The Surface Code and its Decoders

Ben Criger [1]

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Introduction

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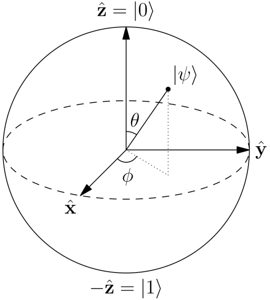

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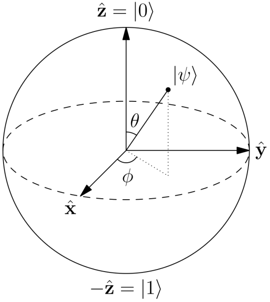

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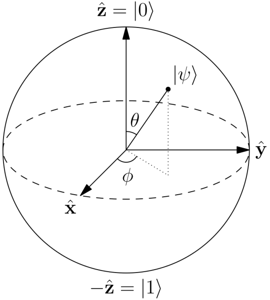

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- relies on operations that are continuous with finite accuracy

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The process of inferring which error happened given a syndrome is called *decoding*.

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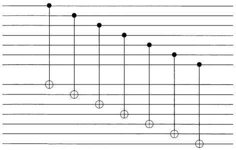

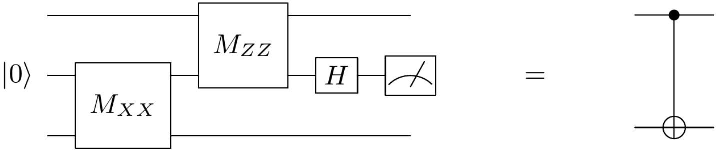

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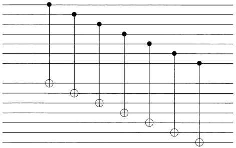
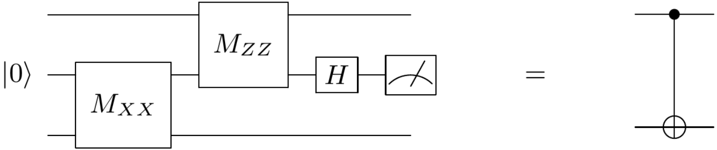


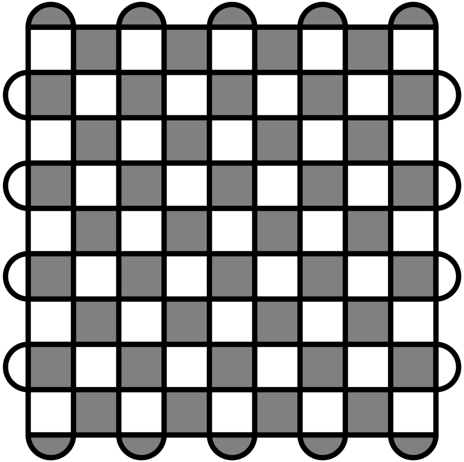


We'll focus on fault-tolerant quantum memory for now.

The Surface Code

Requirements

The stabilisers of the surface code are local, constant-weight, and can be measured with a planar circuit:



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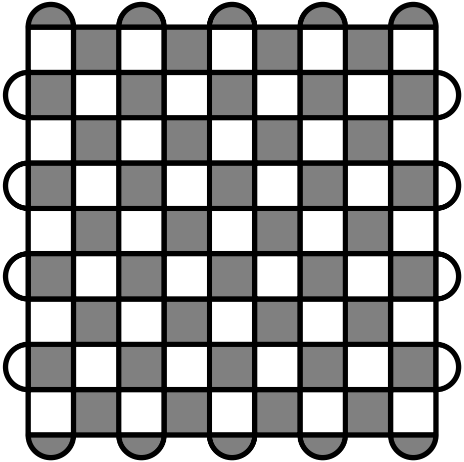

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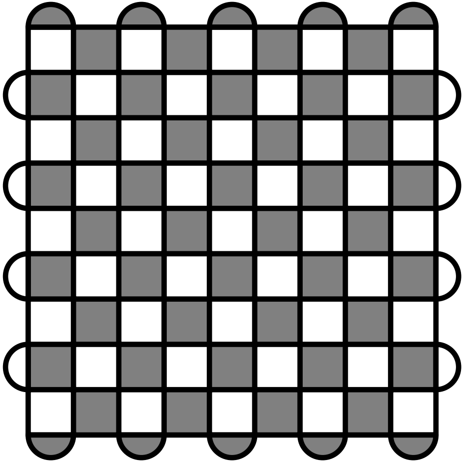

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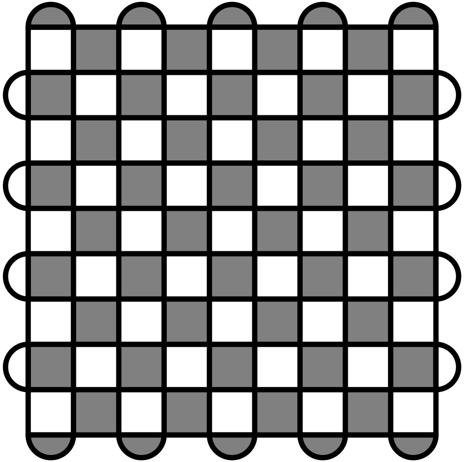

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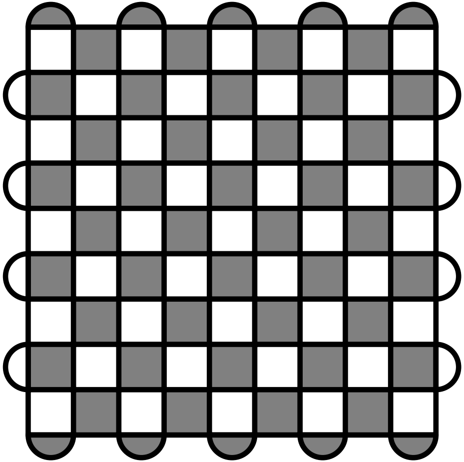

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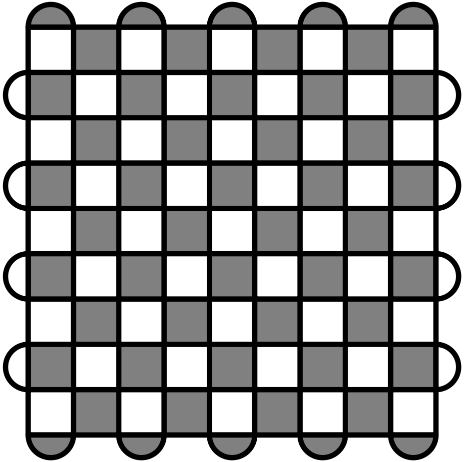

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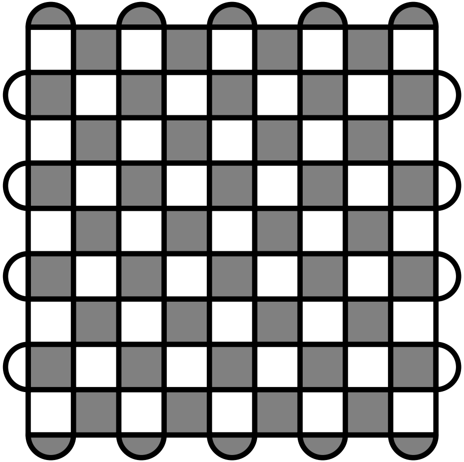

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- On average, can protect against $\sim 1\%$ of operations failing, as long as the failure mode is purely decoherent (not over/under-rotation).

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- The surface code doesn't compare well with the bounds on how well a quantum code can function. k/n can approach a constant as $n \to \infty$, but for the surface code, it decays to 0. This implies that large computations have large overhead.
- Most urgent disadvantage is that frequently-used decoding algorithms take an amount of time that scales (provably) as d^{6-12}, resulting in a backlog.

The Backlog Problem

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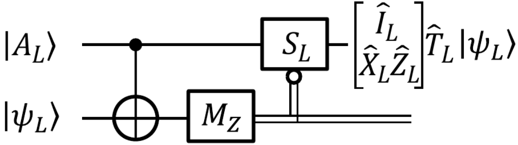
To produce a universal gate set, some gates are performed by *gate teleportation*:



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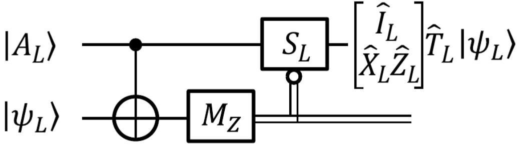

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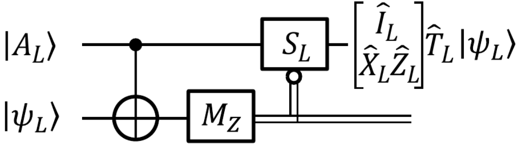

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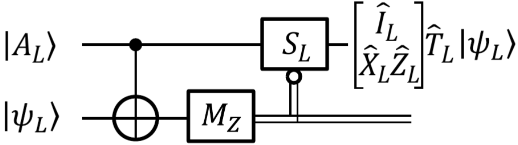

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Repeated feed-forward results in computation time scaling like a power tower.
Exercise: What happens if the decoder takes time $O(d)$?

Fast Decoders for Topological Quantum Codes

Known Decoders

Renormalisation Group

Fast Decoders for Topological Quantum Codes

Guillaume Duclos-Cianci and David Poulin
Département de Physique, Université de Sherbrooke, Québec, Canada
(Dated: February 5, 2010)

We present a family of algorithms, combining real-space renormalization methods and belief propagation, to estimate the free energy of a topologically ordered system in the presence of defects. Such an algorithm is needed to preserve the quantum information stored in the ground space of a topologically ordered system and to decode topological error-correcting codes. For a system of linear size ℓ, our algorithm runs in time $\log \ell$ compared to ℓ^6 needed for the minimum-weight perfect matching algorithm previously used in this context and achieves a higher depolarizing error threshold.

Inherently parallel algorithms exist to reduce the decoding time to $\log(d)$.

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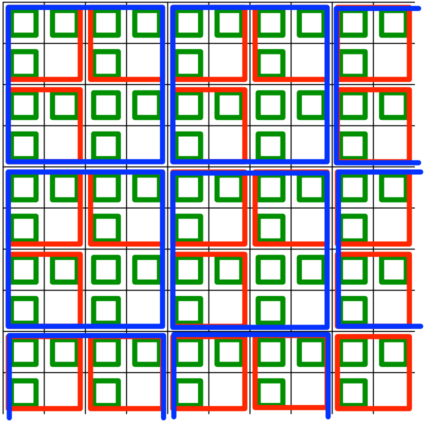
This speedup requires us to use a local lookup table and modified belief propagation to achieve a parallelizable algorithm:



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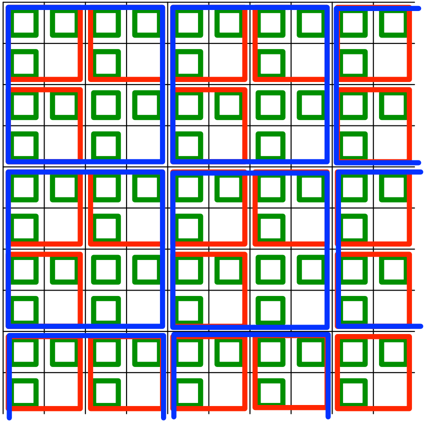

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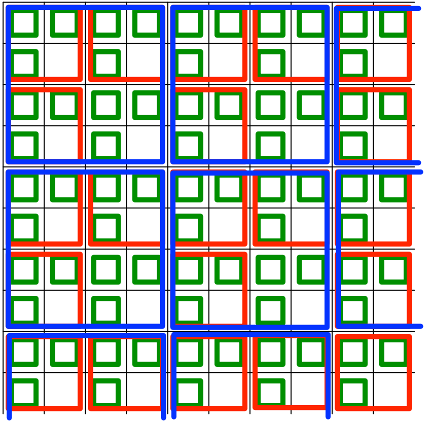

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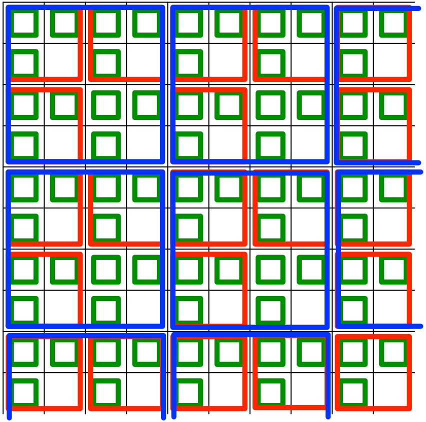

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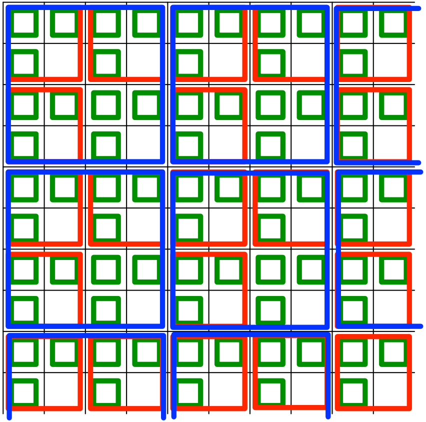

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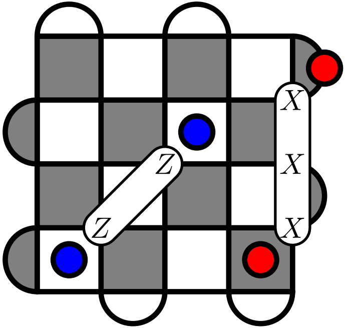
The syndromes of the surface code appear at the ends of chains of errors:



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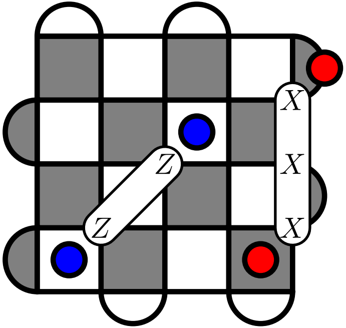

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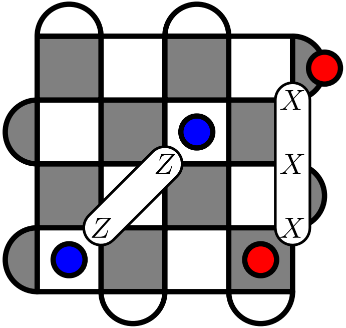

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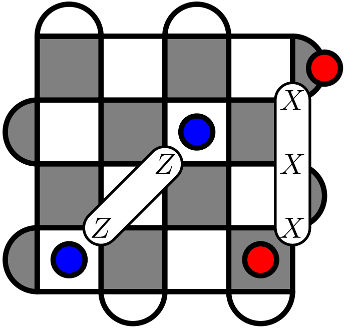

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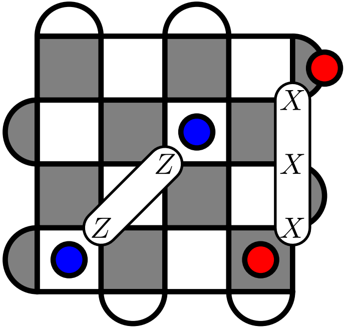

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Our input graphs are extremely structured, though, so there are shortcuts we can take.

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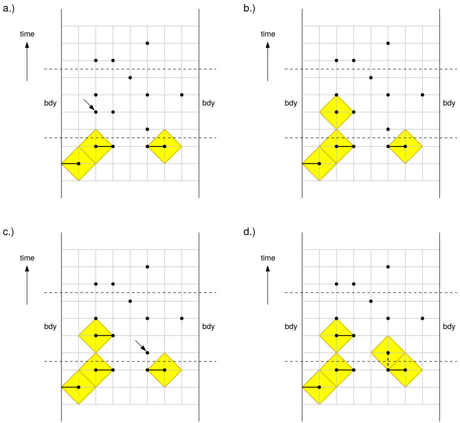
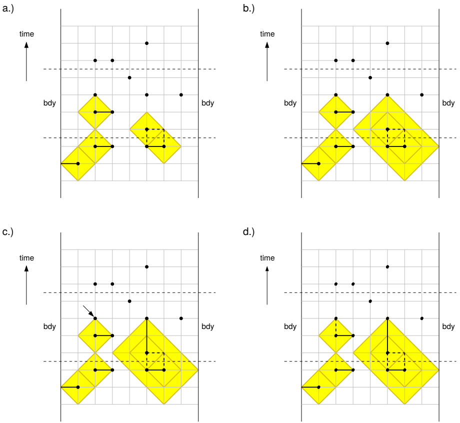
To speed up the decoding, Fowler interleaves the running of the algorithm with the measurement of syndromes:





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Parallel Approximate MWPM

Approximating Minimum Weight Perfect Matchings for Complete Graphs Satisfying the Triangle Inequality

N.W. Holloway, S. Ravindran and A.M. Gibbons*

Department of Computer Science, University of Warwick, Coventry CV4 7AL, UK.

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- Nobody knows how well it works for decoding …

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Neural Networks

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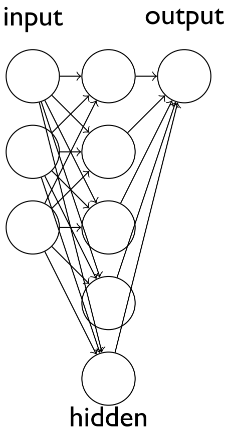

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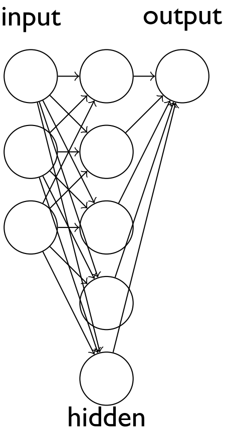

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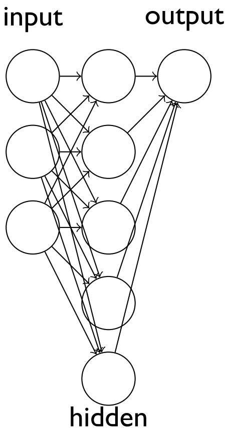

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Unknown Decoders

Neural Networks

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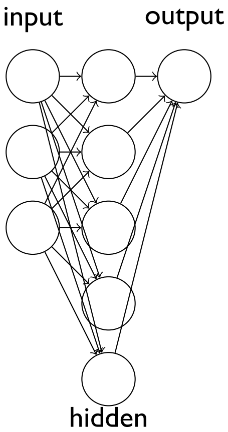

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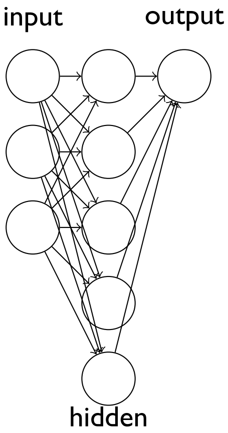

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- Composition implies gradient in $\vec{w}$-space can be simplified using the chain rule.
- For large enough networks, many knowledge-free fits can be obtained.
- Can they decode the surface code? (Our research.)

Unknown Decoders

Neural Networks

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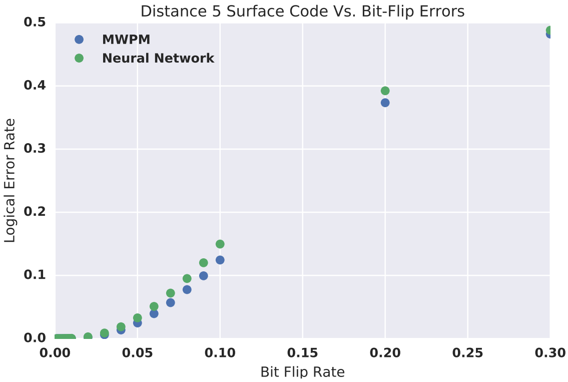
Unknown Decoders

Neural Networks

The distance-5 code has 12-bit syndromes, and the decoder output is a single bit. Using three hidden layers (8, 6, and 4 vertices), we can obtain semi-reasonable performance:



- Ongoing work with Savvas Varsamopoulos (TU Delft) and Xiaotong Ni (MPQ)
- Next steps: Encode space and time symmetries of decoding problem in neural network, move to higher distance.
- Central question: Can a threshold be obtained with a constant number of layers?

Questions?