Symbolic AI

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Photo by Vasilyev Alexandr

Acknowledgements

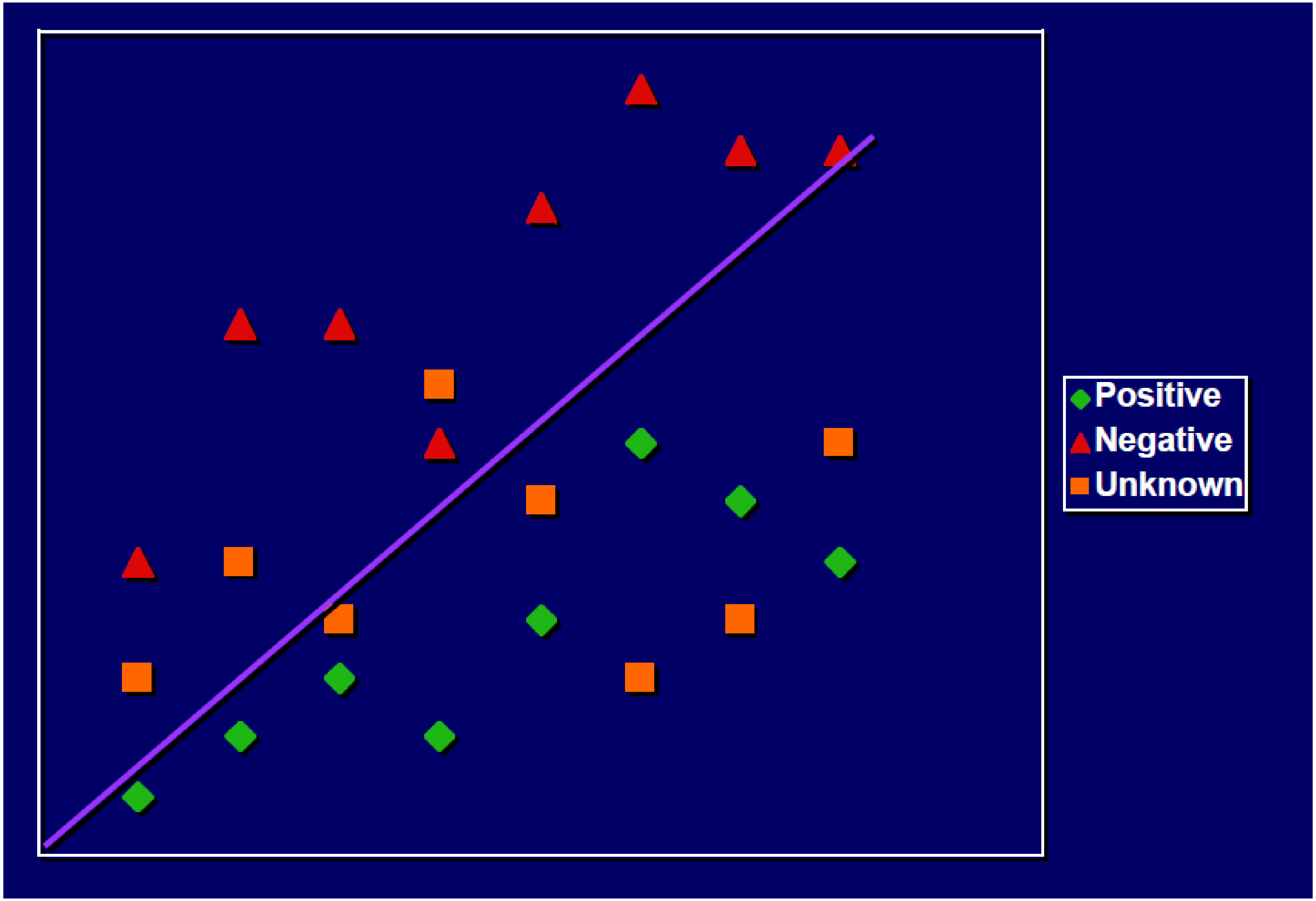
- These slides were based on the slides of:
  - Peter A. Flach, Rule induction tutorial, IDA Spring School 2001.
  - Anoop & Hector, Inductive Logic Programming (for Dummies).
  - Gabor Melli, Scribe Notes on FOIL and Inverted Deduction.
  - CS 5751 Machine Learning, Chapter 10 Learning Sets of Rules.

This Lecture

- Introduction to Inductive Logic Programming
- FOIL

Flach 2001

Linear Classifier



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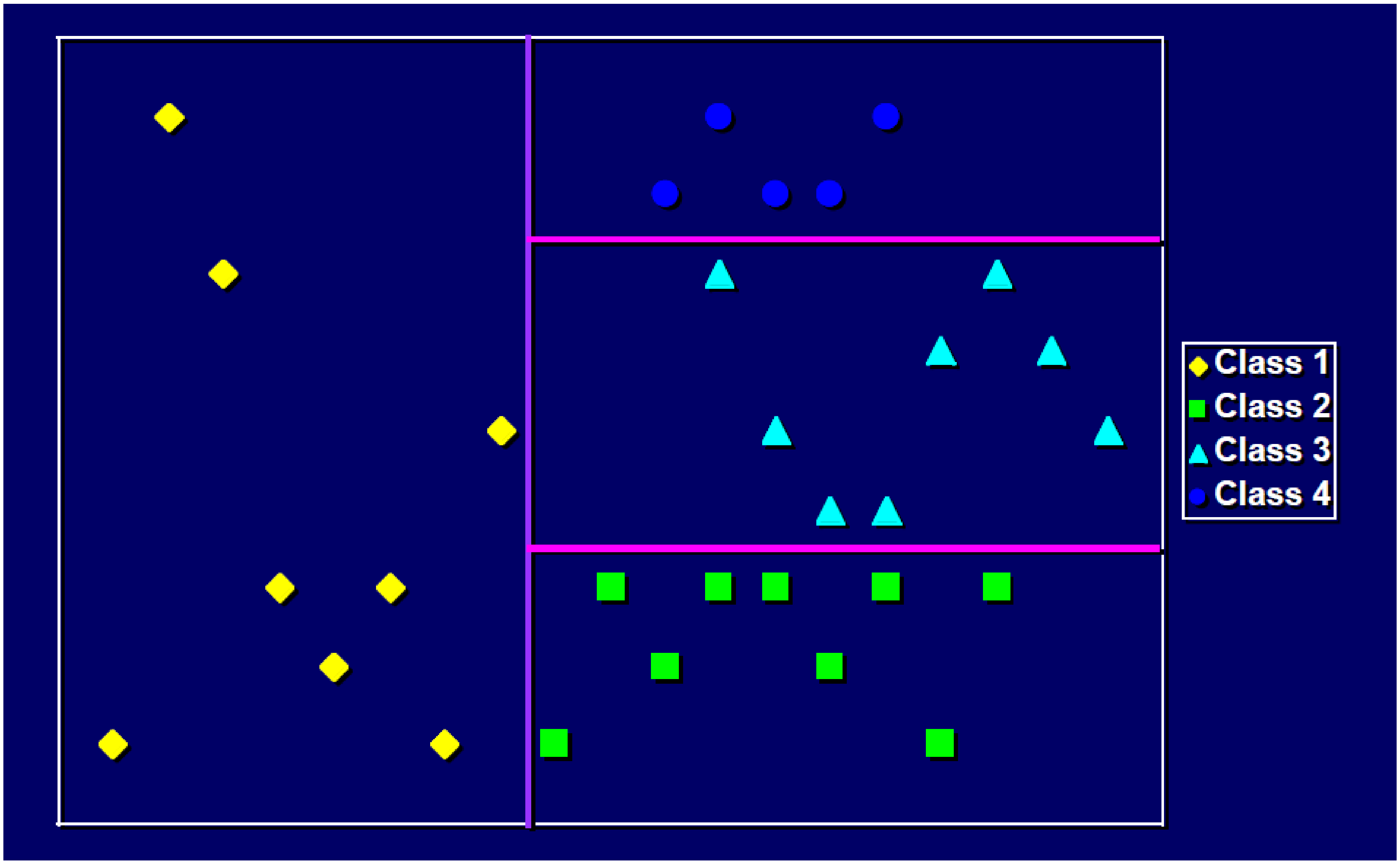
Decision Trees



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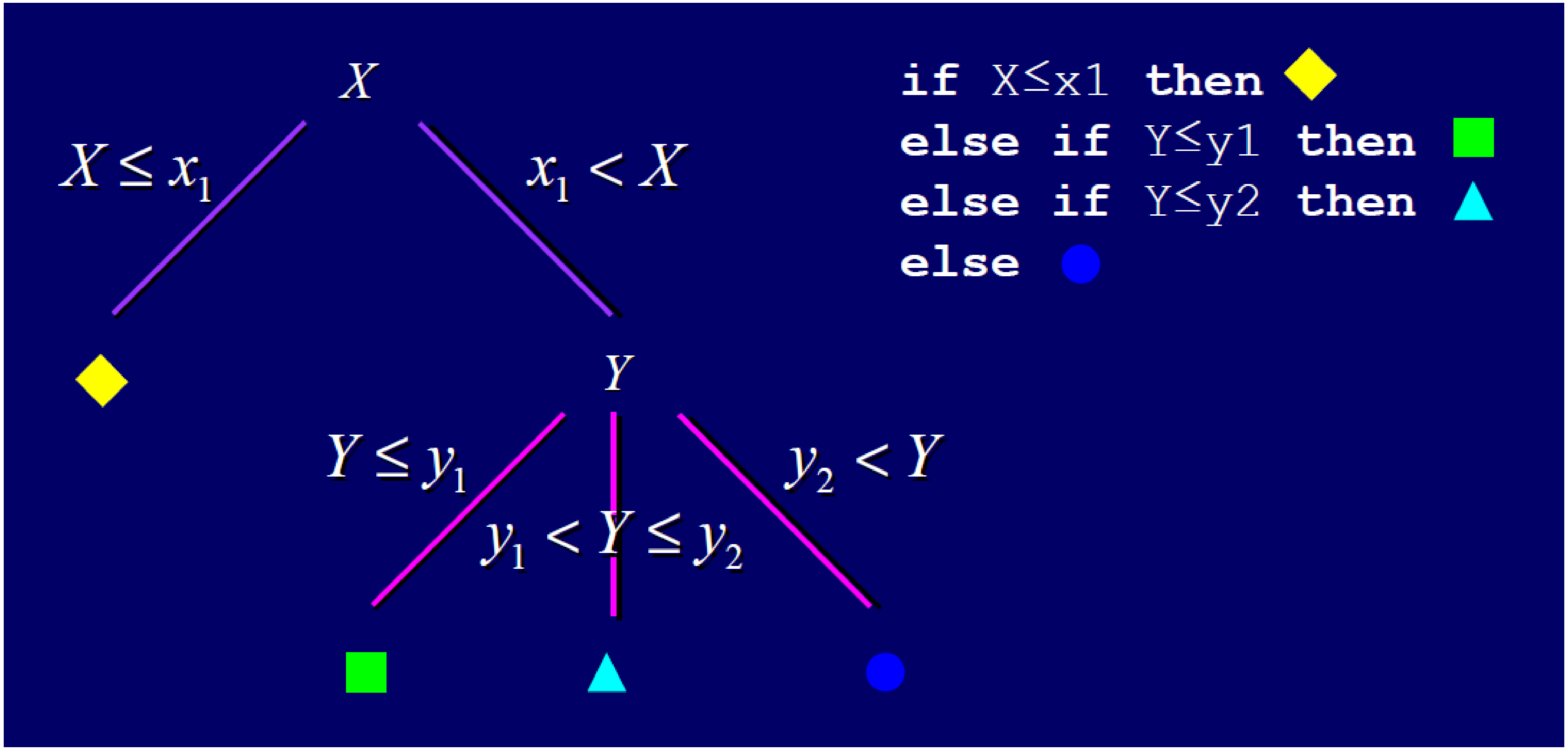
Decision Trees



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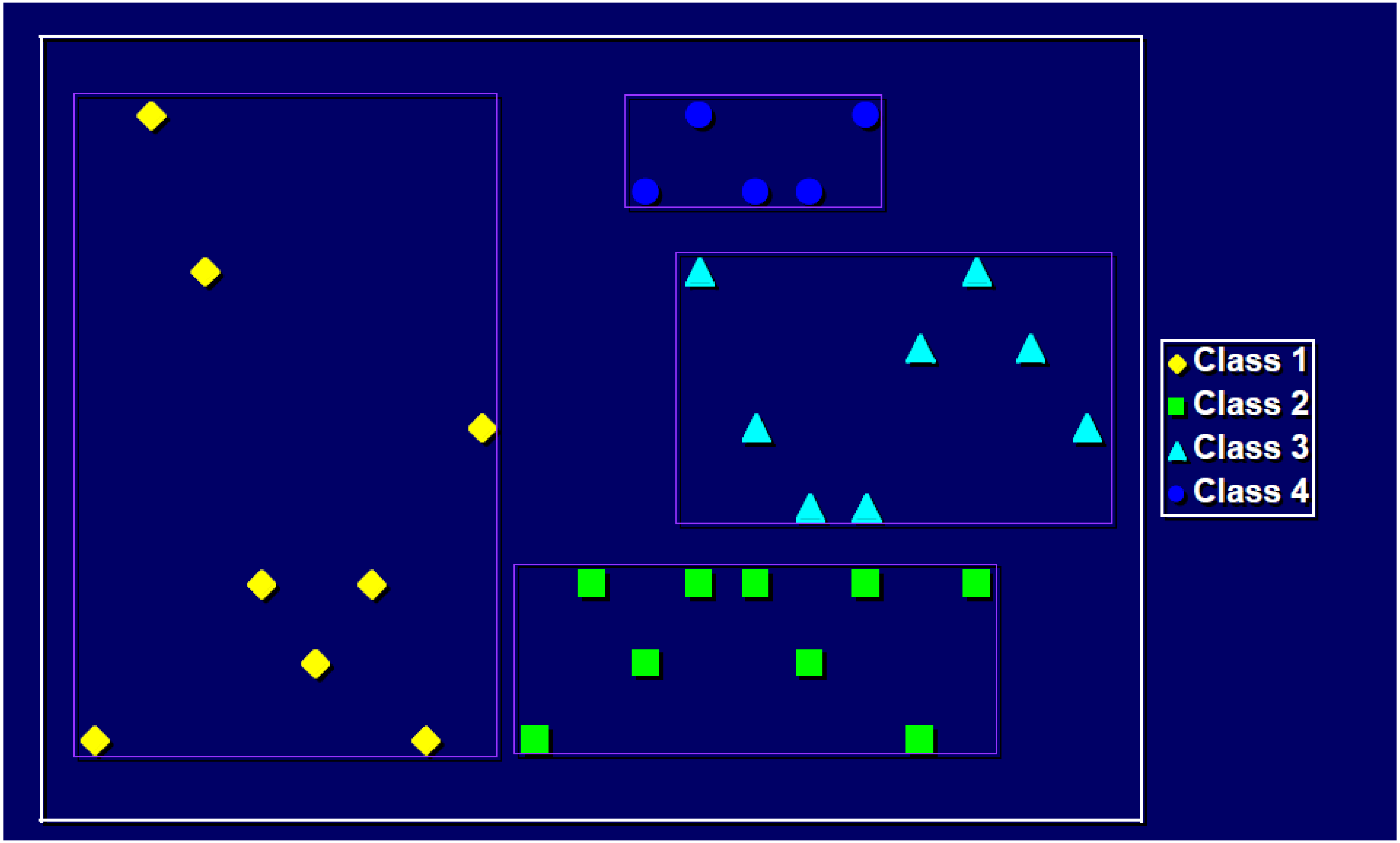
Rules



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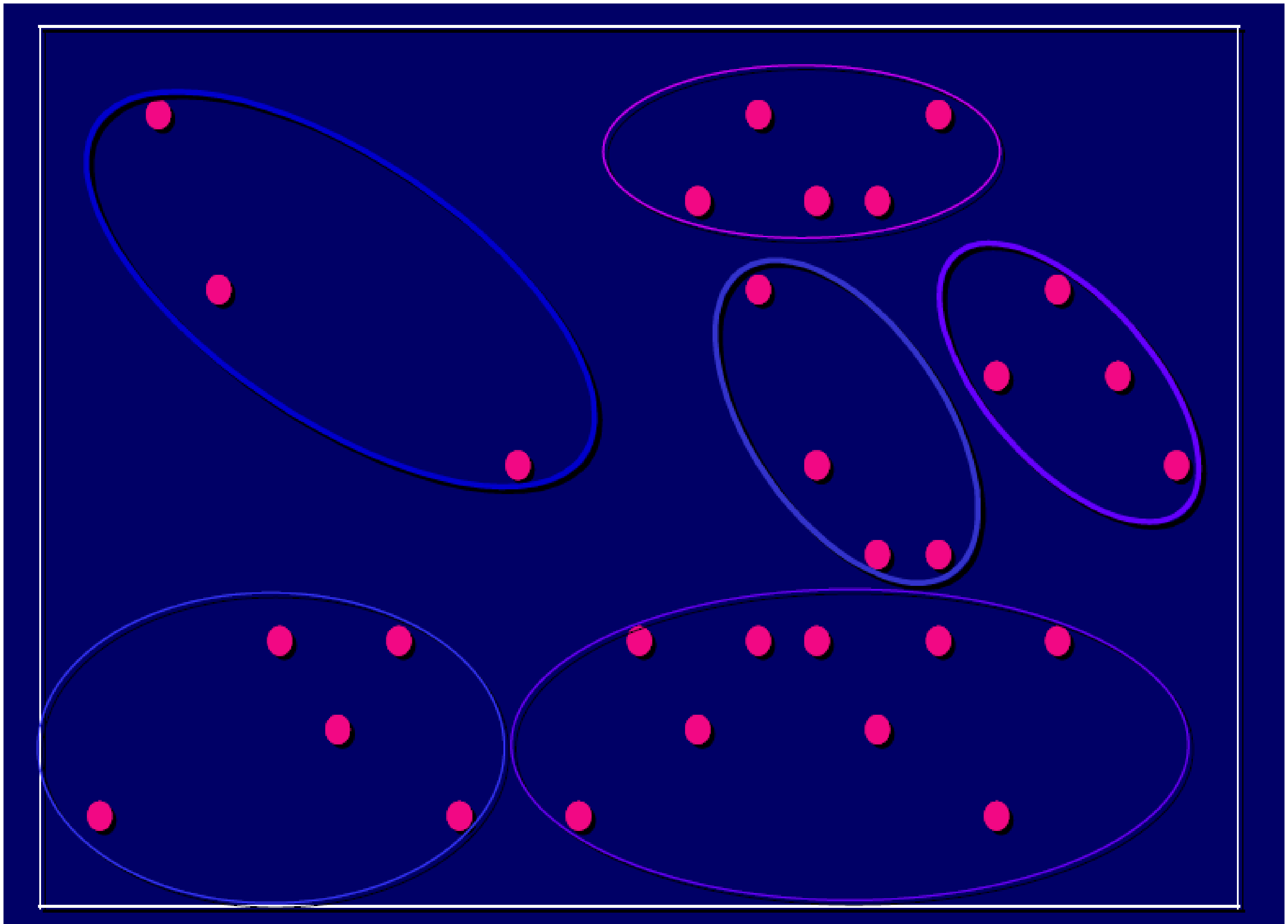
Clustering



ILP: Objective

Given a dataset:

- Positive examples (*E+*) and optionally negative examples (*E-*).
- Additional knowledge about the problem/application domain (Background Knowledge *B*).
- Set of constraints to make the learning process more efficient (*C*).

Goal of an ILP system is to find a set of hypothesis that:

- Explains (covers) the positive examples – Completeness.
- Are consistent with the negative examples – Consistency.

Generalisation & Specialisation

- **<u>Generalising</u>** a concept involves enlarging its extension in order to cover a given instance or subsume another concept.

- **<u>Specialising</u>** a concept involves restricting its extension in order to avoid covering a given instance or subsuming another concept.

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First-order Representations

- **Propositional** representations:
  - datacase is ***fixed-size vector of values***
  - features are those given in the dataset

- **First-order** representations:
  - datacase is ***flexible-size, structured object***
  - sequence, set, graph
  - hierarchical: e.g. set of sequences
  - features need to be **selected** from potentially infinite set

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Deductive Vs Inductive Reasoning

$T \cup B \rightarrow E$ **(deduce)**

```
parent(X,Y) :- mother(X,Y).
parent(X,Y) :- father(X,Y).
```

```
mother(mary,vinni).
mother(mary,andre).
father(carrey,vinni).
father(carry,andre).
```

```
parent(mary,vinni).
parent(mary,andre).
parent(carrey,vinni).
parent(carrey,andre).
```

$E \cup B \rightarrow T$ **(induce)**

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parent(mary,vinni).
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```

```
parent(X,Y) :- mother(X,Y).
parent(X,Y) :- father(X,Y).
```

Relational Pattern

IF Customer(C1,Age1,Income1,TotSpent1,BigSpender1)
  AND MarriedTo(C1,C2)
  AND Customer(C2,Age2,Income2,TotSpent2,BigSpender2)
  AND Income2 $\geq$ 10000
THEN BigSpender1 = Yes

big_spender(C1,Age1,Income1,TotSpent1) $\leftarrow$
  married_to(C1,C2) $\wedge$
  customer(C2,Age2,Income2,TotSpent2,BigSpender2) $\wedge$
  Income2 $\geq$ 10000

Example ILP Problem

Discover the rule that describes whether a person has a granddaughter

TRAIN

Visible

Father	
a_1	a_2
Sharon	Bob
Tom	Bob
Bob	Victor
…	…

Female
a_1
Sharon
…

GrandDaughter	
a_1	a_2
Victor	Sharon
…	…

TEST

Visible

Father	
a_1	a_2
Julie	John
John	Howard

Female
a_1
Julie

Hidden

GrandDaughter	
a_1	a_2
Howard	Julie

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Propositional Learner with simple data transformation

- One of the first challenges that a propositional learner would encounter with this dataset is that the dataset is not structured as a set of fixed length-vectors of attribute-value pairs. This situation is typically resolved by JOINing the relations.

Predictors			**Target**
Father	***Child***	***Child is Fem.***	***Has Gdaugh***
Bob	Sharon	TRUE	FALSE
Victor	Bob	FALSE	TRUE
…	…	…	…

Propositional Learner with simple data transformation

- A propositional learner would not locate a predictive model for this dataset.
- It would not be able to state that *Sharon* is *Victor*'s granddaughter.
- At best it may discover that a child's gender has some influence on the likelihood that that child is a parent, or even a parent to a female child.

Predictors			**Target**
Father	***Child***	***Child is Fem.***	***Has Gdaugh***
Bob	Sharon	TRUE	FALSE
Victor	Bob	FALSE	TRUE
...	...	...	...

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Propositional Leaner with complex data transformation

- The algorithm cannot make the connection in one observation (*Bob* as a father) and another (*Bob* as child).

- A common way to enable a propositional learner to produce a predictive model on this data is <u>to transform the data so that the required relations appear as attributes in the data</u>.

- This transformation is sometimes referred to as 'flattening' the data.

Predictors					Target
Father	*Child*	*Child is Fem.*	*Child's Child*	*C's C is Fem.*	*Has Gdaugh*
Bob	Sharon	TRUE	NULL	NULL	FALSE
Victor	Bob	FALSE	Sharon	TRUE	TRUE
…	…	…		…	…

- Now the search for a rule is trivial. A decision tree would locate the pattern:

IF Child's Child is Female = TRUE
THEN HasGrandDaughter = TRUE.
ELSE HasGrandDaughter = FALSE

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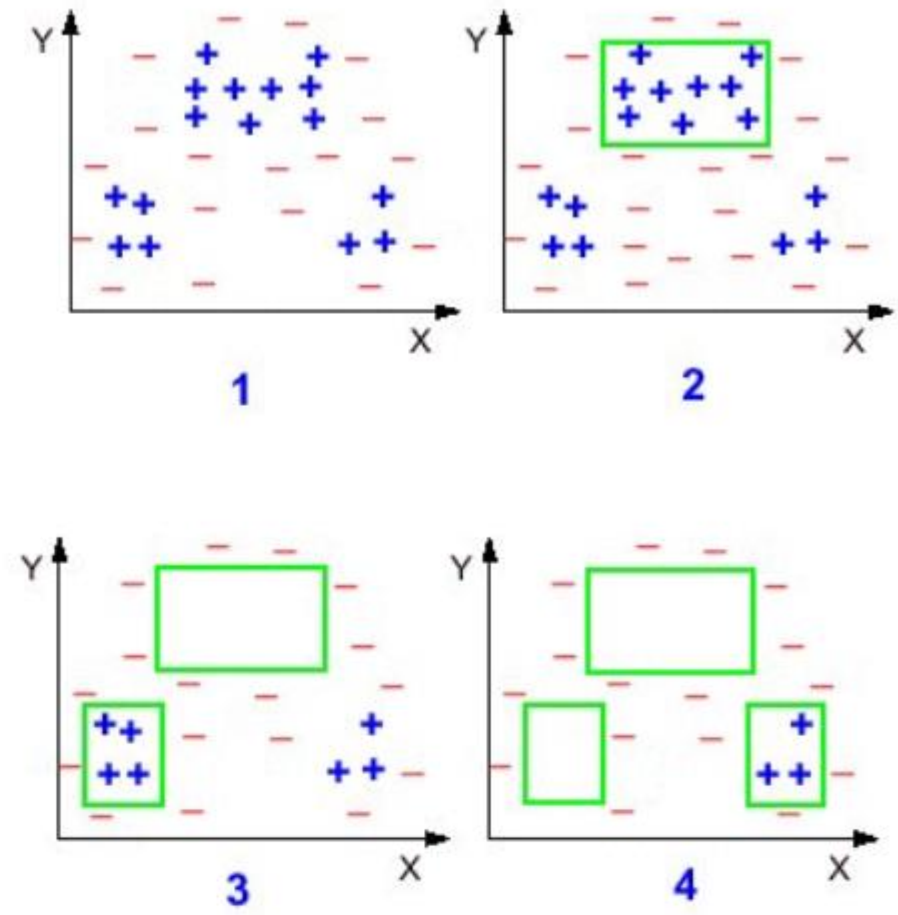
Propositional Sequential Covering

- A <u>covering algorithm</u>, in the context of propositional learning systems, is an algorithm that develops a <u>cover</u> for the set of positive examples.
  - that is, a set of hypotheses that account for all the positive examples but none of the negative examples.

- <u>Sequential covering:</u> it learns one rule at a time and repeat this process to gradually cover the full set of positive examples.

Iterate to Learn Multiple Rules

- Select seed from positive examples to build bottom clause.
- Get some rule "If A $\wedge$ B then P". Now throw away all positive examples that were covered by this rule
- Repeat until there are no more positive examples.



Propositional Sequential Covering

1. Start with an empty **Cover**
2. Use **Learn-One-Rule** to find the best hypothesis.
3. If the Just-Learnt-Rule satisfies the threshold then
   - Put Just-Learnt-Rule to the **Cover**.
   - Remove examples covered by Just-Learnt-Rule.
   - Go to step 2.
4. Sort the **Cover** according to its performance over examples.
5. Return: **Cover**.

Example

Id	Size	Colour	Shape	Weight	Expensive
1	Big	Red	Square	Heavy	Yes
2	Small	Blue	Triangle	Light	Yes
3	Small	Blue	Square	Light	No
4	Big	Green	Triangle	Heavy	No
5	Big	Blue	Square	Light	No
6	Big	Green	Square	Heavy	Yes
7	Small	Red	Triangle	Light	Yes

Expensive = Yes if:

Colour = Red. (covers example 1,7)
Or (Colour = Green & Shape = Square). (covers example 6)
Or (Colour = Blue & Shape = Triangle). (covers example 2)

Complex

- *A* <u>complex</u> is a conjunction of attribute-value specifications. It forms the **condition** part in a rule, like "if **condition** then predict **class**".

Size=Big	Size=Small	Colour=Red
Colour=Green	Colour=Blue	Shape=Square
Shape=Triangle	Weight=Light	Weight=Heavy

- <u>Specialising a complex</u> is making a conjunction of the complex with one more attribute-value pair. For example:

Colour=Green & Shape=Square (specialising Colour=Green or Shape=Square)

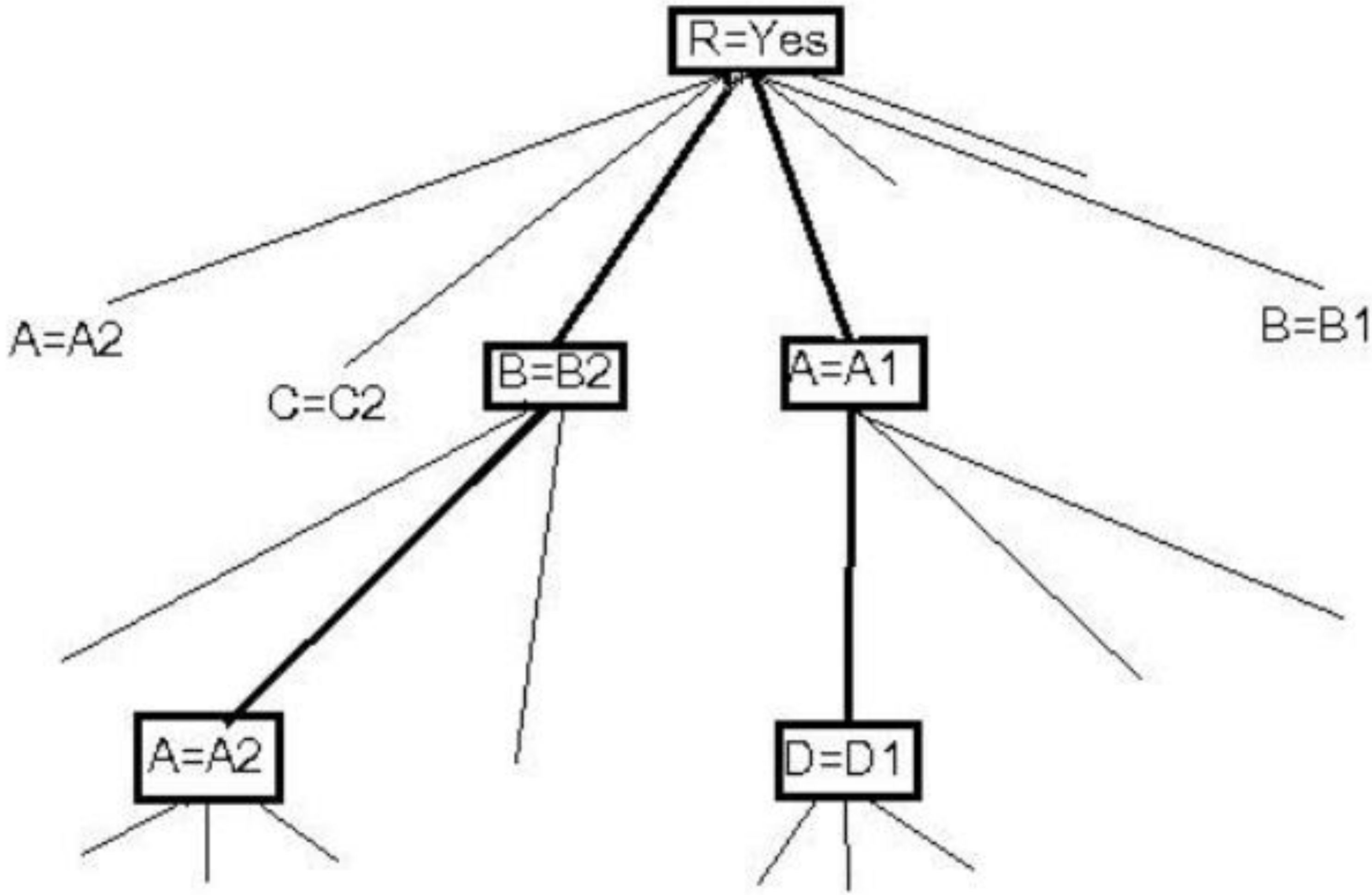
Colour=Blue & Weight=Heavy (specialising Colour=Blue or Weight=Heavy)

Learn-One-Rule using Beam Search

1. Initialize a set of most general complexes.
2. Evaluate performances of those complexes over the example set.
   - Count how many positive and negative examples it covers.
   - Evaluate their performances.
3. Sort complexes according to their performances.
4. If the best complex satisfies some **threshold**, form the hypothesis and **return**.
5. Otherwise, pick k best performing complexes for the next generation.
6. Specializing all k complexes in the set to find new set of less general complexes.
7. Go to step 2.

The number k is the beam factor of the search, meaning the maximum number of complexes to be specialized.

Example



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General to Specific Beam Search Example

- In the first step, 2 best complexes are found, namely A=A1 and B=B2.

- None of them satisfy the threshold, then the next level complexes are expanded and found 2 best complexes, eg. A=A1 & D=D1 and B=B2 & A=A2.

- The procedure keeps going until we find a complex that satisfies the threshold.

Entropy Evaluation Function

- The evaluation is based on the entropy of the set covered by that complex. Here is an example of a hypothesis covering 8 positive and 2 negative examples.

$$p1 = P(\text{positive}) = 8/(2+8) = 0.8;$$

$$p2 = P(\text{negative}) = 2/(2+8) = 0.2;$$

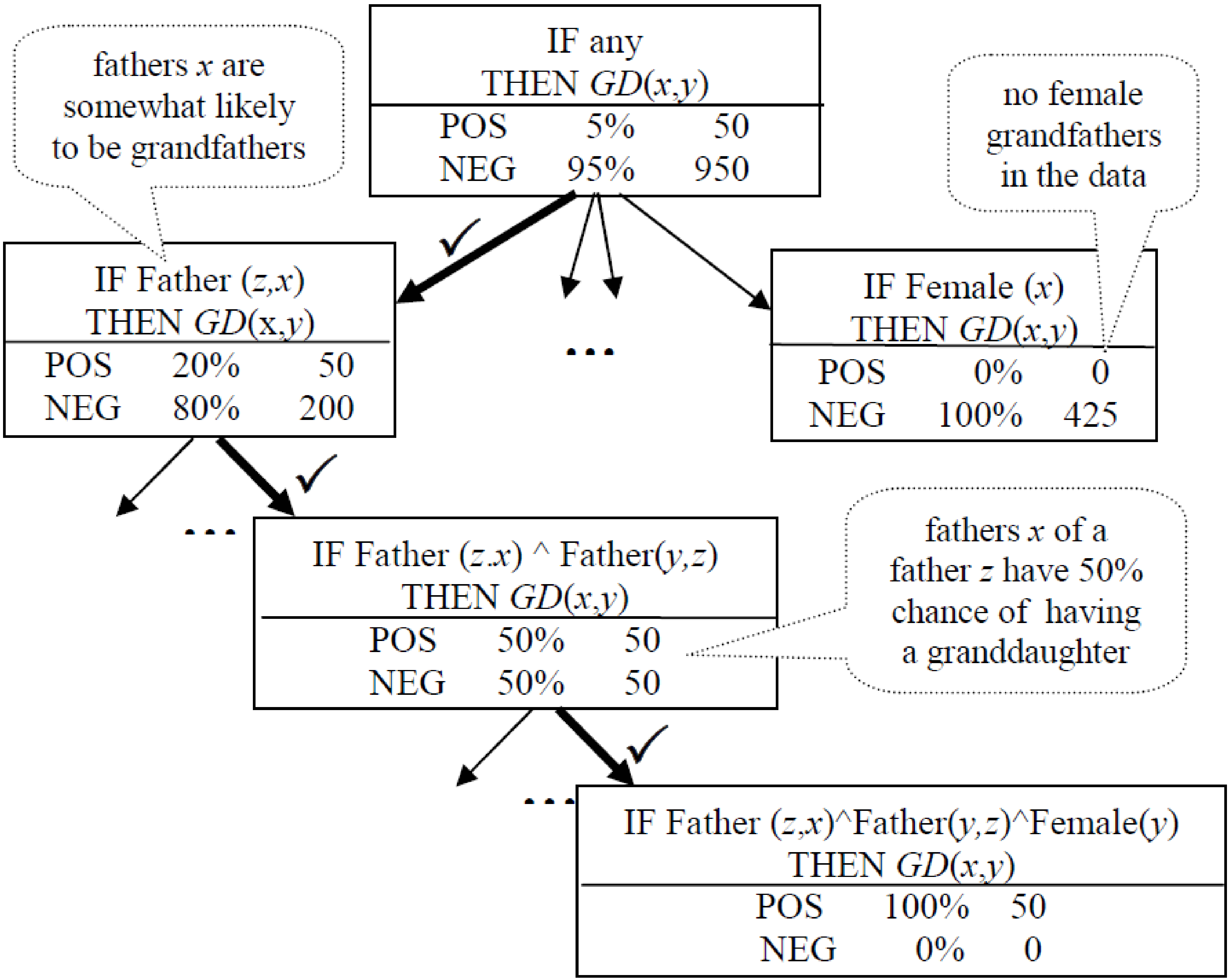
$$\text{Entropy} = - p1 * \log(p1) - p2 * \log(p2) = 0.72.$$

- In this function, the smaller the entropy is, the better the complex.
- In other words, the accuracy function can be defined as (1-Entropy).

The FOIL Algorithm

- The FOIL algorithm is a supervised learning algorithm that produces rules in first-order logic.
- The algorithm is a generalization of the SEQUENTIAL-COVERING and LEARN-ONE-RULE algorithms .
- The main modification is that search can also specialize on predicates with variables.
- The resulting rules differ from Horn clauses in two ways:
  - Negated symbols are allowed within the body.
  - FOIL’s rules will not include function symbols.

Back to the Example


IF any
THEN GD(x,y)
POS 5% 50
NEG 95% 950
fathers x are somewhat likely to be grandfathers
IF Father (z,x)
THEN GD(x,y)
POS 20% 50
NEG 80% 200
...
IF Female (x)
THEN GD(x,y)
POS 0% 0
NEG 100% 425
no female grandfathers in the data
...
IF Father (z,x) ^ Father(y,z)
THEN GD(x,y)
POS 50% 50
NEG 50% 50
fathers x of a father z have 50% chance of having a granddaughter
...
IF Father (z,x)^Father(y,z)^Female(y)
THEN GD(x,y)
POS 100% 50
NEG 0% 0
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FOIL

FOIL(*Target_predicate*, *Predicates*, *Examples*)
Pos $\leftarrow$ positive *Examples*
Neg $\leftarrow$ negative *Examples*
while *Pos* do (*Learn a New Rule*)
    NewRule $\leftarrow$ most general rule possible
    NegExamplesCovered $\leftarrow$ *Neg*
    while *NegExamplesCovered* do
        Add a new literal to specialize *NewRule*
        1. *Candidate_literals* $\leftarrow$ generate candidates
        2. *Best_literal* $\leftarrow \text{argmax}_{L \in \mathit{candidate\_literal}}$ FOIL_GAIN(*L*, *NewRule*)
        3. Add *Best_literal* to *NewRule* preconditions
        4. *NegExamplesCovered* $\leftarrow$ subset of *NegExamplesCovered* that satistifies *NewRule* preconditions
    Learned_rules $\leftarrow$ *Learned_rules* + *NewRule*
    Pos $\leftarrow$ *Pos* - {members of *Pos* covered by *NewRule*}
Return *Learned_rules*

The FOIL Algorithm

- The *outer loop adds new rules* to the output until no more positive examples are covered.
- The *inner loop searches for the next best rule* by incremental specialization.
- The outer loop corresponds to the SEQUENTIAL-CONVERING algorithm, the inner to FIND-A-RULE

Specialising Rules in FOIL

Learning rule: $P(x_1,x_2,\ldots,x_k) \leftarrow L_1 \ldots L_n$

Candidate specializations add new literal of form:

- $Q(v_1,\ldots,v_r)$, where at least one of the v_i in the created literal must already exist as a variable in the rule
- $Equal(x_j,x_k)$, where x_j and x_k are variables already present in the rule
- The negation of either of the above forms of literals

Information Gain in FOIL

$$FOIL\_GAIN(L,R) \equiv t\left(\log_2 \frac{p_1}{p_1+n_1} - \log_2 \frac{p_0}{p_0+n_0}\right)$$

Where

- L is the candidate literal to add to rule R
- p_0 = number of positive bindings of R
- n_0 = number of negative bindings of R
- p_1 = number of positive bindings of $R+L$
- n_1 = number of negative bindings of $R+L$
- t is the number of positive bindings of R also covered by $R+L$

Note

- $-\log_2 \frac{p_0}{p_0+n_0}$ is optimal number of bits to indicate the class of a positive binding covered by R

Applications

First Order Rule for Classifying Web Pages

From (Slattery, 1997)

course(A) $\leftarrow$
  has-word(A,instructor),
  NOT has-word(A,good),
  link-from(A,B)
  has-word(B,assignment),
  NOT link-from(B,C)

Train: 31/31, Test 31/34

Flach 2001

Early diagnosis of rheumatic diseases

- Sample CN2 rule for an 8-class problem :

IF Sex = male AND Age > 46 AND
Number_of_painful_joints > 3 AND
Skin_manifestations = psoriasis

Flach 2001

Application

A molecular compound is carcinogenic if:

(1) it tests positive in the Salmonella assay; or

(2) it tests positive for sex-linked recessive lethal mutation in Drosophila; or

(3) it tests negative for chromosome aberration; or

(4) it has a carbon in a six-membered aromatic ring with a partial charge of -0.13; or

(5) it has a primary amine group and no secondary or tertiary amines; or

(6) it has an aromatic (or resonant) hydrogen with partial charge ≥ 0.168; or

(7) it has an hydroxy oxygen with a partial charge ≥ -0.616 and an aromatic (or resonant) hydrogen; or

(8) it has a bromine; or

(9) it has a tetrahedral carbon with a partial charge ≤ -0.144 and tests positive on Progol's mutagenicity rules.

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Final Considerations

Why ILP is not just Decision Trees.

- Language is First-Order Logic
  - Natural representation for multi-relational settings
  - Thus, a natural representation for *full* databases

- Not restricted to the classification task.
- So then, what is ILP?

Efficiency Issues

- Representational Aspects
- Search
- Evaluation
- Sharing computations
- Memory-wise scalability

Representational Aspects

- Example:
  - Student(string <u>sname</u>, string major, string minor)
  - Course(string <u>cname</u>, string prof, string cred)
  - Enrolled(string <u>sname,</u> string <u>cname</u>)

- In a natural join of these tables there is a one-to-one correspondence between join result and the Enrolled table.

- Data mining tasks on the Enrolled table are really propositional.

Representational Aspects

- Three settings for data mining:
  - Find patterns within individuals represented as tuples (single table, propositional)
    - eg. Which minor is chosen with what major
  - Find patterns within individuals represented as sets of tuples (each individual 'induces' a sub-database)
    - Multiple tables, restricted to some individual
    - eg. Student X taking course A, usually takes course B
  - Find patterns within the whole database
    - Multiple tables

Evaluation

- Evaluating a clause: get some measure of coverage
  - Match each example to the clause:
    - Run multiple logical queries.
  - Query optimization methods from DB community
    - Rel. Algebra operator reordering
    - BUT: queries for DB are set oriented (bottom-up), queries in PROLOG find a single solution (top-down).

Sharing Computations

- Materialization of features
- Propositionalization
- Pre-compute some statistics
  - Joint distribution over attributes of a table
  - Query selectivity
- Store proofs, reuse when evaluating new clauses

Summary

- Rules: easy to understand
  – Sequential covering algorithm
  – generate one rule at a time
  – general to specific - add antecedents
  – specific to general - delete antecedents

- First order logic and covering
  – how to connect variables
  – FOIL

Recommended Reading

QuickFOIL: Scalable Inductive Logic Programming

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