

# Lecture 3 Reasoning in Datalog

COMP24412: Symbolic AI

Giles Reger

February 2019

# Aim and Learning Outcomes

The aim of this lecture is to:

Introduce you to the main concepts around *querying a knowledge base* and how these are concretely realised in the *Datalog* language.

## Learning Outcomes

By the end of this lecture you will be able to:

- 1 Describe what it means to *query* a knowledge base
- 2 Define *matching* and compute matching substitutions
- 3 Apply the *forward chaining* algorithm to find consequences of a knowledge base
- 4 Explain certain optimisations of the algorithm

## In General

Abstraction

Reality

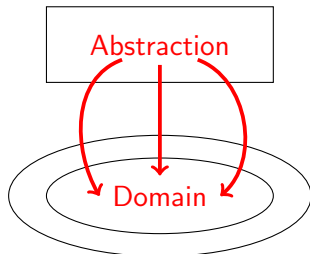
## In General

Abstraction

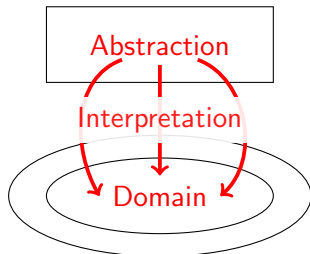


Domain

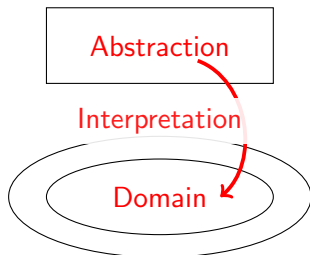
## In General



## In General



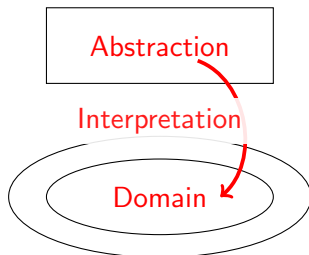
## In General



## Database Semantics

- Closed World
- Domain Closure
- Unique Names

## In General



## Database Semantics

- Closed World
- Domain Closure
- Unique Names

## Datalog

Has Database Semantics

**Fact:** concrete relationship between objects  
e.g. loves(giles, cheese)

**Rule:**  $\underbrace{\text{loves}(X, Y), \text{has}(X, Y)}_{\text{body}} \Rightarrow \underbrace{\text{happy}(X)}_{\text{head}}$

Knowledge Base  $\mathcal{KB}$ : set of facts and rules

Fact  $f$  is a **consequence** of  $\mathcal{KB}$

If all interpretations satisfying  $\mathcal{KB}$  satisfy  $f$   
written  $\mathcal{KB} \models f$



# Properties of Datalog

Given a knowledge base there are a **finite** number of consequences

**Why?**

# Properties of Datalog

Given a knowledge base there are a **finite** number of consequences

**Why?** Each rule has a finite number of instances (finite new facts)

# Properties of Datalog

Given a knowledge base there are a **finite** number of consequences

**Why?** Each rule has a finite number of instances (finite new facts)

Checking if  $f$  is a consequence of  $\mathcal{KB}$  is **decidable**.

- An interpretation can be defined by the facts true in it
- Due to database semantics,  $\mathcal{KB}$  has a single minimal interpretation  $\mathcal{M}$  satisfying it. If a fact is satisfied by this it is a consequence of  $\mathcal{KB}$
- The set of all facts built from  $\mathcal{O}$  and  $\mathcal{R}$  is finite, call this  $\mathcal{A}$
- Clearly  $\mathcal{M} \subseteq \mathcal{A}$ ; we can search all subsets of  $\mathcal{A}$

# Properties of Datalog

Given a knowledge base there are a **finite** number of consequences

**Why?** Each rule has a finite number of instances (finite new facts)

Checking if  $f$  is a consequence of  $\mathcal{KB}$  is **decidable**.

- An interpretation can be defined by the facts true in it
- Due to database semantics,  $\mathcal{KB}$  has a single minimal interpretation  $\mathcal{M}$  satisfying it. If a fact is satisfied by this it is a consequence of  $\mathcal{KB}$
- The set of all facts built from  $\mathcal{O}$  and  $\mathcal{R}$  is finite, call this  $\mathcal{A}$
- Clearly  $\mathcal{M} \subseteq \mathcal{A}$ ; we can search all subsets of  $\mathcal{A}$

Finally, can a Datalog knowledge base be **inconsistent**?

# Properties of Datalog

Given a knowledge base there are a **finite** number of consequences

**Why?** Each rule has a finite number of instances (finite new facts)

Checking if  $f$  is a consequence of  $\mathcal{KB}$  is **decidable**.

- An interpretation can be defined by the facts true in it
- Due to database semantics,  $\mathcal{KB}$  has a single minimal interpretation  $\mathcal{M}$  satisfying it. If a fact is satisfied by this it is a consequence of  $\mathcal{KB}$
- The set of all facts built from  $\mathcal{O}$  and  $\mathcal{R}$  is finite, call this  $\mathcal{A}$
- Clearly  $\mathcal{M} \subseteq \mathcal{A}$ ; we can search all subsets of  $\mathcal{A}$

Finally, can a Datalog knowledge base be **inconsistent**? No, the set of all consequences always exists and defines a satisfying interpretation.

Given a knowledge base we want to ask **queries**

These can be ground e.g. `is ancestor(giles, adam)` true?

Or, more interestingly, they can contain variables e.g. give me all ancestors of giles or more formally all  $X$  such that `ancestor(giles, X)` is true.

**A query is a fact, possibly containing variables.**

# The Semantics of Queries

The **answer** to a query  $q$  of a knowledge base  $\mathcal{KB}$  is the set

$$ans(q) = \{\sigma \mid \mathcal{KB} \models \sigma(q)\}$$

e.g. the set of all substitutions, which when applied to  $q$  produce a ground fact that is a consequence of  $\mathcal{KB}$ .

# The Semantics of Queries

The **answer** to a query  $q$  of a knowledge base  $\mathcal{KB}$  is the set

$$ans(q) = \{\sigma \mid \mathcal{KB} \models \sigma(q)\}$$

e.g. the set of all substitutions, which when applied to  $q$  produce a ground fact that is a consequence of  $\mathcal{KB}$ .

If the query has no answers then  $ans$  is empty. Can this happen?



# The Semantics of Queries

The **answer** to a query  $q$  of a knowledge base  $\mathcal{KB}$  is the set

$$ans(q) = \{\sigma \mid \mathcal{KB} \models \sigma(q)\}$$

e.g. the set of all substitutions, which when applied to  $q$  produce a ground fact that is a consequence of  $\mathcal{KB}$ .

If the query has no answers then  $ans$  is empty. Can this happen?

What will happen if  $q$  is ground?

# The Semantics of Queries

The **answer** to a query  $q$  of a knowledge base  $\mathcal{KB}$  is the set

$$ans(q) = \{\sigma \mid \mathcal{KB} \models \sigma(q)\}$$

e.g. the set of all substitutions, which when applied to  $q$  produce a ground fact that is a consequence of  $\mathcal{KB}$ .

If the query has no answers then  $ans$  is empty. Can this happen?

What will happen if  $q$  is ground? The substitution will be empty

# The Semantics of Queries

The **answer** to a query  $q$  of a knowledge base  $\mathcal{KB}$  is the set

$$ans(q) = \{\sigma \mid \mathcal{KB} \models \sigma(q)\}$$

e.g. the set of all substitutions, which when applied to  $q$  produce a ground fact that is a consequence of  $\mathcal{KB}$ .

If the query has no answers then  $ans$  is empty. Can this happen?

What will happen if  $q$  is ground? The substitution will be empty

Will  $ans(q)$  always be finite?

# The Semantics of Queries

The **answer** to a query  $q$  of a knowledge base  $\mathcal{KB}$  is the set

$$ans(q) = \{\sigma \mid \mathcal{KB} \models \sigma(q)\}$$

e.g. the set of all substitutions, which when applied to  $q$  produce a ground fact that is a consequence of  $\mathcal{KB}$ .

If the query has no answers then  $ans$  is empty. Can this happen?

What will happen if  $q$  is ground? The substitution will be empty

Will  $ans(q)$  always be finite? Yes - there are finite consequences

# Computing the Set of Consequences

Given our initial set of facts  $\mathcal{F}_0$  we want to add *new* consequences until we reach a **fixed-point**

Let our knowledge base  $\mathcal{KB}$  consist of facts  $\mathcal{F}_0$  and rules  $\mathcal{RU}$

Define the *next* set of facts as follows

$$\mathcal{F}_i = \mathcal{F}_{i-1} \cup \left\{ \sigma(head) \mid \begin{array}{l} body \Rightarrow head \in \mathcal{RU} \\ \sigma(body) \in \mathcal{F}_{i-1} \end{array} \right\}$$

This reaches a fixed point when  $\mathcal{F}_j = \mathcal{F}_{j+1}$

As there are finite consequences this will terminate

# Computing the Set of Consequences

Given our initial set of facts  $\mathcal{F}_0$  we want to add *new* consequences until we reach a **fixed-point**

Let our knowledge base  $\mathcal{KB}$  consist of facts  $\mathcal{F}_0$  and rules  $\mathcal{RU}$

Define the *next* set of facts as follows

$$\mathcal{F}_i = \mathcal{F}_{i-1} \cup \left\{ \sigma(head) \mid \begin{array}{l} body \Rightarrow head \in \mathcal{RU} \\ \sigma(body) \in \mathcal{F}_{i-1} \end{array} \right\}$$

This reaches a fixed point when  $\mathcal{F}_j = \mathcal{F}_{j+1}$

As there are finite consequences this will terminate

How do we find  $\sigma$ ? How do we compute  $\mathcal{F}_{i+1}$  efficiently?

# Matching

A fact is ground if it does not contain variables

Given a fact  $f_1$  and a ground fact  $f_2$  we say  $f_2$  **matches**  $f_1$  if there exists a substitution  $\sigma$  such that  $f_2 = \sigma(f_1)$ .

Examples:

Ground fact $f_2$ matches	fact $f_1$ using	substitution $\sigma$
happy(giles)	happy( $X$ )	$\{X \mapsto \text{giles}\}$
loves(giles, cheese)	loves( $X$ , cheese)	$\{X \mapsto \text{giles}\}$
loves(giles, cheese)	loves( $X$ , $Y$ )	$\{X \mapsto \text{giles}, Y \mapsto \text{cheese}\}$
happy(giles)	happy(giles)	$\{\}$

Note that loves(giles, cheese) **does not** match with loves( $X$ ,  $X$ ).

# Computing Matching Substitutions

Match two facts given an existing substitution

```
def match( $f_1 = name_1(args_1)$ ,  $f_2 = name_2(args_2)$ ,  $\sigma$ ):  
    if name1 and name2 are different then return  $\perp$ ;  
    for  $i \leftarrow 0$  to length(args1) do  
        if args1[i] is a variable and args1[i]  $\notin \sigma$  then  
            |  $\sigma = \sigma \cup \{args_1[i] \mapsto args_2[i]\}$   
        else if  $\sigma(args_1[i]) \neq args_2[i]$  then  
            | return  $\perp$   
    end  
    return  $\sigma$ 
```

If names are different, no match. For each parameter of  $f_1$ , if it is an unseen variable then extend  $\sigma$ , otherwise check that things are consistent.

Matching is an instance of **unification**, which we will meet later. In unification both sides can contain variables.



# Computing Matching Substitutions

```
def match( $f_1 = name_1(args_1)$ ,  $f_2 = name_2(args_2)$ ,  $\sigma$ ):  
  if  $name_1$  and  $name_2$  are different then return  $\perp$ ;  
  for  $i \leftarrow 0$  to  $length(args_1)$  do  
    if  $args_1[i]$  is a variable and  $args_1[i] \notin \sigma$  then  
      |  $\sigma = \sigma \cup \{args_1[i] \mapsto args_2[i]\}$   
    else if  $\sigma(args_1[i]) \neq args_2[i]$  then  
      | return  $\perp$   
  end  
  return  $\sigma$ 
```

$match(\text{parent}(X, Y), \text{parent}(\text{giles}, \text{mark}), \{X \mapsto \text{giles}\})$

# Computing Matching Substitutions

```
def match( $f_1 = name_1(args_1)$ ,  $f_2 = name_2(args_2)$ ,  $\sigma$ ):  
    if  $name_1$  and  $name_2$  are different then return  $\perp$ ;  
    for  $i \leftarrow 0$  to  $length(args_1)$  do  
        if  $args_1[i]$  is a variable and  $args_1[i] \notin \sigma$  then  
             $\sigma = \sigma \cup \{args_1[i] \mapsto args_2[i]\}$   
        else if  $\sigma(args_1[i]) \neq args_2[i]$  then  
            return  $\perp$   
    end  
    return  $\sigma$ 
```

$match(\text{parent}(X, Y), \text{parent}(\text{giles}, \text{mark}), \{X \mapsto \text{giles}\})$

- $f_1 = \text{parent}(X, Y)$
- $f_2 = \text{parent}(\text{giles}, \text{mark})$
- $\sigma = \{X \mapsto \text{giles}\}$

# Computing Matching Substitutions

```
def match( $f_1 = name_1(args_1)$ ,  $f_2 = name_2(args_2)$ ,  $\sigma$ ):  
    if name1 and name2 are different then return  $\perp$ ;  
    for  $i \leftarrow 0$  to length( $args_1$ ) do  
        if args1[i] is a variable and args1[i]  $\notin \sigma$  then  
            |  $\sigma = \sigma \cup \{args_1[i] \mapsto args_2[i]\}$   
        else if  $\sigma(args_1[i]) \neq args_2[i]$  then  
            | return  $\perp$   
    end  
    return  $\sigma$ 
```

match(parent( $X$ ,  $Y$ ), parent(giles, mark),  $\{X \mapsto \text{giles}\}$ )

- $f_1 = \text{parent}(X, Y)$
- $f_2 = \text{parent}(\text{giles}, \text{mark})$
- $\sigma = \{X \mapsto \text{giles}\}$

# Computing Matching Substitutions

```
def match( $f_1 = name_1(args_1)$ ,  $f_2 = name_2(args_2)$ ,  $\sigma$ ):  
  if  $name_1$  and  $name_2$  are different then return  $\perp$ ;  
  for  $i \leftarrow 0$  to  $length(args_1)$  do  
    if  $args_1[i]$  is a variable and  $args_1[i] \notin \sigma$  then  
      |  $\sigma = \sigma \cup \{args_1[i] \mapsto args_2[i]\}$   
    else if  $\sigma(args_1[i]) \neq args_2[i]$  then  
      | return  $\perp$   
  end  
  return  $\sigma$ 
```

$match(\text{parent}(X, Y), \text{parent}(\text{giles}, \text{mark}), \{X \mapsto \text{giles}\})$

- $f_1 = \text{parent}(X, Y)$
- $args_1[0] = X$
- $f_2 = \text{parent}(\text{giles}, \text{mark})$
- $args_2[0] = \text{giles}$
- $\sigma = \{X \mapsto \text{giles}\}$

# Computing Matching Substitutions

```
def match( $f_1 = name_1(args_1)$ ,  $f_2 = name_2(args_2)$ ,  $\sigma$ ):  
    if  $name_1$  and  $name_2$  are different then return  $\perp$ ;  
    for  $i \leftarrow 0$  to  $length(args_1)$  do  
        if  $args_1[i]$  is a variable and  $args_1[i] \notin \sigma$  then  
            |  $\sigma = \sigma \cup \{args_1[i] \mapsto args_2[i]\}$   
        else if  $\sigma(args_1[i]) \neq args_2[i]$  then  
            | return  $\perp$   
    end  
    return  $\sigma$ 
```

$match(\text{parent}(X, Y), \text{parent}(\text{giles}, \text{mark}), \{X \mapsto \text{giles}\})$

- $f_1 = \text{parent}(X, Y)$
- $f_2 = \text{parent}(\text{giles}, \text{mark})$
- $\sigma = \{X \mapsto \text{giles}\}$
- $args_1[0] = X$
- $args_2[0] = \text{giles}$
- $\sigma(args_1[0]) = \{X \mapsto \text{giles}\}(X) = \text{giles}$

# Computing Matching Substitutions

```
def match( $f_1 = name_1(args_1)$ ,  $f_2 = name_2(args_2)$ ,  $\sigma$ ):  
    if  $name_1$  and  $name_2$  are different then return  $\perp$ ;  
    for  $i \leftarrow 0$  to  $length(args_1)$  do  
        if  $args_1[i]$  is a variable and  $args_1[i] \notin \sigma$  then  
             $\sigma = \sigma \cup \{args_1[i] \mapsto args_2[i]\}$   
        else if  $\sigma(args_1[i]) \neq args_2[i]$  then  
            return  $\perp$   
    end  
    return  $\sigma$ 
```

$match(\text{parent}(X, Y), \text{parent}(\text{giles}, \text{mark}), \{X \mapsto \text{giles}\})$

- $f_1 = \text{parent}(X, Y)$
- $f_2 = \text{parent}(\text{giles}, \text{mark})$
- $\sigma = \{X \mapsto \text{giles}\}$
- $args_1[1] = Y$
- $args_2[1] = \text{mark}$

# Computing Matching Substitutions

```
def match( $f_1 = name_1(args_1)$ ,  $f_2 = name_2(args_2)$ ,  $\sigma$ ):  
    if  $name_1$  and  $name_2$  are different then return  $\perp$ ;  
    for  $i \leftarrow 0$  to  $length(args_1)$  do  
        if  $args_1[i]$  is a variable and  $args_1[i] \notin \sigma$  then  
            |  $\sigma = \sigma \cup \{args_1[i] \mapsto args_2[i]\}$   
        else if  $\sigma(args_1[i]) \neq args_2[i]$  then  
            | return  $\perp$   
    end  
    return  $\sigma$ 
```

$match(\text{parent}(X, Y), \text{parent}(\text{giles}, \text{mark}), \{X \mapsto \text{giles}\})$

- $f_1 = \text{parent}(X, Y)$
- $f_2 = \text{parent}(\text{giles}, \text{mark})$
- $\sigma = \{X \mapsto \text{giles}, Y \mapsto \text{mark}\}$
- $args_1[1] = Y$
- $args_2[1] = \text{mark}$

# Computing Matching Substitutions

```
def match( $f_1 = name_1(args_1)$ ,  $f_2 = name_2(args_2)$ ,  $\sigma$ ):  
    if  $name_1$  and  $name_2$  are different then return  $\perp$ ;  
    for  $i \leftarrow 0$  to  $length(args_1)$  do  
        if  $args_1[i]$  is a variable and  $args_1[i] \notin \sigma$  then  
             $\sigma = \sigma \cup \{args_1[i] \mapsto args_2[i]\}$   
        else if  $\sigma(args_1[i]) \neq args_2[i]$  then  
            return  $\perp$   
    end  
    return  $\sigma$ 
```

$match(\text{parent}(X, Y), \text{parent}(\text{giles}, \text{mark}), \{X \mapsto \text{giles}\})$

- $f_1 = \text{parent}(X, Y)$
- $f_2 = \text{parent}(\text{giles}, \text{mark})$
- $\sigma = \{X \mapsto \text{giles}, Y \mapsto \text{mark}\}$



# Computing Matching Substitutions

```
def match( $f_1 = name_1(args_1)$ ,  $f_2 = name_2(args_2)$ ,  $\sigma$ ):  
    if  $name_1$  and  $name_2$  are different then return  $\perp$ ;  
    for  $i \leftarrow 0$  to  $length(args_1)$  do  
        if  $args_1[i]$  is a variable and  $args_1[i] \notin \sigma$  then  
             $\sigma = \sigma \cup \{args_1[i] \mapsto args_2[i]\}$   
        else if  $\sigma(args_1[i]) \neq args_2[i]$  then  
            return  $\perp$   
    end  
    return  $\sigma$ 
```

match(parent( $X$ ,  $Y$ ), parent(giles, mark),  $\{X \mapsto \text{bob}\}$ )

- $f_1 = \text{parent}(X, Y)$
- $f_2 = \text{parent}(\text{giles}, \text{mark})$
- $\sigma = \{X \mapsto \text{bob}\}$

# Computing Matching Substitutions

```
def match( $f_1 = name_1(args_1)$ ,  $f_2 = name_2(args_2)$ ,  $\sigma$ ):  
    if name1 and name2 are different then return  $\perp$ ;  
    for  $i \leftarrow 0$  to length( $args_1$ ) do  
        if  $args_1[i]$  is a variable and  $args_1[i] \notin \sigma$  then  
             $\sigma = \sigma \cup \{args_1[i] \mapsto args_2[i]\}$   
        else if  $\sigma(args_1[i]) \neq args_2[i]$  then  
            return  $\perp$   
    end  
    return  $\sigma$ 
```

match(parent( $X$ ,  $Y$ ), parent(giles, mark),  $\{X \mapsto \text{bob}\}$ )

- $f_1 = \text{parent}(X, Y)$
- $f_2 = \text{parent}(\text{giles}, \text{mark})$
- $\sigma = \{X \mapsto \text{bob}\}$

# Computing Matching Substitutions

```
def match( $f_1 = name_1(args_1)$ ,  $f_2 = name_2(args_2)$ ,  $\sigma$ ):  
    if  $name_1$  and  $name_2$  are different then return  $\perp$ ;  
    for  $i \leftarrow 0$  to  $length(args_1)$  do  
        if  $args_1[i]$  is a variable and  $args_1[i] \notin \sigma$  then  
            |  $\sigma = \sigma \cup \{args_1[i] \mapsto args_2[i]\}$   
        else if  $\sigma(args_1[i]) \neq args_2[i]$  then  
            | return  $\perp$   
    end  
    return  $\sigma$ 
```

$match(\text{parent}(X, Y), \text{parent}(\text{giles}, \text{mark}), \{X \mapsto \text{bob}\})$

- $f_1 = \text{parent}(X, Y)$
- $args_1[0] = X$
- $f_2 = \text{parent}(\text{giles}, \text{mark})$
- $args_2[0] = \text{giles}$
- $\sigma = \{X \mapsto \text{bob}\}$

# Computing Matching Substitutions

```
def match( $f_1 = \text{name}_1(\text{args}_1)$ ,  $f_2 = \text{name}_2(\text{args}_2)$ ,  $\sigma$ ):  
    if  $\text{name}_1$  and  $\text{name}_2$  are different then return  $\perp$ ;  
    for  $i \leftarrow 0$  to  $\text{length}(\text{args}_1)$  do  
        if  $\text{args}_1[i]$  is a variable and  $\text{args}_1[i] \notin \sigma$  then  
             $\sigma = \sigma \cup \{\text{args}_1[i] \mapsto \text{args}_2[i]\}$   
        else if  $\sigma(\text{args}_1[i]) \neq \text{args}_2[i]$  then  
            return  $\perp$   
    end  
    return  $\sigma$ 
```

$\text{match}(\text{parent}(X, Y), \text{parent}(\text{giles}, \text{mark}), \{X \mapsto \text{bob}\})$

- $f_1 = \text{parent}(X, Y)$
- $f_2 = \text{parent}(\text{giles}, \text{mark})$
- $\sigma = \{X \mapsto \text{bob}\}$
- $\text{args}_1[0] = X$
- $\text{args}_2[0] = \text{giles}$
- $\sigma(\text{args}_1[0]) = \{X \mapsto \text{bob}\}(X) = \text{bob}$

# Computing Matching Substitutions

```
def match( $f_1 = name_1(args_1)$ ,  $f_2 = name_2(args_2)$ ,  $\sigma$ ):  
    if name1 and name2 are different then return  $\perp$ ;  
    for  $i \leftarrow 0$  to  $length(args_1)$  do  
        if args1[i] is a variable and args1[i]  $\notin \sigma$  then  
            |  $\sigma = \sigma \cup \{args_1[i] \mapsto args_2[i]\}$   
        else if  $\sigma(args_1[i]) \neq args_2[i]$  then  
            | return  $\perp$   
    end  
    return  $\sigma$ 
```

$match(\text{parent}(X, Y), \text{parent}(\text{giles}, \text{mark}), \{X \mapsto \text{bob}\}) = \perp$

# Matching A Rule Body

We lift the matching algorithm to match a list of facts (the rule body) against a set of ground facts (the known consequences).

```
def match(body,  $\mathcal{F}$ ):  
    matches =  $\{\emptyset\}$   
    for  $f_1 \in \textit{body}$  do  
        new =  $\emptyset$   
        for  $\sigma_1 \in \textit{matches}$  do  
            for  $f_2 \in \mathcal{F}$  do  
                 $\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$   
                if  $\sigma_2 \neq \perp$  then new.add( $\sigma_2$ );  
            end  
        end  
        matches = new  
    end  
    return matches
```

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

```
def match(body,  $\mathcal{F}$ ):
    matches =  $\{\emptyset\}$ 
    for  $f_1 \in \text{body}$  do
        new =  $\emptyset$ 
        for  $\sigma_1 \in \text{matches}$  do
            for  $f_2 \in \mathcal{F}$  do
                 $\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$ 
                if  $\sigma_2 \neq \perp$  then new.add( $\sigma_2$ );
            end
        end
        matches = new
    end
    return matches
```

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

```
def match(body,  $\mathcal{F}$ ):
```

```
    matches =  $\{\emptyset\}$ 
```

```
    for  $f_1 \in \text{body}$  do
```

```
        new =  $\emptyset$ 
```

```
        for  $\sigma_1 \in \text{matches}$  do
```

```
            for  $f_2 \in \mathcal{F}$  do
```

```
                 $\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$ 
```

```
                if  $\sigma_2 \neq \perp$  then new.add( $\sigma_2$ );
```

```
            end
```

```
        end
```

```
        matches = new
```

```
    end
```

```
    return matches
```

matches =  $\{\emptyset\}$



# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

```
def match(body,  $\mathcal{F}$ ):
```

```
    matches =  $\{\emptyset\}$ 
```

```
    for  $f_1 \in \text{body}$  do
```

```
        new =  $\emptyset$ 
```

```
        for  $\sigma_1 \in \text{matches}$  do
```

```
            for  $f_2 \in \mathcal{F}$  do
```

```
                 $\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$ 
```

```
                if  $\sigma_2 \neq \perp$  then new.add( $\sigma_2$ );
```

```
            end
```

```
        end
```

```
        matches = new
```

```
    end
```

```
    return matches
```

matches =  $\{\emptyset\}$

$f_1 = \text{parent}(X, Y)$

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

**def**  $\text{match}(\text{body}, \mathcal{F})$ :

$\text{matches} = \{\emptyset\}$

**for**  $f_1 \in \text{body}$  **do**

$\text{new} = \emptyset$

**for**  $\sigma_1 \in \text{matches}$  **do**

**for**  $f_2 \in \mathcal{F}$  **do**

$\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$

**if**  $\sigma_2 \neq \perp$  **then**  $\text{new.add}(\sigma_2)$ ;

**end**

**end**

$\text{matches} = \text{new}$

**end**

**return**  $\text{matches}$

$\text{matches} = \{\emptyset\}$

$f_1 = \text{parent}(X, Y)$

$\text{new} = \emptyset$

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

**def**  $\text{match}(\text{body}, \mathcal{F})$ :

$\text{matches} = \{\emptyset\}$

**for**  $f_1 \in \text{body}$  **do**

$\text{new} = \emptyset$

**for**  $\sigma_1 \in \text{matches}$  **do**

**for**  $f_2 \in \mathcal{F}$  **do**

$\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$

**if**  $\sigma_2 \neq \perp$  **then**  $\text{new.add}(\sigma_2)$ ;

**end**

**end**

$\text{matches} = \text{new}$

**end**

**return**  $\text{matches}$

$\text{matches} = \{\emptyset\}$

$f_1 = \text{parent}(X, Y)$

$\text{new} = \emptyset$

$\sigma_1 = \emptyset$

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

```
def match(body,  $\mathcal{F}$ ):
    matches =  $\{\emptyset\}$ 
    for  $f_1 \in \text{body}$  do
        new =  $\emptyset$ 
        for  $\sigma_1 \in \text{matches}$  do
            for  $f_2 \in \mathcal{F}$  do
                 $\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$ 
                if  $\sigma_2 \neq \perp$  then new.add( $\sigma_2$ );  $f_2 = \text{parent}(\text{giles}, \text{mark})$ 
            end
        end
        matches = new
    end
    return matches
```

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

**def**  $\text{match}(\text{body}, \mathcal{F})$ :

$\text{matches} = \{\emptyset\}$

**for**  $f_1 \in \text{body}$  **do**

$\text{new} = \emptyset$

**for**  $\sigma_1 \in \text{matches}$  **do**

**for**  $f_2 \in \mathcal{F}$  **do**

$\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$

**if**  $\sigma_2 \neq \perp$  **then**  $\text{new.add}(\sigma_2)$ ;  $f_2 = \text{parent}(\text{giles}, \text{mark})$

**end**

**end**

$\text{matches} = \text{new}$

**end**

**return**  $\text{matches}$

$\text{matches} = \{\emptyset\}$

$f_1 = \text{parent}(X, Y)$

$\text{new} = \emptyset$

$\sigma_1 = \emptyset$

$f_2 = \text{parent}(\text{giles}, \text{mark})$

$\sigma_2 = \{X \mapsto \text{giles}, Y \mapsto \text{mark}\}$

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

**def**  $\text{match}(\text{body}, \mathcal{F})$ :

$\text{matches} = \{\emptyset\}$

**for**  $f_1 \in \text{body}$  **do**

$\text{new} = \emptyset$

**for**  $\sigma_1 \in \text{matches}$  **do**

**for**  $f_2 \in \mathcal{F}$  **do**

$\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$

**if**  $\sigma_2 \neq \perp$  **then**  $\text{new.add}(\sigma_2)$ ;  $\sigma_1 = \emptyset$

**end**

**end**

$\text{matches} = \text{new}$

**end**

**return**  $\text{matches}$

$\text{matches} = \{\emptyset\}$

$f_1 = \text{parent}(X, Y)$

$\text{new} =$

$\{ \{X \mapsto \text{giles}, Y \mapsto \text{mark}\} \}$

$f_2 = \text{parent}(\text{giles}, \text{mark})$

$\sigma_2 = \{X \mapsto \text{giles}, Y \mapsto \text{mark}\}$

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

**def**  $\text{match}(\text{body}, \mathcal{F})$ :

$\text{matches} = \{\emptyset\}$

**for**  $f_1 \in \text{body}$  **do**

$\text{new} = \emptyset$

**for**  $\sigma_1 \in \text{matches}$  **do**

**for**  $f_2 \in \mathcal{F}$  **do**

$\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$

**if**  $\sigma_2 \neq \perp$  **then**  $\text{new.add}(\sigma_2)$ ;  $\sigma_1 = \emptyset$

**end**

**end**

$\text{matches} = \text{new}$

**end**

**return**  $\text{matches}$

$\text{matches} = \{\emptyset\}$

$f_1 = \text{parent}(X, Y)$

$\text{new} =$

$\{ \{ X \mapsto \text{giles}, Y \mapsto \text{mark} \} \}$

$f_2 = \text{man}(\text{giles})$

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

**def**  $\text{match}(\text{body}, \mathcal{F})$ :

$\text{matches} = \{\emptyset\}$

**for**  $f_1 \in \text{body}$  **do**

$\text{new} = \emptyset$

**for**  $\sigma_1 \in \text{matches}$  **do**

**for**  $f_2 \in \mathcal{F}$  **do**

$\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$

**if**  $\sigma_2 \neq \perp$  **then**  $\text{new.add}(\sigma_2)$ ;  $\sigma_1 = \emptyset$

**end**

**end**

$\text{matches} = \text{new}$

**end**

**return**  $\text{matches}$

$\text{matches} = \{\emptyset\}$

$f_1 = \text{parent}(X, Y)$

$\text{new} =$

$\{ \{ X \mapsto \text{giles}, Y \mapsto \text{mark} \} \}$

$f_2 = \text{man}(\text{giles})$

$\sigma_2 = \perp$



# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

**def**  $\text{match}(\text{body}, \mathcal{F})$ :

$\text{matches} = \{\emptyset\}$

**for**  $f_1 \in \text{body}$  **do**

$\text{new} = \emptyset$

**for**  $\sigma_1 \in \text{matches}$  **do**

**for**  $f_2 \in \mathcal{F}$  **do**

$\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$

**if**  $\sigma_2 \neq \perp$  **then**  $\text{new.add}(\sigma_2)$ ;  $\sigma_1 = \emptyset$

**end**

**end**

$\text{matches} = \text{new}$

**end**

**return**  $\text{matches}$

$\text{matches} = \{\emptyset\}$

$f_1 = \text{parent}(X, Y)$

$\text{new} =$

$\left\{ \left\{ X \mapsto \text{giles}, Y \mapsto \text{mark} \right\} \right\}$

$f_2 = \text{man}(\text{giles})$

$\sigma_2 = \perp$

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

**def**  $\text{match}(\text{body}, \mathcal{F})$ :

$\text{matches} = \{\emptyset\}$

**for**  $f_1 \in \text{body}$  **do**

$\text{new} = \emptyset$

**for**  $\sigma_1 \in \text{matches}$  **do**

**for**  $f_2 \in \mathcal{F}$  **do**

$\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$

**if**  $\sigma_2 \neq \perp$  **then**  $\text{new.add}(\sigma_2); \sigma_1 = \emptyset$

**end**

**end**

$\text{matches} = \text{new}$

**end**

**return**  $\text{matches}$

$\text{matches} = \{\emptyset\}$

$f_1 = \text{parent}(X, Y)$

$\text{new} =$

$\{ \{ X \mapsto \text{giles}, Y \mapsto \text{mark} \} \}$

$f_2 = \text{parent}(\text{bob}, \text{sara})$

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

**def**  $\text{match}(\text{body}, \mathcal{F})$ :

$\text{matches} = \{\emptyset\}$

**for**  $f_1 \in \text{body}$  **do**

$\text{new} = \emptyset$

**for**  $\sigma_1 \in \text{matches}$  **do**

**for**  $f_2 \in \mathcal{F}$  **do**

$\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$

**if**  $\sigma_2 \neq \perp$  **then**  $\text{new.add}(\sigma_2)$ ;  $\sigma_1 = \emptyset$

**end**

**end**

$\text{matches} = \text{new}$

**end**

**return**  $\text{matches}$

$\text{matches} = \{\emptyset\}$

$f_1 = \text{parent}(X, Y)$

$\text{new} =$

$\{ \{ X \mapsto \text{giles}, Y \mapsto \text{mark} \} \}$

$f_2 = \text{parent}(\text{bob}, \text{sara})$

$\sigma_2 = \{ X \mapsto \text{bob}, Y \mapsto \text{sara} \}$

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

**def**  $\text{match}(\text{body}, \mathcal{F})$ :

$\text{matches} = \{\emptyset\}$

**for**  $f_1 \in \text{body}$  **do**

$\text{new} = \emptyset$

**for**  $\sigma_1 \in \text{matches}$  **do**

**for**  $f_2 \in \mathcal{F}$  **do**

$\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$

**if**  $\sigma_2 \neq \perp$  **then**  $\text{new.add}(\sigma_2)$ ;

**end**

**end**

$\text{matches} = \text{new}$

**end**

**return**  $\text{matches}$

$\text{matches} = \{\emptyset\}$

$f_1 = \text{parent}(X, Y)$

$\text{new} =$

$\left\{ \begin{array}{l} \{X \mapsto \text{giles}, Y \mapsto \text{mark}\} \\ \{X \mapsto \text{bob}, Y \mapsto \text{sara}\} \end{array} \right\}$

$\sigma_1 = \emptyset$

$f_2 = \text{parent}(\text{bob}, \text{sara})$

$\sigma_2 = \{X \mapsto \text{bob}, Y \mapsto \text{sara}\}$

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

**def**  $\text{match}(\text{body}, \mathcal{F})$ :

$\text{matches} = \{\emptyset\}$

**for**  $f_1 \in \text{body}$  **do**

$\text{new} = \emptyset$

**for**  $\sigma_1 \in \text{matches}$  **do**

**for**  $f_2 \in \mathcal{F}$  **do**

$\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$

**if**  $\sigma_2 \neq \perp$  **then**  $\text{new.add}(\sigma_2)$ ;  $\sigma_1 = \emptyset$

**end**

**end**

$\text{matches} = \text{new}$

**end**

**return**  $\text{matches}$

$\text{matches} = \{\emptyset\}$

$f_1 = \text{parent}(X, Y)$

$\text{new} =$

$\left\{ \begin{array}{l} \{X \mapsto \text{giles}, Y \mapsto \text{mark}\} \\ \{X \mapsto \text{bob}, Y \mapsto \text{sara}\} \end{array} \right\}$

$f_2 = \text{man}(\text{bob})$

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

**def**  $\text{match}(\text{body}, \mathcal{F})$ :

$\text{matches} = \{\emptyset\}$

**for**  $f_1 \in \text{body}$  **do**

$\text{new} = \emptyset$

**for**  $\sigma_1 \in \text{matches}$  **do**

**for**  $f_2 \in \mathcal{F}$  **do**

$\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$

**if**  $\sigma_2 \neq \perp$  **then**  $\text{new.add}(\sigma_2)$ ;  $\sigma_1 = \emptyset$

**end**

**end**

$\text{matches} = \text{new}$

**end**

**return**  $\text{matches}$

$\text{matches} = \{\emptyset\}$

$f_1 = \text{parent}(X, Y)$

$\text{new} =$

$\left\{ \begin{array}{l} \{X \mapsto \text{giles}, Y \mapsto \text{mark}\} \\ \{X \mapsto \text{bob}, Y \mapsto \text{sara}\} \end{array} \right\}$

$f_2 = \text{man}(\text{bob})$

$\sigma_2 = \perp$

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

**def**  $\text{match}(\text{body}, \mathcal{F})$ :

$\text{matches} = \{\emptyset\}$

**for**  $f_1 \in \text{body}$  **do**

$\text{new} = \emptyset$

**for**  $\sigma_1 \in \text{matches}$  **do**

**for**  $f_2 \in \mathcal{F}$  **do**

$\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$

**if**  $\sigma_2 \neq \perp$  **then**  $\text{new.add}(\sigma_2)$ ;

**end**

**end**

$\text{matches} = \text{new}$

**end**

**return**  $\text{matches}$

$\text{matches} = \{\emptyset\}$

$f_1 = \text{parent}(X, Y)$

$\text{new} =$

$\left\{ \begin{array}{l} \{X \mapsto \text{giles}, Y \mapsto \text{mark}\} \\ \{X \mapsto \text{bob}, Y \mapsto \text{sara}\} \end{array} \right\}$

$\sigma_1 = \emptyset$

$f_2 = \text{man}(\text{bob})$

$\sigma_2 = \perp$

# Matching A Rule Body

$\text{match}(\text{parent}(X, Y), \text{man}(X), \left\{ \begin{array}{l} \text{parent}(\text{giles}, \text{mark}), \text{man}(\text{giles}) \\ \text{parent}(\text{bob}, \text{sara}), \text{man}(\text{bob}) \end{array} \right\})$

**def**  $\text{match}(\text{body}, \mathcal{F})$ :

$\text{matches} = \{\emptyset\}$

**for**  $f_1 \in \text{body}$  **do**

$\text{new} = \emptyset$

**for**  $\sigma_1 \in \text{matches}$  **do**

**for**  $f_2 \in \mathcal{F}$  **do**

$\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$

**if**  $\sigma_2 \neq \perp$  **then**  $\text{new.add}(\sigma_2)$ ;  $\sigma_1 = \{X \mapsto \text{giles}, Y \mapsto \text{mark}\}$

**end**

**end**

$\text{matches} = \text{new}$

**end**

**return**  $\text{matches}$

$\text{matches} = \left\{ \begin{array}{l} \{X \mapsto \text{giles}, Y \mapsto \text{mark}\} \\ \{X \mapsto \text{bob}, Y \mapsto \text{sara}\} \end{array} \right\}$

$f_1 = \text{parent}(X, Y)$

$\text{new} = \emptyset$

$\sigma_1 = \{X \mapsto \text{giles}, Y \mapsto \text{mark}\}$



# Matching A Rule Body

```
def match(body,  $\mathcal{F}$ ):  
    matches =  $\{\emptyset\}$   
    for  $f_1 \in \textit{body}$  do  
        new =  $\emptyset$   
        for  $\sigma_1 \in \textit{matches}$  do  
            for  $f_2 \in \mathcal{F}$  do  
                 $\sigma_2 = \text{match}(f_1, f_2, \sigma_1)$   
                if  $\sigma_2 \neq \perp$  then new.add( $\sigma_2$ );  
            end  
        end  
        matches = new  
    end  
    return matches
```

Clearly inefficient

The order in which we check elements in the body can effect the complexity as we can get a large set of initial fact on the first item and find that most are inconsistent with the next one

In reality we do something cleverer



# Forward Chaining Algorithm

Compute the next set of consequences whilst there are new consequences.  
Search all consequences for facts matching the query.

```
def forward(facts  $\mathcal{F}_0$ , rules  $\mathcal{RU}$ , query  $q$ ):  
     $\mathcal{F} = \emptyset$ ;  $new = \mathcal{F}_0$   
    do  
         $\mathcal{F} = \mathcal{F} \cup new$ ;  $new = \emptyset$   
        for  $body \Rightarrow head \in \mathcal{RU}$  do  
            for  $\sigma \in \text{match}(body, \mathcal{F})$  do  
                if  $\sigma(head) \notin \mathcal{F}$  then  $new.add(\sigma(head))$   
            end  
        end  
    while  $new \neq \emptyset$   
     $ans = \emptyset$   
    for  $f \in \mathcal{F}$  do  $\sigma = \text{match}(q, f, \emptyset)$ ; if  $\sigma \neq \perp$  then  $ans.add(\sigma)$   
    return  $ans$ 
```

# Forward Chaining Algorithm

Compute the next set of consequences whilst there are new consequences.  
Search all consequences for facts matching the query.

```
def forward(facts  $\mathcal{F}_0$ , rules  $\mathcal{RU}$ , query  $q$ ):  
     $\mathcal{F} = \emptyset$ ;  $new = \mathcal{F}_0$   
    do  
         $\mathcal{F} = \mathcal{F} \cup new$ ;  $new = \emptyset$   
        for  $body \Rightarrow head \in \mathcal{RU}$  do  
            for  $\sigma \in \text{match}(body, \mathcal{F})$  do  
                if  $\sigma(head) \notin \mathcal{F}$  then  $new.add(\sigma(head))$   
            end  
        end  
    while  $new \neq \emptyset$   
     $ans = \emptyset$   
    for  $f \in \mathcal{F}$  do  $\sigma = \text{match}(q, f, \emptyset)$ ; if  $\sigma \neq \perp$  then  $ans.add(\sigma)$   
    return  $ans$ 
```

# Forward Chaining Algorithm

Compute the next set of consequences whilst there are new consequences.  
Search all consequences for facts matching the query.

```
def forward(facts  $\mathcal{F}_0$ , rules  $\mathcal{RU}$ , query  $q$ ):  
     $\mathcal{F} = \emptyset$ ;  $new = \mathcal{F}_0$   
    do  
         $\mathcal{F} = \mathcal{F} \cup new$ ;  $new = \emptyset$   
        for  $body \Rightarrow head \in \mathcal{RU}$  do  
            for  $\sigma \in \text{match}(body, \mathcal{F})$  do  
                if  $\sigma(head) \notin \mathcal{F}$  then  $new.add(\sigma(head))$   
            end  
        end  
    while  $new \neq \emptyset$   
     $ans = \emptyset$   
    for  $f \in \mathcal{F}$  do  $\sigma = \text{match}(q, f, \emptyset)$ ; if  $\sigma \neq \perp$  then  $ans.add(\sigma)$   
    return  $ans$ 
```

# Forward Chaining Algorithm

Compute the next set of consequences whilst there are new consequences.  
Search all consequences for facts matching the query.

```
def forward(facts  $\mathcal{F}_0$ , rules  $\mathcal{RU}$ , query  $q$ ):  
     $\mathcal{F} = \emptyset$ ;  $new = \mathcal{F}_0$   
    do  
         $\mathcal{F} = \mathcal{F} \cup new$ ;  $new = \emptyset$   
        for  $body \Rightarrow head \in \mathcal{RU}$  do  
            for  $\sigma \in \text{match}(body, \mathcal{F})$  do  
                if  $\sigma(head) \notin \mathcal{F}$  then  $new.add(\sigma(head))$   
            end  
        end  
    while  $new \neq \emptyset$   
     $ans = \emptyset$   
    for  $f \in \mathcal{F}$  do  $\sigma = \text{match}(q, f, \emptyset)$ ; if  $\sigma \neq \perp$  then  $ans.add(\sigma)$   
    return  $ans$ 
```

# Forward Chaining Algorithm

Compute the next set of consequences whilst there are new consequences.  
Search all consequences for facts matching the query.

```
def forward(facts  $\mathcal{F}_0$ , rules  $\mathcal{RU}$ , query  $q$ ):  
     $\mathcal{F} = \emptyset$ ;  $new = \mathcal{F}_0$   
    do  
         $\mathcal{F} = \mathcal{F} \cup new$ ;  $new = \emptyset$   
        for  $body \Rightarrow head \in \mathcal{RU}$  do  
            for  $\sigma \in \text{match}(body, \mathcal{F})$  do  
                if  $\sigma(head) \notin \mathcal{F}$  then  $new.add(\sigma(head))$   
            end  
        end  
    while  $new \neq \emptyset$   
     $ans = \emptyset$   
    for  $f \in \mathcal{F}$  do  $\sigma = \text{match}(q, f, \emptyset)$ ; if  $\sigma \neq \perp$  then  $ans.add(\sigma)$   
    return  $ans$ 
```

# Forward Chaining Algorithm

Compute the next set of consequences whilst there are new consequences.  
Search all consequences for facts matching the query.

```
def forward(facts  $\mathcal{F}_0$ , rules  $\mathcal{RU}$ , query  $q$ ):  
     $\mathcal{F} = \emptyset$ ;  $new = \mathcal{F}_0$   
    do  
         $\mathcal{F} = \mathcal{F} \cup new$ ;  $new = \emptyset$   
        for  $body \Rightarrow head \in \mathcal{RU}$  do  
            for  $\sigma \in \text{match}(body, \mathcal{F})$  do  
                if  $\sigma(head) \notin \mathcal{F}$  then  $new.add(\sigma(head))$   
            end  
        end  
    while  $new \neq \emptyset$   
     $ans = \emptyset$   
    for  $f \in \mathcal{F}$  do  $\sigma = \text{match}(q, f, \emptyset)$ ; if  $\sigma \neq \perp$  then  $ans.add(\sigma)$   
    return  $ans$ 
```



# Forward Chaining Algorithm

Compute the next set of consequences whilst there are new consequences.  
Search all consequences for facts matching the query.

```
def forward(facts  $\mathcal{F}_0$ , rules  $\mathcal{RU}$ , query  $q$ ):  
     $\mathcal{F} = \emptyset$ ;  $new = \mathcal{F}_0$   
    do  
         $\mathcal{F} = \mathcal{F} \cup new$ ;  $new = \emptyset$   
        for  $body \Rightarrow head \in \mathcal{RU}$  do  
            for  $\sigma \in \text{match}(body, \mathcal{F})$  do  
                if  $\sigma(head) \notin \mathcal{F}$  then  $new.add(\sigma(head))$   
            end  
        end  
    while  $new \neq \emptyset$   
     $ans = \emptyset$   
    for  $f \in \mathcal{F}$  do  $\sigma = \text{match}(q, f, \emptyset)$ ; if  $\sigma \neq \perp$  then  $ans.add(\sigma)$   
    return  $ans$ 
```

# Forward Chaining Algorithm

Compute the next set of consequences whilst there are new consequences.  
Search all consequences for facts matching the query.

```
def forward(facts  $\mathcal{F}_0$ , rules  $\mathcal{RU}$ , query  $q$ ):  
     $\mathcal{F} = \emptyset$ ;  $new = \mathcal{F}_0$   
    do  
         $\mathcal{F} = \mathcal{F} \cup new$ ;  $new = \emptyset$   
        for  $body \Rightarrow head \in \mathcal{RU}$  do  
            for  $\sigma \in \text{match}(body, \mathcal{F})$  do  
                if  $\sigma(head) \notin \mathcal{F}$  then  $new.add(\sigma(head))$   
            end  
        end  
    while  $new \neq \emptyset$   
     $ans = \emptyset$   
    for  $f \in \mathcal{F}$  do  $\sigma = \text{match}(q, f, \emptyset)$ ; if  $\sigma \neq \perp$  then  $ans.add(\sigma)$   
    return  $ans$ 
```

# Efficient Matching

**Observation:** The current algorithm for matching against known consequences is inefficient; it involves multiple iterations over all known consequences.

**Solution 1:** Use heuristics to select the order in which facts in the body are matched e.g. pick least frequently occurring name first.

**Solution 2:** Store known facts in a data structure that facilitates quick lookup of matching facts. We will see such a data structure for *unification* towards the end of the course

# Incremental Forward Chaining

**Observation:** On each step the only new additions come from rules that are triggered by new facts.

**Solution:** Use the previous set of new facts as an initial filter to identify which rules are relevant and which further facts need to match against existing facts

# Dealing with Irrelevant Facts

**Observation:** We can derive a lot of facts that are irrelevant to the query

**Solution 1:** Rewrite the knowledge base to remove/reduce rules that produce irrelevant facts. Computationally expensive but may be worth it if similar queries executed often. Similar to query optimisation in database.

**Solution 2:** Backward Chaining. Start from the query and work backwards to see which facts support it. This is what Prolog does.

# Summary

**Queries** are facts possibly containing variables

To answer queries we can compute all **consequences** and check these

We can use **forward chaining** to compute consequences

This relies on **matching**, which can be tricky to implement efficiently

**Next time: Prolog!**