Update

I am working towards Dr Wen's suggestions and explanation from 06/08 meeting. As explained, the goal is to look at the eigen values of the hessian of $f(R_x(t\eta))$. This is where some insights given in Chapter 7 of Boumal's book come into picture. In this chapter a geometric toolbox has been developed and consolidated for several embedded manifolds. This includes first and second order geometries including a discussion of the hessian of the functions from a given manifold to $\mathbb{R}$, say $f : St(n,p) \to \mathbb{R}$. I am currently focusing on the Stiefel manifold from this chapter and putting all the pieces together. My immediate goal is to use an objective function and use the polar retraction to come up with an expression for the function $f \circ R_x$. The objective function f I am trying to work with is the from the PCA problem as explained on page 20 of Boumal's book. of A crucial skill needed to tackle these problems was a thorough understanding of the notion of second derivative and connections as well as an understanding of the geodesic equation including the calculation of the Christoffel symbols. These took a long time for me to fully understand and grasp. In fact now I am able to appreciate a discussion of these topics from Boothby's book's second last chapter. Some remaining pieces in the puzzle include the tool box for specific manifolds (chapter 7 of Boumals book) and that is what I am finishing now.