

$$a^{\log_a b} = \frac{b}{a}$$

$$2^{\log_2 N} = N$$

$$T(N) = \log(N) + \log(N) + \log(N) + \dots + \log(N) + \log(N)$$

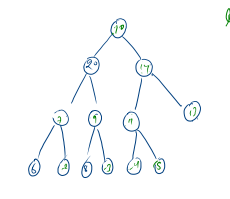
$$= \log(N \cdot N \cdot N \cdot \dots \cdot N) = \log(N^N) = N \log(N)$$

$$= N + N \log(N)$$

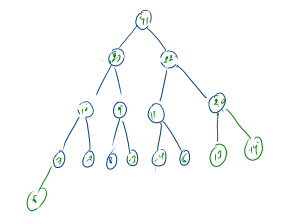
$T(N \log(N) + N)$
 $S: O(N)$ space

14 3 2 3 6 2 8 9 10 11 12 13 20 21 22

10 20 14 2 9 11 12 6 4 8 7 15 12 14 6



0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0 6



$a > b \rightarrow \text{true}$
 $a < b \rightarrow \text{true}$
 $a > b \rightarrow \text{true}$
 $a < b \rightarrow \text{true}$

$$T(N) = 2^0 \times 0 + 2^1 \times 1 + 2^2 \times 2 + 2^3 \times 3 + 2^4 \times 4 + \dots + 2^{n-1} \times (n-1)$$

$$= \frac{2^n}{2} \times 1 + \frac{2^n}{2^2} \times 2 + \frac{2^n}{2^3} \times 3 + \dots + \frac{2^n}{2^n} \times n = 2^n \left(\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \frac{5}{2^5} + \frac{6}{2^6} + \dots + \frac{n}{2^n} \right)$$

$$T(N) = 2^n \geq \frac{\lambda}{2^n}$$

$$T(N) = N \sum_{i=1}^n \lambda^i = N \frac{\lambda}{1-\lambda}$$

$$T(N) \leq N \sum_{i=1}^n \lambda^i$$

$$\sum_{i=0}^n \lambda^i = \frac{1}{1-\lambda}$$

$$\sum_{i=0}^n \lambda^i = \frac{\lambda}{1-\lambda^2} = 2$$

$$S_N = (1 \cdot x + 2x^2 + 3x^3 + 4x^4 + \dots + nx^n)$$

$$x S_N = (0x + 1x^2 + 2x^3 + 3x^4 + \dots + (n-1)x^n + nx^{n+1})$$

$$S_N(1-x) = x + x^2 + x^3 + x^4 + \dots + x^n + nx^{n+1}$$

$$S_N(1-x) = (x) \frac{(1-x^n)}{(1-x)} + nx^{n+1}$$

$$S_N(1-x) = (x) \frac{(1-x^n)}{(1-x)} + nx^{n+1}$$

$$\frac{S_N}{2} = 1 - \frac{1}{2^{n+1}} + \frac{\log_2(N)}{2^{n+1}} \cdot \frac{1}{2}$$

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$$S_N = 2 - \frac{2}{N} + \frac{\log_2(N)}{N}$$

$$S_N = 2 - \left(\frac{2}{N} \right) + \left(\frac{\log_2(N)}{N} \right)$$

$$\frac{S_N}{2} \approx 2$$

$$S_N \approx 4$$