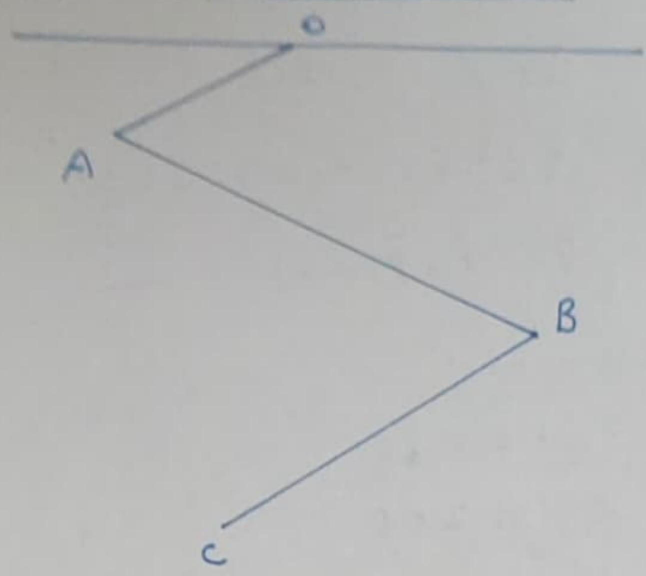
Robot dog calculations:



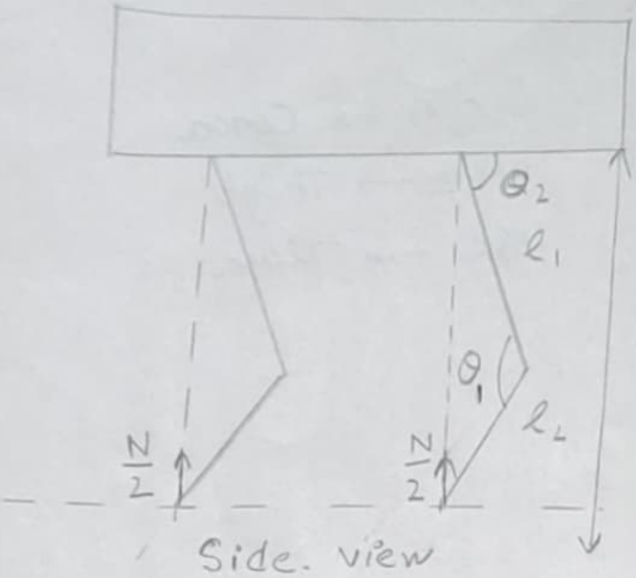
OA → Coxa
AB → Thigh
BC → Tibia

Link lengths:

It was studied that the thigh and the shank (tibia) are nearly kept same or thigh is kept with a little more length for stability. If the shank is given more length, then the quadruped dog might become unstable as while moving forward the centre of mass will shift.
It was seen that keeping this configuration leads to less energy consumption.

Angle between the links:

It has been found that the angle between the thigh and the tibia ranges about 90-120° and the and the minimum angle ranges from 20-40°.
Also it is favoured that the foot of the robot should be placed below the shoulder for better stability



Side. view

$$l_1^2 + l_2^2 \mp 2 l_1 l_2 \cos\theta = 25^2$$

Let $l_1 = l_2 = l$

$$2l^2 \mp 2l^2 \cos\theta = 25^2$$

$$2l^2(1 \mp \cos\theta) = 625$$

Let $\theta = 120^\circ$

$$3l^2 = 625$$

$$l^2 \approx 208$$

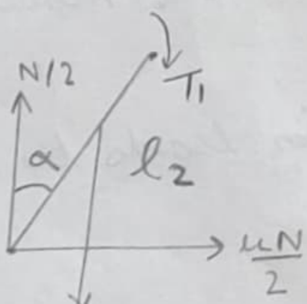
$$l = 14.4 \text{ cm}$$

If $\theta = 90^\circ$,

$$2l^2 = 625$$

$$l^2 = 312.5 \implies l = 17.67 \text{ cm}$$

Also, $l_1 \sin\theta_2 + l_2 \cos(\theta_1 - \theta_2) = 25$

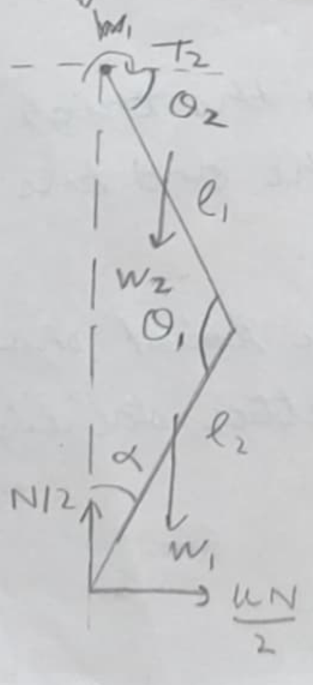


$$T_1 = \frac{N}{2}\sin\alpha\, l_2 - \frac{\mu N}{2}\cos\alpha\, l_2 - W_1 \times \frac{l_2}{2}\sin\alpha$$

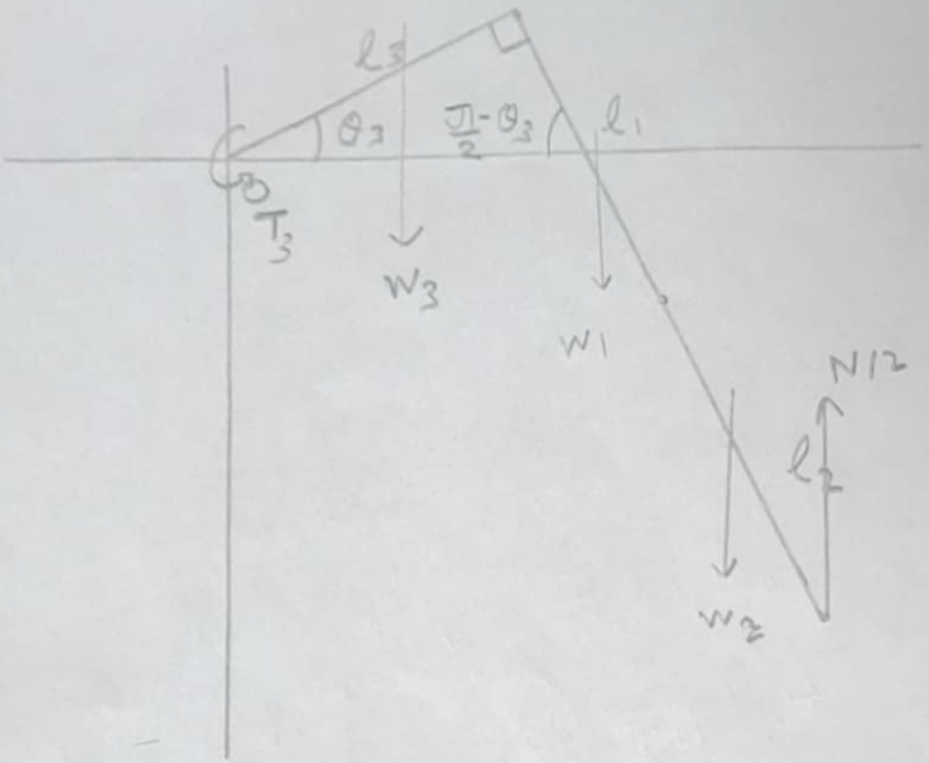
$$\alpha = 180 - \theta_1 - (90 - \theta_2)$$

$$\alpha = 90 - (\theta_1 - \theta_2)$$

$$\therefore T_1 = \frac{N}{2}\cos(\theta_1 - \theta_2)\, l_2 - \frac{\mu N}{2}\sin(\theta_1 - \theta_2) - W_1 \times \frac{l_2}{2}\cos(\theta_1 - \theta_2)$$



$$T_2 = -(l_1 \sin\theta_2 + l_2 \cos(\theta_1 - \theta_2))\frac{\mu N}{2} + W_2 \times \frac{l_1}{2}\cos\theta_2 + W_1 \times \frac{l_2}{2}\cos(\theta_1 - \theta_2)$$

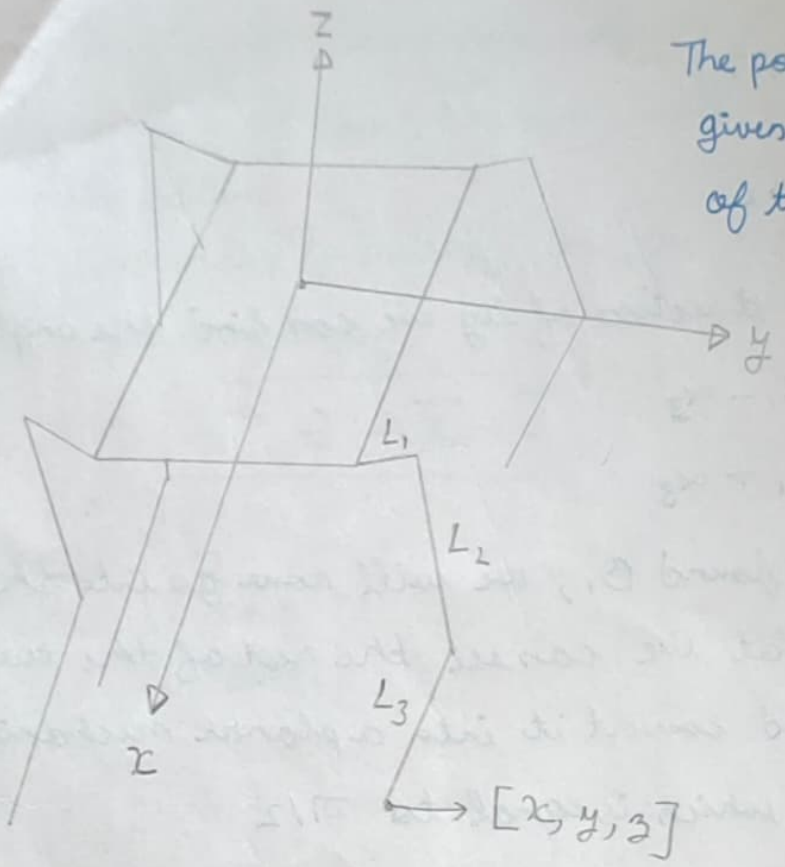


$$\therefore T_3 = \frac{l_3 \cos\theta_3}{2} \times W_3 + \left[\frac{l_1 \sin\theta_3}{2} + l_3 \cos\theta_3\right] W_1$$

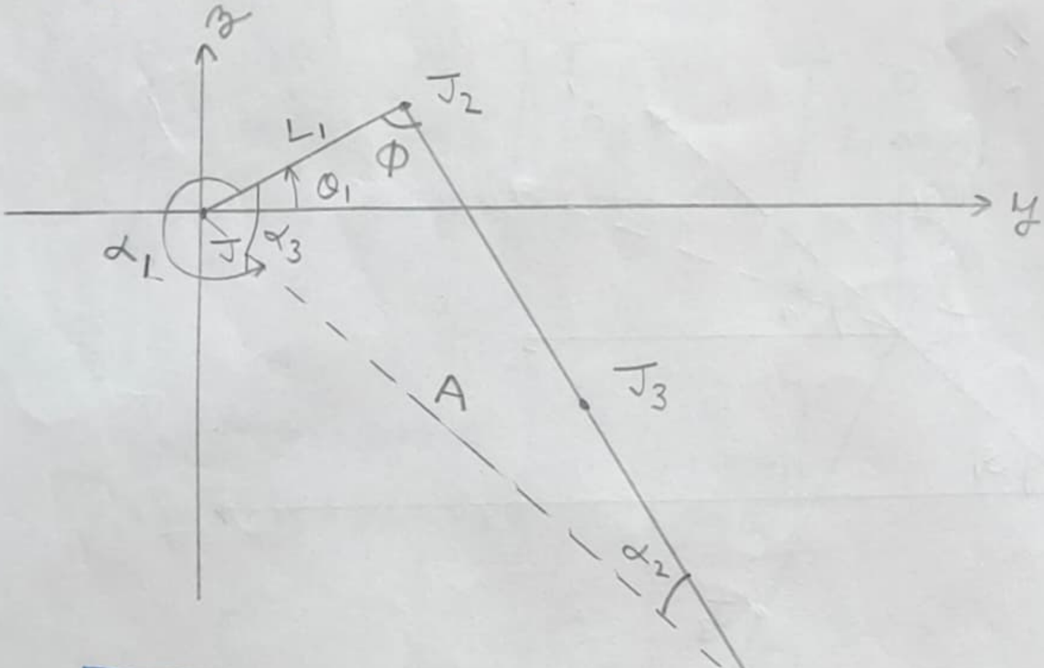
$$+ \frac{l_3 \cos\theta_3}{2} \times W_3 + \left[\left(l_1 + \frac{l_2}{2}\right)\sin\theta_3 + l_3 \cos\theta_3\right] W_2$$

$$- \left(l_3 \cos\theta_3 + (l_1 + l_2)\sin\theta_3\right) N/2$$

The point (x, y, z) has been given w.r.t. the origin of the leg



Considering the y-z plane,



$$A = \sqrt{z^2 + y^2} = |\bar{z}||\bar{y}|$$

$$= \text{norm}(z, y)$$

Since $\dfrac{\sin \alpha_2}{L_1} = \dfrac{\sin \phi}{A}$

$$\alpha_2 = \sin^{-1}\left(\frac{L_1}{A}\sin\phi\right)$$

$$\alpha_3 = \pi - \phi - \alpha_2$$

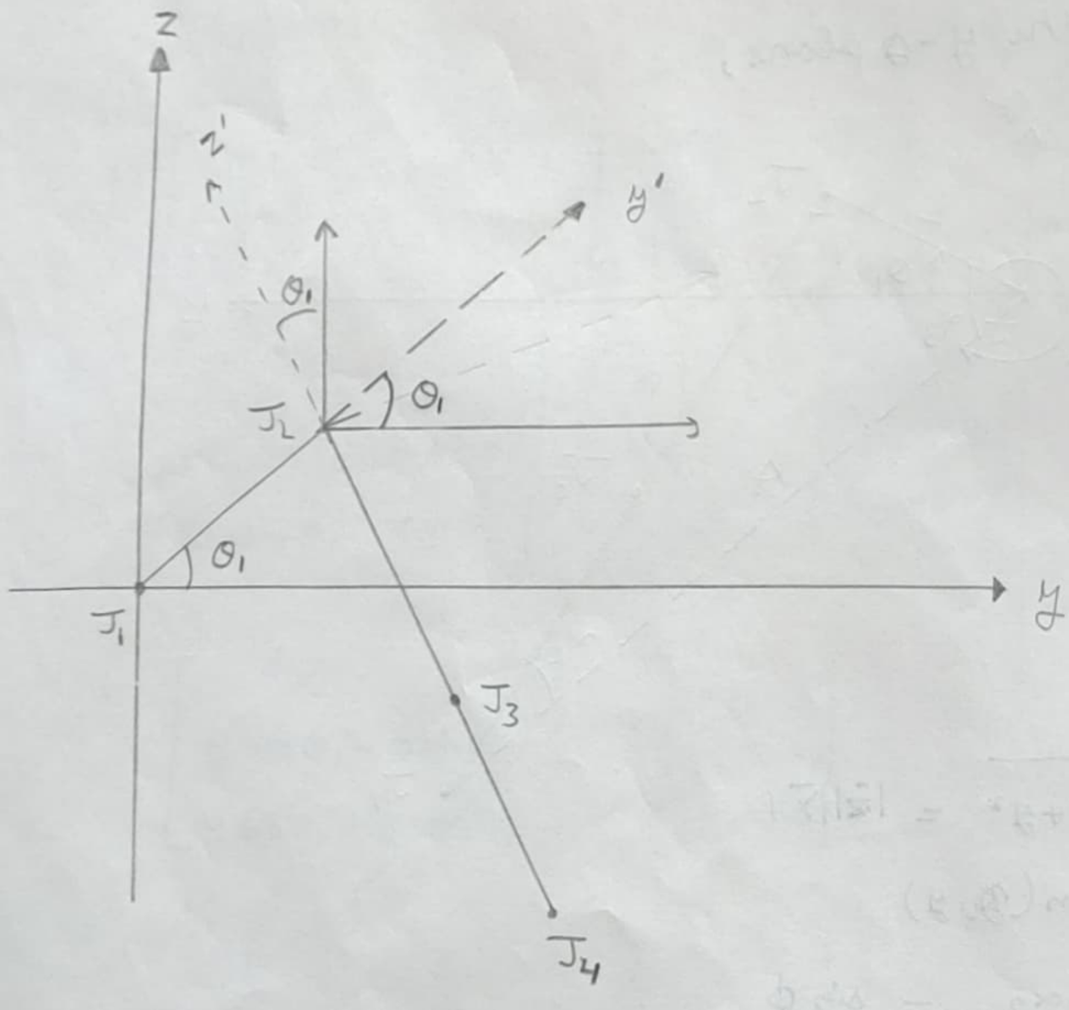
$\therefore$ depending on the direction of leg we can find the angle θ_1

right $\longrightarrow$ $\theta_1 = \alpha_1 - \alpha_3$

left $\longrightarrow$ $\theta_1 = \alpha_1 + \alpha_3$

As we have already found θ_1, we will now go into the plane of L_1 so that we can see the rest of the two links in a plane and convert it into a planar mechanism

ϕ is a constant value which is equal to $\pi/2$



$\therefore$ w.r.t the new coordinate system $xy'z'$,
the coordinates of point (x, y, z) will change in the new coordinate system

$\therefore$ we will have to multiply the point (x, y, z) with a transformation matrix

$$\boxed{\vec{X} = A\vec{x} + \vec{d}}$$

$$d = \begin{bmatrix} 0 \\ L_1 \cos\theta_1 \\ L_1 \sin\theta_1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & +\sin\theta_1 & \cos\theta_1 \end{bmatrix}$$

$$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad X = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ L_1 \cos\theta_1 \\ L_1 \sin\theta_1 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y\cos\theta_1 - z\sin\theta_1 \\ y\sin\theta_1 + z\cos\theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ L_1\cos\theta_1 \\ L_1\sin\theta_1 \end{bmatrix}$$

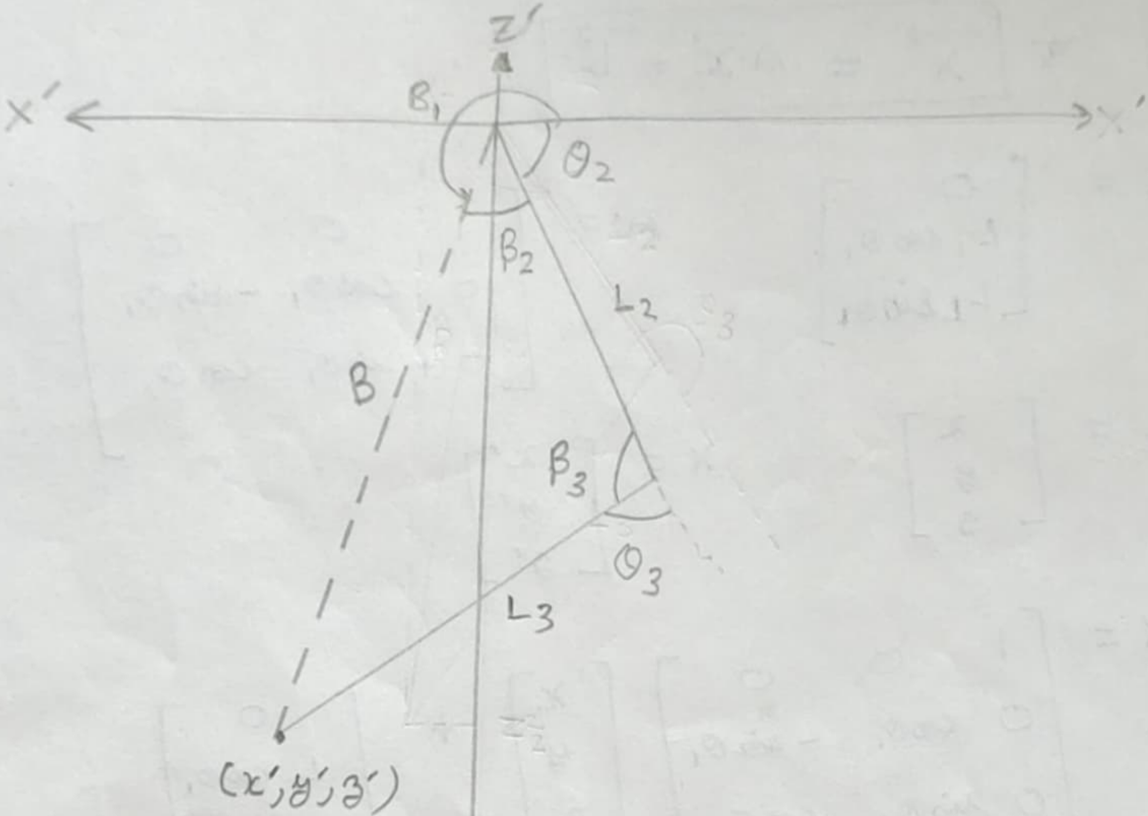
$$X = \begin{bmatrix} x \\ y\cos\theta_1 - z\sin\theta_1 + L_1\cos\theta_1 \\ y\sin\theta_1 + z\cos\theta_2 + L_1\sin\theta_1 \end{bmatrix}$$

$\therefore x' = x$

$$y' = y\cos\theta_1 - z\sin\theta_1 + L_1\cos\theta_1$$

$$z' = y\sin\theta_1 + z\cos\theta_2 + L_1\sin\theta_1$$

$\therefore$ Now we will look from the $x-z$ plane



$$B = \sqrt{(x')^2 + (z')^2} = \text{norm}(x', z')$$

$$\tan\beta_1 = \left(\frac{z'}{x'}\right) \Rightarrow B_1 = \tan^{-1}\left(\frac{z'}{x'}\right)$$

Using the law of cosines,

$$\cancel{B_2 \Rightarrow} \; L_3^2 = L_2^2 + B^2 - 2L_2 B\cos B_2$$

$$B_2 = \cos^{-1}\left(\frac{L_2^2 + B^2 - L_3^2}{2 \times L_2 \times B}\right)$$

Similarly,

$$B_3 = \cos^{-1}\left(\frac{L_2^2 + L_3^2 - B^2}{2 L_2 L_3}\right)$$

$\therefore \; \theta_2 = \pi - \beta_1 - \beta_2$

$\theta_3 = \pi - \beta_3$

Hence we have found θ_1, θ_2 and θ_3 which will be sent to the servos