaginary part is $-i\gamma/2$. Cavity dissipation is often modelled by coupling the field oscillator to a reservoir of external vacuum modes (Louisell 1973, Agarwal 1974). Alternatively, one can recognize the Markov stochastic nature of these dissipative processes by augmenting the Heisenberg or Liouville equations to include both damping and fluctuations, as in the density matrix equation (see Louisell (1973), p. 347)

$$\hbar\frac{\partial}{\partial t}\rho = -\mathrm{i}[H,\rho] + \frac{\hbar\omega}{Q}(2\hat{a}\rho\hat{a}^\dagger - \rho\hat{a}^\dagger\hat{a}\rho - \rho\hat{a}^\dagger\hat{a})$$
$$+\frac{\hbar\gamma}{2}(2\hat{S}_{12}\rho\hat{S}_{21} - \hat{S}_{11}\rho - \rho\hat{S}_{11}). \tag{80}$$

The two dissipative mechanisms have different effects upon the dynamics (Quang *et al.* 1991): revivals of population oscillations are much more sensitive to the cavity damping than to spontaneous emission. Even slight cavity damping is sufficient to destroy revivals (Barnett and Knight 1986 *b*). The field energy decays at rate ω/Q which, as we have seen, is much smaller than the vacuum Rabi frequency in realizable Rydberg-atom masers. But to see a revival one must be able to distinguish between discrete atom–field eigenvalues. If these form a broad overlapping quasi-continuum, then no revival occurs, but the width of the distribution of eigenvalues is essentially unchanged by such modest dissipation, so that the Rabi oscillations collapse as before. The decay rate of a Fock-state component of the field with n photons is $n\omega/Q$, which is a rapid decay typical of the destruction of macroscopic coherence, so that the revival is destroyed. As we noted above, the revival is associated with the constructive interference of two recombining Schrödinger cat states of the field. The environmental destruction of macroscopic coherence between the cat states means that when they recombine at the revival time, they do so *incoherently*. Rabi oscillations are then not reconstructed by interference, and the revival does not occur.

19. Multiatom generalizations

One of the first published extensions of the JCM was by Tavis and Cummings (1968, 1969), who examined a collection of two-state atoms interacting with a single field mode. Such a collection of collisionless atoms, each an example of spin one-half, can form (coupled) states of total spin S. This mathematical analogy with spin was first exploited by Dicke (Dicke (1954), see also Fleck (1966), Barnett and Knight (1984) and references therein). Permissible values for S range from a maximum of $N/2$ (all atoms either excited or unexcited) down to $|N_2 - N_1|/N$. For example, two atoms may form either a singlet or a triplet system. These two systems are independent. In particular, if initially both atoms are excited, then the pair of atoms remain in a triplet state at all times. The Dicke model with spin S involves the Hamiltonian

$$H = \hbar\omega\hat{a}^\dagger\hat{a} + \sum_{k=1}^{2S+1} kE_1\hat{S}_{kk} + \sum_{k=1}^{2S}\frac{\hbar}{2}\Omega_k(\hat{a}^\dagger\hat{S}_{k,k+1} + \hat{S}_{k+1,k}\hat{a}). \tag{81}$$

This model is equivalent to an atom possessing $2S+1$ equidistant energy levels that couple by the Rabi frequencies

$$\Omega_k = [k(2S+1-k)]^{1/2}\Omega_0. \tag{82}$$

This Tavis–Cummings model has been studied in much detail (Tavis and Cummings 1968, 1969, Bonifacio and Preparata 1970, Scharf 1970, Senitzky 1971, Swain 1972, Kumar and Mehta 1980, 1981, Haroche and Raimond 1985, Knight and Quang 1990), and various two-atom models have been examined (Deng 1985, Adam *et al.* 1987, Iqbal *et al.* 1988, Sharma *et al.* 1989 *b*, Jex 1990). Haroche and coworkers (Haroche 1984) have made an extensive study of the collective coupling of many Rydberg atoms to a single cavity field, demonstrating both the Dicke N-dependent enhancement of the Rabi frequency in temporal evolution as well as the collective modification of thermodynamic states. Others have examined collisional effects, as embodied in dipole–dipole interactions between the atoms (Seminara and Leonardi 1990, Joshi *et al.* 1991).

20. Three-level atoms

The various cases of three-state atoms have been treated in great detail by Yoo and Eberly (1985), by Bogolubov *et al.* (1984, 1985 *a,b*, 1986 *a,b*) and subsequently by Li and coworkers (Li and Bei 1984, Li and Peng 1985, Li and Gong 1986, Li *et al.* 1987, 1989 *a*, Liu *et al.* 1987, Zhu *et al.* 1988, Lin *et al.* 1989) and by others (Obada and Abdel-Hafez 1987, Zhu *et al.* 1988, Mahran 1990, Dung *et al.* 1991, Poizat *et al.* 1992). As mentioned in connection with the more general N-state atom, there are three distinct linkages: the ladder (also termed the cascade or Ξ), the vee (V) and the lambda (Λ). The distinction is:

$$\begin{aligned} &\text{Ladder } (\Xi): && E_1 < E_2 < E_3, && (3 \text{ is uppermost}), \\ &\text{Vee } (V): && E_1 > E_2 < E_3, && (2 \text{ is lowest}), \\ &\text{Lambda } (\Lambda): && E_1 < E_2 > E_3, && (2 \text{ is uppermost}). \end{aligned} \tag{83}$$

The model is soluble either for a single mode field or for a pair of fields, as in the Hamiltonian for the ladder

$$H = \sum_{v=1}^{2} \hbar\omega_v \hat{a}_v^\dagger \hat{a}_v + \sum_{k=1}^{2} E_k \hat{S}_{kk} + \frac{\hbar}{2}\Omega_1(\hat{a}_1^\dagger \hat{S}_{12} + \hat{S}_{21}\hat{a}_1) + \frac{\hbar}{2}\Omega_2(\hat{a}_2^\dagger \hat{S}_{23} + \hat{S}_{32}\hat{a}_2). \tag{84}$$

In each case the balance of excitation with photon number, as required by the three-state RWA, means that we need deal only with independent 3×3 Hamiltonians and we can write a field–atom state-vector in the form

$$\Psi(n,t) = \exp(-in\omega t - iE_1 t/\hbar) \sum_{k=1}^{3} \phi_k(n) C_k(n,t). \tag{85}$$

For the single-mode ladder configuration a choice for the three required field–atom basis states is

$$\phi_1(n) = |n\rangle\psi_1, \qquad \phi_2(n) = |n-1\rangle\psi_2, \qquad \phi_3(n) = |n-2\rangle\psi_3, \tag{86}$$

while for the vee configuration a choice is

$$\phi_1(n) = |n-1\rangle\psi_1, \qquad \phi_2(n) = |n\rangle\psi_2, \qquad \phi_3(n) = |n-1\rangle\psi_3, \tag{87}$$

and for the lambda we may take

$$\phi_1(n) = |n\rangle\psi_1, \qquad \phi_2(n) = |n-1\rangle\psi_2, \qquad \phi_3(n) = |n\rangle\psi_3. \tag{88}$$

The three-level models, interacting with a coherent state cavity field, exhibit the same complicated phenomena of collapse and revival as do two-state atoms (Cardimona *et al.* 1989, Cardimona 1990). However, when the single-photon detuning of the intermediate state becomes large the system becomes effectively a two-state system subject to a two-photon interaction. In this limit the pattern of collapse and revival becomes periodic (Yoo and Eberly 1985, Ashraf *et al.* 1990, Phoenix and Knight 1990). The three-level JCM continues to receive attention, including effects of damping (e.g. Adam *et al.* (1989, 1990)).

21. Multilevel atoms: ladder

The generalization of the JCM to an N state atom interacting with a single field mode has the form

$$H=\hbar\omega\hat{a}^{\dagger}\hat{a}+\sum_{k=1}^{N} E_k\hat{S}_{kk}+\sum_{j<k=1}^{N}\frac{\hbar}{2}\Omega_{jk}(\hat{a}^{\dagger}\hat{S}_{jk}+\hat{S}_{kj}\hat{a}). \tag{89}$$

The particular case of $N=4$ has been discussed several times (Buck and Sukumar 1984, Kozierowski and Shumovsky 1987, Liu *et al.* 1988) as have general N level $(N-1)$ mode models (Li and Zhu 1985, Abdel-Hafez *et al.* 1987 *a,b*, Kozierowski and Shumovsky 1987). A variety of linkage patterns have been considered. One of these is the ladder configuration, in which single-photon transitions connect only adjacent levels in a sequence of increasing energy, $E_k<E_{k+1}$

$$H=\hbar\omega\hat{a}^{\dagger}\hat{a}+\sum_{k=1}^{N} E_k\hat{S}_{kk}+\sum_{k=1}^{N-1}\frac{\hbar}{2}\Omega_k(\hat{a}^{\dagger}\hat{S}_{k,k+1}+\hat{S}_{k+1,k}\hat{a}). \tag{90}$$

The ladder configuration of excitation, together with the rotating wave approximation and the assumption of an initially precise atom-excitation state, provides a fixed correspondence between the photon number and the excitation number: each incremental excitation of the atom reduces the photon number by one. By starting in a definite atomic state we need deal at one time only with the an N-state expansion

$$\Psi(t)=\sum_n f_n\Psi(n,t)=\sum_n \exp(-\mathrm{i}n\omega t)\sum_{k=1}^{N}\phi_k(n)f_nC_k(n,t), \tag{91}$$

in which the field-atom states are those used with the traditional JCM,

$$\phi_k(n)=|n-k+1\rangle\psi_k. \tag{92}$$

The probability of finding the atom in state k at time t, given that at time $t=0$ the field and atom each had precisely determined energy, is

$$P_k(n,t)=|f_nC_k(n,t)|^2. \tag{93}$$

The probability amplitudes are obtained as solutions to the time-dependent Schrödinger equation for the N state Hamiltonian

$$H(n)=\sum_{k=1}^{N}(E_k-k\hbar\omega)\hat{S}_{kk}+\sum_{k=1}^{N-1}\frac{\hbar}{2}\Omega_k(n)(\hat{S}_{k,k+1}+\hat{S}_{k+1,k}), \tag{94}$$

where

$$\Omega_k(n)=(n-k+1)^{1/2}\Omega_k. \tag{95}$$

To complete the definition of the Hamiltonian we must specify the $N-1$ parameters Ω_k. The simplest choice is for equal values, $\Omega_k = \Omega_1$ for all k. This was the choice of Li *et al.* (1989*b*,*c*, 1992). Another reasonable choice is that of the Tavis–Cummings model in which the distributions are those of spin $S=(N-1)/2$.

Multilevel atoms, interacting with a coherent state of a single cavity mode, exhibit collapse and revival phenomena similar to that of two-level atoms. However, an N-level atom does not generally exhibit periodic population oscillations. The multilevel spin system is exceptional: the behaviour in the semiclassical limit (or for a photon number state) is exactly periodic (see Cook and Shore (1979)).

Figure 9 shows examples of the population inversion $w(t) = P_5(t) - P_1(t)$ for a five-level spin system (i.e. $S=2$, as might occur with four two-state atoms in the Tavis–Cummings model) initially excited to the uppermost level and entering a cavity in which the field is a coherent state. Each transition interacts reasonantly with the same mode. Upon entering a vacuum the populations undergo periodic (but not sinusoidal) changes. Consequently the inversion is periodic. As the initial mean photon number increases the superposed Rabi frequencies cause the periodicities to collapse. As with the two-level collapse, there occurs a quiescent interval

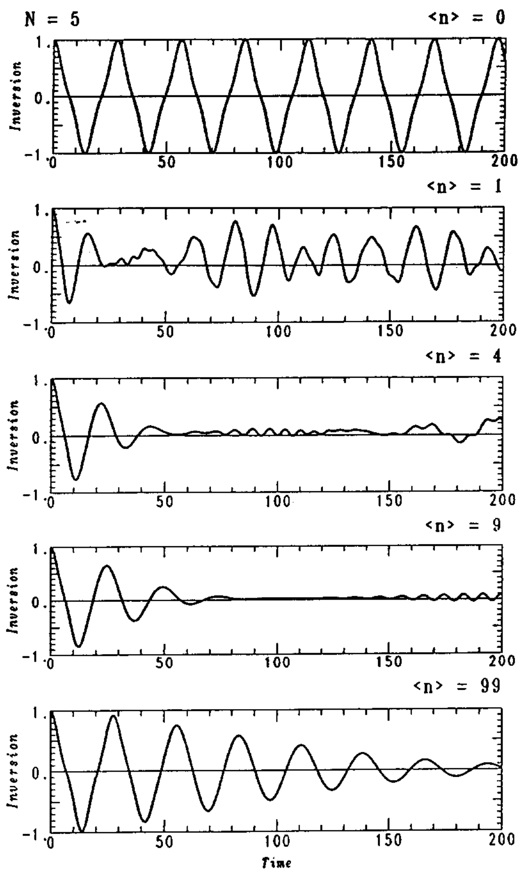

Figure 9. The population inversion $w(t) = P_5(t) - P_1(t)$ for a five-level atom initially in level 5, interacting with a cavity coherent state. Mean photon numbers of the initial field are $\langle n \rangle = 0$, 1, 4, 9 and 99. Times are units of the inverse of the mean Rabi frequency.

followed by a revival. The photon number variation is larger with a multilevel ladder than with a two-level atom. Whereas the photon number can change by only unity for two levels, it can change by as much as $N-1$ for an N-level ladder.

22. Multilevel atoms: generalized vee and lambda

Another class of N-state Hamiltonians are those of a generalized vee, in which the lowest atom state, here taken to be ψ_1 (rather than ψ_2), is linked by one-photon transition to a succession of higher-lying states:

$$H=\hbar\omega\hat{a}^\dagger\hat{a}+\sum_{k=1}^{N} E_k\hat{S}_{kk}+\sum_{k=2}^{N}\frac{\hbar}{2}\Omega_k(\hat{a}^\dagger\hat{S}_{1,k}+\hat{S}_{k,1}\hat{a}). \tag{96}$$

The statevector can be expanded as before, but now the N field–atom basis states are

$$\phi_k(n)=\begin{cases}|n\rangle\psi_1, & k=1\\ |n-1\rangle\psi_k, & k>1.\end{cases} \tag{97}$$

A third possibility, a generalized lambda linkage, has the uppermost state, here taken to be ψ_N (rather than ψ_2), interacting with each lower-lying state,

$$H=\hbar\omega\hat{a}^\dagger\hat{a}+\sum_{k=1}^{N} E_k\hat{S}_{kk}+\sum_{k=2}^{N}\frac{\hbar}{2}\Omega_k(\hat{a}^\dagger\hat{S}_{k,N}+\hat{S}_{N,k}\hat{a}). \tag{98}$$

The needed field-atom basis states can be taken to be

$$\phi_k(n)=\begin{cases}|n\rangle\psi_k, & k<N,\\ |n-1\rangle\psi_N, & k=N.\end{cases} \tag{99}$$

With each of these systems the energies E_k and coupling constants Ω_k appear as possibly adjustable parameters, as do the initial conditions. By finding the N eigenvalues and eigenvectors of $\boldsymbol{H}(n)$ one obtains analytic (or numerical) solutions to the generalized Jaynes-Cummings model. In special cases (i.e. particular choices of parameters) it may be possible to find useful analytic expressions, but there is no appreciable difficulty in constructing numerical solutions for modest values of N.

23. Generalized interactions

The basic JCM requires that each atomic excitation accompany the loss of a single photon. It is natural to generalize by considering intensity dependent or multiphoton transitions between the two atomic states, as in the form

$$H=\hbar\omega\hat{a}^\dagger\hat{a}+\sum_{k=1}^{2} E_k\hat{S}_{kk}+\frac{\hbar}{2}\Omega_1(\hat{R}^\dagger\hat{S}_{12}+\hat{S}_{21}\hat{R}). \tag{100}$$

The usual JCM takes $\hat{R}=\hat{a}$. A simple generalization, to an interaction proportional to photon number, is (Knight 1986, Phoenix and Knight 1990, Schoendorff and Risken 1990, Cardimona *et al.* 1991, Puri and Agarwal 1992)

$$\hat{R}=\hat{a}^\dagger\hat{a}. \tag{101}$$

Such a situation, applied to degenerate atomic states, $E_2=E_1$, models a degenerate Raman coupling (it might be termed a zero-photon interaction, because the change in atomic excitation leaves the photon number unchanged). This model leads to the same set of paired equations as does the JCM, but now for product states

$$\phi_1(n)=|n\rangle\psi_1, \qquad \phi_2(n)=|n\rangle\psi_2. \tag{102}$$

In place of equation (11) one has the expression

$$\Omega(n)=n\Omega_1. \tag{103}$$

The succession of Rabi frequencies for resonant excitation are all multiples of a common parameter Ω_1, and so the superpositions of different photon numbers can lead to exact periodicities: one observes a succession of identical collapses and revivals of Rabi oscillations in populations. When the initial field is a coherent state or a thermal state and the atom is in a definite energy state the sums over photon distributions are readily done (Knight 1986), to give simple expressions for the ensemble averages. The periodicities of the two-photon interaction, like the revivals of the standard JCM, offer direct evidence for the discreteness of photons.

Such periodic revivals are also obtained in the intensity-dependent interaction of Buck and Sukumar (1981), considered by Singh (1982) and by Buzek (1989, 1990) and Buzek and Jex (1989 *a*),

$$\hat{R}=\hat{a}(\hat{a}^\dagger\hat{a})^{1/2}. \tag{104}$$

Again the analytic solution can be summed exactly for an initial coherent-state field, and one finds the same regular collapses and revivals as are shown by the zero-photon model.

Multiphoton interactions of the form

$$\hat{R}=(\hat{a})^m, \tag{105}$$

(Buck and Sukumar 1981, Sebabwe Abdalla *et al.* 1986, Aliskenderov *et al.* 1987, Gerry 1988, Ashraf *et al.* 1990, Mahran and Obada 1990, Abdel-Hafez and Obada, 1991, Rosenhouse-Dantsker 1991) also lead to pairs of equations when treated in the rotating wave approximation. The two coupled field-atom product states are

$$\phi_1(n)=|n\rangle\psi_1, \qquad \phi_2(n)=|n-m\rangle\psi_2, \tag{106}$$

and the Rabi frequencies are

$$\Omega(n)=[(n+m)!/n!]^{1/2}\Omega_1. \tag{107}$$

The simplest examples are two-photon transitions (e.g. Alsing and Zubairy (1987), Davidovich *et al.* (1987), Gerry (1988), Puri and Agarwal (1988), who include damping; Mir and Razmi (1991), Zhou and Peng (1991)). Analytic solutions to the general interaction

$$\hat{R}=(\hat{a}^\dagger)^m(\hat{a})^n \tag{108}$$

have been presented by Kochetov (1987). Other discussions of multiphoton interactions in the JCM include Zubairy and Yeh (1980), Shumovsky *et al.* (1985), Buzek and Jex (1989 *b*), Drobny and Jex (1992), Meng *et al.* (1992), and Zhou *et al.* (1992). Sukumar and Buck (1981) have considered a multi*phonon* counterpart to the JCM.

Whilst such multiphoton generalizations of the JCM are of intrinsic theoretical interest, we should note that whenever intermediate states are eliminated to construct an effective multiphoton coupling, Stark shifts of energy levels are of importance (see Alsing and Zubairy (1987), Alsing *et al.* (1992). These shifts are proportional to photon numbers and they act as intensity-dependent detunings. It has been argued (Moya-Cessa *et al.* 1991) that such shifts are responsible for observable asymmetries in the one-atom maser spectra (Meschede *et al.* 1985).

24. Multiple modes

The extension of the JCM to treat three-level systems leads to interest in two-mode fields, one associated with each of the two transitions. For example the three-level ladder configuration, with mode frequencies ω_P and ω_S, has the Hamiltonian

$$H=\hbar\omega_P\hat{a}_P^\dagger\hat{a}_P+\hbar\omega_S\hat{a}_S^\dagger\hat{a}_S+\sum_k E_k\hat{S}_{kk}$$
$$+\frac{\hbar}{2}\Omega_P(\hat{a}_P^\dagger\hat{S}_{12}+\hat{S}_{21}\hat{a}_P)+\frac{\hbar}{2}\Omega_S(\hat{a}_S^\dagger\hat{S}_{23}+\hat{S}_{32}\hat{a}_S). \tag{109}$$

The lambda configuration differs only in the interchange of $\hat{S}_{32}$ and $\hat{S}_{23}$:

$$H=\hbar\omega_P\hat{a}_P^\dagger\hat{a}_P+\hbar\omega_S\hat{a}_S^\dagger\hat{a}_S+\sum_k E_k\hat{S}_{kk}$$
$$+\frac{\hbar}{2}\Omega_P(\hat{a}_P^\dagger\hat{S}_{12}+\hat{S}_{21}\hat{a}_P)+\frac{\hbar}{2}\Omega_S(\hat{a}_S^\dagger\hat{S}_{32}+\hat{S}_{23}\hat{a}_S). \tag{110}$$

More generally, one can consider a Hamiltonian involving several field modes, as in the form

$$H=\sum_v\hbar\omega_v\hat{a}_v^\dagger\hat{a}_v+\sum_k E_k\hat{S}_{kk}+\sum_{jkv}\frac{\hbar}{2}\Omega_{jkv}(\hat{R}_{jkv}^\dagger\hat{S}_{jk}+\hat{S}_{kj}\hat{R}_{kjv}). \tag{111}$$

Models of this form (e.g. Parker and Stroud (1986)), including three-dimensional modes, (Seke (1985 *a,b*) and Swain (1972 *a*)) have been considered. In the limit of a continuum distribution of modes, this Hamiltonian leads to the conventional predictions of exponentially decaying spontaneous emission, Lamb shifts, and other aspects of quantum electrodynamics (see Ackerhalt and Eberly (1974)). More in keeping with the spirit of the original JCM are restrictions to a few modes. The two-level atom interacting with two modes has been examined by Gou (1989), Parkins (1990), Joshi and Puri (1990) and Abdel-Hafez (1992) amongst others. In particular, such models show that the interaction of a two-level atom enclosed by a cavity and driven by a classical field (see Alsing and Carmichael (1991), Alsing *et al.* (1992)) is very different from the behaviour of the atom in free space. In particular, the fluorescence can be strongly suppressed (Alsing *et al.* 1992).

In a three-level atom, one can study the effect of intermode correlations on the two coupled transitions. Correlated modes of interest include the two-mode squeezed vacuum state (Lai *et al.* 1991 *a,b*) in which each mode contains precisely the same number of quanta initially, yet individual modes have a Bose–Einstein thermal photon distribution. Of course correlations between the modes are built up as a consequence of the JCM evolution. Curiously, Lai *et al.* (1991 *a,b*) find that an initial two-mode squeezed vacuum (or an initially uncorrelated two-mode thermal state) leads to correlation between the modes. An initially anticorrelated state is the SU(2) coherent state of two modes, whose marginal statistics in each mode are binomial. Periodicities, collapses and revivals of a three-level JCM driven by an SU(2) coherent state have been examined (Lai *et al.* 1991 *a*).

25. Raman process

When the detuning of the middle level is large, then it becomes a virtual level and can be adiabatically eliminated from the dynamics. This elimination holds both in the semiclassical regime and in the fully quantized Jaynes–Cummings regime (see Yoo and Eberly (1985)). By eliminating the middle (virtual) level one obtains a two-level atom whose transitions occur via two-photon transitions. For example, adiabatic elimination with the lambda configuration leads to the Raman process, in which destruction of a pump photon of frequency ω_P accompanies the creation of a Stokes photon of frequency ω_S. The resulting Raman Hamiltonian fits the pattern of an effective Hamiltonian (Walls 1970, Yoo and Eberly 1985, Boone and Swain 1989, Abdulla *et al.* 1990, Gerry and Eberly 1990, Cardimona *et al.* 1991, Cirac and Sanchez-Soto 1991, Mahran 1992, Toor and Zubairy 1992)

$$H=\sum_{v=P,S}\hbar\omega_v\hat{a}_v^\dagger\hat{a}_v+\sum_{k=1}^{2}E_k\hat{S}_{kk}+\frac{\hbar}{2}\Omega_{12}(\hat{R}^\dagger\hat{S}_{12}+\hat{S}_{21}\hat{R}), \tag{112}$$

involving the two-photon field operator

$$\hat{R}=\hat{a}_S^\dagger\hat{a}_P, \tag{113}$$

and an effective two-photon Rabi frequency $\Omega_{12}=\Omega_1\Omega_2/2\Delta$. The two field-atom basis states are

$$\phi_1(n,m)=|n\rangle_P|m\rangle_S\psi_1, \qquad \phi_2(n,m)=|n-1\rangle_P|m+1\rangle_S\psi_2. \tag{114}$$

Application to the two-photon ladder requires that the Hamiltonian involve the operator

$$\hat{R}=\hat{a}_S\hat{a}_P, \tag{115}$$

and the basis states

$$\phi_1(n,m)=|n\rangle_P|m\rangle_S\psi_1, \qquad \phi_2(n,m)=|n-1\rangle_P|m+1\rangle_S\psi_2. \tag{116}$$

The Raman coupled JCM has been discussed by Knight (1986 *a*), Phoenix and Knight (1990), Schoendorff and Risken (1990), and Puri and Agarwal (1992).

26. Nonlinear optical processes

Extension of the JCM to other nonlinear optical processes has been considered. For a Kerr medium one has (Agarwal and Puri 1989, Buzek and Jex 1990, Werner and Risken 1991 *a,b*, Bernat and Jex 1992, Gora and Jedrzejek 1992, Joshi and Puri 1992)

$$H=\hbar\omega\hat{a}^\dagger\hat{a}+\hbar\chi(\hat{a}^\dagger)^2(\hat{a})^2+\sum_{k=1}^{2}E_k\hat{S}_{kk}+\frac{\hbar}{2}\Omega_1(\hat{a}^\dagger\hat{S}_{12}+\hat{S}_{21}\hat{a}). \tag{117}$$

This model of a two-photon alteration to field energy is equivalent to a dynamic Stark shift of atomic energies. The nonlinearity parametrized by χ makes the collapse and revival phenomena much more distinct (Buzek and Jex 1990, Werner and Risken 1991 *a,b*). This behaviour is simple to explain: the detuned flopping frequency from equation (14) for the coupling λ_n is dominated for large λ_n by intensity-dependent detunings (i.e. $\lambda_n \propto n$). This leads to near-periodic revivals.

Moya-Cessa *et al.* (1991) have, as noted above, considered the effect of a.c. Stark shifts dependent on cavity photon number in the JCM. They see a strong influence on the collapse-revival dynamics and an asymmetric distortion of the transition lineshape which depends on photon statistics. Early one-atom micromaser experiments by Meschede *et al.* (1985) in thermal fields showed closely related asymmetric, shifted and power-broadened lineshapes. When such experiments are repeated at substantially lower temperatures, the thermal asymmetries noted by Moya-Cessa *et al.* (1991) become much less significant, as is appropriate for the near-Poisson nature of the cavity field statistics (Rempe *et al.* 1989).

27. The JCM and quantum non-demolition

Traditional methods for measuring electromagnetic energy (i.e. photon numbers) do so by destroying photons to produce detector pulses or photocurrents. When considering quantum fields in cavities it is natural to enquire whether it is possible to determine the photon number without changing that number. Such measurements are indeed possible. They offer examples of quantum non-demolition (QND) measurements that avoid back-action of the variable being measured (Braginsky *et al.* 1980, Caves *et al.* 1980, Braginsky and Khalili 1992, Meystre 1992).

In a QND measurement of an observable (say A), the inevitable disturbance generated by the measurement is restricted to the observable which is canonically conjugate to A. The subsequent evolution of A is unaffected by such a measurement. For the photon number observable the conjugate variable is the phase of the electromagnetic field (see Barnett and Pegg (1989), Pegg and Barnett (1989)). One QND method for determining photon numbers relies on the existence of a phase (or energy) shift which depends on photon number and which can be measured non-absorptively in an interference experiment.

A strong optical field propagating in a dispersive medium changes the refractive index of the medium by an amount which depends on the photon number. It shifts the phase of a second, probe, field, and this phase shift can be measured in an interference experiment (Levenson *et al.* 1986). Brune *et al.* (1992) have shown how this technique can be employed in a modified JCM to detect very small photon numbers, and how a special case can generate quantum mechanical superpositions of macroscopically different radiation field states (Schrödinger cats). The technique proposed by Brune *et al.* employs highly excited Rydberg states of alkali atoms to manipulate microwave radiation stored in a superconducting high Q cavity. The frequency of the stored radiation is close to the resonance frequency between two states e and i (see figure 10), but it differs by a detuning δ. The detuning is sufficiently large that no real transitions are possible from e to i. Instead, the field induces in state e a Stark shift that depends linearly on the number of photons in the cavity. The nearest state f is so far from resonance that it has negligible radiation-induced Stark shift. However, Rydberg states have such enormous dipole moments that for the states investigated by Brune *et al.* the Stark shift per photon is $4 \times 10^5\,\mathrm{s}^{-1}$. This is larger than the inherent linewidth of the transition. Therefore the Rydberg atoms can sense the difference between a field of (say) four or five photons without changing that number. The field phase (parametrized, for example, with Pegg–Barnett operators (see Pegg and Barnett (1989)) undergoes a diffusive evolution during the measured process.

Interference forms the basis of the QND measurement. The probe atoms are created in a beam which propagates through the microwave cavity (see figure 11). Just prior to entry into this cavity they are excited into a superposition of the levels e and f by a separate microwave field that is resonant with the e–f transition. This superposition traverses the microwave cavity containing the trapped quantum field. The trapped field has negligible effect on the component f, but the component e acquires a phase change proportional to the photon number and the time of flight through the cavity. After the atom leaves the cavity a second field, again resonant with the e–f transition, is applied. The combination of two e–f excitations results in Ramsey interference fringes in the probability of the atom being found in state e, due to the interference between two amplitudes (excitation may occur in either the first or the second excitation region). The fringes can be shifted by the phase acquired in the microwave cavity between the two excitation regions. This shift depends on the cavity photon number and the atom velocity.

The fringes responsible for the QND decimation are caused by interference between two amplitudes. In one, the atom traverses the cavity in state f, experiences no phase shift due to the cavity field, and then gets excited to state e. In the other, the atom is excited to state e, acquires a phase shift as it travels through the cavity, and remains in state e until detected.

Brune *et al.* (1992) discuss the effects of a broad atomic velocity distribution (i.e. various times of flight) and of initial field statistics, and they show how the measurements can be used to prepare field states of precisely known photon number.

Consider a cavity field prepared initially as a coherent superposition of different photon numbers. After one atomic interaction the photon distribution becomes modulated by the Ramsey fringe pattern. In other words, the measurement decimates the number distribution. If the next atom to pass through the apparatus has a different velocity it will have a slightly different fringe pattern, resulting in a different decimation. After a small number of atomic interactions, a

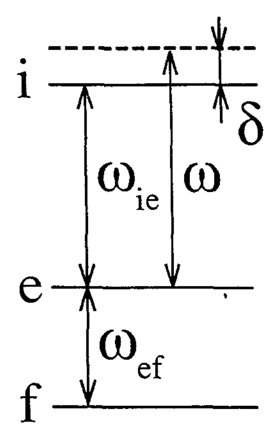

Figure 10. The relevant Rydberg levels employed by Brune *et al.* in their QND scheme, showing the two basic states e and i, and the perturbing state f. Taken from Brune *et al.* (1992).

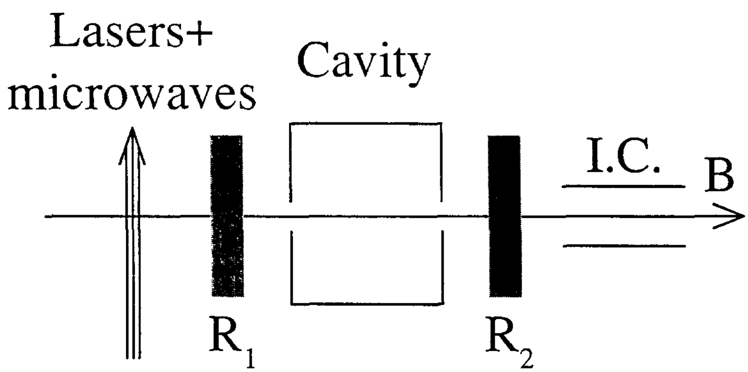

Figure 11. Interaction geometry of the QND experiment of Brune *et al.* Atoms in an atomic beam B are excited by lasers and by two microwave regions $R1$ and $R2$, forming a Ramsey excitation scheme. The atoms are detected at an ionization counter IC. Between the Ramsey regions the atoms traverse a high-Q superconducting cavity which stores the photons being measured. Taken from Brune *et al.* (1992).

cavity field is generated in a pure number state, picked at random from the initial distribution.

If the atoms are chosen carefully to have the same velocity (e.g. through laser cooling), then each injected atom can maintain the superposition of two phase-shifted fields. The field superposition maintained by such atoms is a quantum superposition of macroscopically distinct fields. Conventional Schrödinger cats die very rapidly when exposed to a dissipative environment: their decay depends on how macroscopically distinct are the components. These cats are fed 'quantum food' in the form of injected atoms in superposition states (see Meystre (1992)), and the cats reside in a highly non-dissipative cavity.

Each atomic interaction in the cavity and the Ramsey excitation zones is followed by a detection. The state of the field can be inferred directly from the detected state of the atom. Measurement of an excited atom at the end of all the interactions allows us to infer the existence of a superposition of two field states in the cavity: one state is phase shifted from the other unchanged original (apart from normalization). The next atom, injected with a different velocity, will cause each of these components of the superposition to split further into two more phase-shifted states. The effect of numerous such splittings is to effect a phase diffusion.

Brune *et al.* (1992) estimate that in the cavities currently available, the field damping time is 0·1 s and that Schrödinger cat states with photon numbers as large as 100 could be generated and observed through a modified interference technique. These numbers are substantially larger than in previous optical cat proposals. Sherman and Kurizki (1992) have shown how *conditional* measurements can be used to generate Schrödinger cats in the JCM.

28. Conclusions

The Jaynes–Cummings model continues today to fulfil in unanticipated ways the objectives of its originators: to permit examination of basic properties of quantum electrodynamics. The early observations all emerged from studies of two-state atoms: Rabi oscillations of population as energy shifted between field and matter; the collapse and subsequent revival of these oscillations; the appearance of chaos in an appropriate semiclassical limit; the non-classical statistical properties of the cavity field; and the recent recognition of the formation of macroscopic quantum states. The extension to three-state atoms and pairs of cavity modes introduced additional properties, though none yet rival the surprises of the two-state model. We can hope that studies of still more general extensions, involving multiple excitations of multilevel atoms, may in time reveal still further surprises dwelling within the established framework of quantum electrodynamics.

Acknowledgments

We dedicate this paper to the memory of our good friend and colleague, Dr Jay Ackerhalt (1947–1992), from whom we first learned of the beauty and complexity of the JCM. His many contributions to the study of the Jaynes-Cummings model and the Heisenberg equations have done much to further general understanding of quantum optics.

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