Filter Design Assignment Elliptic

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Filter Number=84

Group 26

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1 Student Details

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2 Analog Lowpass Elliptic

We will have to get an analogous lowpass transfer function for both bandpass and band-stop requirements. Therefore, we need to design using **Elliptic** approximation. Since, the tolerance(δ) in both passband and stopband is 0.15. The transfer function is of the form: -

$$H_{analog,LPF}.S_L/ = \frac{H_o}{D_o.S_L/} \frac{s_L^2 + a_{0I}}{s_L^2 + b_{1i}s_L + b_{0i}} \tag{1}$$

Where,

$$r = \begin{cases} \frac{n-1}{2} & \text{for odd n} \\ \frac{n}{2} & \text{for even n} \end{cases}$$

$$D_0(s) = \begin{cases} s + \sigma_0 & \text{for odd n} \\ 1 & \text{for even n} \end{cases}$$

The transfer function coefficient and multiplier H_o can be computed using the following formulae sequence:

$$k' = \sqrt{1-k^2}$$

$$q_0 = \frac{1}{2}\left(\frac{1-\sqrt{k'}}{1+\sqrt{k'}}\right)$$

$$q = q_0 + 2q_0^5 + 15q_0^9 + 150q_0^{13}$$

$$D = \frac{10^{0.1A_a}-1}{10^{0.1A_p}-1}$$

$$n \geq \frac{\log 16D}{\log(1/q)}$$

$$\Lambda = \frac{1}{2n}\ln\frac{10^{0.05A_p}+1}{10^{0.05A_p}-1}$$

$$\sigma_0 = \left|\frac{2q^{1/4}\sum_{m=0}^{\infty}(-1)^m q^{m(m+1)}\sinh[(2m+1)\Lambda]}{1+2\sum_{m=1}^{\infty}(-1)^m q^{m^2}\cosh 2m\Lambda}\right|$$

$$W = \sqrt{(1+k\sigma_0^2)\left(1+\frac{\sigma_0^2}{k}\right)}$$

$$\Omega_i = \frac{2q^{1/4}\sum_{m=0}^{\infty}(-1)^m q^{m(m+1)}\sin\frac{(2m+1)\pi\mu}{n}}{1+2\sum_{m=1}^{\infty}(-1)^m q^{m^2}\cos\frac{2m\pi\mu}{n}}$$

where

$$\mu = \begin{cases} i & \text{for odd } n \\ i-\frac{1}{2} & \text{for even } n \end{cases} \qquad i = 1, 2, \ldots, r$$

$$V_i = \sqrt{(1-k\Omega_i^2)\left(1-\frac{\Omega_i^2}{k}\right)}$$

$$a_{0i} = \frac{1}{\Omega_i^2}$$

$$b_{0i} = \frac{(\sigma_0 V_i)^2 + (\Omega_i W)^2}{\left(1+\sigma_0^2\Omega_i^2\right)^2}$$

$$b_{1i} = \frac{2\sigma_0 V_i}{1+\sigma_0^2\Omega_i^2}$$

$$H_0 = \begin{cases} \sigma_0 \prod_{i=1}^{r}\frac{b_{0i}}{a_{0i}} & \text{for odd } n \\ 10^{-0.05A_p}\prod_{i=1}^{r}\frac{b_{0i}}{a_{0i}} & \text{for even } n \end{cases}$$

3 Filter-1(Bandpass) Details

3.1 Un-normalized Discrete Time filter Specifications

Filter Number = 84
Therefore,
m = 4
q(m) = 0.1*4 = 0
r(m) = m - 10*q(m) = 4
BL(m) = 10 + 5*q(m) + 13*r(m) = 10 + 52 = 62 Khz
BH(m) = 75 + 62 = 137 kHz

Here, we have a analog signal with bandwidth of 280Khz. Where we sample it with 600Khz.

Here we have a bandpass filter with BL and BH (Khz)being the pass band frequencies for the filter to be designed.

Hence, specifications here would be:

- **PassBand**: 62 to 137 Khz
- **Transitionband**: 5Khz
- **StopBand**: 0 to 57Khz and 142Khz to 300 Khz (As sampling frequency is 600 kHz)
- **Tolerance**: 0.15 in magnitude for both Passband and Stopband
- **Passband Nature**: Equiripple
- **Stopband Nature**; Equiripple

3.2 Normalized Digital Filter Specifications

As we are sampling this signal at a frequency of 600 KHz. It is then normalized from 0 to 1π. By the following transformation:

$$\omega = \frac{\Omega ¡ 2 ¡ \pi/}{\Omega_s.Sampling\_Rate/} \tag{2}$$

Therefore the corresponding normalized discrete filter specifications are :-

- **PassBand**: 0.207π to 0.457π
- **Transitionband**: 0.0167π
- **StopBand**: $0 * 0.1899\pi$ to $0.4733\pi * 1\pi$
- **Tolerance**: 0.15 in magnitude for both Passband and Stopband
- **Passband Nature**: Equiripple
- **Stopband Nature**; Equiripple

3.3 After applying bilinear transformation

The bilinear transformation is given as:

$$\Omega = tan.\frac{\omega}{2}/ \tag{3}$$

From this transformation we end with the following new values:

Discrete	Analog
0.207π	0.3365
0.457π	0.8723
0.0167π	0.02618
$0 * 0.1899\pi$	0 - 0.3076
$0.4733\pi * 1\pi$	0.9195 *

Hence the corresponding frequencies will be as follows:

- **StopBand**: 0.3365 to 0.8723
- **Transitionband**: 0.02618
- **PassBand**: 0 to 0.3076 and 0.9195 to
- **Tolerance**: 0.15 in magnitude for both Passband and Stopband
- **Passband Nature**: Equiripple
- **Stopband Nature**: Equiripple

4 Elliptical Bandpass Filter

We need to use the same specifications which was there for the 1st filter

$$B = 0.5358$$
$$\Omega_0 = 0.5418$$
$$\Omega_{LP} = 1$$
$$\Omega_{LS} = 1.1203$$

Consider G_p = 0.85 and G_s = 0.15
The value of K = $\frac{W_p}{W_s}$ = 0.8926
The value of K1 = $\frac{\sqrt{1^{\cdot} G_p^2 * 1}}{\sqrt{1^{\cdot} G_s^2 * 1}}$ =0.094 The minimum value of N after solving the elliptical integral of K,K1

$$\frac{K_{1p}K}{K_pK_1} = 4$$

Next we calculate the analog low pass transfer function

$$\frac{0.85\left(0.3196s^2+1.0\right)\left(0.8835s^2+1.0\right)}{\left(1.6s^2+1.131s+1.0\right)\left(1.0s^2+0.06192s+1.0\right)}$$

We now use the inverse transformation to convert the equivalent Elliptic low-pass transfer function to get the Band-pass transfer function using the relation:-

$$s_L = \frac{s^2+\Omega_0^2}{B\Omega}$$

The analog band-pass transfer function:-
Numerator:-
$1.7\text{e+}21\,s^8 + 3.978\text{e+}21\,s^6 + 3.113\text{e+}21\,s^4 + 9.372\text{e+}20\,s^2 + 9.437\text{e+}19$

The denominator is:-

$$\begin{aligned} &2.0\text{e+}21\,s^8 \\ &+1.373\text{e+}21\,s^7 \\ &+5.651\text{e+}21\,s^6 \\ &+2.469\text{e+}21\,s^5 \\ &+4.895\text{e+}21\,s^4 \\ &+1.198\text{e+}21\,s^3 \\ &+1.331\text{e+}21\,s^2 \\ &+1.571\text{e+}20\,s \\ &+1.116\text{e+}20 \end{aligned}$$

We now make use of the Bilinear Transformation to convert the analog bandpass filter into a discrete bandpass filter in the normalized angular frequency domain. The transform is as follows

$$s = \frac{1*z^{*1}}{1+z^{*1}}$$

Hence the Discrete Bandpass transfer function is:-
Numerator:-

$$
\begin{aligned}
&1.965e{+}6\, z^8 \\
&- 5.002e{+}6\, z^7 \\
&+1.149e{+}7\, z^6 \\
&-1.555e{+}7\, z^5 \\
&+1.903e{+}7\, z^4 \\
&-1.555e{+}7\, z^3 \\
&+1.149e{+}7\, z^2 \\
&-5.002e{+}6\, z \\
&+1.965e{+}6
\end{aligned}
$$

Denominator is :-

$$
\begin{aligned}
&3.837e{+}6\, z^8 \\
&-8.446e{+}6\, z^7 \\
&+1.631e{+}8\, z^6 \\
&- 1.958e{+}7\, z^5 \\
&+2.146e{+}7\, z^4 \\
&- 1.582e{+}7\, z^3 \\
&+1.067e{+}7\, z^2 \\
&- 4.511e{+}6\, z \\
&+1.758e{+}6
\end{aligned}
$$

The frequency response is:-

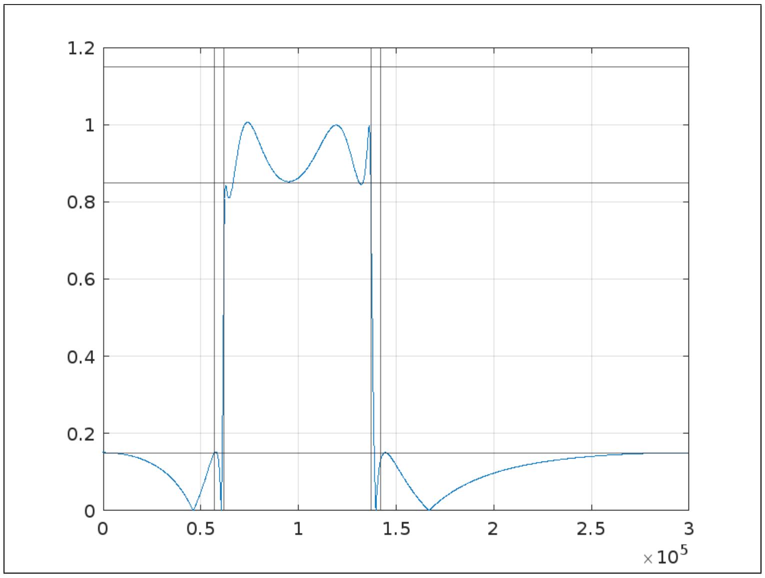

Figure 1: Frequency response

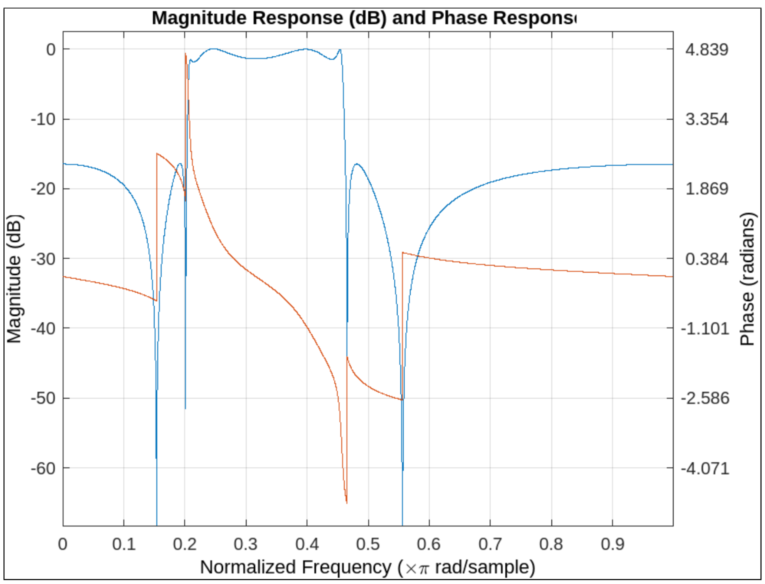

Figure 2: Magnitude and phase response

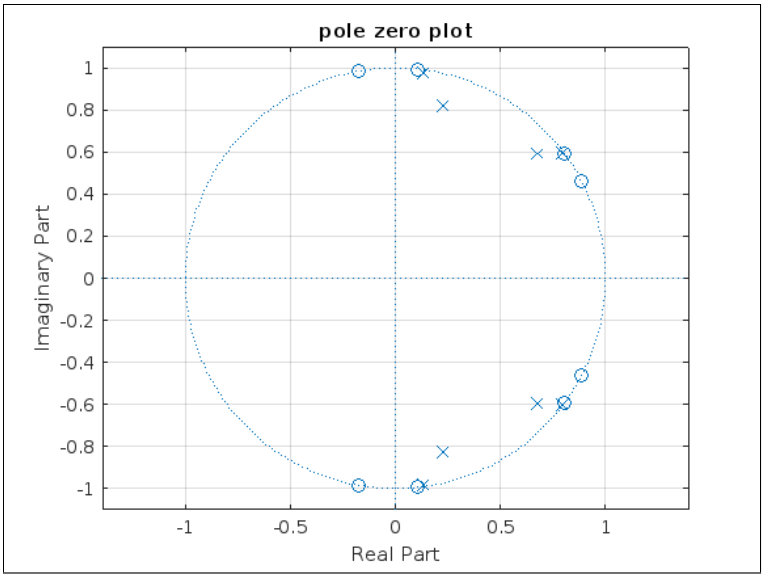

Figure 3: Pole zero plot

5 Filter Details BandStop

Calculations and detailed step-wise description is given below.

5.1 Un-normalized discrete time filter specs

Filter Number = 84 Therefore, m = 4 q(m) = 0 r(m) = 4 BL(m) = 64 BH(m) = 104

Here, we have a analog signal with bandwidth of 200Khz. Where we sample it with 425Khz.

Here we have a bandstop filter with BL and BH (Khz)being the stop band frequencies for the filter to be designed.

Hence, specifications here would be:

- **StopBand**: 64 to 104 Khz
- **Transitionband**: 5Khz
- **PassBand**: 0 to 59Khz and 109Khz to 200Khz
- **Tolerance**: 0.15 in magnitude for both Passband and Stopband

- **Passband Nature**: Equiripple
- **Stopband Nature**; Equiripple

5.2 Normalized Digital Filter Specifications

As we are sampling this signal at a frequency of 425 KHz. It is then normalized from 0 to 2π. By the following transformation:

$$\omega = \frac{\Omega \text{¡} 2 \text{¡} \pi/}{\Omega_s.Sampling\_Rate/} \tag{4}$$

Therefore the corresponding normalized discrete filter specifications are :-

- **StopBand**: 0.325π to 0.4894π
- **Transitionband**: 0.0235π
- **PassBand**: $0 * 0.301\pi$ to $0.5129\pi * 1\pi$
- **Tolerance**: 0.15 in magnitude for both Passband and Stopband
- **Passband Nature**: Equiripple
- **Stopband Nature**; Equiripple

5.3 After applying bilinear transformation

The bilinear transformation is given as:

$$\Omega = tan.\frac{\omega}{2}/ \tag{5}$$

From this transformation we end with the following new values:

Discrete	Analog
0.325π	0.5594
0.4894π	0.96727
0.0235π	0.0369
$0 * 0.301\pi$	$0 * 0.5118$
$0.5129\pi * 1\pi$	$1.041 *$

Hence the corresponding frequencies will be as follows:

- **StopBand**: 0.5118 to 0.96727
- **Transitionband**: 0.0369
- **PassBand**: 0 to 0.466 and 1.041 to
- **Tolerance**: 0.15 in magnitude for both Passband and Stopband
- **Passband Nature**: Equiripple
- **Stopband Nature**; Equiripple

5.4 Frequency Transformation and Relevant Parameters

We need to transform the analog bandstop filter to an analog low-pass filter so that we can apply **Equiripple** filter design on it as both passband and stopband have equiripple nature. This is performed by using the following transformation.

$$\Omega_L = \frac{B \text{ ¡ } \Omega}{\Omega_o^2 * \Omega^2} \tag{6}$$

Where the two parameters above depend on the band pass frequencies in the following way:

$$\Omega_o = \sqrt{\Omega_{P_1} \text{ ¡ } \Omega_{P_2}} = \sqrt{0.466 \text{ ¡ } 1.041} = 0.57544$$

$$B = \Omega_{P_2} * \Omega_{P_1} = 1.041 * 0.466 = 0.69670$$

Ω	Ω_L
0^+	0^+
0.466	$+1.\Omega_{L_{S_1}}/$
0.5118	$1.3184.\Omega_{L_{P_1}}/$
0.7299	
0.72994	$*$
0.96727	$*1.23631.\Omega_{L_{S_2}}/$
1.041	$*1.\Omega_{L_{P_2}}/$
	0^*

5.5 Frequency Transformed Lowpass Analog filter specification

- **Passband Edge**: $1.\Omega_{LP}/$
- **Stopband Edge**: $\min.\Omega_{LS1},*\Omega_{LS2}/ = \min.1.23621, 1.321/ = 1.23621.\Omega_{LS}/$
- **Tolerance**: 0.15 in magnitude for both Passband and Stopband
- **Passband Nature**: Equirriple
- **Stopband Nature**: Equirriple

6 Elliptical Bandstop filter

We need to use the same specifications which was there for the 2nd filter

$$B = 0.57544$$
$$\Omega_0 = 0.6967$$
$$\Omega_{LP} = 1$$
$$\Omega_{LS} = 1.23621$$

Consider $G_p = 0.85$ and $G_s = 0.15$
The value of K = $\frac{W_p}{W_s} = 0.8726$
The value of K1 = $\frac{\sqrt{1^{\cdot}G_p^2*1}}{\sqrt{1^{\cdot}G_s^2*1}}$ =0.094 The minimum value of N after solving the elliptical integral of K,K1

$$\frac{K_{1p}K}{K_pK_1} = 4$$

Next we calculate the analog low pass transfer function

$$\frac{0.6295\,s^2 + 1.0}{(1.605\,s + 1.0)\,(0.9994\,s^2 + 0.2305\,s + 1.0)}$$

We now use the inverse transformation to convert the equivalent Elliptic lowpass transfer function to get the Bandpstop transfer function using the relation:-

$$s_L = \frac{B\Omega}{s^2 + \Omega_0^2}$$

The analog bandstop transfer function:-
Numerator:-

$$1.0\text{e+}15\, s^6 + 1.665\text{e+}15\, s^4 + 8.08\text{e+}14\, s^2 + 1.144\text{e+}14$$

The denominator is:-

$$1.0\text{e+}15\, s^6 + 1.056\text{e+}15\, s^5 + 1.91\text{e+}15\, s^4 + 1.331\text{e+}15\, s^3 + 9.269\text{e+}14\, s^2 + 2.488\text{e+}14\, s + 1.144\text{e+}14$$

We now make use of the Bilinear Transformation to convert the analog bandstop filter into a discrete bandstop filter in the normalized angular frequency domain. The transform is as follows

$$s = \frac{1 * z^{*1}}{1 + z^{*1}}$$

Hence the Discrete Bandpass transfer function is:-
Numerator is:-

$$3.587\text{e+}5\, z^6\text{-}7.027\text{e+}5\, z^5 + 1.424\text{e+}6\, z^4\text{-}1.429\text{e+}6\, z^3 + 1.424\text{e+}6\, z^2\text{-}7.027\text{e+}5\, z + 3.587\text{e+}5$$

Denominator:-

$$6.587\text{e+}5\, z^6\text{-}1.051\text{e+}6\, z^5 + 1.641\text{e+}6\, z^4\text{-}1.378\text{e+}6\, z^3 + 1.135\text{e+}6\, z^2\text{-}4.05\text{e+}5\, z + 1.315\text{e+}6$$

The frequency response is:-

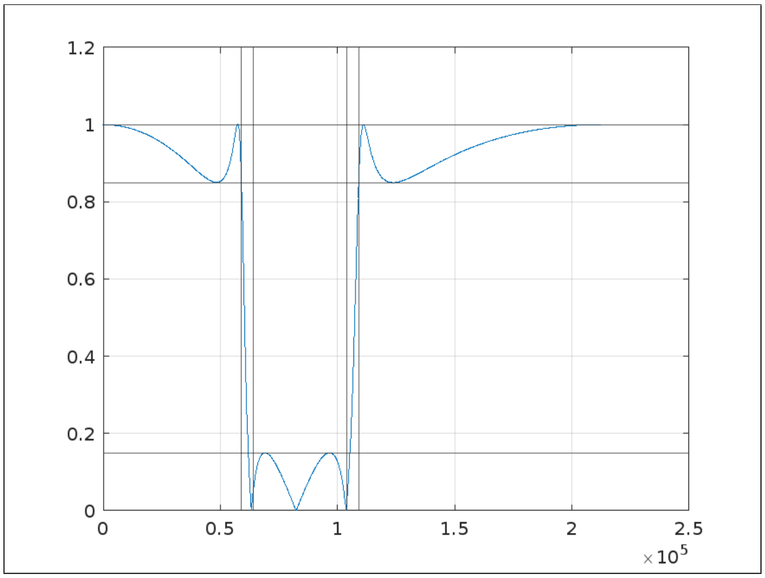

Figure 4: Frequency response

The magnitude and phase responses are

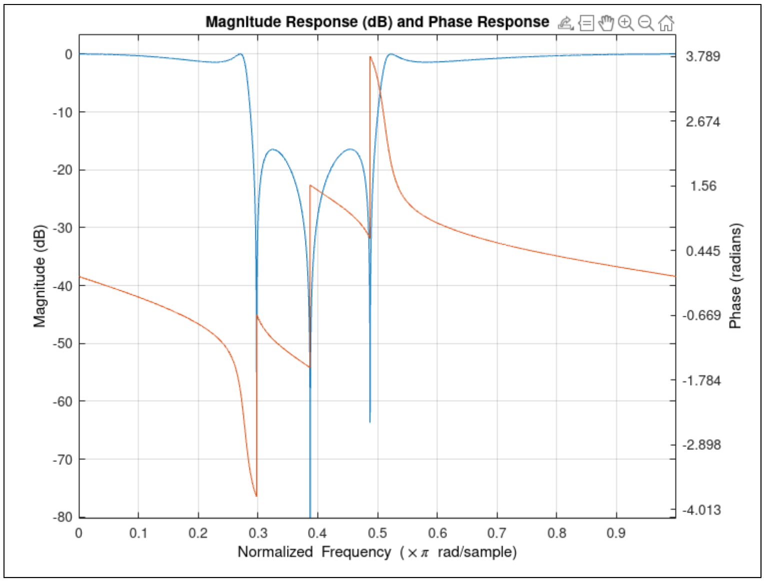

Figure 5: Magnitude and phase response

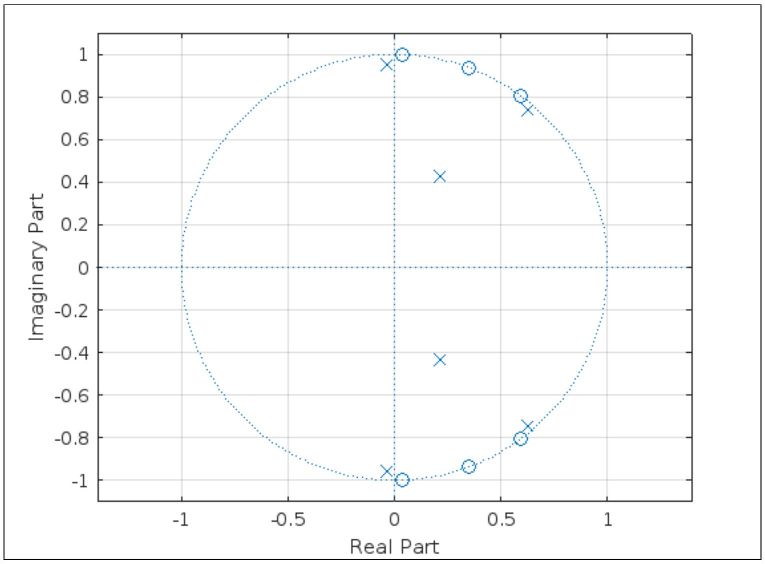

Figure 6: Pole zero plot

7 Comparison between Elliptical, Butterworth, Chebyshev filters

- The elliptic filter has the sharpest transition from passband to stopband or vice versa for the same specifications.

$$Elliptic > Chebyshev > Butterworth > KaiserFIR$$

- We can see that the least resources required were in Elliptical filter, as compared to Butterworth and Chebyshev. The increasing order of resources:-

$$Elliptic.3/ < Chebyshev.5/ < Butterworth.22/ < KaiserFIR.100+/$$

- The FIR realisation gives the most linear phase response in the passband region. The increasing order of linearity

$$Elliptic < Chebyshev < Butterworth < KaiserFIR$$

All the codes are in the zip file along with this report