S412 Empirical Asset Pricing

Assignment 4: Report

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Study Program : MSc International Economics

1 Introduction

The Consumption-Based Asset Pricing Model (CBM) links asset prices to the intertemporal marginal rate of substitution in consumption. Although elegant in theory, it has struggled empirically, often failing to explain observed equity premia or yielding implausibly high risk aversion estimates.

One proposed reason is the use of filtered NIPA consumption data, which may obscure co-movements relevant for pricing. Kroencke (2014) suggests reversing these filters improves empirical performance.

This report tests the CBM using U.S. data from 1949 to 2018. In Section 1, we estimate the model using annual filtered and unfiltered consumption data. In Section 2, we extend the analysis to quarterly data and 25 test portfolios to assess cross-sectional fit. For each case, we estimate risk aversion via GMM, test relevant hypotheses, evaluate the implied risk-free rate, and assess model fit.

2 Methodology

We begin our analysis with annual data, focusing on the excess return of the market portfolio as the test asset. This part is divided into two specifications: one using filtered consumption growth, and another using unfiltered consumption growth. We then extend the analysis to quarterly data, where we evaluate the model across 25 value-weighted portfolios. The quarterly analysis is again divided into filtered and unfiltered consumption specifications.

2.1 Annual Model: Single Asset GMM

We begin with a standard consumption-based asset pricing model (CBM) where the representative agent has power utility. Under this assumption, the stochastic discount factor (SDF) takes the form:

$$m_{t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma}$$

and the model implies the Euler condition:

$$\mathbb{E}\left[m_{t+1} R^{e,m}_{t+1} \right] = 0$$

The moment condition is estimated using Generalized Method of Moments (GMM), fixing the discount factor at $\beta = 0.95$. We use U.S. annual data from 1949 to 2018, and treat the excess return on the market portfolio as the test asset. Two specifications of the consumption growth series are evaluated: a filtered version based on NIPA aggregates, and an unfiltered version motivated by measurement concerns (Kroencke, 2014). In both cases, the risk aversion parameter γ is estimated using one-step GMM with Newey-West standard errors and a constant instrument vector. We also compute the model-implied risk-free rate and evaluate the correlation between the estimated SDF and market excess returns.

2.2 Quarterly Model: Cross-Sectional GMM

We extend the model to a cross-sectional setting with 25 size and value-sorted portfolios, using quarterly U.S. data from 1949Q1 onward. The augmented CBM includes two additional parameters: an intercept α, capturing pricing error orthogonal to the SDF, and a mean constraint μ on the SDF. Following standard approaches, we define the moment conditions as:

$$\mathbb{E}\left[\begin{bmatrix} R_t^e - \alpha + \frac{(m_t-\mu)R_t^e}{\mu} \\ m_t - \mu \end{bmatrix}\right] = 0$$

These conditions capture the pricing equation across assets and a normalization condition to anchor the level of the SDF. The GMM estimation is implemented using a custom weighting matrix that emphasizes the mean-zero constraint (with weight $\tau = 1500$), and the estimation is conducted using one-step GMM with Newey-West standard errors. As in the annual setting, we perform the estimation separately using both filtered and unfiltered consumption growth data.

For both specifications, we evaluate identification quality, test hypotheses on γ and α, and assess model fit via the cross-sectional R^2, the GMM J-statistic, and scatter plots comparing realized and model-implied mean excess returns.

3 Results

3.1 Annual Model: Single Asset

We begin with the estimation of the risk aversion parameter γ using annual U.S. data and the market excess return as the test asset. As outlined earlier, we evaluate two specifications: filtered and unfiltered consumption growth.

Filtered Consumption Growth

Using filtered consumption (NIPA column 2), the model yields a high risk aversion estimate of $\hat{\gamma} = 70.72$ with a standard error of 31.34. The 95% confidence interval is wide [9.29, 132.14], and the one-sided t-statistic for $H_0 : \gamma \leq 10$ is 1.94 (p = 0.0263), rejecting the null at the 5% level.

Although statistically significant, the magnitude of $\hat{\gamma}$ is economically implausible. The model implies a risk-free rate of 0.3951, which exceeds observed short-term rates—consistent with the risk-free rate puzzle. The correlation between the estimated SDF and excess returns is -0.2288, indicating weak countercyclical behavior.

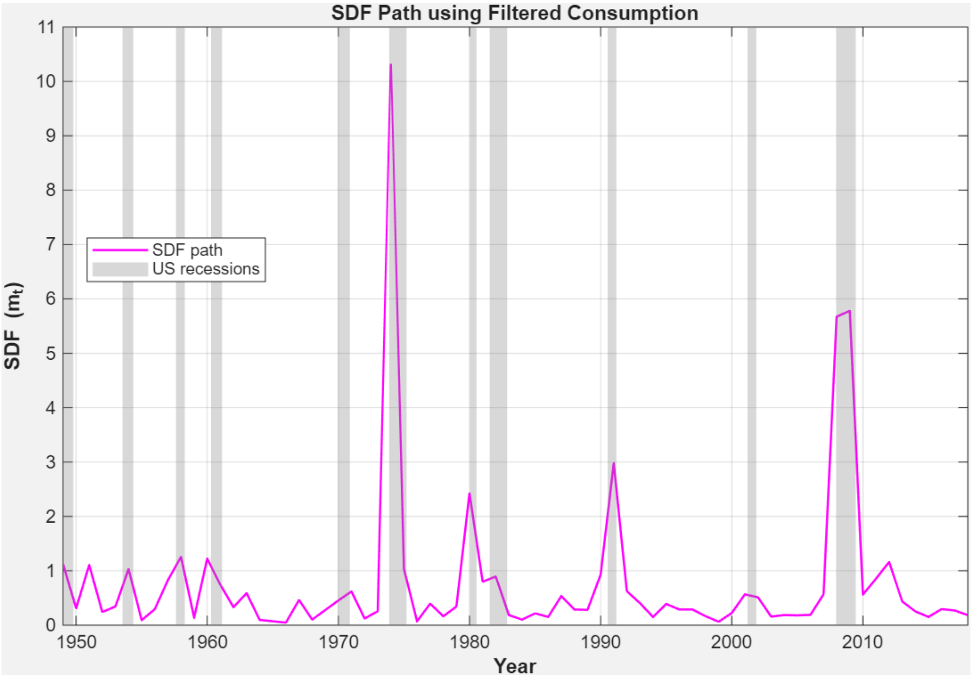

Figure 1: SDF over Time using Filtered Consumption Growth

Unfiltered Consumption Growth

Replacing the filtered series with unfiltered consumption (column 3) reduces the estimated risk aversion to $\hat{\gamma} = 29.83$ (SE = 12.12, CI = [6.07, 53.58]). The t-statistic drops to 1.64 (p = 0.0509), failing to reject the null at 5%.

While less significant, the estimate is more economically plausible. The implied risk-free rate falls to 0.1521, much closer to observed values. The SDF also exhibits stronger negative correlation with excess returns at -0.3331. These results support the notion that unfiltered consumption may be better suited for capturing asset-pricing-relevant variation.

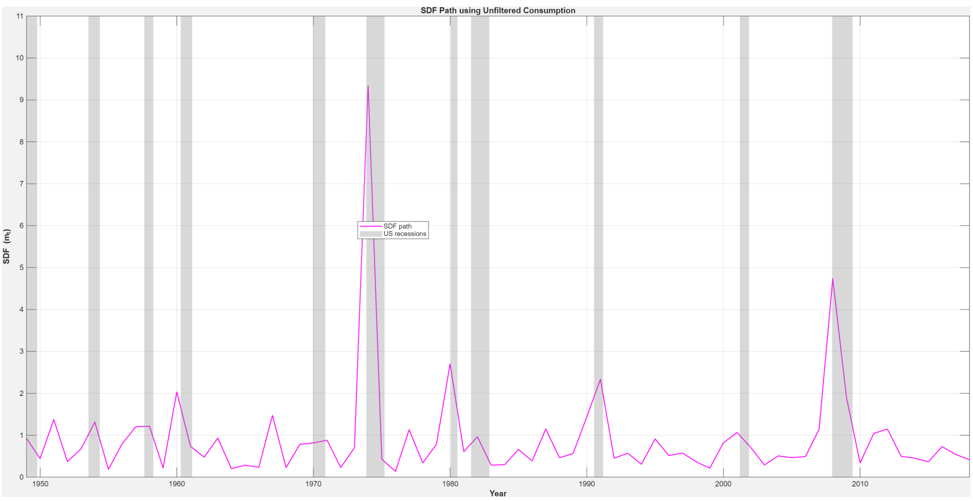

Figure 2: SDF over Time using Unfiltered Consumption Growth

In summary, unfiltered consumption delivers a more reasonable estimate of risk aversion, a more realistic implied risk-free rate, and stronger comovement with returns, suggesting improved performance of the CBM.

3.2 Quarterly Model: Multiple Portfolios

We now estimate the extended CBM using quarterly data and 25 excess return series. The model includes three parameters: the risk aversion γ, an intercept α, and the mean of the SDF μ. Estimation is performed via one-step GMM using a custom weighting matrix and Newey-West standard errors.

Filtered Consumption Growth

With filtered quarterly consumption growth, the model estimates:

$$\hat{\alpha} = 0.0173, \quad \hat{\mu} = 0.7699, \quad \hat{\gamma} = 47.93$$

$$\text{Std. Errors: } 0.0588,\ 0.1081,\ 23.35 \quad 95\% \text{ CI for } \gamma : [2.17,\ 93.69]$$

The t-statistic for testing $H_0 : \gamma \leq 10$ is 1.6247 (p = 0.0521), marginally insignificant. The intercept is not significant either (t = 0.2945, p = 0.7684). The model underpredicts mean excess returns across most portfolios. The cross-sectional $R^2 = 0.3383$ and the GMM J-statistic is 151.25, indicating poor fit at conventional levels.

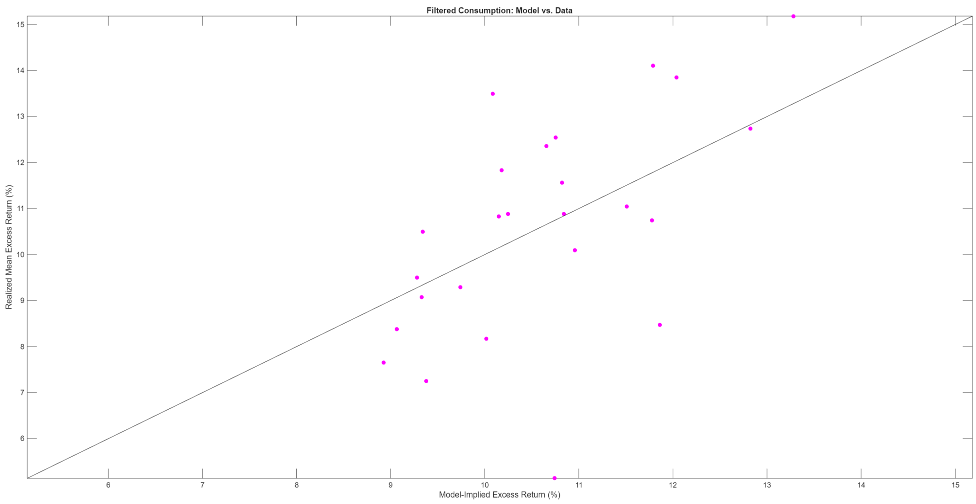

Figure 3: Model-Implied vs. Realized Mean Excess Returns (Filtered Consumption)

Unfiltered Consumption Growth

Replacing filtered with unfiltered consumption results in the following estimates:

$$\hat{\alpha} = 0.0229, \quad \hat{\mu} = 0.8933, \quad \hat{\gamma} = 32.78$$

$$\text{Std. Errors: } 0.0430,\ 0.1286,\ 15.05 \quad 95\% \text{ CI for } \gamma : [3.28,\ 62.28]$$

Test statistics indicate weak significance: for $\gamma \leq 10$, t = 1.5133 (p = 0.0651); for $\alpha = 0$, $t = 0.5336$ (p = 0.5936). However, model fit improves modestly. The R^2 increases to 0.3405 and the J-statistic falls to 146.56. The scatter plot shows a tighter clustering around the 45-degree line, indicating improved explanatory power.

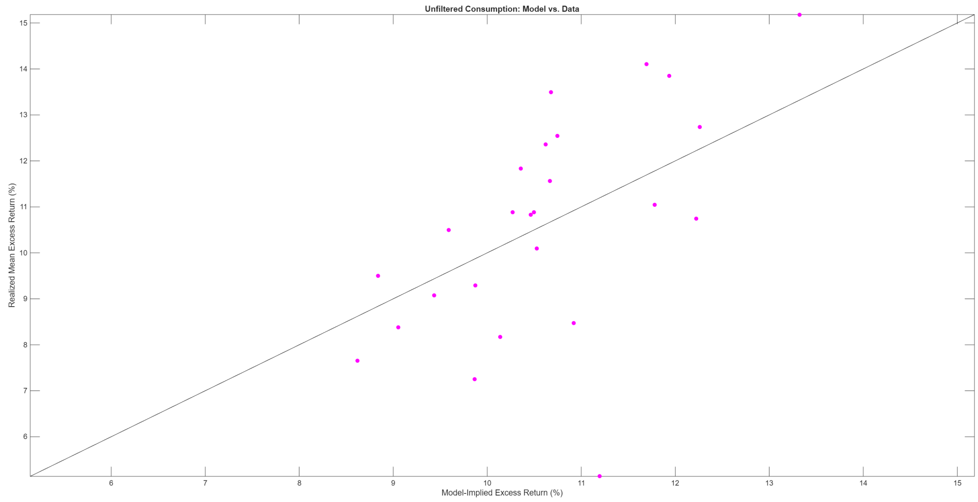

Figure 4: Model-Implied vs. Realized Mean Excess Returns (Unfiltered Consumption)

The shift from filtered to unfiltered consumption enhances empirical performance. Estimates become more reasonable, and the model better explains cross-sectional return variation, albeit without fully resolving the J-test rejection.

4 Conclusion

This study evaluates the empirical performance of the Classical Consumption-Based Asset Pricing Model (CBM) using both annual and quarterly U.S. data. For annual data, we find that the model delivers statistically significant but economically implausible estimates of the risk aversion parameter γ when using filtered consumption. Switching to unfiltered consumption data improves the plausibility of the estimates, lowers the implied risk-free rate, and strengthens the correlation between the SDF and excess returns.

In the more demanding quarterly setting with 25 size and value-sorted portfolios, the extended CBM yields moderate cross-sectional explanatory power. Once again, the use of unfiltered consumption improves the overall fit of the model. The estimated risk aversion parameter becomes smaller and more reasonable, and the scatter plots of model-implied versus realized returns indicate a closer alignment.

Despite these improvements, both specifications result in high J-statistics, suggesting the model remains rejected at conventional significance levels. However, the use of unfiltered consumption data, motivated by concerns about measurement filtering does enhance the empirical performance of the CBM, especially in matching the moments and co-movements between consumption and returns.

Future research could explore whether further refinements in the measurement of consumption or alternative utility specifications could help resolve the remaining asset pricing puzzles highlighted in this study.