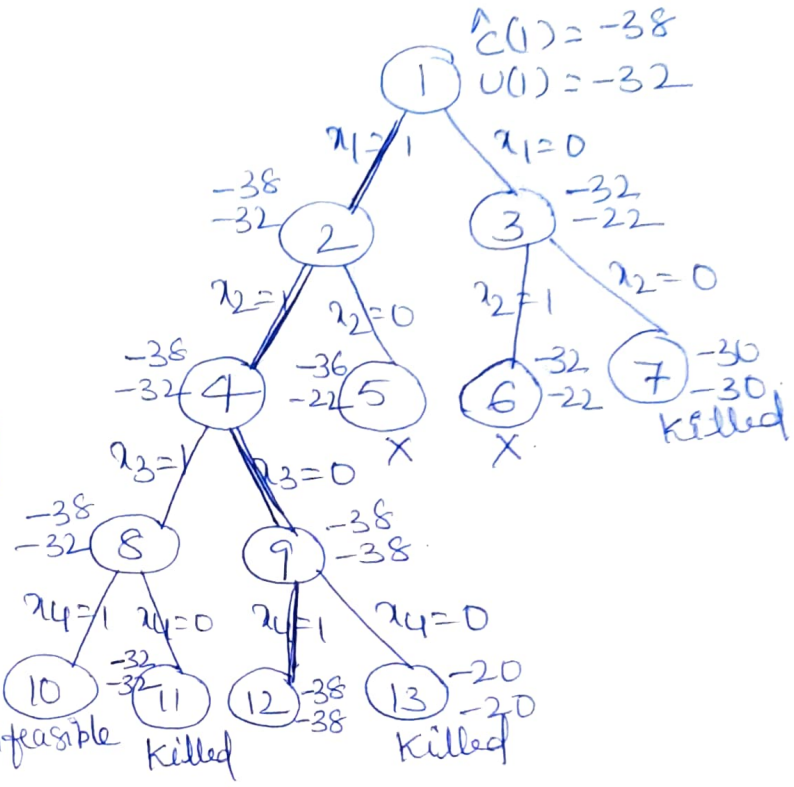
0/1 Knapsack Problem (Using FIFOBB)

Example :-

$n = 4, (P_1, P_2, P_3, P_4) = (10, 10, 12, 18)$

$m = 15, (w_1, w_2, w_3, w_4) = (2, 4, 6, 9)$.



At Node 1 :-

$2+4+6 = 12 \leq m$

$\therefore u(1) = (10+10+12)$

$= -32$

$\hat{c}(1) = -32 + [-3 * \frac{18}{9}^{2}]$

$= -32 + (-6)$

$\hat{c}(1) = -38$

Upper is initialized to $u(1) = -32$

Nodes 2 and 3 are generated and added to the queue. The value of "upper" remains unchanged. ~~Node 2 becomes the next E-node~~

2	3

At Node 2 :- $x_1 = 1$

$2+4+6 = 10 \leq m$

$u(2) = (10+10+12) = -32$

$\hat{c}(2) = -32 + [-3 * \frac{18}{9}]$

$= -32 + (-6) = -38$

At Node 3 :- $x_1 = 0$

$4+6 = 10 \leq m$

$u(3) = -(10+12) = -22$

$\hat{c}(3) = -22 + [-5 * \frac{18}{9}^{2}]$

$= -22 + (-10)$

$\hat{c}(3) = -32$

Node (2) becomes next E-node as it satisfies $\hat{c} <$ upper.

$\hat{c}(2) <$ upper.

$-38 < -32$ ✓

Its children nodes 4 & 5 are generated and to the queue

~~3~~	~~4~~	5	6

Node (3) becomes next E-node as it satisfies $\hat{c}(3) <$ upper. Its children nodes are generated 6 & 7.

At Node 4:—

$x_1 = 1, x_2 = 1$

$u(4) = -32$

$\hat{c}(4) = -38$.

$\hat{c}(4) <$ upper.

$-38 < -32$ ✓.

So added to queue.

At Node 5:—

$x_1 = 1, x_2 = 0$.

$2 + 6 = 8 \leq m$.

$u(5) = -(10 + 12) = -22$

$\hat{c}(5) = -22 + (-7 * \frac{18^2}{9})$

$\hat{c}(5) = -36$

$\hat{c}(5) <$ upper.

$-36 < -32$ ✓.

So added queue

At Node 6:—

$x_1 = 0, x_2 = 1$.

$w + 6 = 10 \leq m$

$u(6) = -(10 + 12) = -22$

$\hat{c}(6) = -22 + (-5 * \frac{18^2}{9})$

$= -22 + (-10)$

$\hat{c}(6) = -32$.

$\hat{c}(6) <$ upper

$-32 < -32$

So added to queue.

At Node 7 :- $x_1 = 0, x_2 = 6$

$6 + 9 = 15 \leq m$ ✓

$u(7) = -(12 + 18) = -30$

$\hat{c}(7) = -30 + (-0 \times 0)$

$\hat{c}(7) = -30$

$\hat{c}(7) < \text{upper}$

$-30 \not< -32$

hence node (7) is immediately killed.

~~7~~	5	6

the Next E-node is (4), expanded.

nodes (8) & (9) are generated.

~~At Node 8~~ :-

At Node 8 :-

$x_1 = 1, x_2 = 1, x_3 = 1$

$2 + 4 + 6 = 12 \leq m$

$\therefore u(8) = -32$

$\hat{c}(8) = -38$

At Node 9 :-

$x_1 = 1, x_2 = 1, x_3 = 0$

$2 + 4 + 9 = 15 \leq 15$

$u(9) = -(10 + 10 + 18) = -38$

$\hat{c}(9) = -38$

* Now upper is updated to $u(9) = -38$

5	6	8	9

next E-node is (5).

bcz $\hat{c}(5) < upper$

$-36 \not< -38$ (since upper is updated to -38)

So not generated.

next E-node is (6) bcz-

$\hat{c}(6) < upper$

$-32 \not< -38$

So not generated.

8	9

Next E-node (8) is ~~generated~~ expanded

hence nodes (10) & (11) are generated.

At Node 10 :-

$x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 1.$

$2 + 4 + 6 + 9 = 21 > m$

hence it is infeasible.

At Node 11 :-

$x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 0$

$u(11) = -32$

$\hat{c}(11) = -32$

$\hat{c}(11) < upper$

$-32 \not< -38$ So killed.

9

Next E-node is (9) i.e, bound. So Nodes 12 & 13 are generated.

At Node 12 :-

$x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 1$

$2 + 4 + 9 = 15 \leq m$.

$\therefore u(12) = -38$

$\hat{c}(12) = -38$.

12	13

At Node 13 :-

$x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 0$

$2 + 4 = 6 \leq m$

$\therefore u(13) = -20$

$\hat{c}(13) = -20$

$\hat{c}(13) <$ upper

$-20 \not< -38$ So killed.

The path from root is the answer node (12) is solution

$\therefore (x_1 x_2 x_3 x_4) = (1, 1, 0, 1)$

LCBB :- based on least cost
we will select the path.

FIFOBB :- based on $\hat{c}$ & upper values:
if $\hat{c} >$ upper, the node is killed.
else placed into the queue of
live nodes.