In [1]:
```
import pandas as pd
import numpy as np
import pandas_datareader as pdr
from scipy.optimize import minimize
import matplotlib.pyplot as plt
```

1

In [2]:
```
l1=['AAPL','AMZN','META','GOOGL','IBM','INTC','MSFT','ORCL','WMT']
```

In [3]:
```
df1 = pdr.get_data_yahoo(l1, start='2019-01-01',end='2019-12-31')["Adj Close"]
df1
```

Out[3]:

Symbols	AAPL	AMZN	META	GOOGL	IBM	INTC	MSFT	ORC
Date								
2019-01-02	38.168343	76.956497	135.679993	52.734001	91.668594	42.586578	97.147690	42.29615
2019-01-03	34.366497	75.014000	131.740005	51.273499	89.838562	40.243778	93.573830	41.88460
2019-01-04	35.833584	78.769501	137.949997	53.903500	93.347450	42.713211	97.925873	43.68980
2019-01-07	35.753819	81.475502	138.050003	53.796001	94.007866	42.912209	98.050781	44.38196
2019-01-08	36.435398	82.829002	142.529999	54.268501	95.344566	43.183578	98.761726	44.78416
...	...	...	...	...	...	...	...	
2019-12-24	69.738731	89.460503	205.119995	67.221497	112.550056	55.080463	153.433289	50.83708
2019-12-26	71.122368	93.438499	207.789993	68.123497	112.491669	55.460583	154.690887	50.96074
2019-12-27	71.095375	93.489998	208.100006	67.732002	112.791870	55.701633	154.973633	50.85609
2019-12-30	71.517334	92.344498	204.410004	66.985497	110.740646	55.275146	153.638000	50.13311
2019-12-31	72.039871	92.391998	205.250000	66.969498	111.766243	55.488400	153.745239	50.39947

252 rows × 9 columns

1.1

In [6]:
```
df2=np.log(df1/df1.shift(1)).dropna()
df2
```

Out[6]:

Date	AAPL	AMZN	META	GOOGL	IBM	INTC	MSFT	ORCL	W
2019-01-03	-0.104924	-0.025566	-0.029469	-0.028086	-0.020166	-0.056584	-0.037482	-0.009778	-0.005
2019-01-04	0.041803	0.048851	0.046061	0.050021	0.038314	0.059553	0.045460	0.042197	0.006
2019-01-07	-0.002228	0.033777	0.000725	-0.001996	0.007050	0.004648	0.001275	0.015719	0.011
2019-01-08	0.018884	0.016476	0.031937	0.008745	0.014119	0.006304	0.007225	0.009021	0.006
2019-01-09	0.016839	0.001713	0.011857	-0.003433	0.007151	0.005640	0.014198	-0.002091	-0.003
...	...	...	...	...	...	...	...	...	
2019-12-24	0.000950	-0.002116	-0.005154	-0.004601	-0.004214	0.003034	-0.000190	-0.005040	0.004
2019-12-26	0.019646	0.043506	0.012933	0.013329	-0.000519	0.006877	0.008163	0.002430	0.000
2019-12-27	-0.000380	0.000551	0.001491	-0.005763	0.002665	0.004337	0.001826	-0.002056	0.000
2019-12-30	0.005918	-0.012328	-0.017891	-0.011083	-0.018353	-0.007686	-0.008656	-0.014318	-0.001
2019-12-31	0.007280	0.000514	0.004101	-0.000239	0.009219	0.003851	0.000698	0.005299	-0.004

251 rows × 9 columns

1.2

In [7]:
```
frontier_y=np.linspace(start=0.2,stop=0.5,num=100)
frontier_y
```

Out[7]:
```
array([0.2       , 0.2030303 , 0.20606061, 0.20909091, 0.21212121,
       0.21515152, 0.21818182, 0.22121212, 0.22424242, 0.22727273,
       0.23030303, 0.23333333, 0.23636364, 0.23939394, 0.24242424,
       0.24545455, 0.24848485, 0.25151515, 0.25454545, 0.25757576,
       0.26060606, 0.26363636, 0.26666667, 0.26969697, 0.27272727,
       0.27575758, 0.27878788, 0.28181818, 0.28484848, 0.28787879,
       0.29090909, 0.29393939, 0.2969697 , 0.3       , 0.3030303 ,
       0.30606061, 0.30909091, 0.31212121, 0.31515152, 0.31818182,
       0.32121212, 0.32424242, 0.32727273, 0.33030303, 0.33333333,
       0.33636364, 0.33939394, 0.34242424, 0.34545455, 0.34848485,
       0.35151515, 0.35454545, 0.35757576, 0.36060606, 0.36363636,
       0.36666667, 0.36969697, 0.37272727, 0.37575758, 0.37878788,
       0.38181818, 0.38484848, 0.38787879, 0.39090909, 0.39393939,
       0.3969697 , 0.4       , 0.4030303 , 0.40606061, 0.40909091,
       0.41212121, 0.41515152, 0.41818182, 0.42121212, 0.42424242,
       0.42727273, 0.43030303, 0.43333333, 0.43636364, 0.43939394,
       0.44242424, 0.44545455, 0.44848485, 0.45151515, 0.45454545,
       0.45757576, 0.46060606, 0.46363636, 0.46666667, 0.46969697,
       0.47272727, 0.47575758, 0.47878788, 0.48181818, 0.48484848,
       0.48787879, 0.49090909, 0.49393939, 0.4969697 , 0.5       ])
```

1.3

In [8]:
```
# objective function
def std(weight):
    std=np.dot(weight.T, np.dot(df2.cov()*250, weight))**0.5
    return std
```

1.4

In [9]:
```
# sum of the weights minus one is zero
def sum_weight(weight):
    return np.sum(weight)-1
```

In [10]:
```
def ret(weight):
    ret=np.dot(df2.mean(),weight)*250
    return ret
```

1.5

In [11]:
```
# lower and upper bounds
bound = ((0, 1), (0, 1), (0, 1), (0, 1),(0, 1), (0, 1), (0, 1), (0, 1),(0,1))
```

1.6

In [12]:
```
a1=np.array([1/9]*9)
a1
```

Out[12]:
```
array([0.11111111, 0.11111111, 0.11111111, 0.11111111, 0.11111111,
       0.11111111, 0.11111111, 0.11111111, 0.11111111])
```

1.7

In [13]:
```
frontier_x=[]
```

In [14]:
```
# for each return, find the portfolio with the minimum std
for a_return in frontier_y:
    constraint = ({'type':'eq','fun': sum_weight},{'type':'eq','fun': lambda weig
    min_std=minimize(fun=std,x0=a1,method='SLSQP',bounds=bound,constraints=constr
    frontier_x.append(round(min_std["fun"],6))
```

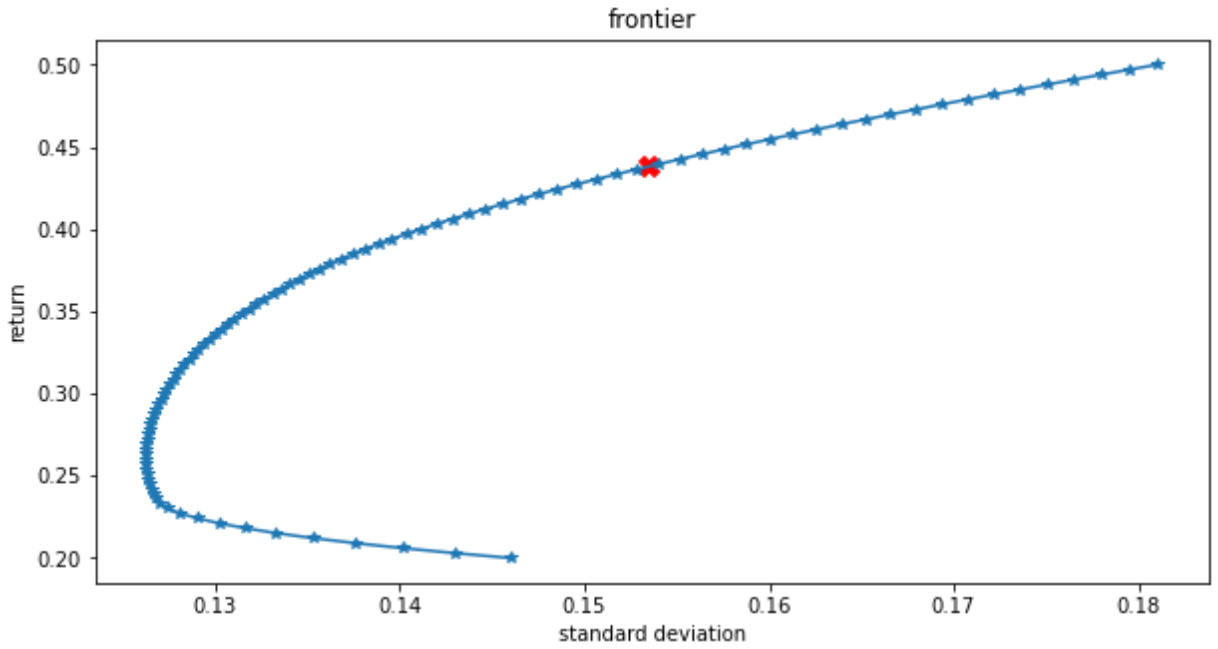
In [16]:
```
frontier_x
```

Out[16]:
```
[0.145994,
 0.142931,
 0.140114,
 0.137559,
 0.135281,
 0.133294,
 0.131606,
 0.130176,
 0.128993,
 0.128063,
 0.127395,
 0.126989,
 0.126778,
 0.126621,
 0.126515,
 0.12643,
 0.126357,
 0.126297,
 0.126253,
 0.126224,
 0.126211,
 0.126213,
 0.126227,
 0.126253,
 0.126291,
 0.126341,
 0.126404,
 0.126478,
 0.126564,
 0.126663,
 0.126773,
 0.126895,
 0.127029,
 0.127175,
 0.127333,
 0.127503,
 0.127684,
 0.127876,
 0.128085,
 0.128308,
 0.128547,
 0.128801,
 0.129071,
 0.129355,
 0.129655,
 0.129969,
 0.1303,
 0.130648,
 0.131013,
 0.131394,
 0.131792,
 0.132208,
 0.132639,
 0.133087,
 0.133551,
```

```
0.134031,
0.134526,
0.135042,
0.13559,
0.136171,
0.136785,
0.137431,
0.138109,
0.138818,
0.139557,
0.140326,
0.141125,
0.141954,
0.142811,
0.143696,
0.144609,
0.145548,
0.146515,
0.147508,
0.148526,
0.149569,
0.150638,
0.151729,
0.152844,
0.153982,
0.155144,
0.156328,
0.157535,
0.158765,
0.160015,
0.161287,
0.16258,
0.163892,
0.165225,
0.166576,
0.167946,
0.169334,
0.17074,
0.172164,
0.173604,
0.175061,
0.176534,
0.178023,
0.179527,
0.181046]
```

1.8

In [17]:
```
plt.figure(figsize = (10, 5))
plt.plot(frontier_x,frontier_y,marker="*")
plt.scatter(x=0.153483,y=0.438073,color="red",s=100,marker="X")
plt.xlabel('standard deviation')
plt.ylabel('return')
plt.title('frontier');
```



1.9

In [18]:
```
min_std=min(frontier_x)
min_std
```

Out[18]: 0.126211

In [19]:
```
frontier_x.index(min_std)
```

Out[19]: 20

1.10

In [20]:
```
plt.figure(figsize = (10, 5))
plt.plot(frontier_x[20:],frontier_y[20:],marker="*")
plt.scatter(x=0.153483,y=0.438073,color="red",s=100,marker="X")
plt.xlabel('standard deviation')
plt.ylabel('return')
plt.title('efficient frontier');
```