Introduction to Colour Codes

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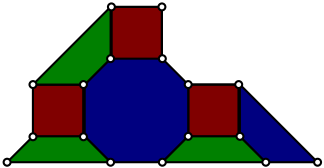
The Basics



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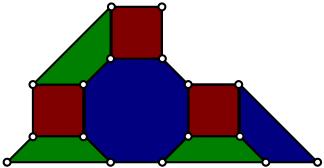

- Color codes are stabiliser codes

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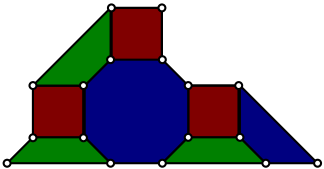

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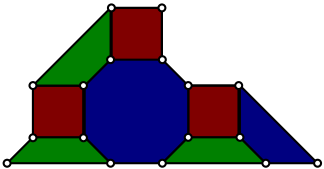

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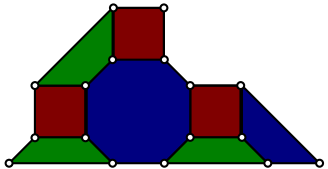
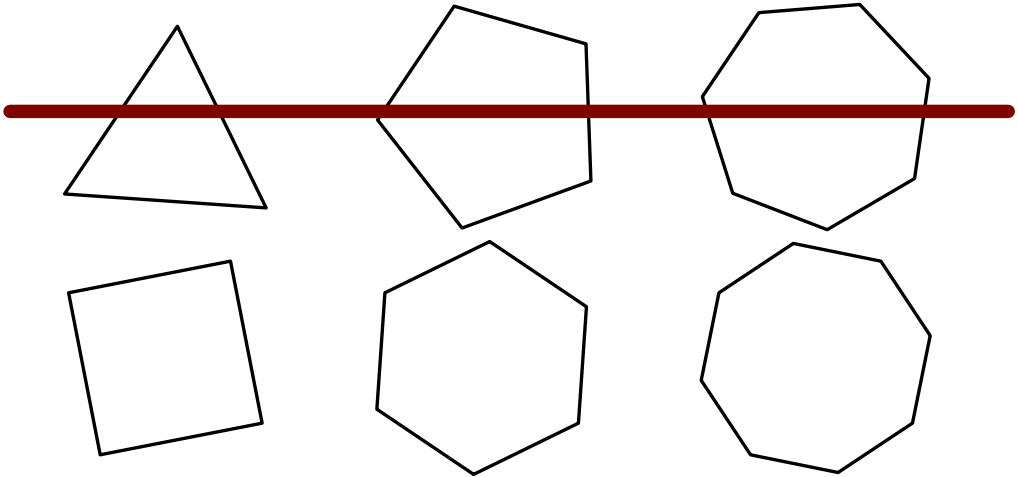


- Color codes are stabiliser codes
- qubits are placed on vertices of a tiling (construction coming up)
- stabilisers are constant-weight, transversal X and Z on every face
- One logical qubit encoded in $\mathcal{O}(d^2)$ physical qubits

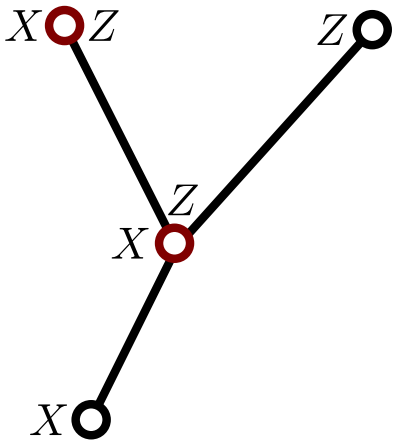
Commuting Stabilisers on Faces

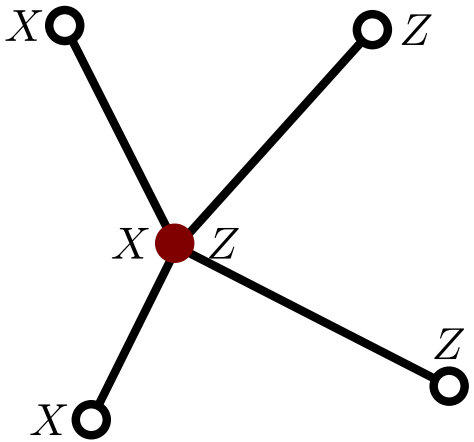
In order for X and Z stabilisers on the same face to commute, the face has to be *even-weight*:



Commuting Stabilisers on Faces

In order for X and Z stabilisers on different faces to commute, the tiling has to have degree three (it must be *trivalent*):





Commuting Stabilisers on Faces

Face-3-colorable tilings satisfy this requirement:

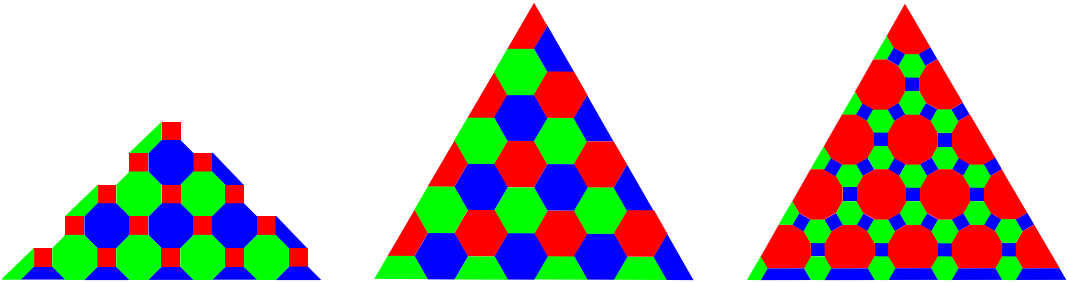
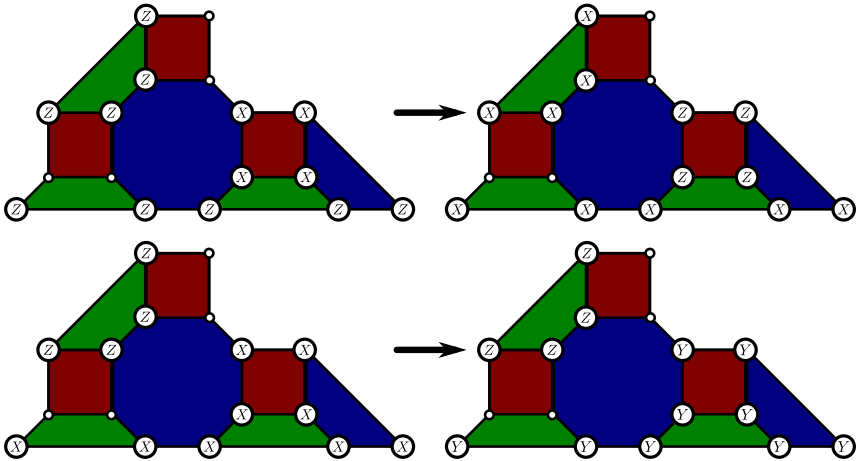


Figure: Three admissible tilings, with $n = \frac{1}{2}d^2 + d - \frac{1}{2}$, $n = \frac{3}{4}d^2 + \frac{1}{4}$, and $n = \frac{3}{2}d^2 - 3d + \frac{5}{2}$ respectively.

Minimum-weight logicals are supported on sides of the triangle.

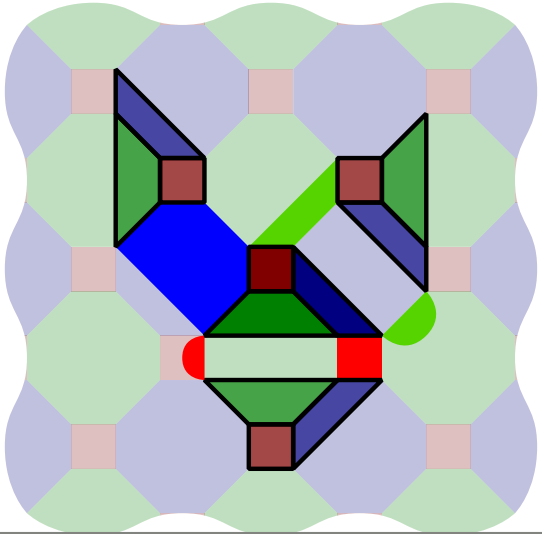
Single-Qubit Gates

Color codes admit a transversal Hadamard and phase gate:



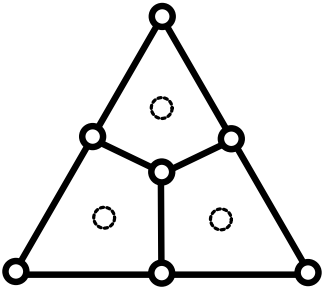
Multi-Qubit Gates

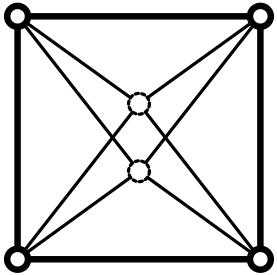
CNOT gates (and other multi-qubit Cliffords) can be accomplished by lattice surgery:



Pros & Cons

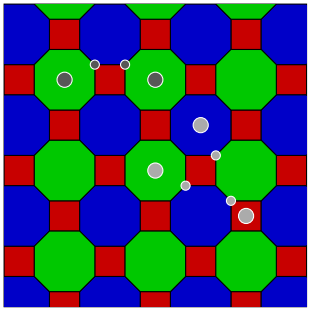
Fewer physical qubits per logical qubit at the same distance, but higher stabiliser weight, and you risk non-planarity:





Decoding

Syndromes can appear in *triples*, making ‘matching’ an NP-complete problem:



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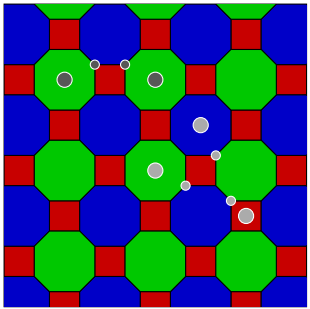

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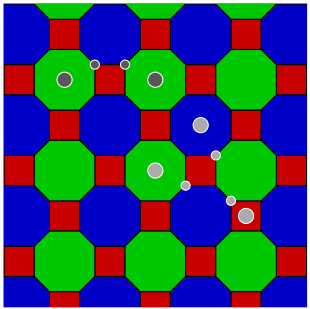

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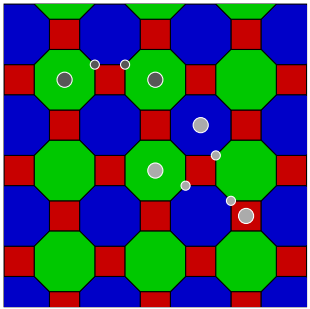

- statistical physics simulations show that thresholds at or above the surface code's can be obtained
- efficient decoders can also be produced using RG, or a reduction to multiple matching problems.
- no efficient decoder has obtained the optimal threshold.

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- They use fewer data qubits, and permit easier logical gates, at the cost of higher stabiliser weight.
- Efficient decoders do not yet hit the optimal threshold.