



# Question Bank

# Math

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## Nonlinear Functions (key)

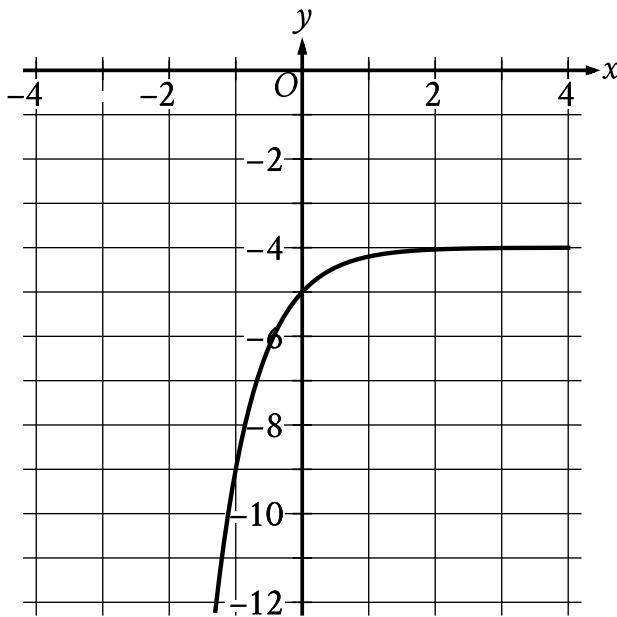




## Question ID 6abec9a8

1.1

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 6abec9a8**

What is the  $y$ -intercept of the graph shown?

- A.  $(-1, -9)$
- B.  $(0, -5)$
- C.  $(0, -4)$
- D.  $(0, 0)$

**ID: 6abec9a8 Answer**

Correct Answer: B

Rationale

Choice B is correct. The  $y$ -intercept of a graph in the  $xy$ -plane is the point  $(x, y)$  on the graph where  $x = 0$ . At  $x = 0$ , the corresponding value of  $y$  is  $-5$ . Therefore, the  $y$ -intercept of the graph shown is  $(0, -5)$ .

Choice A is incorrect and may result from conceptual errors.

Choice C is incorrect. This is the  $y$ -intercept of a graph in the  $xy$ -plane that intersects the  $y$ -axis at  $y = -4$ , not  $y = -5$ .

Choice D is incorrect. This is the  $y$ -intercept of a graph in the  $xy$ -plane that intersects the  $y$ -axis at  $y = 0$ , not  $y = -5$ .

Question Difficulty: Easy

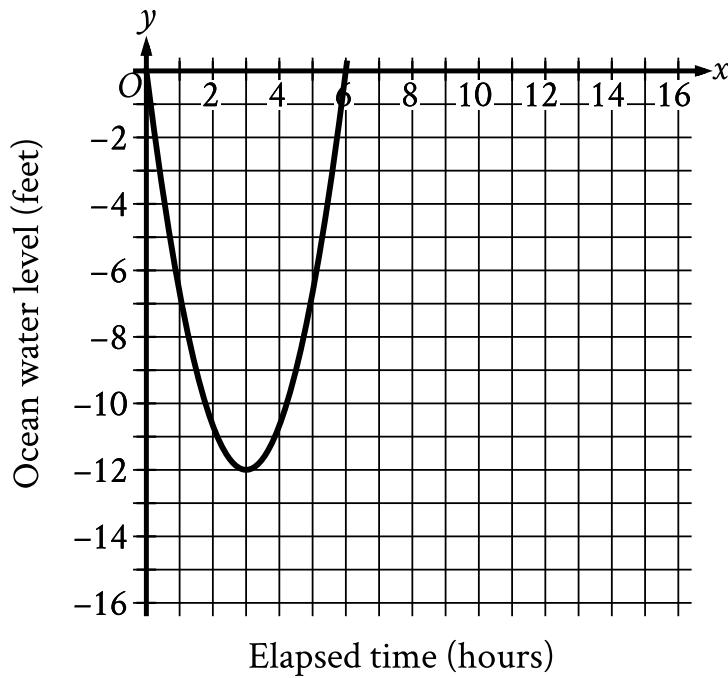




# Question ID 1ee962ec

1.2

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 1ee962ec**

Scientists recorded data about the ocean water levels at a certain location over a period of 6 hours. The graph shown models the data, where  $y = 0$  represents sea level. Which table gives values of  $x$  and their corresponding values of  $y$  based on the model?

A.

$x$	$y$
0	-12
0	3
3	6

B.

$x$	$y$
0	0
3	12
0	-6

C.

$x$	$y$
0	0
3	-12

6	0
<b>x</b>	<b>y</b>
0	0
12	3
-6	0



**ID: 1ee962ec Answer**

Correct Answer: C

Rationale

Choice C is correct. Each point  $(x, y)$  on the graph represents an elapsed time  $x$ , in hours, and the corresponding ocean water level  $y$ , in feet, at a certain location based on the model. The graph shown passes through the points  $(0, 0)$ ,  $(3, -12)$ , and  $(6, 0)$ . Thus, the table in choice C gives the values of  $x$  and their corresponding values of  $y$  based on the model.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Easy

# Question ID 788bfd56



1.3

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 788bfd56**

The function  $f$  is defined by  $f(x) = 4 + \sqrt{x}$ . What is the value of  $f(144)$ ?

- A. 0
- B. 16
- C. 40
- D. 76

**ID: 788bfd56 Answer**

Correct Answer: B

Rationale

Choice B is correct. The value of  $f(144)$  is the value of  $f(x)$  when  $x = 144$ . It's given that the function  $f$  is defined by  $f(x) = 4 + \sqrt{x}$ . Substituting 144 for  $x$  in this equation yields  $f(144) = 4 + \sqrt{144}$ . Since the positive square root of 144 is 12, it follows that this equation can be rewritten as  $f(144) = 4 + 12$ , or  $f(144) = 16$ . Therefore, the value of  $f(144)$  is 16.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This is the value of  $f(1,296)$ , not  $f(144)$ .

Choice D is incorrect. This is the value of  $f(5,184)$ , not  $f(144)$ .

Question Difficulty: Easy



# Question ID b39d74a0

1.4

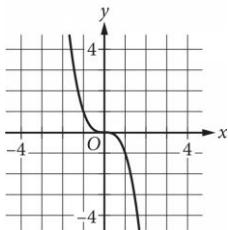
Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: b39d74a0**

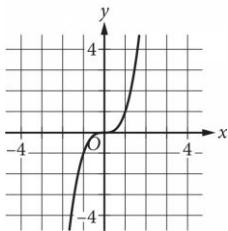
x	y
0	0
1	1
2	8
3	27

The table shown includes some values of  $x$  and their corresponding values of  $y$ . Which of the following graphs in the  $xy$ -plane could represent the relationship between  $x$  and  $y$ ?

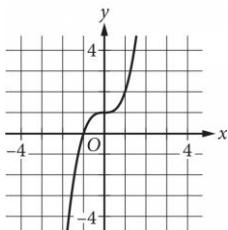
A.



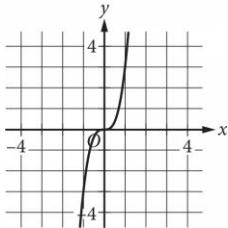
B.



C.



D.



**ID: b39d74a0 Answer**

Correct Answer: B

Rationale

Choice B is correct. Each pair of values shown in the table gives the ordered pair of coordinates for a point that lies on the graph that represents the relationship between  $x$  and  $y$  in the  $xy$ -plane:  $(0,0)$ ,  $(1,1)$ ,  $(2,8)$ , and  $(3,27)$ .

. Only the graph in choice B passes through the points listed in the table that are visible in the given choices.

Choices A, C, and D are incorrect. None of these graphs passes through the point  $(1,1)$ .

Question Difficulty: Easy

# Question ID 5377d9cf



1.5

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 5377d9cf**

If  $f(x) = \frac{x^2 - 6x + 3}{x - 1}$ ,

what is  $f(-1)$ ?

- A. -5
- B. -2
- C. 2
- D. 5

**ID: 5377d9cf Answer**

Correct Answer: A

Rationale

Choice A is correct. Substituting -1 for x in the equation that defines f gives  $f(-1) = \frac{(-1)^2 - 6(-1) + 3}{(-1) - 1}$ .

Simplifying the expressions in the numerator and denominator yields  $\frac{1+6+3}{-2} = \frac{10}{-2}$ , which is equal to  $-\frac{10}{2}$  or -5.

Choices B, C, and D are incorrect and may result from misapplying the order of operations when substituting -1 for x.

Question Difficulty: Easy



# Question ID 75915e3c

1.6

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 75915e3c**

$$f(x) = 2(3^x)$$

For the function  $f$  defined above, what is the value of  $f(2)$ ?

- A. 9
- B. 12
- C. 18
- D. 36

**ID: 75915e3c Answer**

Correct Answer: C

Rationale

Choice C is correct. The value of  $f(2)$  is found by evaluating the expression  $2(3^x)$  when  $x = 2$ . Substituting 2 for  $x$  in the given equation yields  $f(2) = 2(3^2)$ . Simplifying  $3^2$  in the equation results in  $f(2) = 2(9)$ . Evaluating the right-hand side of the equation yields  $f(2) = 18$ . Therefore, the value of  $f(2)$  is 18.

Choice A is incorrect and may result from evaluating the expression as  $(3^2)$ . Choice B is incorrect and may result from evaluating the expression as  $2(3 \cdot 2)$ . Choice D is incorrect and may result from evaluating the expression as  $(2 \cdot 3)^2$ .

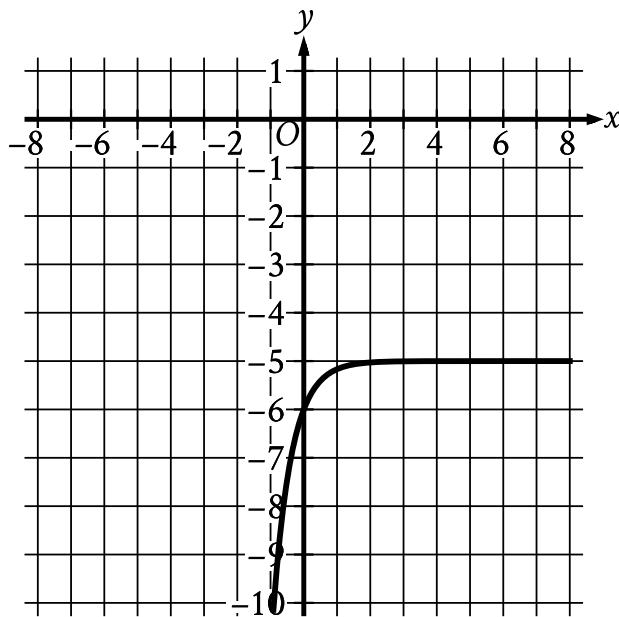
Question Difficulty: Easy



# Question ID 7160cbb3

1.7

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 7160cbb3**

What is the  $y$ -intercept of the graph shown?

- A.  $(0, -6)$
- B.  $(-6, 0)$
- C.  $(0, 0)$
- D.  $(-5, -5)$

**ID: 7160cbb3 Answer**

Correct Answer: A

Rationale

Choice A is correct. The  $y$ -intercept of a graph in the  $xy$ -plane is the point  $(x, y)$  on the graph where  $x = 0$ . For the graph shown, at  $x = 0$ , the corresponding value of  $y$  is  $-6$ . Therefore, the  $y$ -intercept of the graph shown is  $(0, -6)$ .

Choice B is incorrect and may result from conceptual errors.

Choice C is incorrect and may result from conceptual errors.

Choice D is incorrect and may result from conceptual errors.

Question Difficulty: Easy



# Question ID 72ae8a87



1.8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 72ae8a87**

The function  $f(x) = 200,000(1.21)^x$  gives a company's predicted annual revenue, in dollars,  $x$  years after the company started selling light bulbs online, where  $0 < x \leq 10$ . What is the best interpretation of the statement " $f(5)$  is approximately equal to 518,748" in this context?

- A. 5 years after the company started selling light bulbs online, its predicted annual revenue is approximately 518,748 dollars.
- B. 5 years after the company started selling light bulbs online, its predicted annual revenue will have increased by a total of approximately 518,748 dollars.
- C. When the company's predicted annual revenue is approximately 518,748 dollars, it is 5 times the predicted annual revenue for the previous year.
- D. When the company's predicted annual revenue is approximately 518,748 dollars, it is 5% greater than the predicted annual revenue for the previous year.

**ID: 72ae8a87 Answer**

Correct Answer: A

Rationale

Choice A is correct. It's given that the function  $f(x) = 200,000(1.21)^x$  gives a company's predicted annual revenue, in dollars,  $x$  years after the company started selling light bulbs online. It follows that  $f(x)$  represents the company's predicted annual revenue, in dollars,  $x$  years after the company started selling light bulbs online. Since the value of  $f(5)$  is the value of  $f(x)$  when  $x = 5$ , it follows that " $f(5)$  is approximately equal to 518,748" means that  $f(x)$  is approximately equal to 518,748 when  $x = 5$ . Therefore, the best interpretation of the statement " $f(5)$  is approximately equal to 518,748" in this context is 5 years after the company started selling light bulbs online, its predicted annual revenue is approximately 518,748 dollars.

Choice B is incorrect and may result from conceptual errors.

Choice C is incorrect and may result from conceptual errors.

Choice D is incorrect and may result from conceptual errors.

Question Difficulty: Easy



## Question ID 09f58996

1.9

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 09f58996**

The function  $f$  is defined by  $f(x) = 6 + \sqrt{x}$ . What is the value of  $f(36)$ ?

**ID: 09f58996 Answer**

Correct Answer: 12

Rationale

The correct answer is **12**. The value of  $f(36)$  is the value of  $f(x)$  when  $x = 36$ . Substituting  $36$  for  $x$  in the given equation yields  $f(36) = 6 + \sqrt{36}$ , which is equivalent to  $f(36) = 6 + 6$ , or  $f(36) = 12$ . Thus, the value of  $f(36)$  is **12**.

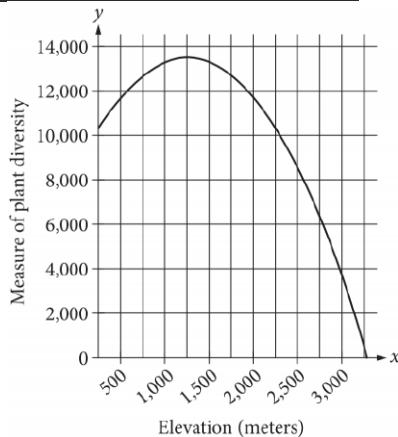
Question Difficulty: Easy



## Question ID ebe4bde0

1.10

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: ebe4bde0**

The quadratic function graphed above models a particular measure of plant diversity as a function of the elevation in a region of Switzerland. According to the model, which of the following is closest to the elevation, in meters, at which plant diversity is greatest?

- A. 13,500
- B. 3,000
- C. 1,250
- D. 250

**ID: ebe4bde0 Answer**

Correct Answer: C

Rationale

Choice C is correct. Each point  $(x, y)$  on the graph represents the elevation  $x$ , in meters, and the corresponding measure of plant diversity  $y$  in a region of Switzerland. Therefore, the point on the graph with the greatest  $y$ -coordinate represents the location that has the greatest measure of plant diversity in the region. The greatest  $y$ -coordinate of any point on the graph is approximately 13,500. The  $x$ -coordinate of that point is approximately 1,250. Therefore, the closest elevation at which the plant diversity is the greatest is 1,250 meters.

Choice A is incorrect. This value is closest to the greatest  $y$ -coordinate of any point on the graph and therefore represents the greatest measure of plant diversity, not the elevation where the greatest measure of plant diversity occurs. Choice B is incorrect. At an elevation of 3,000 meters the measure of plant diversity is approximately 4,000. Because there are points on the graph with greater  $y$ -coordinates, 4,000 can't be the greatest measure of plant diversity, and 3,000 meters isn't the elevation at which the greatest measure of plant diversity occurs. Choice D is incorrect. At an elevation of 250 meters, the measure of plant diversity is approximately 11,000. Because there are points with greater  $y$ -coordinates, 11,000 can't be the

greatest measure of plant diversity and 250 meters isn't the elevation at which the greatest measure of plant diversity occurs.



Question Difficulty: Easy



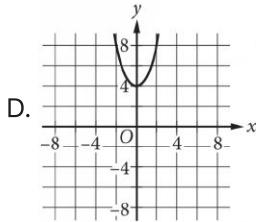
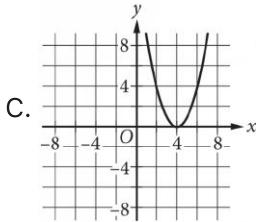
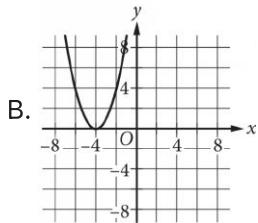
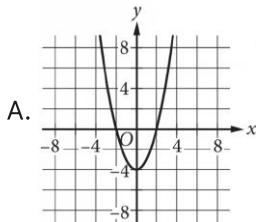
# Question ID d46da42c

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: d46da42c**

$$f(x) = x^2 + 4$$

The function  $f$  is defined as shown. Which of the following graphs in the  $xy$ -plane could be the graph of  $y = f(x)$ ?

**ID: d46da42c Answer**

Correct Answer: D

Rationale

Choice D is correct. For the quadratic function  $f(x) = x^2 + 4$ , the vertex of the graph is  $(0, 4)$ , and because the  $x^2$  term is positive, the vertex is the minimum of the function. Choice D is the only option that meets these conditions.

Choices A, B, and C are incorrect. The vertex of each of these graphs doesn't correspond to the minimum of the given function.



Question Difficulty: Easy



## Question ID 79ba511a

1.12

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 79ba511a**

The function  $f$  is defined by  $f(x) = x^3 + 15$ . What is the value of  $f(2)$ ?

- A. 20
- B. 21
- C. 23
- D. 24

**ID: 79ba511a Answer**

Correct Answer: C

Rationale

Choice C is correct. The value of  $f(2)$  is the value of  $f(x)$  when  $x = 2$ . Substituting 2 for  $x$  in the given function yields  $f(2) = (2)^3 + 15$ , or  $f(2) = 8 + 15$ , which is equivalent to  $f(2) = 23$ . Therefore, the value of  $f(2)$  is 23.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect. This is the value of  $f(2)$  when  $f(x) = x(3) + 15$ , rather than  $f(x) = x^3 + 15$ .

Choice D is incorrect and may result from conceptual or calculation errors.

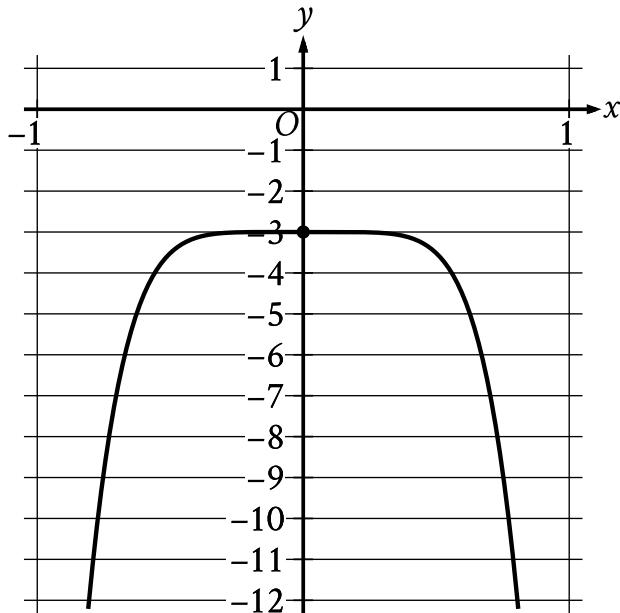
Question Difficulty: Easy



# Question ID 50418728

1.13

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 50418728**

The graph of the polynomial function  $f$ , where  $y = f(x)$ , is shown. The  $y$ -intercept of the graph is  $(0, y)$ . What is the value of  $y$ ?

**ID: 50418728 Answer**

Correct Answer: -3

Rationale

The correct answer is  $-3$ . The  $y$ -intercept of the graph of a function in the  $xy$ -plane is the point where the graph crosses the  $y$ -axis. The graph of the polynomial function shown crosses the  $y$ -axis at the point  $(0, -3)$ . It's given that the  $y$ -intercept of the graph is  $(0, y)$ . Thus, the value of  $y$  is  $-3$ .

Question Difficulty: Easy



## Question ID ee05c84e

1.14

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: ee05c84e**

$$f(x) = (x + 0.25x)(50 - x)$$

The function  $f$  is defined above. What is the value of  $f(20)$ ?

- A. 250
- B. 500
- C. 750
- D. 2,000

**ID: ee05c84e Answer**

Correct Answer: C

Rationale

Choice C is correct. Adding the like terms  $x$  and  $0.25x$  yields the equation  $f(x) = (1.25x)(50 - x)$ . Substituting 20 for  $x$  yields  $f(20) = (1.25(20))(50 - 20)$ . The product  $1.25(20)$  is equal to 25, and the difference  $50 - 20$  is equal to 30. Substituting these values in the given equation gives  $f(20) = (25)(30)$ , and multiplying 25 by 30 results in  $f(20) = 750$ .

Choices A, B, and D are incorrect and may result from conceptual or computational errors when finding the value of  $f(20)$ .

Question Difficulty: Easy

# Question ID f89af023



2.1

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: f89af023**

A rectangular volleyball court has an area of 162 square meters. If the length of the court is twice the width, what is the width of the court, in meters?

- A. 9
- B. 18
- C. 27
- D. 54

**ID: f89af023 Answer**

Correct Answer: A

Rationale

Choice A is correct. It's given that the volleyball court is rectangular and has an area of 162 square meters. The formula for the area of a rectangle is  $A = \ell \cdot w$ , where  $A$  is the area,  $\ell$  is the length, and  $w$  is the width of the rectangle. It's also given that the length of the volleyball court is twice the width, thus  $\ell = 2w$ . Substituting the given value into the formula for the area of a rectangle and using the relationship between length and width for this rectangle yields  $162 = (2w)(w)$ . This equation can be rewritten as  $162 = 2w^2$ . Dividing both sides of this equation by 2 yields  $81 = w^2$ . Taking the square root of both sides of this equation yields  $\pm 9 = w$ . Since the width of a rectangle is a positive number, the width of the volleyball court is 9 meters.

Choice B is incorrect because this is the length of the rectangle. Choice C is incorrect because this is the result of using 162 as the perimeter rather than the area. Choice D is incorrect because this is the result of calculating  $w$  in the equation  $162 = 2w + w$  instead of  $162 = (2w)(w)$ .

Question Difficulty: Medium



## Question ID e53add44

2.2

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: e53add44**

$$S(n) = 38,000a^n$$

The function  $S$  above models the annual salary, in dollars, of an employee  $n$  years after starting a job, where  $a$  is a constant. If the employee's salary increases by 4% each year, what is the value of  $a$ ?

- A. 0.04
- B. 0.4
- C. 1.04
- D. 1.4

**ID: e53add44 Answer**

Correct Answer: C

Rationale

Choice C is correct. A model for a quantity  $S$  that increases by a certain percentage per time period  $n$  is an exponential function in the form  $S(n) = I\left(1 + \frac{r}{100}\right)^n$ , where  $I$  is the initial value at time  $n = 0$  for  $r\%$  annual increase. It's given that the annual increase in an employee's salary is 4%, so  $r = 4$ . The initial value can be found by substituting 0 for  $n$  in the given function, which yields  $S(0) = 38,000$ . Therefore,  $I = 38,000$ .

Substituting these values for  $r$  and  $I$  into the form of the exponential function  $S(n) = I\left(1 + \frac{r}{100}\right)^n$  yields  $S(n) = 38,000\left(1 + \frac{4}{100}\right)^n$ , or  $S(n) = 38,000(1.04)^n$ . Therefore, the value of  $a$  in the given function is 1.04.

Choices A, B, and D are incorrect and may result from incorrectly representing the annual increase in the exponential function.

Question Difficulty: Medium



## Question ID 926c246b

2.3

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

### ID: 926c246b

$$D = 5,640(1.9)^t$$

The equation above estimates the global data traffic  $D$ , in terabytes, for the year that is  $t$  years after 2010. What is the best interpretation of the number 5,640 in this context?

- A. The estimated amount of increase of data traffic, in terabytes, each year
- B. The estimated percent increase in the data traffic, in terabytes, each year
- C. The estimated data traffic, in terabytes, for the year that is  $t$  years after 2010
- D. The estimated data traffic, in terabytes, in 2010

### ID: 926c246b Answer

Correct Answer: D

#### Rationale

Choice D is correct. Since  $t$  represents the number of years after 2010, the estimated data traffic, in terabytes, in 2010 can be calculated using the given equation when  $t = 0$ . Substituting 0 for  $t$  in the given equation yields

$$D = 5,640(1.9)^0, \text{ or } 5,640(1) = 5,640.$$

Thus, 5,640 represents the estimated data traffic, in terabytes, in 2010. Choice A is incorrect. Since the equation is exponential, the amount of increase of data traffic each year isn't constant. Choice B is incorrect. According to the equation, the percent increase in data traffic each year is 90%. Choice C is incorrect. The estimated data traffic, in terabytes, for the year that is  $t$  years after 2010 is represented by  $D$ , not the number 5,640.

Question Difficulty: Medium



## Question ID 50e40f08

2.4

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 50e40f08**

$$f(x) = (x + 6)(x - 4)$$

If the given function  $f$  is graphed in the  $xy$ -plane, where  $y = f(x)$ , what is the  $x$ -coordinate of an  $x$ -intercept of the graph?

**ID: 50e40f08 Answer**

Correct Answer: -6, 4

Rationale

The correct answer is either **-6** or **4**. The  $x$ -intercepts of a graph in the  $xy$ -plane are the points where  $y = 0$ . Thus, for an  $x$ -intercept of the graph of  $y = f(x)$ ,  $0 = f(x)$ . Substituting  $0$  for  $f(x)$  in the equation  $f(x) = (x + 6)(x - 4)$  yields  $0 = (x + 6)(x - 4)$ . By the zero product property,  $x + 6 = 0$  and  $x - 4 = 0$ . Subtracting **6** from both sides of the equation  $x + 6 = 0$  yields  $x = -6$ . Adding **4** to both sides of the equation  $x - 4 = 0$  yields  $x = 4$ . Therefore, the  $x$ -coordinates of the  $x$ -intercepts of the graph of  $y = f(x)$  are **-6** and **4**. Note that **-6** and **4** are examples of ways to enter a correct answer.

Question Difficulty: Medium



## Question ID be0c419e

2.5

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: be0c419e**

Immanuel purchased a certain rare coin on January 1. The function  $f(x) = 65(1.03)^x$ , where  $0 \leq x \leq 10$ , gives the predicted value, in dollars, of the rare coin  $x$  years after Immanuel purchased it. What is the best interpretation of the statement " $f(8)$  is approximately equal to 82" in this context?

- A. When the rare coin's predicted value is approximately 82 dollars, it is 8% greater than the predicted value, in dollars, on January 1 of the previous year.
- B. When the rare coin's predicted value is approximately 82 dollars, it is 8 times the predicted value, in dollars, on January 1 of the previous year.
- C. From the day Immanuel purchased the rare coin to 8 years after Immanuel purchased the coin, its predicted value increased by a total of approximately 82 dollars.
- D. 8 years after Immanuel purchased the rare coin, its predicted value is approximately 82 dollars.

**ID: be0c419e Answer**

Correct Answer: D

Rationale

Choice D is correct. It's given that the function  $f(x) = 65(1.03)^x$  gives the predicted value, in dollars, of a certain rare coin  $x$  years after Immanuel purchased it. It follows that  $f(x)$  represents the predicted value, in dollars, of the coin  $x$  years after Immanuel purchased it. Since the value of  $f(8)$  is the value of  $f(x)$  when  $x = 8$ , it follows that " $f(8)$  is approximately equal to 82" means that  $f(x)$  is approximately equal to 82 when  $x = 8$ . Therefore, the best interpretation of the statement " $f(8)$  is approximately equal to 82" in this context is 8 years after Immanuel purchased the rare coin, its predicted value is approximately 82 dollars.

Choice A is incorrect and may result from conceptual errors.

Choice B is incorrect and may result from conceptual errors.

Choice C is incorrect and may result from conceptual errors.

Question Difficulty: Medium



# Question ID a31417d1

2.6

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: a31417d1**

From 2005 through 2014, the number of music CDs sold in the United States declined each year by approximately 15% of the number sold the preceding year. In 2005, approximately 600 million CDs were sold in the United States. Of the following, which best models  $C$ , the number of millions of CDs sold in the United States,  $t$  years after 2005?

- A.  $C = 600(0.15)^t$
- B.  $C = 600(0.85)^t$
- C.  $C = 600(1.15)^t$
- D.  $C = 600(1.85)^t$

**ID: a31417d1 Answer**

Correct Answer: B

Rationale

Choice B is correct. A model for a quantity  $C$  that decreases by a certain percentage per time period  $t$  is an

exponential equation in the form  $C = I \left(1 - \frac{r}{100}\right)^t$ , where  $I$  is the initial value at time  $t = 0$  for  $r\%$  annual decline. It's given that  $C$  is the number of millions of CDs sold in the United States and that  $t$  is the number of years after 2005. It's also given that 600 million CDs were sold at time  $t = 0$ , so  $I = 600$ . This number declines

by 15% per year, so  $r = 15$ . Substituting these values into the equation produces  $C = 600 \left(1 - \frac{15}{100}\right)^t$ , or  $C = 600(0.85)^t$ .

Choice A is incorrect and may result from errors made when representing the percent decline. Choices C and D are incorrect. These equations model exponential increases in CD sales, not exponential decreases.

Question Difficulty: Medium



## Question ID c4cd5bcc

2.7

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: c4cd5bcc**

In the  $xy$ -plane, the  $y$ -coordinate of the  $y$ -intercept of the graph of the function  $f$  is  $c$ . Which of the following must be equal to  $c$ ?

- A.  $f(0)$
- B.  $f(1)$
- C.  $f(2)$
- D.  $f(3)$

**ID: c4cd5bcc Answer**

Correct Answer: A

Rationale

Choice A is correct. A  $y$ -intercept is the point in the  $xy$ -plane where the graph of the function crosses the  $y$ -axis, which is where  $x = 0$ . It's given that the  $y$ -coordinate of the  $y$ -intercept of the graph of function  $f$  is  $c$ . It follows that the coordinate pair representing the  $y$ -intercept must be  $(0, c)$ . Therefore,  $c$  must equal  $f(0)$ .

Choices B, C, and D are incorrect because  $f(1)$ ,  $f(2)$ , and  $f(3)$  would represent the  $y$ -value of the coordinate where  $x = 1$ ,  $x = 2$ , and  $x = 3$ , respectively.

Question Difficulty: Medium



## Question ID 78d5f91a

2.8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 78d5f91a**

$$f(x) = x^3 + 3x^2 - 6x - 1$$

For the function  $f$  defined above, what is the value of  $f(-1)$ ?

A. **-11**

B. **-7**

C. **7**

D. **11**

**ID: 78d5f91a Answer**

Correct Answer: C

Rationale

Choice C is correct. Substituting  $-1$  for  $x$  in the given function  $f$  gives  $f(-1) = (-1)^3 + 3(-1)^2 - 6(-1) - 1$ , which simplifies to  $f(-1) = -1 + 3(1) - 6(-1) - 1$ . This further simplifies to  $f(-1) = -1 + 3 + 6 - 1$ , or  $f(-1) = 7$ .

Choice A is incorrect and may result from correctly substituting  $-1$  for  $x$  in the function but incorrectly simplifying the resulting expression to  $f(-1) = -1 - 3 - 6 - 1$ , or  $-11$ . Choice B is incorrect and may result from arithmetic errors. Choice D is incorrect and may result from correctly substituting  $-1$  for  $x$  in the function but incorrectly simplifying the expression to  $f(-1) = 1 + 3 + 6 + 1$ , or  $11$ .

Question Difficulty: Medium



# Question ID d675744f

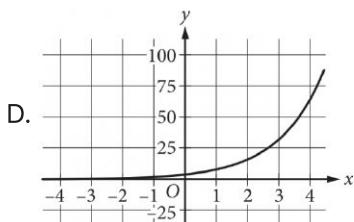
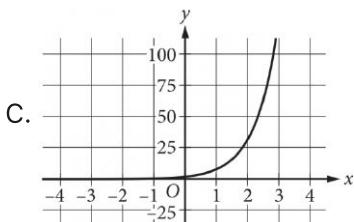
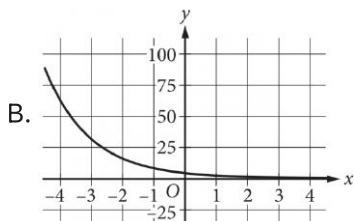
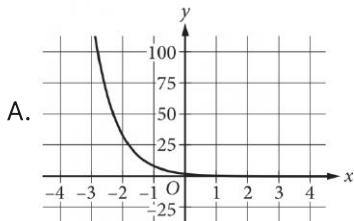
2.9

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: d675744f**

$$y = 4(2^x)$$

Which of the following is the graph in the  $xy$ -plane of the given equation?

**ID: d675744f Answer**

Correct Answer: D

Rationale

Choice D is correct. The  $y$ -intercept of the graph of an equation is the point  $(0, b)$ , where  $b$  is the value of  $y$  when  $x = 0$ . For the given equation,  $y = 4$  when  $x = 0$ . It follows that the  $y$ -intercept of the graph of the given equation is  $(0, 4)$ . Additionally, for the given equation, the value of  $y$  doubles for each increase of 1 in the value of  $x$ . Therefore, the graph contains the points  $(1, 8)$ ,  $(2, 16)$ ,  $(3, 32)$ , and  $(4, 64)$ . Only the graph shown in choice D passes through these points.

Choices A and B are incorrect because these are graphs of decreasing, not increasing, exponential functions.  
Choice C is incorrect because the value of  $y$  increases by a growth factor greater than 2 for each increase of 1 in the value of  $x$ .

Question Difficulty: Medium



## Question ID f44a29a8

2.10

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: f44a29a8**

An object's kinetic energy, in joules, is equal to the product of one-half the object's mass, in kilograms, and the square of the object's speed, in meters per second. What is the speed, in meters per second, of an object with a mass of 4 kilograms and kinetic energy of 18 joules?

- A. 3
- B. 6
- C. 9
- D. 36

**ID: f44a29a8 Answer**

Correct Answer: A

Rationale

Choice A is correct. It's given that an object's kinetic energy, in joules, is equal to the product of one-half the object's mass, in kilograms, and the square of the object's speed, in meters per second. This relationship can be represented by the equation  $K = \frac{1}{2}mv^2$ , where K is the kinetic energy, m is the mass, and v is the speed.

Substituting a mass of 4 kilograms for m and a kinetic energy of 18 joules for K results in the equation  $18 = \left(\frac{1}{2}\right)(4)v^2$ , or  $18 = 2v^2$ . Dividing both sides of this equation by 2 yields  $9 = v^2$ . Taking the square root of both sides yields  $v = -3$  and  $v = 3$ . Since speed can't be expressed as a negative number, the speed of the object is 3 meters per second.

Choice B is incorrect and may result from computation errors. Choice C is incorrect. This is the value of  $v^2$  rather than v. Choice D is incorrect. This is the value of  $4v^2$  rather than v.

Question Difficulty: Medium



# Question ID d71f6dbf

2.11

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: d71f6dbf**

The height, in feet, of an object  $x$  seconds after it is thrown straight up in the air can be modeled by the function  $h(x) = -16x^2 + 20x + 5$ . Based on the model, which of the following statements best interprets the equation  $h(1.4) = 1.64$ ?

- A. The height of the object 1.4 seconds after being thrown straight up in the air is 1.64 feet.
- B. The height of the object 1.64 seconds after being thrown straight up in the air is 1.4 feet.
- C. The height of the object 1.64 seconds after being thrown straight up in the air is approximately 1.4 times as great as its initial height.
- D. The speed of the object 1.4 seconds after being thrown straight up in the air is approximately 1.64 feet per second.

**ID: d71f6dbf Answer**

Correct Answer: A

Rationale

Choice A is correct. The value 1.4 is the value of  $x$ , which represents the number of seconds after the object was thrown straight up in the air. When the function  $h$  is evaluated for  $x = 1.4$ , the function has a value of 1.64, which is the height, in feet, of the object.

Choices B and C are incorrect and may result from misidentifying seconds as feet and feet as seconds. Additionally, choice C may result from incorrectly including the initial height of the object as the input  $x$ . Choice D is incorrect and may result from misidentifying height as speed.

Question Difficulty: Medium



# Question ID 6676f055

2.12

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 6676f055**

$$f(\theta) = -0.28(\theta - 27)^2 + 880$$

An engineer wanted to identify the best angle for a cooling fan in an engine in order to get the greatest airflow. The engineer discovered that the function above models the airflow  $f(\theta)$ , in cubic feet per minute, as a function of the angle of the fan  $\theta$ , in degrees. According to the model, what angle, in degrees, gives the greatest airflow?

- A. -0.28
- B. 0.28
- C. 27
- D. 880

**ID: 6676f055 Answer**

Correct Answer: C

Rationale

Choice C is correct. The function  $f$  is quadratic, so it will have either a maximum or a minimum at the vertex of the graph. Since the coefficient of the quadratic term ( $-0.28$ ) is negative, the vertex will be at a maximum. The equation  $f(\theta) = -0.28(\theta - 27)^2 + 880$  is given in vertex form, so the vertex is at  $\theta = 27$ . Therefore, an angle of 27 degrees gives the greatest airflow.

Choices A and B are incorrect and may be the result of misidentifying which value in a quadratic equation in vertex form represents the vertex. Choice D is incorrect. This choice identifies the maximum value of  $f(\theta)$  rather than the value of  $\theta$  for which  $f(\theta)$  is maximized.

Question Difficulty: Medium



# Question ID dd8ac009

2.13

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: dd8ac009**

Time (years)	Total amount (dollars)
0	670.00
1	674.02
2	678.06

Sara opened a savings account at a bank. The table shows the exponential relationship between the time  $t$ , in years, since Sara opened the account and the total amount  $d$ , in dollars, in the account. If Sara made no additional deposits or withdrawals, which of the following equations best represents the relationship between  $t$  and  $d$ ?

- A.  $d = 0.006(1 + 670)^t$
- B.  $d = 670(1 + 0.006)^t$
- C.  $d = 0.006(670t)$
- D.  $d = 670(0.006 + t)$

**ID: dd8ac009 Answer**

Correct Answer: B

Rationale

Choice B is correct. It's given that the relationship between  $t$  and  $d$  is exponential. The table shows that the value of  $d$  increases as the value of  $t$  increases. Therefore, the relationship between  $t$  and  $d$  can be represented by an increasing exponential equation of the form  $d = a(1 + b)^t$ , where  $a$  and  $b$  are positive constants. The table shows that when  $t = 0$ ,  $d = 670$ . Substituting 0 for  $t$  and 670 for  $d$  in the equation  $d = a(1 + b)^t$  yields  $670 = a(1 + b)^0$ , which is equivalent to  $670 = a(1)$ , or  $670 = a$ . Substituting 670 for  $a$  in the equation  $d = a(1 + b)^t$  yields  $d = 670(1 + b)^t$ . The table also shows that when  $t = 1$ ,  $d = 674.02$ . Substituting 1 for  $t$  and 674.02 for  $d$  in the equation  $d = 670(1 + b)^t$  yields  $674.02 = 670(1 + b)^1$ , or  $674.02 = 670(1 + b)$ . Dividing both sides of this equation by 670 yields  $1.006 = 1 + b$ . Subtracting 1 from both sides of this equation yields  $b = 0.006$ . Substituting 0.006 for  $b$  in the equation  $d = 670(1 + b)^t$  yields  $d = 670(1 + 0.006)^t$ . Therefore, of the choices, choice B best represents the relationship between  $t$  and  $d$ .

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Medium



## Question ID 281a4f3b

2.14

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

### ID: 281a4f3b

A certain college had 3,000 students enrolled in 2015. The college predicts that after 2015, the number of students enrolled each year will be 2% less than the number of students enrolled the year before. Which of the following functions models the relationship between the number of students enrolled,  $f(x)$ , and the number of years after 2015,  $x$ ?

- A.  $f(x) = 0.02(3,000)^x$
- B.  $f(x) = 0.98(3,000)^x$
- C.  $f(x) = 3,000(0.02)^x$
- D.  $f(x) = 3,000(0.98)^x$

### ID: 281a4f3b Answer

Correct Answer: D

#### Rationale

Choice D is correct. Because the change in the number of students decreases by the same percentage each year, the relationship between the number of students and the number of years can be modeled with a decreasing exponential function in the form  $f(x) = a(1 - r)^x$ , where  $f(x)$  is the number of students,  $a$  is the number of students in 2015,  $r$  is the rate of decrease each year, and  $x$  is the number of years since 2015. It's given that 3,000 students were enrolled in 2015 and that the rate of decrease is predicted to be 2%, or 0.02. Substituting these values into the decreasing exponential function yields  $f(x) = 3,000(1 - 0.02)^x$ , which is equivalent to  $f(x) = 3,000(0.98)^x$ .

Choices A, B, and C are incorrect and may result from conceptual errors when translating the given information into a decreasing exponential function.

Question Difficulty: Medium



## Question ID 100030d9

2.15

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 100030d9**

A rubber ball bounces upward one-half the height that it falls each time it hits the ground. If the ball was originally dropped from a distance of 20.0 feet above the ground, what was its maximum height above the ground, in feet, between the third and fourth time it hit the ground?

**ID: 100030d9 Answer**

### Rationale

The correct answer is 2.5. After hitting the ground once, the ball bounces to  $20.0 \div 2 = 10.0$  feet. After hitting the ground a second time, the ball bounces to  $10.0 \div 2 = 5.0$  feet. After hitting the ground for the third time, the ball bounces to  $5.0 \div 2 = 2.5$  feet. Note that 2.5 and  $5/2$  are examples of ways to enter a correct answer.

Question Difficulty: Medium



## Question ID c7a187a7

2.16

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: c7a187a7**

$$f(x) = x^2 - 18x - 360$$

If the given function  $f$  is graphed in the  $xy$ -plane, where  $y = f(x)$ , what is an  $x$ -intercept of the graph?

- A.  $(-12, 0)$
- B.  $(-30, 0)$
- C.  $(-360, 0)$
- D.  $(12, 0)$

**ID: c7a187a7 Answer**

Correct Answer: A

Rationale

Choice A is correct. It's given that  $y = f(x)$ . The  $x$ -intercepts of a graph in the  $xy$ -plane are the points where  $y = 0$ . Thus, for an  $x$ -intercept of the graph of function  $f$ ,  $0 = f(x)$ . Substituting 0 for  $f(x)$  in the equation  $f(x) = x^2 - 18x - 360$  yields  $0 = x^2 - 18x - 360$ . Factoring the right-hand side of this equation yields  $0 = (x + 12)(x - 30)$ . By the zero product property,  $x + 12 = 0$  and  $x - 30 = 0$ . Subtracting 12 from both sides of the equation  $x + 12 = 0$  yields  $x = -12$ . Adding 30 to both sides of the equation  $x - 30 = 0$  yields  $x = 30$ . Therefore, the  $x$ -intercepts of the graph of  $y = f(x)$  are  $(-12, 0)$  and  $(30, 0)$ . Of these two  $x$ -intercepts, only  $(-12, 0)$  is given as a choice.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Medium



## Question ID e1391dd6

2.17

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: e1391dd6**

According to Moore's law, the number of transistors included on microprocessors doubles every 2 years. In 1985, a microprocessor was introduced that had 275,000 transistors. Based on this information, in which of the following years does Moore's law estimate the number of transistors to reach 1.1 million?

- A. 1987
- B. 1989
- C. 1991
- D. 1994

**ID: e1391dd6 Answer****Rationale**

Choice B is correct. Let  $x$  be the number of years after 1985. It follows that  $\frac{x}{2}$  represents the number of 2-year periods that will occur within an  $x$ -year period. According to Moore's law, every 2 years, the number of transistors included on microprocessors is estimated to double. Therefore,  $x$  years after 1985, the number of transistors will double  $\frac{x}{2}$  times. Since the number of transistors included on a microprocessor was 275,000, or .275 million, in 1985, the estimated number of transistors, in millions, included  $x$  years after 1985 can be

$$0.275 \cdot 2^{\frac{x}{2}}$$

modeled as . The year in which the number of transistors is estimated to be 1.1 million is

$$1.1 = 0.275 \cdot 2^{\frac{x}{2}}$$

represented by the value of  $x$  when . Dividing both sides of this equation by .275 yields

$$4 = 2^{\frac{x}{2}} \quad 2^2 = 2^{\frac{x}{2}}$$

, which can be rewritten as

. Since the exponential equation has equal bases on each side, it

follows that the exponents must also be equal:  $2 = \frac{x}{2}$ . Multiplying both sides of the equation  $2 = \frac{x}{2}$  by 2 yields  $x = 4$ . Therefore, according to Moore's law, 4 years after 1985, or in 1989, the number of transistors included on microprocessors is estimated to reach 1.1 million.

Alternate approach: According to Moore's law, 2 years after 1985 (in 1987), the number of transistors included on a microprocessor is estimated to be  $2 \cdot 275,000$ , or 550,000, and 2 years after 1987 (in 1989), the number of

transistors included on microprocessors is estimated to be 2·550,000, or 1,100,000. Therefore, the year that Moore's law estimates the number of transistors on microprocessors to reach 1.1 million is 1989.

Choices A, C, and D are incorrect. According to Moore's law, the number of transistors included on microprocessors is estimated to reach 550,000 in 1987, 2.2 million in 1991, and about 6.2 million in 1994.

Question Difficulty: Medium



## Question ID 5bf0f84a

2.18

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

### ID: 5bf0f84a

$$h(t) = -16t^2 + 110t + 72$$

The function above models the height  $h$ , in feet, of an object above ground  $t$  seconds after being launched straight up in the air. What does the number 72 represent in the function?

- A. The initial height, in feet, of the object
- B. The maximum height, in feet, of the object
- C. The initial speed, in feet per second, of the object
- D. The maximum speed, in feet per second, of the object

### ID: 5bf0f84a Answer

Correct Answer: A

#### Rationale

Choice A is correct. The variable  $t$  represents the seconds after the object is launched. Since  $h(0) = 72$ , this means that the height, in feet, at 0 seconds, or the initial height, is 72 feet.

Choices B, C, and D are incorrect and may be the result of misinterpreting the function in context.

Question Difficulty: Medium



## Question ID 70ebd3d0

2.19

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

### ID: 70ebd3d0

$$N(d) = 115(0.90)^d$$

The function  $N$  defined above can be used to model the number of species of brachiopods at various ocean depths  $d$ , where  $d$  is in hundreds of meters. Which of the following does the model predict?

- A. For every increase in depth by 1 meter, the number of brachiopod species decreases by 115.
- B. For every increase in depth by 1 meter, the number of brachiopod species decreases by 10%.
- C. For every increase in depth by 100 meters, the number of brachiopod species decreases by 115.
- D. For every increase in depth by 100 meters, the number of brachiopod species decreases by 10%.

### ID: 70ebd3d0 Answer

Correct Answer: D

#### Rationale

Choice D is correct. The function  $N$  is exponential, so it follows that  $N(d)$  changes by a fixed percentage for each increase in  $d$  by 1. Since  $d$  is measured in hundreds of meters, it also follows that the number of brachiopod species changes by a fixed percentage for each increase in ocean depth by 100 meters. Since the base of the exponent in the model is 0.90, which is less than 1, the number of brachiopod species decreases as the ocean depth increases. Specifically, the number of brachiopod species at a depth of  $d + 100$  meters is 90% of the number of brachiopod species at a depth of  $d$  meters. This means that for each increase in ocean depth by 100 meters, the number of brachiopod species decreases by 10%.

Choices A and C are incorrect. These describe situations where the number of brachiopod species are decreasing linearly rather than exponentially. Choice B is incorrect and results from interpreting the decrease in the number of brachiopod species as 10% for every 1-meter increase in ocean depth rather than for every 100-meter increase in ocean depth.

Question Difficulty: Medium



## Question ID 97158b3a

2.20

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 97158b3a**

The area  $A$ , in square centimeters, of a rectangular painting can be represented by the expression  $w(w + 29)$ , where  $w$  is the width, in centimeters, of the painting. Which expression represents the length, in centimeters, of the painting?

- A.  $w$
- B. 29
- C.  $(w + 29)$
- D.  $w(w + 29)$

**ID: 97158b3a Answer**

Correct Answer: C

Rationale

Choice C is correct. It's given that the expression  $w(w + 29)$  represents the area, in square centimeters, of a rectangular painting, where  $w$  is the width, in centimeters, of the painting. The area of a rectangle can be calculated by multiplying its length by its width. It follows that the length, in centimeters, of the painting is represented by the expression  $(w + 29)$ .

Choice A is incorrect. This expression represents the width, in centimeters, of the painting, not its length, in centimeters.

Choice B is incorrect. This is the difference between the length, in centimeters, and the width, in centimeters, of the painting, not its length, in centimeters.

Choice D is incorrect. This expression represents the area, in square centimeters, of the painting, not its length, in centimeters.

Question Difficulty: Medium



# Question ID dba7432e

2.21

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: dba7432e**

x	f(x)
0	5
1	$\frac{5}{2}$
2	$\frac{5}{4}$
3	$\frac{5}{8}$

The table above gives the values of the function  $f$  for some values of  $x$ .

Which of the following equations could define  $f$ ?

- A.  $f(x) = 5(2^{x+1})$
- B.  $f(x) = 5(2^x)$
- C.  $f(x) = 5(2^{-(x+1)})$
- D.  $f(x) = 5(2^{-x})$

**ID: dba7432e Answer**

Correct Answer: D

Rationale

Choice D is correct. Each choice has a function with coefficient 5 and base 2, so the exponents must be analyzed. When the input value of  $x$  increases, the output value of  $f(x)$  decreases, so the exponent must be negative. An exponent of  $-x$  yields the values in the table:  $5 = 5(2^{-0})$ ,  $\frac{5}{2} = 5(2^{-1})$ ,  $\frac{5}{4} = 5(2^{-2})$ , and  $\frac{5}{8} = 5(2^{-3})$ .

Choices A and B are incorrect and may result from choosing equations that yield an increasing, rather than decreasing, output value of  $f(x)$  when the input value of  $x$  increases. Choice C is incorrect and may result from choosing an equation that doesn't yield the values in the table.

Question Difficulty: Medium



## Question ID f5e8ccf1

2.22

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

### ID: f5e8ccf1

$$f(x) = (x+4)(x-1)(2x-3)$$

The function  $f$  is defined above. Which of the following is NOT an  $x$ -intercept of the graph of the function in the  $xy$ -plane?

A.  $(-4, 0)$

B.  $\left(-\frac{2}{3}, 0\right)$

C.  $(1, 0)$

D.  $\left(\frac{3}{2}, 0\right)$

### ID: f5e8ccf1 Answer

Correct Answer: B

#### Rationale

Choice B is correct. The graph of the function  $f$  in the  $xy$ -plane has  $x$ -intercepts at the points  $(x, y)$ , where  $y = f(x) = 0$ . Substituting 0 for  $f(x)$  in the given equation yields  $0 = (x+4)(x-1)(2x-3)$ . By the zero product property, if  $0 = (x+4)(x-1)(2x-3)$ , then  $x+4=0$ ,  $x-1=0$ , or  $2x-3=0$ . Solving each of these linear equations for  $x$ , it follows that  $x = -4$ ,  $x = 1$ , and  $x = \frac{3}{2}$ , respectively. This means that the graph of the function  $f$  in the  $xy$ -plane has three  $x$ -intercepts:  $(-4, 0)$ ,  $(1, 0)$ , and  $\left(\frac{3}{2}, 0\right)$ . Therefore,  $\left(-\frac{2}{3}, 0\right)$  isn't an  $x$ -intercept of the graph of the function  $f$ .

Alternate approach: Substitution may be used. Since by definition an  $x$ -intercept of any graph is a point in the form  $(k, 0)$  where  $k$  is a constant, and since all points in the options are in this form, it need only be checked

whether the points in the options lie on the graph of the function  $f$ . Substituting  $-\frac{2}{3}$  for  $x$  and 0 for  $f(x)$  in the given equation yields  $0 = \left(-\frac{2}{3} + 4\right)\left(-\frac{2}{3} - 1\right)\left(2\left(-\frac{2}{3}\right) - 3\right)$ , or  $0 = \frac{650}{27}$ . Therefore, the point  $\left(-\frac{2}{3}, 0\right)$  doesn't lie on the graph of the function  $f$  and can't be an  $x$ -intercept of the graph.

Choices A, C, and D are incorrect because each of these points is an  $x$ -intercept of the graph of the function  $f$  in the  $xy$ -plane. By definition, an  $x$ -intercept is a point on the graph of the form  $(k, 0)$ , where  $k$  is a constant.

Substituting  $-4$  for  $x$  and 0 for  $f(x)$  in the given equation yields  $0 = (-4+4)(-4-1)(2(-4)-3)$ , or  $0 = 0$ .

Since this is a true statement, the point  $(-4, 0)$  lies on the graph of the function  $f$  and is an  $x$ -intercept of the graph. Performing similar substitution using the points  $(1, 0)$  and  $\left(\frac{3}{2}, 0\right)$  also yields the true statement  $0 = 0$ , illustrating that these points also lie on the graph of the function  $f$  and are  $x$ -intercepts of the graph.

Question Difficulty: Medium



## Question ID 5c00c2c1

2.23

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 5c00c2c1**

There were no jackrabbits in Australia before 1788 when 24 jackrabbits were introduced. By 1920 the population of jackrabbits had reached 10 billion. If the population had grown exponentially, this would correspond to a 16.2% increase, on average, in the population each year. Which of the following functions best models the population  $p(t)$  of jackrabbits  $t$  years after 1788?

- A.  $p(t) = 1.162(24)^t$
- B.  $p(t) = 24(2)^{1.162t}$
- C.  $p(t) = 24(1.162)^t$
- D.  $p(t) = (24, \cdot, 1.162)^t$

**ID: 5c00c2c1 Answer**

Correct Answer: C

Rationale

Choice C is correct. This exponential growth model can be written in the form  $p(t) = A(1+r)^t$ , where  $p(t)$  is the population  $t$  years after 1788,  $A$  is the initial population, and  $r$  is the yearly growth rate, expressed as a decimal. Since there were 24 jackrabbits in Australia in 1788,  $A = 24$ . Since the number of jackrabbits increased by an average of 16.2% each year,  $r = 0.162$ . Therefore, the equation that best models this situation is  $p(t) = 24(1.162)^t$ .

Choices A, B, and D are incorrect and may result from misinterpreting the form of an exponential growth model.

Question Difficulty: Medium

# Question ID 91e7ea5e



3.1

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 91e7ea5e**

$$h(x) = 2(x - 4)^2 - 32$$

The quadratic function  $h$  is defined as shown. In the  $xy$ -plane, the graph of  $y = h(x)$  intersects the  $x$ -axis at the points  $(0, 0)$  and  $(t, 0)$ , where  $t$  is a constant. What is the value of  $t$ ?

- A. 1
- B. 2
- C. 4
- D. 8

**ID: 91e7ea5e Answer**

Correct Answer: D

Rationale

Choice D is correct. It's given that the graph of  $y = h(x)$  intersects the  $x$ -axis at  $(0, 0)$  and  $(t, 0)$ , where  $t$  is a constant. Since this graph intersects the  $x$ -axis when  $y = 0$  or when  $h(x) = 0$ , it follows that  $h(0) = 0$  and  $h(t) = 0$ . If  $h(t) = 0$ , then  $0 = 2(t - 4)^2 - 32$ . Adding 32 to both sides of this equation yields  $32 = 2(t - 4)^2$ . Dividing both sides of this equation by 2 yields  $16 = (t - 4)^2$ . Taking the square root of both sides of this equation yields  $4 = |t - 4|$ . Adding 4 to both sides of this equation yields  $8 = t$ . Therefore, the value of  $t$  is 8.

Choices A, B, and C are incorrect and may result from calculation errors.

Question Difficulty: Hard



## Question ID a9084ca4

3.2

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: a9084ca4**

$$f(x) = 9,000(0.66)^x$$

The given function  $f$  models the number of advertisements a company sent to its clients each year, where  $x$  represents the number of years since 1997, and  $0 \leq x \leq 5$ . If  $y = f(x)$  is graphed in the  $xy$ -plane, which of the following is the best interpretation of the  $y$ -intercept of the graph in this context?

- A. The minimum estimated number of advertisements the company sent to its clients during the 5 years was 1,708.
- B. The minimum estimated number of advertisements the company sent to its clients during the 5 years was 9,000.
- C. The estimated number of advertisements the company sent to its clients in 1997 was 1,708.
- D. The estimated number of advertisements the company sent to its clients in 1997 was 9,000.

**ID: a9084ca4 Answer**

Correct Answer: D

Rationale

Choice D is correct. The  $y$ -intercept of a graph in the  $xy$ -plane is the point where  $x = 0$ . For the given function  $f$ , the  $y$ -intercept of the graph of  $y = f(x)$  in the  $xy$ -plane can be found by substituting 0 for  $x$  in the equation  $y = 9,000(0.66)^x$ , which gives  $y = 9,000(0.66)^0$ . This is equivalent to  $y = 9,000(1)$ , or  $y = 9,000$ . Therefore, the  $y$ -intercept of the graph of  $y = f(x)$  is  $(0, 9,000)$ . It's given that the function  $f$  models the number of advertisements a company sent to its clients each year. Therefore,  $f(x)$  represents the estimated number of advertisements the company sent to its clients each year. It's also given that  $x$  represents the number of years since 1997. Therefore,  $x = 0$  represents 0 years since 1997, or 1997. Thus, the best interpretation of the  $y$ -intercept of the graph of  $y = f(x)$  is that the estimated number of advertisements the company sent to its clients in 1997 was 9,000.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard



## Question ID b8f13a3a

3.3

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: b8f13a3a**

Function  $f$  is defined by  $f(x) = -a^x + b$ , where  $a$  and  $b$  are constants. In the  $xy$ -plane, the graph of  $y = f(x) - 12$  has a  $y$ -intercept at  $(0, -\frac{75}{7})$ . The product of  $a$  and  $b$  is  $\frac{320}{7}$ . What is the value of  $a$ ?

**ID: b8f13a3a Answer**

Correct Answer: 20

Rationale

The correct answer is 20. It's given that  $f(x) = -a^x + b$ . Substituting  $-a^x + b$  for  $f(x)$  in the equation  $y = f(x) - 12$  yields  $y = -a^x + b - 12$ . It's given that the  $y$ -intercept of the graph of  $y = f(x) - 12$  is  $(0, -\frac{75}{7})$ . Substituting 0 for  $x$  and  $-\frac{75}{7}$  for  $y$  in the equation  $y = -a^x + b - 12$  yields  $-\frac{75}{7} = -a^0 + b - 12$ , which is equivalent to  $-\frac{75}{7} = -1 + b - 12$ , or  $-\frac{75}{7} = b - 13$ . Adding 13 to both sides of this equation yields  $\frac{16}{7} = b$ . It's given that the product of  $a$  and  $b$  is  $\frac{320}{7}$ , or  $ab = \frac{320}{7}$ . Substituting  $\frac{16}{7}$  for  $b$  in this equation yields  $(a)(\frac{16}{7}) = \frac{320}{7}$ . Dividing both sides of this equation by  $\frac{16}{7}$  yields  $a = 20$ .

Question Difficulty: Hard



## Question ID 7902bed0

3.4

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 7902bed0**

A machine launches a softball from ground level. The softball reaches a maximum height of **51.84** meters above the ground at **1.8** seconds and hits the ground at **3.6** seconds. Which equation represents the height above ground  $h$ , in meters, of the softball  $t$  seconds after it is launched?

- A.  $h = -t^2 + 3.6$
- B.  $h = -t^2 + 51.84$
- C.  $h = -16(t + 51.84)^2 - 3.6$
- D.  $h = -16(t - 1.8)^2 + 51.84$

**ID: 7902bed0 Answer**

Correct Answer: D

Rationale

Choice D is correct. An equation representing the height above ground  $h$ , in meters, of a softball  $t$  seconds after it is launched by a machine from ground level can be written in the form  $h = -a(t - b)^2 + c$ , where  $a$ ,  $b$ , and  $c$  are positive constants. In this equation,  $b$  represents the time, in seconds, at which the softball reaches its maximum height of  $c$  meters above the ground. It's given that this softball reaches a maximum height of **51.84** meters above the ground at **1.8** seconds; therefore,  $b = 1.8$  and  $c = 51.84$ . Substituting **1.8** for  $b$  and **51.84** for  $c$  in the equation  $h = -a(t - b)^2 + c$  yields  $h = -a(t - 1.8)^2 + 51.84$ . It's also given that this softball hits the ground at **3.6** seconds; therefore,  $h = 0$  when  $t = 3.6$ . Substituting **0** for  $h$  and **3.6** for  $t$  in the equation  $h = -a(t - 1.8)^2 + 51.84$  yields  $0 = -a(3.6 - 1.8)^2 + 51.84$ , which is equivalent to  $0 = -a(1.8)^2 + 51.84$ , or  $0 = -3.24a + 51.84$ . Adding **3.24a** to both sides of this equation yields **3.24a = 51.84**. Dividing both sides of this equation by **3.24** yields **a = 16**. Substituting **16** for  $a$  in the equation  $h = -a(t - 1.8)^2 + 51.84$  yields  $h = -16(t - 1.8)^2 + 51.84$ . Therefore,  $h = -16(t - 1.8)^2 + 51.84$  represents the height above ground  $h$ , in meters, of this softball  $t$  seconds after it is launched.

Choice A is incorrect. This equation represents a situation where the maximum height is **3.6** meters above the ground at **0** seconds, not **51.84** meters above the ground at **1.8** seconds.

Choice B is incorrect. This equation represents a situation where the maximum height is **51.84** meters above the ground at **0** seconds, not **1.8** seconds.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard



## Question ID 4a0d0399

3.5

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 4a0d0399**

The function  $f$  is defined by  $f(x) = a^x + b$ , where  $a$  and  $b$  are constants. In the  $xy$ -plane, the graph of  $y = f(x)$  has an  $x$ -intercept at  $(2, 0)$  and a  $y$ -intercept at  $(0, -323)$ . What is the value of  $b$ ?

**ID: 4a0d0399 Answer**

Correct Answer: -324

## Rationale

The correct answer is  $-324$ . It's given that the function  $f$  is defined by  $f(x) = a^x + b$ , where  $a$  and  $b$  are constants. It's also given that the graph of  $y = f(x)$  has a  $y$ -intercept at  $(0, -323)$ . It follows that  $f(0) = -323$ . Substituting  $0$  for  $x$  and  $-323$  for  $f(x)$  in  $f(x) = a^x + b$  yields  $-323 = a^0 + b$ , or  $-323 = 1 + b$ . Subtracting  $1$  from each side of this equation yields  $-324 = b$ . Therefore, the value of  $b$  is  $-324$ .

Question Difficulty: Hard



## Question ID 9654add7

3.6

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 9654add7**

$$f(x) = -500x^2 + 25,000x$$

The revenue  $f(x)$ , in dollars, that a company receives from sales of a product is given by the function  $f$  above, where  $x$  is the unit price, in dollars, of the product. The graph of  $y = f(x)$  in the  $xy$ -plane intersects the  $x$ -axis at 0 and  $a$ . What does  $a$  represent?

- A. The revenue, in dollars, when the unit price of the product is \$0
- B. The unit price, in dollars, of the product that will result in maximum revenue
- C. The unit price, in dollars, of the product that will result in a revenue of \$0
- D. The maximum revenue, in dollars, that the company can make

**ID: 9654add7 Answer**

Correct Answer: C

Rationale

Choice C is correct. By definition, the  $y$ -value when a function intersects the  $x$ -axis is 0. It's given that the graph of the function intersects the  $x$ -axis at 0 and  $a$ , that  $x$  is the unit price, in dollars, of a product, and that  $f(x)$ , where  $y = f(x)$ , is the revenue, in dollars, that a company receives from the sales of the product. Since the value of  $a$  occurs when  $y = 0$ ,  $a$  is the unit price, in dollars, of the product that will result in a revenue of \$0.

Choice A is incorrect. The revenue, in dollars, when the unit price of the product is \$0 is represented by  $f(x)$ , when  $x = 0$ . Choice B is incorrect. The unit price, in dollars, of the product that will result in maximum revenue is represented by the  $x$ -coordinate of the maximum of  $f$ . Choice D is incorrect. The maximum revenue, in dollars, that the company can make is represented by the  $y$ -coordinate of the maximum of  $f$ .

Question Difficulty: Hard



# Question ID 263f9937

3.7

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	3

**ID: 263f9937**

## Growth of a Culture of Bacteria

Day	Number of bacteria per milliliter at end of day
1	$2.5 \times 10^5$
2	$5.0 \times 10^5$
3	$1.0 \times 10^6$

A culture of bacteria is growing at an exponential rate, as shown in the table above. At this rate, on which day would the number of bacteria per milliliter reach  $5.12 \times 10^8$ ?

- A. Day 5
- B. Day 9
- C. Day 11
- D. Day 12

**ID: 263f9937 Answer**

Correct Answer: D

### Rationale

Choice D is correct. The number of bacteria per milliliter is doubling each day. For example, from day 1 to day 2, the number of bacteria increased from  $2.5 \times 10^5$  to  $5.0 \times 10^5$ . At the end of day 3 there are  $10^6$  bacteria per milliliter. At the end of day 4, there will be  $10^6 \times 2$  bacteria per milliliter. At the end of day 5, there will be  $(10^6 \times 2) \times 2$ , or  $10^6 \times (2^2)$  bacteria per milliliter, and so on. At the end of day d, the number of bacteria will be  $10^6 \times (2^{d-3})$ . If the number of bacteria per milliliter will reach  $5.12 \times 10^8$  at the end of day d, then the equation  $10^6 \times (2^{d-3}) = 5.12 \times 10^8$  must hold. Since  $5.12 \times 10^8$  can be rewritten as  $512 \times 10^6$ , the equation is equivalent to  $2^{d-3} = 512$ . Rewriting 512 as  $2^9$  gives  $d - 3 = 9$ , so  $d = 12$ . The number of bacteria per milliliter would reach  $5.12 \times 10^8$  at the end of day 12.

Choices A, B, and C are incorrect. Given the growth rate of the bacteria, the number of bacteria will not reach  $5.12 \times 10^8$  per milliliter by the end of any of these days.

Question Difficulty: Hard



## Question ID 18e35375

3.8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 18e35375**

$$f(x) = (x - 14)(x + 19)$$

The function  $f$  is defined by the given equation. For what value of  $x$  does  $f(x)$  reach its minimum?

- A.  $-266$
- B.  $-19$
- C.  $-\frac{33}{2}$
- D.  $-\frac{5}{2}$

**ID: 18e35375 Answer**

Correct Answer: D

Rationale

Choice D is correct. It's given that  $f(x) = (x - 14)(x + 19)$ , which can be rewritten as  $f(x) = x^2 + 5x - 266$ . Since the coefficient of the  $x^2$ -term is positive, the graph of  $y = f(x)$  in the  $xy$ -plane opens upward and reaches its minimum value at its vertex. The  $x$ -coordinate of the vertex is the value of  $x$  such that  $f(x)$  reaches its minimum. For an equation in the form  $f(x) = ax^2 + bx + c$ , where  $a$ ,  $b$ , and  $c$  are constants, the  $x$ -coordinate of the vertex is  $-\frac{b}{2a}$ . For the equation  $f(x) = x^2 + 5x - 266$ ,  $a = 1$ ,  $b = 5$ , and  $c = -266$ . It follows that the  $x$ -coordinate of the vertex is  $-\frac{5}{2(1)}$ , or  $-\frac{5}{2}$ . Therefore,  $f(x)$  reaches its minimum when the value of  $x$  is  $-\frac{5}{2}$ .

Alternate approach: The value of  $x$  for the vertex of a parabola is the  $x$ -value of the midpoint between the two  $x$ -intercepts of the parabola. Since it's given that  $f(x) = (x - 14)(x + 19)$ , it follows that the two  $x$ -intercepts of the graph of  $y = f(x)$  in the  $xy$ -plane occur when  $x = 14$  and  $x = -19$ , or at the points  $(14, 0)$  and  $(-19, 0)$ . The midpoint between two points,  $(x_1, y_1)$  and  $(x_2, y_2)$ , is  $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ . Therefore, the midpoint between  $(14, 0)$  and  $(-19, 0)$  is  $\left(\frac{14+(-19)}{2}, \frac{0+0}{2}\right)$ , or  $(-\frac{5}{2}, 0)$ . It follows that  $f(x)$  reaches its minimum when the value of  $x$  is  $-\frac{5}{2}$ .

Choice A is incorrect. This is the  $y$ -coordinate of the  $y$ -intercept of the graph of  $y = f(x)$  in the  $xy$ -plane.

Choice B is incorrect. This is one of the  $x$ -coordinates of the  $x$ -intercepts of the graph of  $y = f(x)$  in the  $xy$ -plane.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard



## Question ID 9afe2370

3.9

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

### ID: 9afe2370

The population  $P$  of a certain city  $y$  years after the last census is modeled by the equation below, where  $r$  is a constant and  $P_0$  is the population when  $y = 0$ .

$$P = P_0(1+r)^y$$

If during this time the population of the city decreases by a fixed percent each year, which of the following must be true?

- A.  $r < -1$
- B.  $-1 < r < 0$
- C.  $0 < r < 1$
- D.  $r > 1$

### ID: 9afe2370 Answer

Correct Answer: B

#### Rationale

Choice B is correct. The term  $(1 + r)$  represents a percent change. Since the population is decreasing, the percent change must be between 0% and 100%. When the percent change is expressed as a decimal rather than as a percent, the percentage change must be between 0 and 1. Because  $(1 + r)$  represents percent change, this can be expressed as  $0 < 1 + r < 1$ . Subtracting 1 from all three terms of this compound inequality results in  $-1 < r < 0$ .

Choice A is incorrect. If  $r < -1$ , then after 1 year, the population  $P$  would be a negative value, which is not possible. Choices C and D are incorrect. For any value of  $r > 0$ ,  $1 + r > 1$ , and the exponential function models growth for positive values of the exponent. This contradicts the given information that the population is decreasing.

Question Difficulty: Hard



# Question ID 0121a235

3.10

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 0121a235**

x	$p(x)$
-2	5
-1	0
0	-3
1	-1
2	0

The table above gives selected values of a polynomial function  $p$ . Based on the values in the table, which of the following must be a factor of  $p$ ?

- A.  $(x - 3)$
- B.  $(x + 3)$
- C.  $(x - 1)(x + 2)$
- D.  $(x + 1)(x - 2)$

**ID: 0121a235 Answer**

Correct Answer: D

Rationale

Choice D is correct. According to the table, when  $x$  is  $-1$  or  $2$ ,  $p(x) = 0$ . Therefore, two  $x$ -intercepts of the graph of  $p$  are  $(-1, 0)$  and  $(2, 0)$ . Since  $(-1, 0)$  and  $(2, 0)$  are  $x$ -intercepts, it follows that  $(x + 1)$  and  $(x - 2)$  are factors of the polynomial equation. This is because when  $x = -1$ , the value of  $x + 1$  is 0. Similarly, when  $x = 2$ , the value of  $x - 2$  is 0. Therefore, the product  $(x + 1)(x - 2)$  is a factor of the polynomial function  $p$ .

Choice A is incorrect. The factor  $x - 3$  corresponds to an  $x$ -intercept of  $(3, 0)$ , which isn't present in the table.

Choice B is incorrect. The factor  $x + 3$  corresponds to an  $x$ -intercept of  $(-3, 0)$ , which isn't present in the table.

Choice C is incorrect. The factors  $x - 1$  and  $x + 2$  correspond to  $x$ -intercepts  $(1, 0)$  and  $(-2, 0)$ , respectively, which aren't present in the table.

Question Difficulty: Hard



## Question ID 70753f99

3.11

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 70753f99**

The function  $f$  is defined by  $f(x) = (x+3)(x+1)$ . The graph of  $f$  in the  $xy$ -plane is a parabola. Which of the following intervals contains the  $x$ -coordinate of the vertex of the graph of  $f$ ?

- A.  $-4 < x < -3$
- B.  $-3 < x < 1$
- C.  $1 < x < 3$
- D.  $3 < x < 4$

**ID: 70753f99 Answer**

Correct Answer: B

Rationale

Choice B is correct. The graph of a quadratic function in the  $xy$ -plane is a parabola. The axis of symmetry of the parabola passes through the vertex of the parabola. Therefore, the vertex of the parabola and the midpoint of the segment between the two  $x$ -intercepts of the graph have the same  $x$ -coordinate. Since

$f(-3) = f(-1) = 0$ , the  $x$ -coordinate of the vertex is  $\frac{(-3)+(-1)}{2} = -2$ . Of the shown intervals, only the interval in choice B contains  $-2$ . Choices A, C, and D are incorrect and may result from either calculation errors or misidentification of the graph's  $x$ -intercepts.

Question Difficulty: Hard



## Question ID 58dcc59f

3.12

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 58dcc59f**

A landscaper is designing a rectangular garden. The length of the garden is to be 5 feet longer than the width. If the area of the garden will be 104 square feet, what will be the length, in feet, of the garden?

**ID: 58dcc59f Answer**

### Rationale

The correct answer is 13. Let  $w$  represent the width of the rectangular garden, in feet. Since the length of the garden will be 5 feet longer than the width of the garden, the length of the garden will be  $w + 5$  feet. Thus the area of the garden will be  $w(w + 5)$ . It is also given that the area of the garden will be 104 square feet. Therefore,  $w(w + 5) = 104$ , which is equivalent to  $w^2 + 5w - 104 = 0$ . Factoring this equation results in  $(w + 13)(w - 8) = 0$ . Therefore,  $w = 8$  and  $w = -13$ . Because width cannot be negative, the width of the garden must be 8 feet. This means the length of the garden must be  $8 + 5 = 13$  feet.

Question Difficulty: Hard



## Question ID 84dd43f8

3.13

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 84dd43f8**

For the function  $f$ ,  $f(0) = 86$ , and for each increase in  $x$  by 1, the value of  $f(x)$  decreases by 80%. What is the value of  $f(2)$ ?

**ID: 84dd43f8 Answer**

Correct Answer: 3.44, 86/25

**Rationale**

The correct answer is 3.44. It's given that  $f(0) = 86$  and that for each increase in  $x$  by 1, the value of  $f(x)$  decreases by 80%. Because the output of the function decreases by a constant percentage for each 1-unit increase in the value of  $x$ , this relationship can be represented by an exponential function of the form  $f(x) = a(b)^x$ , where  $a$  represents the initial value of the function and  $b$  represents the rate of decay, expressed as a decimal. Because  $f(0) = 86$ , the value of  $a$  must be 86. Because the value of  $f(x)$  decreases by 80% for each 1-unit increase in  $x$ , the value of  $b$  must be  $(1 - 0.80)$ , or 0.2. Therefore, the function  $f$  can be defined by  $f(x) = 86(0.2)^x$ . Substituting 2 for  $x$  in this function yields  $f(2) = 86(0.2)^2$ , which is equivalent to  $f(2) = 86(0.04)$ , or  $f(2) = 3.44$ . Either 3.44 or 86/25 may be entered as the correct answer.

Alternate approach: It's given that  $f(0) = 86$  and that for each increase in  $x$  by 1, the value of  $f(x)$  decreases by 80%. Therefore, when  $x = 1$ , the value of  $f(x)$  is  $(100 - 80)\%$ , or 20%, of 86, which can be expressed as  $(0.20)(86)$ . Since  $(0.20)(86) = 17.2$ , the value of  $f(1)$  is 17.2. Similarly, when  $x = 2$ , the value of  $f(x)$  is 20% of 17.2, which can be expressed as  $(0.20)(17.2)$ . Since  $(0.20)(17.2) = 3.44$ , the value of  $f(2)$  is 3.44. Either 3.44 or 86/25 may be entered as the correct answer.

Question Difficulty: Hard



## Question ID 59d1f4b5

3.14

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 59d1f4b5**

$$M = 1,800(1.02)^t$$

The equation above models the number of members,  $M$ , of a gym  $t$  years after the gym opens. Of the following, which equation models the number of members of the gym  $q$  quarter years after the gym opens?

$$M = 1,800(1.02)^{\frac{q}{4}}$$

A.

B.  $M = 1,800(1.02)^{4q}$

C.  $M = 1,800(1.005)^{4q}$

D.  $M = 1,800(1.082)^q$

**ID: 59d1f4b5 Answer**

Correct Answer: A

Rationale

Choice A is correct. In 1 year, there are 4 quarter years, so the number of quarter years,  $q$ , is 4 times the number of years,  $t$ ; that is,  $q = 4t$ . This is equivalent to  $t = \frac{q}{4}$ , and substituting this into the expression for  $M$  in terms of  $t$  gives

$$M = 1,800(1.02)^{\frac{q}{4}}$$

Choices B and D are incorrect and may be the result of incorrectly using  $t = 4q$  instead of  $q = 4t$ . (Choices B and D are nearly the same since  $1.02^{4q}$  is equivalent to  $(1.02^4)^q$ , which is approximately  $1.082^q$ .) Choice C is incorrect and may be the result of incorrectly using  $t = 4q$  and unnecessarily dividing 0.02 by 4.

Question Difficulty: Hard



## Question ID 01668cd6

3.15

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 01668cd6**

The functions  $f$  and  $g$  are defined by the given equations, where  $x \geq 0$ . Which of the following equations displays, as a constant or coefficient, the maximum value of the function it defines, where  $x \geq 0$ ?

- I.  $f(x) = 33(0.4)^{x+3}$
- II.  $g(x) = 33(0.16)(0.4)^{x-2}$

- A. I only
- B. II only
- C. I and II
- D. Neither I nor II

**ID: 01668cd6 Answer**

Correct Answer: B

Rationale

Choice B is correct. Functions  $f$  and  $g$  are both exponential functions with a base of 0.40. Since 0.40 is less than 1, functions  $f$  and  $g$  are both decreasing exponential functions. This means that  $f(x)$  and  $g(x)$  decrease as  $x$  increases. Since  $f(x)$  and  $g(x)$  decrease as  $x$  increases, the maximum value of each function occurs at the least value of  $x$  for which the function is defined. It's given that functions  $f$  and  $g$  are defined for  $x \geq 0$ . Therefore, the maximum value of each function occurs at  $x = 0$ . Substituting 0 for  $x$  in the equation defining  $f$  yields  $f(0) = 33(0.4)^{0+3}$ , which is equivalent to  $f(0) = 33(0.4)^3$ , or  $f(0) = 2.112$ . Therefore, the maximum value of  $f$  is 2.112. Since the equation  $f(x) = 33(0.4)^{x+3}$  doesn't display the value 2.112, the equation defining  $f$  doesn't display the maximum value of  $f$ . Substituting 0 for  $x$  in the equation defining  $g$  yields  $g(0) = 33(0.16)(0.4)^{0-2}$ , which can be rewritten as  $g(0) = 33(0.16)\left(\frac{1}{0.4^2}\right)$ , or  $g(0) = 33(0.16)\left(\frac{1}{0.16}\right)$ , which is equivalent to  $g(0) = 33$ . Therefore, the maximum value of  $g$  is 33. Since the equation  $g(x) = 33(0.16)(0.4)^{x-2}$  displays the value 33, the equation defining  $g$  displays the maximum value of  $g$ . Thus, only equation II displays, as a constant or coefficient, the maximum value of the function it defines.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard



## Question ID 635f54ee

3.16

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	3

**ID: 635f54ee**

The surface area of a cube is  $6\left(\frac{a}{4}\right)^2$ , where  $a$  is a positive constant. Which of the following gives the perimeter of one face of the cube?

A.  $\frac{a}{4}$

B.  $a$

C.  $4a$

D.  $6a$

**ID: 635f54ee Answer**

Correct Answer: B

Rationale

Choice B is correct. A cube has 6 faces of equal area, so if the total surface area of a cube is  $6\left(\frac{a}{4}\right)^2$ , then the area of one face is  $\left(\frac{a}{4}\right)^2$ . Likewise, the area of one face of a cube is the square of one of its edges; therefore, if the area of one face is  $\left(\frac{a}{4}\right)^2$ , then the length of one edge of the cube is  $\frac{a}{4}$ . Since the perimeter of one face of a cube is four times the length of one edge, the perimeter is  $4\left(\frac{a}{4}\right) = a$ .

Choice A is incorrect because if the perimeter of one face of the cube is  $\frac{a}{4}$ , then the total surface area of the

cube is  $6\left(\frac{a}{4}\right)^2 = 6\left(\frac{a}{16}\right)^2$ , which is not  $6\left(\frac{a}{4}\right)^2$ . Choice C is incorrect because if the perimeter of one face of

the cube is  $4a$ , then the total surface area of the cube is  $6\left(\frac{4a}{4}\right)^2 = 6a^2$ , which is not  $6\left(\frac{a}{4}\right)^2$ . Choice D is incorrect because if the perimeter of one face of the cube is  $6a$ , then the total surface area of the cube is  $6\left(\frac{6a}{4}\right)^2 = 6\left(\frac{3a}{2}\right)^2$ , which is not  $6\left(\frac{a}{4}\right)^2$ .

Question Difficulty: Hard





## Question ID de39858a

3.17

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: de39858a**

The function  $h$  is defined by  $h(x) = a^x + b$ , where  $a$  and  $b$  are positive constants. The graph of  $y = h(x)$  in the  $xy$ -plane passes through the points  $(0, 10)$  and  $(-2, \frac{325}{36})$ . What is the value of  $ab$ ?

- A.  $\frac{1}{4}$
- B.  $\frac{1}{2}$
- C. 54
- D. 60

**ID: de39858a Answer**

Correct Answer: C

Rationale

Choice C is correct. It's given that the function  $h$  is defined by  $h(x) = a^x + b$  and that the graph of  $y = h(x)$  in the  $xy$ -plane passes through the points  $(0, 10)$  and  $(-2, \frac{325}{36})$ . Substituting 0 for  $x$  and 10 for  $h(x)$  in the equation  $h(x) = a^x + b$  yields  $10 = a^0 + b$ , or  $10 = 1 + b$ . Subtracting 1 from both sides of this equation yields  $9 = b$ . Substituting  $-2$  for  $x$  and  $\frac{325}{36}$  for  $h(x)$  in the equation  $h(x) = a^x + 9$  yields  $\frac{325}{36} = a^{-2} + 9$ . Subtracting 9 from both sides of this equation yields  $\frac{1}{36} = a^{-2}$ , which can be rewritten as  $a^2 = 36$ . Taking the square root of both sides of this equation yields  $a = 6$  and  $a = -6$ , but because it's given that  $a$  is a positive constant,  $a$  must equal 6. Because the value of  $a$  is 6 and the value of  $b$  is 9, the value of  $ab$  is  $(6)(9)$ , or 54.

Choice A is incorrect and may result from finding the value of  $a^{-2}b$  rather than the value of  $ab$ .

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from correctly finding the value of  $a$  as 6, but multiplying it by the  $y$ -value in the first ordered pair rather than by the value of  $b$ .

Question Difficulty: Hard



## Question ID 1178f2df

3.18

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 1178f2df**

x	y
21	-8
23	8
25	-8

The table shows three values of  $x$  and their corresponding values of  $y$ , where  $y = f(x) + 4$  and  $f$  is a quadratic function. What is the  $y$ -coordinate of the  $y$ -intercept of the graph of  $y = f(x)$  in the  $xy$ -plane?

**ID: 1178f2df Answer**

Correct Answer: -2112

Rationale

The correct answer is  $-2,112$ . It's given that  $f$  is a quadratic function. It follows that  $f$  can be defined by an equation of the form  $f(x) = a(x - h)^2 + k$ , where  $a$ ,  $h$ , and  $k$  are constants. It's also given that the table shows three values of  $x$  and their corresponding values of  $y$ , where  $y = f(x) + 4$ . Substituting  $a(x - h)^2 + k$  for  $f(x)$  in this equation yields  $y = a(x - h)^2 + k + 4$ . This equation represents a quadratic relationship between  $x$  and  $y$ , where  $k + 4$  is either the maximum or the minimum value of  $y$ , which occurs when  $x = h$ . For quadratic relationships between  $x$  and  $y$ , the maximum or minimum value of  $y$  occurs at the value of  $x$  halfway between any two values of  $x$  that have the same corresponding value of  $y$ . The table shows that  $x$ -values of **21** and **25** correspond to the same  $y$ -value, **-8**. Since **23** is halfway between **21** and **25**, the maximum or minimum value of  $y$  occurs at an  $x$ -value of **23**. The table shows that when  $x = 23$ ,  $y = 8$ . It follows that  $h = 23$  and  $k + 4 = 8$ . Subtracting 4 from both sides of the equation  $k + 4 = 8$  yields  $k = 4$ . Substituting **23** for  $h$  and **4** for  $k$  in the equation  $y = a(x - h)^2 + k + 4$  yields  $y = a(x - 23)^2 + 4 + 4$ , or  $y = a(x - 23)^2 + 8$ . The value of  $a$  can be found by substituting any  $x$ -value and its corresponding  $y$ -value for  $x$  and  $y$ , respectively, in this equation. Substituting **25** for  $x$  and **-8** for  $y$  in this equation yields  $-8 = a(25 - 23)^2 + 8$ , or  $-8 = a(2)^2 + 8$ . Subtracting 8 from both sides of this equation yields  $-16 = a(2)^2$ , or  $-16 = 4a$ . Dividing both sides of this equation by 4 yields  $-4 = a$ . Substituting  $-4$  for  $a$ , **23** for  $h$ , and **4** for  $k$  in the equation  $f(x) = a(x - h)^2 + k$  yields  $f(x) = -4(x - 23)^2 + 4$ . The  $y$ -intercept of the graph of  $y = f(x)$  in the  $xy$ -plane is the point on the graph where  $x = 0$ . Substituting **0** for  $x$  in the equation  $f(x) = -4(x - 23)^2 + 4$  yields  $f(0) = -4(0 - 23)^2 + 4$ , or  $f(0) = -4(-23)^2 + 4$ . This is equivalent to  $f(0) = -2,112$ , so the  $y$ -intercept of the graph of  $y = f(x)$  in the  $xy$ -plane is  $(0, -2,112)$ . Thus, the  $y$ -coordinate of the  $y$ -intercept of the graph of  $y = f(x)$  in the  $xy$ -plane is  $-2,112$ .

Question Difficulty: Hard



## Question ID 84e8cc72

3.19

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 84e8cc72**

A quadratic function models the height, in feet, of an object above the ground in terms of the time, in seconds, after the object is launched off an elevated surface. The model indicates the object has an initial height of **10** feet above the ground and reaches its maximum height of **1,034** feet above the ground **8** seconds after being launched. Based on the model, what is the height, in feet, of the object above the ground **10** seconds after being launched?

- A. **234**
- B. **778**
- C. **970**
- D. **1,014**

**ID: 84e8cc72 Answer**

Correct Answer: C

Rationale

Choice C is correct. It's given that a quadratic function models the height, in feet, of an object above the ground in terms of the time, in seconds, after the object is launched off an elevated surface. This quadratic function can be defined by an equation of the form  $f(x) = a(x - h)^2 + k$ , where  $f(x)$  is the height of the object  $x$  seconds after it was launched, and  $a$ ,  $h$ , and  $k$  are constants such that the function reaches its maximum value,  $k$ , when  $x = h$ . Since the model indicates the object reaches its maximum height of **1,034** feet above the ground **8** seconds after being launched,  $f(x)$  reaches its maximum value, **1,034**, when  $x = 8$ . Therefore,  $k = 1,034$  and  $h = 8$ . Substituting **8** for  $h$  and **1,034** for  $k$  in the function  $f(x) = a(x - h)^2 + k$  yields  $f(x) = a(x - 8)^2 + 1,034$ . Since the model indicates the object has an initial height of **10** feet above the ground, the value of  $f(x)$  is **10** when  $x = 0$ . Substituting **0** for  $x$  and **10** for  $f(x)$  in the equation  $f(x) = a(x - 8)^2 + 1,034$  yields  $10 = a(0 - 8)^2 + 1,034$ , or  $10 = 64a + 1,034$ . Subtracting **1,034** from both sides of this equation yields  $64a = -1,024$ . Dividing both sides of this equation by **64** yields  $a = -16$ . Therefore, the model can be represented by the equation  $f(x) = -16(x - 8)^2 + 1,034$ . Substituting **10** for  $x$  in this equation yields  $f(10) = -16(10 - 8)^2 + 1,034$ , or  $f(10) = 970$ . Therefore, based on the model, **10** seconds after being launched, the height of the object above the ground is **970** feet.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard



## Question ID 4b642eef

3.20

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 4b642eef**

The total distance  $d$ , in meters, traveled by an object moving in a straight line can be modeled by a quadratic function that is defined in terms of  $t$ , where  $t$  is the time in seconds. At a time of 10.0 seconds, the total distance traveled by the object is 50.0 meters, and at a time of 20.0 seconds, the total distance traveled by the object is 200.0 meters. If the object was at a distance of 0 meters when  $t = 0$ , then what is the total distance traveled, in meters, by the object after 30.0 seconds?

**ID: 4b642eef Answer****Rationale**

The correct answer is 450. The quadratic equation that models this situation can be written in the form  $d = at^2 + bt + c$ , where  $a$ ,  $b$ , and  $c$  are constants. It's given that the distance,  $d$ , the object traveled was 0 meters when  $t = 0$  seconds. These values can be substituted into the equation to solve for  $a$ ,  $b$ , and  $c$ :

$0 = a(0)^2 + b(0) + c$ . Therefore,  $c = 0$ , and it follows that  $d = at^2 + bt$ . Since it's also given that  $d$  is 50 when  $t$  is 10 and  $d$  is 200 when  $t$  is 20, these values for  $d$  and  $t$  can be substituted to create a system of two linear equations:  $50 = a(10)^2 + b(10)$  and  $200 = a(20)^2 + b(20)$ , or  $10a + b = 5$  and  $20a + b = 10$ . Subtracting the

first equation from the second equation yields  $10a = 5$ , or  $a = \frac{1}{2}$ . Substituting  $\frac{1}{2}$  for  $a$  in the first equation

and solving for  $b$  yields  $b = 0$ . Therefore, the equation that represents this situation is  $d = \frac{1}{2}t^2$ . Evaluating

this function when  $t = 30$  seconds yields  $d = \frac{1}{2}(30)^2 = 450$ , or  $d = 450$  meters.

Question Difficulty: Hard



## Question ID 9f2ecade

3.21

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

### ID: 9f2ecade

$$h(x) = x^3 + ax^2 + bx + c$$

The function  $h$  is defined above, where  $a$ ,  $b$ , and  $c$  are integer constants. If the zeros of the function are  $-5$ ,  $6$ , and  $7$ , what is the value of  $c$ ?

### ID: 9f2ecade Answer

#### Rationale

The correct answer is  $210$ . Since  $-5$ ,  $6$ , and  $7$  are zeros of the function, the function can be rewritten as  $h(x) = (x + 5)(x - 6)(x - 7)$ . Expanding the function yields  $h(x) = x^3 - 8x^2 - 23x + 210$ . Thus,  $a = -8$ ,  $b = -23$ , and  $c = 210$ . Therefore, the value of  $c$  is  $210$ .

Question Difficulty: Hard



## Question ID 6f5540a5

3.22

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 6f5540a5**

Kao measured the temperature of a cup of hot chocolate placed in a room with a constant temperature of 70 degrees Fahrenheit ( $^{\circ}\text{F}$ ). The temperature of the hot chocolate was 185 $^{\circ}\text{F}$  at 6:00 p.m. when it started cooling. The temperature of the hot chocolate was 156 $^{\circ}\text{F}$  at 6:05 p.m. and 135 $^{\circ}\text{F}$  at 6:10 p.m. The hot chocolate's temperature continued to decrease. Of the following functions, which best models the temperature  $T(m)$ , in degrees Fahrenheit, of Kao's hot chocolate  $m$  minutes after it started cooling?

- A.  $T(m) = 185(1.25)^m$
- B.  $T(m) = 185(0.85)^m$

C.

$$T(m) = (185 - 70)(0.75)^{\frac{m}{5}}$$

D.

$$T(m) = 70 + 115(0.75)^{\frac{m}{5}}$$

**ID: 6f5540a5 Answer**

Correct Answer: D

Rationale

Choice D is correct. The hot chocolate cools from 185 $^{\circ}\text{F}$  over time, never going lower than the room temperature, 70 $^{\circ}\text{F}$ . Since the base of the exponent in this function, 0.75, is less than 1,  $T(m)$  decreases as time increases. Using the function, the temperature, in  $^{\circ}\text{F}$ , at 6:00 p.m. can be estimated as  $T(0)$  and is equal to

$$70 + 115(0.75)^{\frac{0}{5}} = 185$$

The temperature, in  $^{\circ}\text{F}$ , at 6:05 p.m. can be estimated as  $T(5)$  and is equal to

$$70 + 115(0.75)^{\frac{5}{5}}$$

, which is approximately 156 $^{\circ}\text{F}$ . Finally, the temperature, in  $^{\circ}\text{F}$ , at 6:10 p.m. can be estimated as

$$70 + 115(0.75)^{\frac{10}{5}}$$

$T(10)$  and is equal to

, which is approximately 135 $^{\circ}\text{F}$ . Since these three given values of  $m$


$$T(m) = 70 + 115(0.75)^{\frac{m}{5}}$$

and their corresponding values for  $T(m)$  can be verified using the function  $T(m) = 70 + 115(0.75)^{\frac{m}{5}}$ , this is the best function out of the given choices to model the temperature of Kao's hot chocolate after  $m$  minutes.

Choice A is incorrect because the base of the exponent, 1.25, results in the value of  $T(m)$  increasing over time rather than decreasing. Choice B is incorrect because when  $m$  is large enough,  $T(m)$  becomes less than 70.

Choice C is incorrect because the maximum value of  $T(m)$  at 6:00 p.m. is 115°F, not 185°F.

Question Difficulty: Hard



## Question ID b73ee6cf

3.23

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

### ID: b73ee6cf

The population of a town is currently 50,000, and the population is estimated to increase each year by 3% from the previous year. Which of the following equations can be used to estimate the number of years,  $t$ , it will take for the population of the town to reach 60,000?

- A.  $50,000 = 60,000(0.03)^t$
- B.  $50,000 = 60,000(3)^t$
- C.  $60,000 = 50,000(0.03)^t$
- D.  $60,000 = 50,000(1.03)^t$

### ID: b73ee6cf Answer

Correct Answer: D

#### Rationale

Choice D is correct. Stating that the population will increase each year by 3% from the previous year is equivalent to saying that the population each year will be 103% of the population the year before. Since the initial population is 50,000, the population after  $t$  years is given by  $50,000(1.03)^t$ . It follows that the equation  $60,000 = 50,000(1.03)^t$  can be used to estimate the number of years it will take for the population to reach 60,000.

Choice A is incorrect. This equation models how long it will take the population to decrease from 60,000 to 50,000, which is impossible given the growth factor. Choice B is incorrect and may result from misinterpreting a 3% growth as growth by a factor of 3. Additionally, this equation attempts to model how long it will take the population to decrease from 60,000 to 50,000. Choice C is incorrect and may result from misunderstanding how to model percent growth by multiplying the initial amount by a factor greater than 1.

Question Difficulty: Hard



## Question ID 7eed640d

3.24

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

### ID: 7eed640d

$$h(x) = -16x^2 + 100x + 10$$

The quadratic function above models the height above the ground  $h$ , in feet, of a projectile  $x$  seconds after it had been launched vertically. If  $y = h(x)$  is graphed in the  $xy$ -plane, which of the following represents the real-life meaning of the positive  $x$ -intercept of the graph?

- A. The initial height of the projectile
- B. The maximum height of the projectile
- C. The time at which the projectile reaches its maximum height
- D. The time at which the projectile hits the ground

### ID: 7eed640d Answer

Correct Answer: D

#### Rationale

Choice D is correct. The positive  $x$ -intercept of the graph of  $y = h(x)$  is a point  $(x,y)$  for which  $y = 0$ . Since  $y = h(x)$  models the height above the ground, in feet, of the projectile, a  $y$ -value of 0 must correspond to the height of the projectile when it is 0 feet above ground or, in other words, when the projectile is on the ground. Since  $x$  represents the time since the projectile was launched, it follows that the positive  $x$ -intercept,  $(x,0)$ , represents the time at which the projectile hits the ground.

Choice A is incorrect and may result from misidentifying the  $y$ -intercept as a positive  $x$ -intercept. Choice B is incorrect and may result from misidentifying the  $y$ -value of the vertex of the graph of the function as an  $x$ -intercept. Choice C is incorrect and may result from misidentifying the  $x$ -value of the vertex of the graph of the function as an  $x$ -intercept.

Question Difficulty: Hard

# Question ID 43926bd9



3.25

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: 43926bd9**

x	f(x)
1	a
2	$a^5$
3	$a^9$

For the exponential function  $f$ , the table above shows several values of  $x$  and their corresponding values of  $f(x)$ , where  $a$  is a constant greater than 1. If  $k$  is a constant and  $f(k) = a^{29}$ , what is the value of  $k$ ?

**ID: 43926bd9 Answer****Rationale**

The correct answer is 8. The values of  $f(x)$  for the exponential function  $f$  shown in the table increase by a factor of  $a^4$  for each increase of 1 in  $x$ . This relationship can be represented by the equation  $f(x) = a^{4x+b}$ , where  $b$  is a constant. It's given that when  $x=2$ ,  $f(x)=a^5$ . Substituting 2 for  $x$  and  $a^5$  for  $f(x)$  into  $f(x) = a^{4x+b}$  yields  $a^5 = a^{4(2)+b}$ . Since  $4(2)+b=5$ , it follows that  $b=-3$ . Thus, an equation that defines the function  $f$  is  $f(x) = a^{4x-3}$ . It follows that the value of  $k$  such that  $f(k) = a^{29}$  can be found by solving the equation  $4k-3=29$ , which yields  $k=8$ .

**Question Difficulty:** Hard

# Question ID a7711fe8



3.26

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	

**ID: a7711fe8**

What is the minimum value of the function  $f$  defined by  $f(x) = (x - 2)^2 - 4$ ?

- A.  $-4$
- B.  $-2$
- C.  $2$
- D.  $4$

**ID: a7711fe8 Answer**

Correct Answer: A

Rationale

Choice A is correct. The given quadratic function  $f$  is in vertex form,  $f(x) = (x - h)^2 + k$ , where  $(h, k)$  is the vertex of the graph of  $y = f(x)$  in the  $xy$ -plane. Therefore, the vertex of the graph of  $y = f(x)$  is  $(2, -4)$ . In addition, the  $y$ -coordinate of the vertex represents either the minimum or maximum value of a quadratic function, depending on whether the graph of the function opens upward or downward. Since the leading coefficient of  $f$  (the coefficient of the term  $(x - 2)^2$ ) is 1, which is positive, the graph of  $y = f(x)$  opens upward.

It follows that at  $x = 2$ , the minimum value of the function  $f$  is  $-4$ .

Choice B is incorrect and may result from making a sign error and from using the  $x$ -coordinate of the vertex. Choice C is incorrect and may result from using the  $x$ -coordinate of the vertex. Choice D is incorrect and may result from making a sign error.

Question Difficulty: Hard



## Question ID 1a722d7d

3.27

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	3

**ID: 1a722d7d**

Let the function  $p$  be defined as  $p(x) = \frac{(x-c)^2 + 160}{2c}$ , where  $c$  is a constant. If  $p(c) = 10$ , what is the value of  $p(12)$ ?

- A. 10.00
- B. 10.25
- C. 10.75
- D. 11.00

**ID: 1a722d7d Answer**

Correct Answer: D

Rationale

Choice D is correct. The value of  $p(12)$  depends on the value of the constant  $c$ , so the value of  $c$  must first be determined. It is given that  $p(c) = 10$ . Based on the definition of  $p$ , it follows that:

$$p(c) = \frac{(c-c)^2 + 160}{2c} = 10$$

$$\frac{160}{2c} = 10$$

$$2c = 16$$

$$c = 8$$

This means that  $p(x) = \frac{(x-8)^2 + 160}{16}$  for all values of  $x$ . Therefore:

$$p(12) = \frac{(12-8)^2 + 160}{16}$$

$$= \frac{16 + 160}{16}$$

$$= 11$$

Choice A is incorrect. It is the value of  $p(8)$ , not  $p(12)$ . Choices B and C are incorrect. If one of these values were correct, then  $x = 12$  and the selected value of  $p(12)$  could be substituted into the equation to solve for  $c$ . However, the values of  $c$  that result from choices B and C each result in  $p(c) < 10$ .

Question Difficulty: Hard