Week 5A - more FOL

- literal = atom $\vee \neg$atom
- clause = $(a_1 \wedge a_2 \wedge \ldots \wedge a_n) \longrightarrow (b_1 \vee b_2 \vee \ldots \vee b_n)$
- refutation: $\underbrace{\Gamma \models \phi}_{\text{"}\phi \text{ is entailed by } \Gamma\text{"}}$ iff $\underbrace{\Gamma \cup \{\neg \phi\}}_{\text{i.e. this MUST be inconsistent}} \vdash$ false
  - saturation: adding more and more rules to $\Gamma \cup \{\neg \phi\}$ until
    - nothing more to add
    - we have derived false } in which case, ϕ is entailed by Γ (confirm)
  - paramodulation: $\dfrac{C \vee s = t \qquad l[u] \vee D}{(l[t] \vee C \vee D)\theta}$
    - lifts idea behind resolution to EQUALITY
  - demodulation: for UNIT equalities

- Equality resolution: $\forall x \cdot f(x) \neq f(a)$ # FOL functions never partial
  - NEVER true: can make x and a equal $\therefore$ disequality is false

- Transformation to Clausal Form:

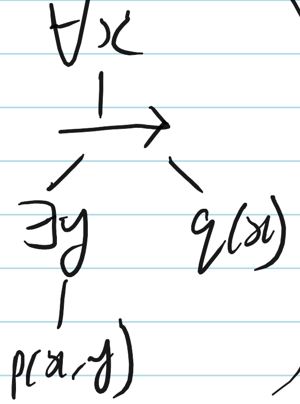
1. Rectify formula
2. Transform to Negation Normal Form
3. Eliminate quantifiers
4. Transform to CNF

- <u>rectifying</u> = making each quantifier bind different variables && all <u>bound</u> variables distinct from free ones

- Transform into NNF:
  - $\forall x((\exists y . p(x,y)) \rightarrow q(x)) \wedge \underbrace{p(a,b) \wedge \neg \exists x . q(x)}$

    $\Downarrow$

    $p(a,b) \wedge \forall z . \neg q(z)$



  - $\underbrace{\forall x . (\neg(\exists y . p(x,y)) \vee q(x))}$

    $\Downarrow$

    $\forall x . (\forall y . \neg p(x,y) \vee q(x))$

- getting rid of existential quantifiers:
  - Skolemisation:

    $\forall x_1, x_2, \ldots, x_n \; F \Longrightarrow F$

    $\exists x_1, x_2, \ldots, x_n$