

Algos: P vs NP

→ Termination Analysis; examples like Yes, No, (?)

→ 1936: "There's a program I can tell if another program (input) halts or thinks"

→ unbounded memory usage

→ Tractable: done easily (e.g. in polynomial time)
intractable: done \rightarrow easily (e.g. in exp. time)
as inputs grow in size

→ polynomial: with input size n , $O(n^k)$

→ $O(1)$, $O(\log n)$, $O(n)$, $O(n^2)$, $O(n^3)$...

→ if $f(n)$ takes microseconds

$$\rightarrow f(n) = \sqrt{n} \quad (\because 1 \text{ second}) \rightarrow n = 1 \times 10^{12}$$

$$\rightarrow f(n) = \log n \quad (\because 1 \text{ second}) \rightarrow n = 2^{10^6}$$

→ "NP = ~~P~~" polynomial/tractable problems

	Computational Challenge	Walking Problem
Polynomial	<u>NP</u> , <u>Decidable</u>	<u>Undecidable</u>

→ NP = non-deterministic polynomial

→ multiple decisions at once

→ set of all possible decisions from a given state

Hardest NP problems

- NP-complete: decision problems which are
 - NP
 - as "hard" as ANY NP problem
- ① decision problems have binary output
 - Complexity: $O(2^n)$
 - (Circuit V Formula) Satisfiability, program equivalence checking
- if problem is NP-complete, then it is intractable
UNLESS: $P = NP \Leftrightarrow$ (i.e. any NP problem solved in polynomial time)
- Shortest simple path: Dijkstra algo
 - Polynomial
- LONGEST: NP-Complete
- Euler tour: visiting all edges of graph exactly once
- Hamiltonian cycle: each vertex visited exactly once
 - Solution: since \exists (node!) possible permutations, $O(\text{node}!)$ runtime
- $(\neg A \vee B) \wedge A \wedge \neg B \rightarrow$ unsatisfiable
 - Enumeration: one approach to check
 - Deduction
$$\begin{array}{c} \xrightarrow{\quad} \\ A \rightarrow B \end{array} \quad \begin{array}{c} A \\ \hline \neg B \end{array}$$
$$\frac{A \quad \neg B}{\perp}$$
- IF a THEN b ELSE c $\rightarrow (a \rightarrow b) \vee (\neg a \rightarrow c)$

conjunctive normal form?

→ 3-CNF: AND of clauses, each clause has 3 distinct literals

→ reducibility: