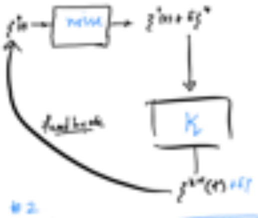
$i|\dot\psi\rangle = H[\epsilon(t)]|\psi\rangle$

$i|\dot\chi\rangle = H^{\dagger}[\epsilon(t)]|\chi\rangle$

$\xi(t) = \frac{1}{\lambda}\,\mathrm{Im}\langle\chi|\mu|\psi\rangle$

Start with $\xi^{k}(t) = \vec{\xi}^{\,k}$



Option #2

$\delta J^{k}[\vec{\xi}] \geq 0$?

$\xi_1^{k+1} = \xi_p^{k+1} + \Omega$

$\langle\langle(\xi_1^{k+1})^2\rangle\rangle = \langle\langle\xi_p^{k+1^2}\rangle\rangle + \langle\langle\Omega^2\rangle\rangle + \cancel{\langle\langle 2\,\Omega\,\xi_p^{k+1}\rangle\rangle}$ = ZERO!

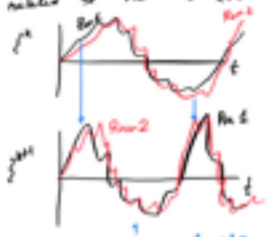
$\langle\langle(\xi^{k+1})^2\rangle\rangle = \xi_p^{k+1^2} + \langle\langle\Omega^2\rangle\rangle$

$\delta J^{k} \doteq \alpha\int_0^T dt\,(\xi_1^{k+1^2} - \xi_1^{k^2})$

$\doteq \alpha\int_0^T dt\left\{[\xi_p^{k+1} + \delta^{k+1}]^2 - [\xi_p^{k} + \delta^{k}]^2\right\}$

$\langle\langle\delta J^{k}\rangle\rangle \doteq \alpha\int_0^T dt\left\{(\xi_p^{k+1^2} - \xi_p^{k^2}) + \langle\langle\delta^{k+1^2} - \delta^{k^2}\rangle\rangle\right\}$

I assumed that "noise" is not related to "run". $\Rightarrow \langle\langle\delta^{k+1^2}\rangle\rangle = \langle\langle\delta^{k^2}\rangle\rangle$



$\delta J_1^{k} = \alpha\int_0^T dt\,[\xi_1^{k+1^2} - \xi_1^{k^2}]$

$\delta J_2^{k} = \alpha\int_0^T dt\,[\xi_2^{k+1^2} - \xi_2^{k^2}]$

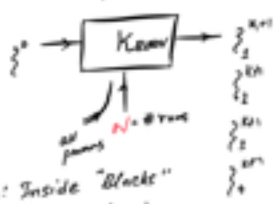
average in δJ?

$\langle\langle\delta J^{k}\rangle\rangle_{\text{Run}} = \frac{\alpha}{N}\int_0^T dt\,[\xi_1^{k+1^2} + \xi_2^{k+1^2} \cdots] - [\xi_1^{k^2} + \xi_2^{k^2} \cdots]$

N = # run

$\langle\langle\delta J^{k}\rangle\rangle_{\text{Run}} = \alpha\int_0^T dt\left\{\langle\langle\xi^{k+1^2}\rangle\rangle_{\text{Run}} - \langle\langle\xi^{k^2}\rangle\rangle_{\text{Run}}\right\}$

I think this will be uniform conv.



Qn: Inside "Blocks" {given run} is Krotov uniform conv?

N = # Repetitions

k = KROTOV ITERATION!

$$\begin{bmatrix} \xi_1^1 & \xi_1^2 & \xi_1^3 & \cdots \end{bmatrix}$$

$$\begin{bmatrix} \xi_2^1 & \xi_2^2 & \cdots \end{bmatrix}$$

One more thing to check

OPEN SYSTEM KROTOV

$$|\dot{\psi}\rangle\rangle = -i \Lambda |\psi\rangle\rangle \longrightarrow (1)$$

$$|\dot{\chi}\rangle\rangle = -i \Lambda^{+} |\chi\rangle\rangle \longrightarrow (2)$$

I have in Hilbert space,

$$\dot{\xi} = -i[H(\epsilon(t)), \xi] + \sum_k L_k \rho L_k^{+} - \frac{1}{2}\{L_k^{+}L_k, \xi\}$$

This corresponds to eq(1) above.

What is

$$\dot{\chi} = -i[H(t), \chi] + \sum L_k^{+} \rho L_k - \frac{1}{2}\{\cdot, \xi\}$$

Is it ~~this~~? YES !

See proof

If $\dot{\xi} = A\rho B$

$$|\dot{\xi}\rangle\rangle = \underbrace{B^{T} \otimes A}\, |\rho\rangle\rangle$$

$$|\dot{\rho}\rangle\rangle = X |\rho\rangle\rangle$$

Now instead of I had

$$|\dot{\rho}\rangle = X^{+} |\rho\rangle$$

$$= B^{T} \otimes A^{+} |\rho\rangle$$

then the corresponding Hilbert.

$$\dot{\xi} = A^{+} \xi (B^{T})^{*}$$

$$= A^{+} \xi B^{+}$$

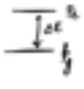
Hence $|\dot{\chi}\rangle = -iA^{+}|\chi\rangle$

$$\Rightarrow \boxed{\dot{\chi} = -i[H, \chi] + \sum L_k^{+} \chi L_k - \frac{1}{2}\{L_k L_k^{+}, \chi\}}$$

$$\dot{\rho} = -i[H, \rho] + \gamma(n+1)\left[a\rho a^{+} - \frac{1}{2}\{a^{+}a, \rho\}\right] + \gamma n \left[a^{+}\rho a - \frac{1}{2}\{a a^{+}, \rho\}\right]$$

→ Thermal Lindbladian with adiabatic control!

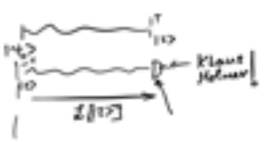
$$\dot{\chi} = -i[H, \chi] + \gamma(n+1)\left[a^{+}\chi a - \frac{1}{2}\{a a^{+}, \chi\}\right] + \gamma n \left[a \chi a^{+} - \frac{1}{2}\{a^{+}a, \chi\}\right]$$



$$\frac{\rho_e}{\rho_g} = e^{-\beta \Delta E} \text{ for } \rho$$

$$\frac{\rho_g}{\rho_e} = e^{-\beta \Delta E} \text{ for } \chi$$

Bottleneck in Implementing of Evolution in NISQ.



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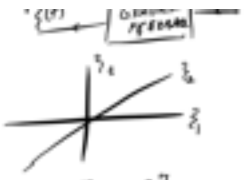


$\xi(t) = \xi_1 \cos[\xi_2 t + \xi_3]$

→ **CRAB!**

16/May

$$\dot{\rho} = -i[H,\rho] + \sum_k \left[L_k \rho L_k^\dagger - \tfrac{1}{2}\{L_k^\dagger L_k, \rho\} \right]$$

$\rho(0) = \rho_0$

$$\dot{\tilde{\rho}} = -i[H,\tilde{\rho}] + \sum_k \left[L_k^\dagger \tilde{\rho} L_k - \tfrac{1}{2}\{L_k L_k^\dagger, \tilde{\rho}\} \right]$$

$\tilde{\rho}(t) = \tilde{E}$.

Qn #1: Can you "evolve" costate eqn "in reverse"?

There are two distinct issues w/ the costate evolution eqn.

ⓐ The whole evolution is using adjoint Lindblad operators.

ⓑ The whole evolution is happening backward in time.

Point ⓐ

On Point ⓐ, I suspect that ∃ Lindblad eqn. s.t. the costate Lindblad eqn is not physical.

If this is true, then not all costate eqns are simulable.

→ Important : "Not all" ⟹ Some.

Consider L_k s.t. $\boxed{\{L_k L_k^\dagger\} = \{L_k^\dagger L_k\}}$

ex: σ_z, n, $a \to a^\dagger$

$$\dot{\tilde{\rho}} = -\gamma(\bar{n}+1)\left[a\rho a^\dagger - \tfrac{1}{2}\{n, \rho\}\right]$$
$$-\gamma\bar{n}\left[a^\dagger \rho a - \tfrac{1}{2}\{a a^\dagger, \rho\}\right]$$

$$\boxed{\frac{p(\infty)}{q(\infty)} = e^{-\beta\hbar\omega}}$$

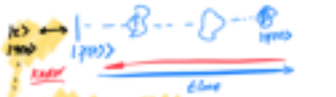
$p(\infty) + q(\infty) = 1$

Inverted thermal map or ...

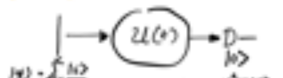
So there is still some program possible of costate Lindbladians.

Point B — The evolution is happening backwards in time.

A model of BACKWARD TIME PROPAGATION



Heisenberg interpretation

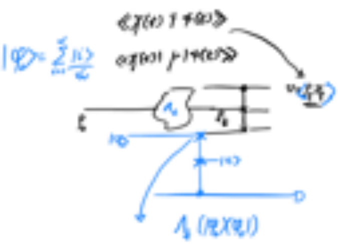


$|\psi\rangle = \sum_{i=1} \frac{|i\rangle}{\sqrt{d}}$

Like first step of ...

$|\psi_1\rangle = |\psi_{\rm tar}\rangle\langle\psi_{\rm tar}|\psi_1\rangle + (1 - |\psi_{\rm tar}\rangle\langle\psi_{\rm tar}|)|\psi_1\rangle$

$\boxed{|\psi_1\rangle = \alpha_{\rm tar}|\psi_{\rm tar}\rangle + \beta_{\rm tar}|\psi_{\rm tar}^\perp\rangle}$

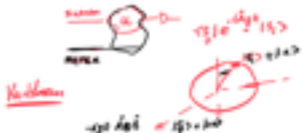


$\mathrm{Tr}[\ldots \rho a^\dagger]$

$\mathrm{Tr}[\,a^{-1}|\psi_0\rangle\langle\psi_0|\,a \quad 1\,]$

$\Lambda(|\psi_0\rangle\langle\psi_0|)\,\xi(t) \;\; \Lambda^{+}_{t_0}[\xi(t)]$

$\Lambda_{t_0}(|\psi_0\rangle\langle\psi_0|)\,\xi(t)$



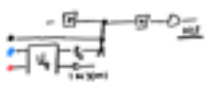
Heisenberg

$\langle\xi_f|\,e^{-igt\,\hat{A}\otimes\hat{B}}\,|\xi_i\rangle|m_i\rangle$

$= \langle\xi_f|\,(1-igt\,\hat{A}\otimes\hat{B})\,|\xi_i\rangle|m_i\rangle$

$= \langle\xi_f|\xi_i\rangle\left(1-igt\,\frac{\langle\xi_f|\hat{A}|\xi_i\rangle}{\langle\xi_f|\xi_i\rangle}\,\hat{B}\right)|m_i\rangle$

$= \langle\xi_f|\xi_i\rangle\left[e^{-igt\,A_w\hat{B}}\right]|m_i\rangle$



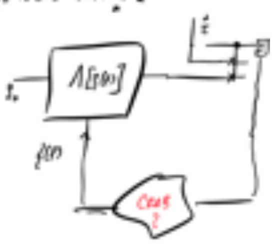
$|\dot{\psi}\rangle = -i(H_0 + \xi(t)H_1)|\psi\rangle$

$|\psi(T)\rangle = |\phi\rangle$

Noise Reduction

1. Closed
2. Controlled fields, $H \to \xi(t)$
3. Time dependent, Time dep. Lindbladian

$\dot{\rho} = -i[H(\xi(t)),\rho] + \sum_k \mathcal{L}_k[A_k(\xi(t))]\rho$

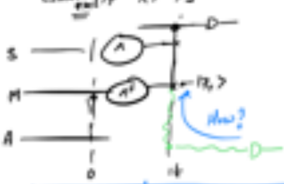


$\xi(t) = \sum_{k=1}^{d} a_k \sin[\omega_k^1 t] + b_k \cos[\omega_k^2 t]$

$\omega_k^1, \omega_k^2 \in$ uniform random $[-\pi, \pi]$

$|\psi\rangle = \frac{|0^s 0^m 0^a\rangle + |1^s 1^m 1^a\rangle}{\sqrt{2}}$

$\mathrm{CSWAP}\,[|\psi^{\mathrm{ini}}\rangle\langle\psi^{\mathrm{tar}}|]$



Section 3 Lindbladian work

$\left|\langle\psi_f|\,e^{iHt}\,e^{-iH't}\,|\psi_0\rangle\right|$

$e \quad e \quad e \qquad |\psi_0\rangle|+\rangle$