UNIT-4

Reliability Design Problem ①

The problem is to design a system that is composed of several devices connected in series.

Let r_i be the reliability of device D_i i.e the probability that device 'i' will fn properly.

Then the reliability of entire system is πr_i (since they are connected in series).

For Ex if $n=10$ and $r_i = .99$
thus $\pi r_i = 0.904$.

Hence it is desirable to duplicate devices. Multiple copies of the same ~~typ~~ device type are connected in parallel.

→ if stage 'i' contains m_i copies of device D_i then the probability that all m_i have malfn is $(1-r_i)^{m_i}$

$(1-r_i) \rightarrow$ gives probability of malfn.

Therefore $(1-r_i)^{m_i} \rightarrow$ gives the probability of malfn of multiple devices in stage 'i'.

$\therefore (1-r_i)^{m_i} \rightarrow$ probability of malfn at stage 'i'

Hence $1-(1-r_i)^{m_i} \rightarrow$ probability of devices in stage 'i' fn properly.

$\therefore 1-(1-r_i)^{m_i} \rightarrow$ is the reliability of stage 'i'.

Thus if $r_i = .99$, $m_i = 2$

$(1-.99)^2 = (1-.01)^2 = (.99)^2$

=

→ Let us assume that the reliability of stage 'i' is given by a fn $\phi_i(m_i)$

→ Hence the reliability of the system of stages is $\prod_{1 \le i \le n} \phi_i(m_i)$.

* Our problem is to use device duplication to maximize reliability. This maximization is to be carried out under cost constraint *

Let c_i be the cost of each unit of device i and let 'c' be the maximum allowable cost of the system being designed

Hence

$$\text{maximize } \overbrace{\prod_{1 \le i \le n} \underbrace{\phi_i(m_i)}_{\text{reliability of stage } i}}^{\text{reliability of entire system}}$$

$$\text{Subject to } \overbrace{\sum_{1 \le i \le n} \underbrace{c_i m_i}_{\text{cost of stage } i}}^{\text{cost of entire system}} \le c$$

A dynamic programming solution can be obtained. Since we can assume each $c_i > 0$, each m_i must be in the range $1 \le m_i \le u_i$ where

$$u_i = \left\lfloor \frac{\left(c + c_i - \sum_1^n c_j\right)}{c_i} \right\rfloor$$

u_i → max ~~allowable~~ no. of allowable devices in stage 'i' (i.e. Upperbound)

→ The <u>dominance rule</u>: (f_1, x_1) dominates (f_2, x_2) iff $f_1 \geq f_2$ & $x_1 \leq x_2$ holds. Hence dominated tuples can be discarded.

<u>Example</u>: design a 3 stage system with D_1, D_2 & D_3 device types. The costs are \$30, \$15, \$20 respectively. The cost of the system is to be no more than \$105. The reliability of each device type is 0.9, 0.8 & 0.5 respectively.

$c_1 = 30$ $\quad r_1 = 0.9$ $\quad c = 105$

$c_2 = 15$ $\quad r_2 = 0.8$

$c_3 = 20$ $\quad r_3 = 0.5$

We assume that if stage 'i' has m_i devices of type i in parallel then

$\phi_i(m_i) = 1 - (1 - r_i)^{m_i}$ → is the reliability of stage 'i' i.e the probability of devices in stage 'i' fn properly.

$$u_i = \left\lfloor \frac{\left(c + c_i - \sum_1^n c_j\right)}{c_i} \right\rfloor$$

u_i → max no. of allocatable devices in stage 'i'.

$$u_1 = \frac{105 + 30 - (30 + 15 + 20)}{30} = \frac{135 - 65}{30} = \frac{70}{30}$$

$$= \lfloor 2.3 \rfloor = \underline{\underline{2}}$$

$$u_2 = \frac{105+15-(30+15+20)}{15} = \frac{120-65}{15} = \frac{55}{15} = \frac{11}{3} = \lfloor 3.6 \rfloor = \underline{\underline{3}}$$

$$u_3 = \frac{105+20-(30+15+20)}{20} = \frac{125-65}{20} = \frac{60}{20} = \underline{\underline{3}}$$

$S^0 = \{(1,0)\}$ We use S^i to represent the set of all undominated tuples (f, x) that result from the various decision sequences for $m_1, m_2 \ldots m_i$.

We can obtain S^i from S^{i-1} by trying out all possible values for m_i and combining the resulting ~~tupe~~ tuples together.

Using S_j^i to represent all tuples obtainable from S^{i-1} by choosing $m_i = j$.

Considering 1 device in stage 1

reliability $= (1-(1-0.9)^1)$

$= (1-0.1) = 0.9$

$\underline{S_1^1} = \{(\underbrace{0.9}_{\text{reliability}}, \underbrace{30}_{\text{cost}})\}$ → considering one device D_1 whose cost is 30 & reliability is 0.9 in stage 1

$\boxed{\text{i.e } m_1 = 1}$

Now consider $m_1 = 2$ [since $u_1 = 2$].

S_2^1

$\phi_i m_i$ reliability of stage 1 $= (1-(1-0.9)^2) = (1-(0.1)^2) = 0.99$

$\therefore S_2^1 = \{(0.99, 60)\}$

reliability if $m_1 = 2$ — cost - since 2 devices (30+30)

$S^1 = \{S_1^1 + S_2^1\}$

$\therefore S^1 = \{(0.9, 30), (0.99, 60)\}$

Consider 1 device (D_2 type) in stage 2

S_1^2 = reliability $= (1-(1-0.8)^1)$

$= (1-(0.2)) = 0.8$

$\therefore S_1^2 = \{(0.8, 15)\}$

Consider 2 devices (D_2 type) in stage 2

S_2^2 = reliability $= (1-(1-0.8)^2)$

$= (1-0.04) = 0.96$

$\therefore S_2^2 = \{(0.96, 30)\}$

reliability if $m_2 = 2$ — cost - since 2 devices (15+15)

Consider 3 devices (D_2 type) in stage 3

[Since $U_2 = 3$]

S_3^2 = reliability $= (1-(1-0.8)^3)$

$= (1-(0.2)^3) = (1-0.008)$

$= 0.992$

$Q_i^{m_i}$ reliability of stage 2 $= \therefore S_3^2 = \{(0.992, 45)\}$

reliability if $m_2 = 3$ — cost - since 3 devices 15+15+15

$\therefore S^2 = \{(0.8, 15), (0.96, 30), (0.992, 45)\}$

Consider 1 device (D_3 type) in stage 3.

S^3_1 = reliability = $(1-(1-0.5)^1)$

$= (1 - 0.5) = 0.5$

$\therefore S^3_1 = \{(0.5, 20)\}$

Consider 2 devices (D_3 type) in stage 3

S^3_2 = reliability = $(1-(1-0.5)^2)$

$= (1-(0.5)^2)$

$= (1 - 0.25) = 0.75$

$\therefore S^3_2 = \{(0.75, 40)\}$

Consider 3 devices (D_3 type) in stage 3.

[Since $U_3 = 3$]

$\phi_i m_i$ = reliability of stage 3. $= S^3_3$ = reliability = $(1-(\cancel{1} 0.5)^3)$

$= (1 - (0.5)^3)$

$= 1 - 0.125 = \underline{0.875}$

$\therefore S^3_3 = \{(0.875, 60)\}$

Multiply each device (tuple) in S^2 stage with S^1 set.

$S^1 * S^2_1 = \{(0.8 * 0.9, 15+30), \cancel{(0.8*0.99}$ $\cancel{0.8 * 15+60}$ $(0.8 * 0.99, 15+60)\}$

$= \{(0.72, 45), (0.792, \cancel{45}\ 75)\}$

$$S^1 \times S^2_2 = \{(0.96 \times 0.9,\ 30+30),\ (0.96 \times 0.99,\ 30+60)\}$$

$$= \{(0.864, 60),\ (0.95\cancel{04}, 90)\}$$

[Since, total cost should be below 105
i.e we cannot accomidate atleast one device in Stage 3
Since D_3 cost is 20
$90+20 = 110 > 105$]

$$S^1 \times S^2_3 = \{(0.992 \times 0.9,\ 45+30),\ (0.992 \times 0.99,\ 45+60)\}$$

$$= \{(0.8928, 75),\ (0.98\cancel{208, 105})\}$$

Now write all the remaining tuples in increasing order of their costs

$$\{(0.72, 45), (0.864, 60), (0.\cancel{792, 75}), (0.8928, 75)\}$$

Since as cost increases reliability also should increase

<u>reliability upto stage 2</u>

$$\therefore S^2 = \{(0.72, 45),\ (0.864, 60),\ (0.8928, 75)\}$$

Multiply each device (tuple) in S^3 stage with S^2 set.

$$S^2 \times S^3_1 = \{(0.5 \times 0.72,\ 20+45),\ (0.5 \times 0.864,\ 20+60),$$
$$(0.5 \times 0.8928,\ 20+75)\}$$

$$= \{(0.36, 65),\ (0.4320, 80),\ (0.44640, 95)\}$$

$$S_2^2 \times S_2^3 = \{(0.75 \times 0.72,\ 40+45),\ (0.75 \times 0.864,\ 40+60),\ (0.75 \times 0.8928,\ 40+75)\}$$

$$= \{(0.54,\ 85),\ (0.648,\ 100),\ (\cancel{0.33375},\ 105)\}$$

as cost increases reliability should also increase.

$$S_3^2 \times S_3^3 = \{(0.875 \times 0.72,\ 60+45),\ (0.875 \times 0.864,\ 60+60),\ (0.875 \times 0.8928,\ 60+75)\}$$

$$= \{(0.63,\ 105),\ \cancel{(0.756,\ 120)},\ \cancel{(0.781,\ 135)}\}$$

[since 120 & 135 > 105]

Now write remaining tuples in the increasing order of their costs.

$$\{(0.36,\ 65),\ (0.4320,\ 80),\ (0.54,\ 85),\ \cancel{(0.4464,\ 95)},\ (0.648,\ 100),\ \cancel{(0.63,\ 105)}\}$$

<u>Reliability upto stage 3</u>

$$\therefore S^3 = \{(0.36,\ 65),\ (0.4320,\ 80),\ (0.54,\ 85),\ (0.648,\ 100)\}$$

Among all these tuples choose the tuple with best reliability under cost constraint i.e. $\underline{(0.648,\ 100)}$

$\therefore$ The best-design has reliability of 0.648 and cost of 100.

Tracing back through S^i's we get

$\boxed{m_3 = 2,\ m_2 = 2,\ m_1 = 1}$ i.e

$(0.648, 100) \in S_2^3$ So $\underline{m_3 = 2}$

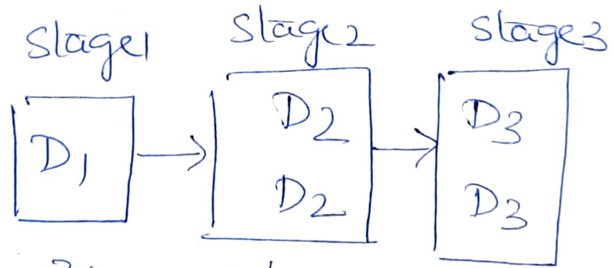
$(0.648, 100)$ came from tuple $(0.864, 60)$

$(0.864, 60) \in S_2^2$ So $\underline{m_2 = 2}$

$(0.864, 60)$ came from tuple $(0.9, 30) \in S_1^1$

So $\underline{m_1 = 1}$

$\therefore$



30 $\quad$ 30(15+15) $\quad$ 40(20+20) = Total $\underline{100}$

which is not more than 105

$\sum_{1}^{n} c_j \le c$