Karnaugh Map

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Contents

***Abstract*—This manual explains Karnaugh maps (K-map) by finding the logic functions for the incrementing decoder.**

1 Incrementing Decoder

The incrementing decoder takes the numbers $0, 1, \ldots, 9$ in binary as inputs and generates the consecutive number as output. The corresponding truth table is available in Table. 0.

Z	Y	X	W	**D**	**C**	**B**	**A**
0	0	0	0	**0**	**0**	**0**	**1**
0	0	0	1	**0**	**0**	**1**	**0**
0	0	1	0	**0**	**0**	**1**	**1**
0	0	1	1	**0**	**1**	**0**	**0**
0	1	0	0	**0**	**1**	**0**	**1**
0	1	0	1	**0**	**1**	**1**	**0**
0	1	1	0	**0**	**1**	**1**	**1**
0	1	1	1	**1**	**0**	**0**	**0**
1	0	0	0	**1**	**0**	**0**	**1**
1	0	0	1	**0**	**0**	**0**	**0**

TABLE 0

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2 Karnaugh Map

Using Boolean logic, output A in Table 0 can be expressed in terms of the inputs W, X, Y, Z as

$$A = W'X'Y'Z' + W'XY'Z' + W'X'YZ' + W'XYZ' + W'X'Y'Z \tag{1}$$

1) K-Map for A: The expression in (1) can be minimized using the K-map in Fig. 1. In Fig. 1, the *implicants* in boxes 0,2,4,6 result in $W'Z'$. The implicants in boxes 0,8 result in $W'X'Y'$. Thus, after minimization using Fig. 2, (1) can be expressed as

$$A = W'Z' + W'X'Y' \tag{2}$$

Using the fact that

$$\begin{aligned} X + X' &= 1 \\ XX' &= 0, \end{aligned} \tag{3}$$

derive (2) from (1) algebraically.
Solution:

$$A = W'X'Y'Z' + W'XY'Z' + W'X'YZ' + W'XYZ' + W'X'Y'Z \tag{4}$$

$$A = W'X'Y' + W'X'Z' + W'Y'Z' + W'XYZ' \tag{5}$$

$$A = W'X'Y' + W'Z'(X' + Y' + XY) \tag{6}$$

$$A = W'X'Y' + W'Z'((XY)' + XY) \tag{7}$$

$$A = W'Z' + W'X'Y' \tag{8}$$

2) K-Map for B: From Table 0, using boolean logic,

$$B = WX'Y'Z' + W'XY'Z' + WX'YZ' + W'XYZ' \tag{9}$$

Show that (11) can be reduced to

$ZY \backslash XW$	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	0	0	0	0
10	1	0	0	0

Fig. 1: K-map for A.

$ZY \backslash XW$	00	01	11	10
00	0	1	0	1
01	0	1	0	1
11	0	0	0	0
10	0	0	0	0

Fig. 2: K-map for B.

$$B = WX'Z' + W'XZ' \tag{10}$$

using Fig. 2.

3) Derive (10) from (11) algebraically using (3).
Solution:

$$B = WX'Y'Z' + W'XY'Z' + WX'YZ' + W'XYZ' \tag{11}$$

$$B = WX'Z' + W'XZ' \tag{12}$$

4) K-Map for C: From Table 0, using boolean logic,

$$C = WXY'Z' + W'X'YZ' + WX'YZ' + W'XYZ' \tag{13}$$

Show that (13) can be reduced to

$ZY \backslash XW$	00	01	11	10
00	0	0	1	0
01	1	1	0	1
11	0	0	0	0
10	0	0	0	0

Fig. 4: K-map for C.

$$C = WXY'Z' + X'YZ' + W'YZ' \tag{14}$$

using Fig. 4.

5) Derive (14) from (13) algebraically using (3).
Solution:

$$C = WXY'Z' + W'X'YZ' + WX'YZ' + W'XYZ' \tag{15}$$

$$C = WXY'Z' + X'YZ' + W'YZ' \tag{16}$$

6) K-Map for D: From Table 0, using boolean logic,

$$D = WXYZ' + W'X'Y'Z \tag{17}$$

7) Minimize (17) using Fig. 6.
8) Download the code in

```
wget https://raw.githubusercontent.
com/gadepall/arduino/master
/7447/codes/inc_dec/inc_dec.ino
```

and modify it using the K-Map equations for A,B,C and D. Execute and verify.
9) Display Decoder: Table 9 is the truth table for the display decoder. Use K-maps to obtain the minimized expressions for a, b, c, d, e, f, g in

$ZY \backslash XW$	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	0	0	0	0
10	1	0	0	0

Fig. 6: K-map for D.

terms of A, B, C, D without don't care conditions.

D	C	B	A	**a**	**b**	**c**	**d**	**e**	**f**	**g**	Decimal
0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	1	1	0	0	1	1	1	1	1
0	0	1	0	0	0	1	0	0	1	0	2
0	0	1	1	0	0	0	0	1	1	0	3
0	1	0	0	0	1	0	0	1	0	0	4
0	1	0	1	1	0	0	1	1	0	0	5
0	1	1	0	0	1	0	0	1	0	0	6
0	1	1	1	0	0	0	1	1	1	1	7
1	0	0	0	0	0	0	0	0	0	0	8
1	0	0	1	0	0	0	0	1	0	0	9

TABLE 9: Truth table for display decoder.

Solution:
Without DON'T CARE:
from Fig. 9

$$a = D'C'B'A + D'CB'A' \tag{18}$$

from Fig. 9

$$b = D'CB'A + D'CBA' \tag{19}$$

from Fig. 9

$$c = D'C'BA' \tag{20}$$

from Fig. 9

$$d = D'CB'A' + D'CBA + C'B'A \tag{21}$$

from Fig. 9

$$e = D'A + C'B'A + D'CB' \tag{22}$$

from Fig. 9

$$f = D'BA + D'C'A + D'C'B \tag{23}$$

from Fig. 9

$$g = D'C'B' + D'CBA \tag{24}$$

$DC \backslash BA$	00	01	11	10
00	0	1	0	0
01	1	0	0	0
11	0	0	0	0
10	0	0	0	0

Fig. 9: K-map for a.

$DC \backslash BA$	00	01	11	10
00	0	0	0	0
01	0	1	0	1
11	0	0	0	0
10	0	0	0	0

Fig. 9: K-map for b.

$DC \backslash BA$	00	01	11	10
00	0	0	0	1
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

Fig. 9: K-map for c.

$DC \backslash BA$	00	01	11	10
00	0	1	1	0
01	1	1	1	0
11	0	0	0	0
10	0	1	0	0

Fig. 9: K-map for e.

$DC \backslash BA$	00	01	11	10
00	0	1	0	0
01	1	0	1	0
11	0	0	0	0
10	0	1	0	0

Fig. 9: K-map for d.

$DC \backslash BA$	00	01	11	10
00	0	1	1	1
01	0	0	1	0
11	0	0	0	0
10	0	0	0	0

Fig. 9: K-map for f.

$DC \backslash BA$	00	01	11	10
00	1	1	0	0
01	0	0	1	0
11	0	0	0	0
10	0	0	0	0

Fig. 9: K-map for g.