Introduction to Lattice Surgery

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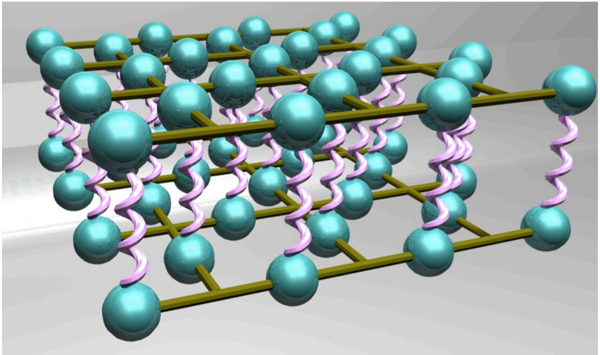
Why Do We Need Lattice Surgery?

Gates By Measurement

Proper Lattice Surgery

Why Do We Need Lattice Surgery?

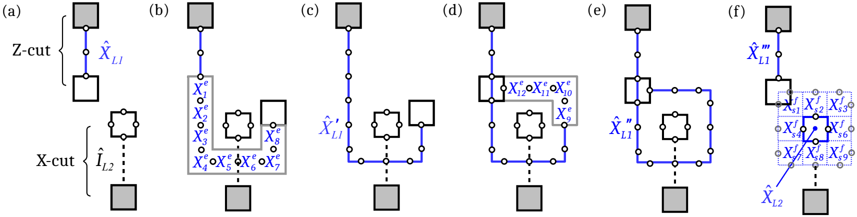
Transversality is a great criterion *for one-qubit gates*. For CNOT gates, though, transversality introduces another dimension:



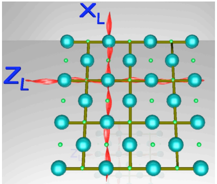
What do we do?

Braiding Recap

By *changing the measurement schedule*, we can implement unitaries which we otherwise couldn't:



However, we'd much rather do this on non-defect codes:



To see how, we'll need to recap (introduce?) some stabilizer calculus.

Stabilizer Calculus Recap

We all know what happens when we act a Clifford gate on an *n*-qubit Pauli:

$$H_j : \begin{array}{ccc} X_j & \rightarrow & Z_j \\ Z_j & \rightarrow & X_j \end{array} \quad P_j : \begin{array}{ccc} X_j & \rightarrow & Y_j \\ Z_j & \rightarrow & Z_j \end{array} \quad CNOT_{jk} : \begin{array}{ccc} X_j & \rightarrow & X_j X_k \\ Z_k & \rightarrow & Z_j Z_k \end{array}$$

What happens when we measure a Pauli?

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- Measurement collapses the state into an eigenstate of the measured operator, so the stabilizer property is preserved.

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- If $(-1)^b M$ (where M is the measured Pauli) is in the stabilizer, the bit b is returned.

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- If $(-1)^b M$ (where M is the measured Pauli) is in the stabilizer, the bit b is returned.
- If M is not in the stabilizer, but commutes with the stabilizer, M enters the stabiliser, and the dimension of the stabilised subspace is reduced by $1/2$.

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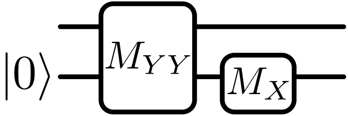
What happens when we measure a Pauli?

- Measurement collapses the state into an eigenstate of the measured operator, so the stabilizer property is preserved.
- If $(-1)^b M$ (where M is the measured Pauli) is in the stabilizer, the bit b is returned.
- If M is not in the stabilizer, but commutes with the stabilizer, M enters the stabiliser, and the dimension of the stabilised subspace is reduced by $1/2$.
- If M does not commute with the stabilizer, a new stabilizer is formed from M and the largest set of stabilizers which commute with M.

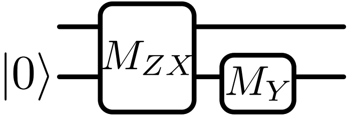
H, P, $CNOT$ By Measurement

It may help to see some examples (without classical correction):

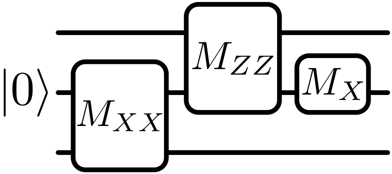
H



P



$CNOT$



Logical Gates By Measurement

To perform Clifford operations, then, all we have to do is measure the logical Paulis of some stabilizer code.