

# Text Classification and Naïve Bayes

The Task of Text  
Classification

# Is this spam?

**Subject: Important notice!**

**From:** Stanford University <newsforum@stanford.edu>

**Date:** October 28, 2011 12:34:16 PM PDT

**To:** undisclosed-recipients::

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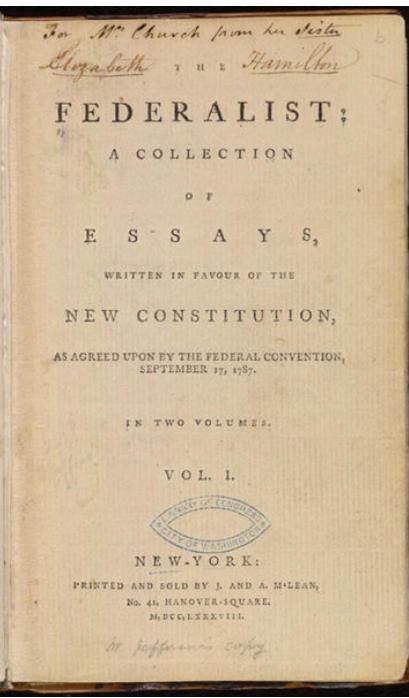
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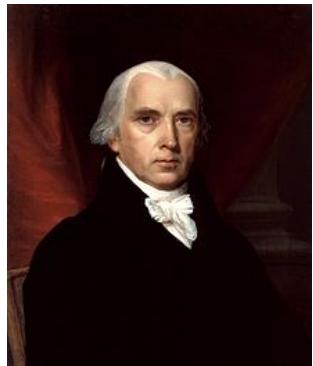
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# Who wrote which Federalist papers?

- 1787-8: anonymous essays try to convince New York to ratify U.S Constitution: Jay, Madison, Hamilton.
- Authorship of 12 of the letters in dispute
- 1963: solved by Mosteller and Wallace using Bayesian methods



James Madison



Alexander Hamilton

# Male or female author?

1. By 1925 present-day Vietnam was divided into three parts under French colonial rule. The southern region embracing Saigon and the Mekong delta was the colony of Cochin-China; the central area with its imperial capital at Hue was the protectorate of Annam...
2. Clara never failed to be astonished by the extraordinary felicity of her own name. She found it hard to trust herself to the mercy of fate, which had managed over the years to convert her greatest shame into one of her greatest assets...

# Positive or negative movie review?

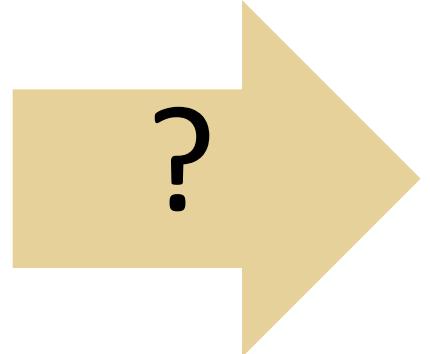


- unbelievably disappointing
- Full of zany characters and richly applied satire, and some great plot twists
- this is the greatest screwball comedy ever filmed
- It was pathetic. The worst part about it was the boxing scenes.



# What is the subject of this article?

## MEDLINE Article



## MeSH Subject Category Hierarchy

- Antagonists and Inhibitors
- Blood Supply
- Chemistry
- Drug Therapy
- Embryology
- Epidemiology
- ...

# Text Classification

- Assigning subject categories, topics, or genres
- Spam detection
- Authorship identification
- Age/gender identification
- Language Identification
- Sentiment analysis
- ...

# Text Classification: definition

- *Input:*
  - a document  $d$
  - a fixed set of classes  $C = \{c_1, c_2, \dots, c_J\}$
- *Output:* a predicted class  $c \in C$

# Classification Methods: Hand-coded rules

- Rules based on combinations of words or other features
  - spam: black-list-address OR (“dollars” AND “have been selected”)
- Accuracy can be high
  - If rules carefully refined by expert
- But building and maintaining these rules is expensive

# Classification Methods: Supervised Machine Learning

- *Input:*
  - a document  $d$
  - a fixed set of classes  $C = \{c_1, c_2, \dots, c_J\}$
  - A training set of  $m$  hand-labeled documents  $(d_1, c_1), \dots, (d_m, c_m)$
- *Output:*
  - a learned classifier  $\gamma: d \rightarrow c$

# Classification Methods: Supervised Machine Learning

- Any kind of classifier
  - Naïve Bayes
  - Logistic regression
  - Support-vector machines
  - k-Nearest Neighbors
  - ...

# Text Classification and Naïve Bayes

The Task of Text  
Classification

# Text Classification and Naïve Bayes

Naïve Bayes (I)

# Naïve Bayes Intuition

- Simple (“naïve”) classification method based on Bayes rule
- Relies on very simple representation of document
  - Bag of words

# The Bag of Words Representation

I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun... It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet!



# The bag of words representation

Y  
(

seen	2
sweet	1
whimsical	1
recommend	1
happy	1
...	...

) = C



# Text Classification and Naïve Bayes

Naïve Bayes (I)

# Text Classification and Naïve Bayes

Formalizing the  
Naïve Bayes  
Classifier

# Bayes' Rule Applied to Documents and Classes

- For a document  $d$  and a class  $c$

$$P(c \mid d) = \frac{P(d \mid c)P(c)}{P(d)}$$

# Naïve Bayes Classifier (I)

$$c_{MAP} = \operatorname{argmax}_{c \in C} P(c | d)$$

MAP is “maximum a posteriori” = most likely class

$$= \operatorname{argmax}_{c \in C} \frac{P(d | c)P(c)}{P(d)}$$

Bayes Rule

$$= \operatorname{argmax}_{c \in C} P(d | c)P(c)$$

Dropping the denominator

# Naïve Bayes Classifier (II)

$$c_{MAP} = \operatorname{argmax}_{c \in C} P(d | c)P(c)$$

$$= \operatorname{argmax}_{c \in C} P(x_1, x_2, \dots, x_n | c)P(c)$$

Document d  
represented as  
features  
x1..xn

# Naïve Bayes Classifier (IV)

$$c_{MAP} = \operatorname{argmax}_{c \in C} P(x_1, x_2, \dots, x_n | c)P(c)$$

$O(|X|^n \cdot |C|)$  parameters

Could only be estimated if a very, very large number of training examples was available.

How often does this class occur?

We can just count the relative frequencies in a corpus

# Multinomial Naïve Bayes Independence Assumptions

$$P(x_1, x_2, \dots, x_n | c)$$

- **Bag of Words assumption:** Assume position doesn't matter
- **Conditional Independence:** Assume the feature probabilities  $P(x_i | c_j)$  are independent given the class  $c$ .

$$P(x_1, \dots, x_n | c) = P(x_1 | c) \cdot P(x_2 | c) \cdot P(x_3 | c) \cdot \dots \cdot P(x_n | c)$$

# Multinomial Naïve Bayes Classifier

$$c_{MAP} = \operatorname{argmax}_{c \in C} P(x_1, x_2, \dots, x_n | c)P(c)$$

$$c_{NB} = \operatorname{argmax}_{c \in C} P(c_j) \prod_{x \in X} P(x | c)$$

# Applying Multinomial Naive Bayes Classifiers to Text Classification

positions  $\leftarrow$  all word positions in test document

$$c_{NB} = \operatorname{argmax}_{c_j \in C} P(c_j) \prod_{i \in positions} P(x_i | c_j)$$

# Text Classification and Naïve Bayes

Formalizing the  
Naïve Bayes  
Classifier

# Text Classification and Naïve Bayes

Naïve Bayes:  
Learning

# Learning the Multinomial Naïve Bayes Model

- First attempt: maximum likelihood estimates
  - simply use the frequencies in the data

$$\hat{P}(c_j) = \frac{\text{doccount}(C = c_j)}{N_{\text{doc}}}$$

$$\hat{P}(w_i | c_j) = \frac{\text{count}(w_i, c_j)}{\sum_{w \in V} \text{count}(w, c_j)}$$

# Parameter estimation

$$\hat{P}(w_i \mid c_j) = \frac{\text{count}(w_i, c_j)}{\sum_{w \in V} \text{count}(w, c_j)}$$

fraction of times word  $w_i$  appears  
among all words in documents of topic  $c_j$

- Create mega-document for topic  $j$  by concatenating all docs in this topic
  - Use frequency of  $w$  in mega-document

# Problem with Maximum Likelihood

- What if we have seen no training documents with the word ***fantastic*** and classified in the topic **positive (*thumbs-up*)**?

$$\hat{P}(\text{"fantastic"} \mid \text{positive}) = \frac{\text{count}(\text{"fantastic"}, \text{positive})}{\sum_{w \in V} \text{count}(w, \text{positive})} = 0$$

- Zero probabilities cannot be conditioned away, no matter the other evidence!

$$c_{MAP} = \operatorname{argmax}_c \hat{P}(c) \prod_i \hat{P}(x_i \mid c)$$

# Laplace (add-1) smoothing for Naïve Bayes

$$\hat{P}(w_i | c) = \frac{\text{count}(w_i, c) + 1}{\sum_{w \in V} (\text{count}(w, c) + 1)}$$

$$= \frac{\text{count}(w_i, c) + 1}{\left( \sum_{w \in V} \text{count}(w, c) \right) + |V|}$$

# Multinomial Naïve Bayes: Learning

- From training corpus, extract *Vocabulary*

- Calculate  $P(c_j)$  terms

- For each  $c_j$  in  $C$  do
  - $docs_j \leftarrow$  all docs with class =  $c_j$

$$P(c_j) \leftarrow \frac{|docs_j|}{|\text{total \# documents}|}$$

- Calculate  $P(w_k | c_j)$  terms

- $Text_j \leftarrow$  single doc containing all  $docs_j$
- Foreach word  $w_k$  in *Vocabulary*
  - $n_k \leftarrow$  # of occurrences of  $w_k$  in  $Text_j$

$$P(w_k | c_j) \leftarrow \frac{n_k + \alpha}{n + \alpha |\text{Vocabulary}|}$$

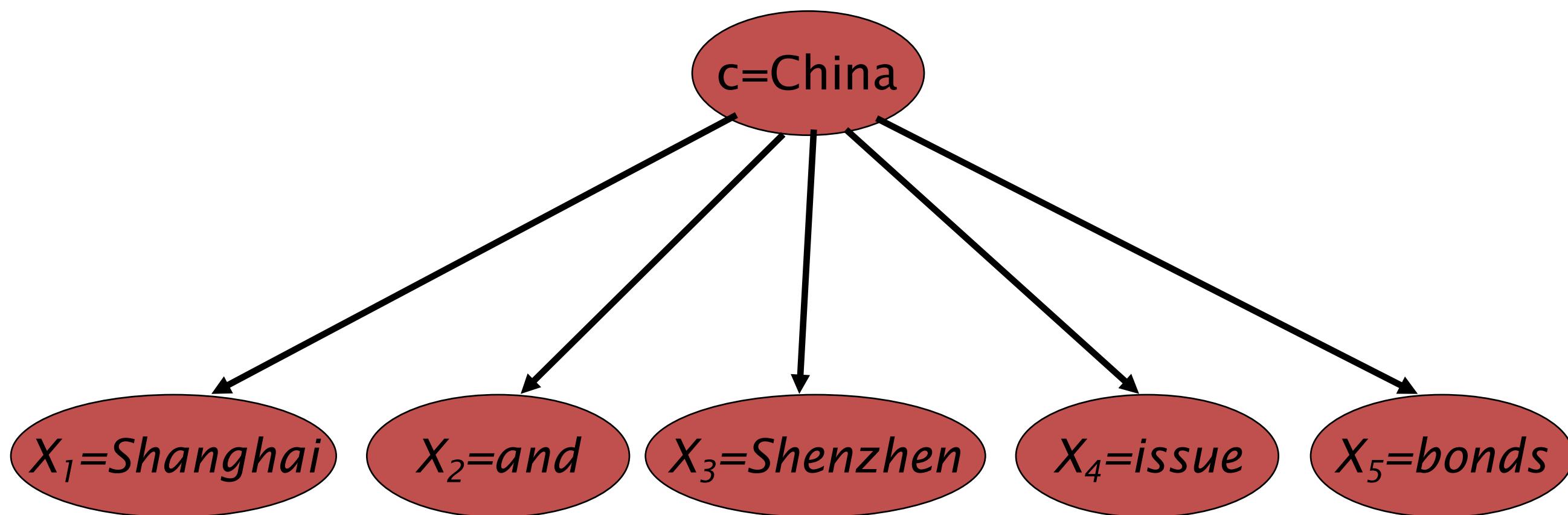
# Text Classification and Naïve Bayes

Naïve Bayes:  
Learning

# Text Classification and Naïve Bayes

Naïve Bayes:  
Relationship to  
Language Modeling

# Generative Model for Multinomial Naïve Bayes



# Naïve Bayes and Language Modeling

- Naïve bayes classifiers can use any sort of feature
  - URL, email address, dictionaries, network features
- But if, as in the previous slides
  - We use **only** word features
  - we use **all** of the words in the text (not a subset)
- Then
  - Naïve bayes has an important similarity to language modeling.

# Each class = a unigram language model

- Assigning each word:  $P(\text{word} | c)$
- Assigning each sentence:  $P(s | c) = \prod P(\text{word} | c)$

Class *pos*

0.1	I		I	<u>love</u>	<u>this</u>	<u>fun</u>	<u>film</u>
0.1	love			0.1	0.1	.05	0.01
0.01	this						0.1
0.05	fun						
0.1	film						
							$P(s   pos) = 0.0000005$

# Naïve Bayes as a Language Model

- Which class assigns the higher probability to s?

Model pos		Model neg			
0.1	I	0.2	I		
0.1	love	0.001	love		
0.01	this	0.01	this		
0.05	fun	0.005	fun		
0.1	film	0.1	film		

$P(s|pos) > P(s|neg)$

# Text Classification and Naïve Bayes

Naïve Bayes:  
Relationship to  
Language Modeling

# Text Classification and Naïve Bayes

Multinomial Naïve  
Bayes: A Worked  
Example

$$\hat{P}(c) = \frac{N_c}{N}$$

$$\hat{P}(w|c) = \frac{\text{count}(w,c)+1}{\text{count}(c)+|V|}$$

	Doc	Words	Class
Training	1	Chinese Beijing Chinese	c
	2	Chinese Chinese Shanghai	c
	3	Chinese Macao	c
	4	Tokyo Japan Chinese	j
Test	5	Chinese Chinese Chinese Tokyo Japan	?

Priors:

$$P(c) = \frac{3}{4}$$

$$P(j) = \frac{1}{4}$$

Choosing a class:

$$P(c|d5) \propto 3/4 * (3/7)^3 * 1/14 * 1/14 \\ \approx 0.0003$$

Conditional Probabilities:

$$P(\text{Chinese}|c) = (5+1) / (8+6) = 6/14 = 3/7$$

$$P(\text{Tokyo}|c) = (0+1) / (8+6) = 1/14$$

$$P(\text{Japan}|c) = (0+1) / (8+6) = 1/14$$

$$P(\text{Chinese}|j) = (1+1) / (3+6) = 2/9$$

$$P(\text{Tokyo}|j) = (1+1) / (3+6) = 2/9$$

$$P(\text{Japan}|j) = (1+1) / (3+6) = 2/9$$

$$P(j|d5) \propto 1/4 * (2/9)^3 * 2/9 * 2/9 \\ \approx 0.0001$$

# Naïve Bayes in Spam Filtering

- SpamAssassin Features:

- Mentions Generic Viagra
- Online Pharmacy
- Mentions millions of (dollar) ((dollar) NN,NNN,NNN.NN)
- Phrase: impress ... girl
- From: starts with many numbers
- Subject is all capitals
- HTML has a low ratio of text to image area
- One hundred percent guaranteed
- Claims you can be removed from the list
- 'Prestigious Non-Accredited Universities'
- [http://spamassassin.apache.org/tests\\_3\\_3\\_x.html](http://spamassassin.apache.org/tests_3_3_x.html)

# Summary: Naive Bayes is Not So Naive

- Very Fast, low storage requirements
- Robust to Irrelevant Features
  - Irrelevant Features cancel each other without affecting results
- Very good in domains with many equally important features
  - Decision Trees suffer from *fragmentation* in such cases – especially if little data
- Optimal if the independence assumptions hold: If assumed independence is correct, then it is the Bayes Optimal Classifier for problem
- A good dependable baseline for text classification
  - **But we will see other classifiers that give better accuracy**

# Text Classification and Naïve Bayes

Multinomial Naïve  
Bayes: A Worked  
Example