

Hands On - 4

Problem 0.

1. List out the function call stack

$\text{fib}(5) \rightarrow \text{fib}(4) \rightarrow \text{fib}(3) \rightarrow \text{fib}(2) \rightarrow \text{fib}(1) \rightarrow \text{fib}(0) \rightarrow$
 $\text{fib}(1) \rightarrow \text{fib}(0)$

$\text{fib}(2) \rightarrow \text{fib}(1) \rightarrow \text{fib}(0) \rightarrow \text{fib}(0) \rightarrow \text{fib}(3) \rightarrow \text{fib}(2) \rightarrow$
 $\text{fib}(1) \rightarrow \text{fib}(0) \rightarrow \text{fib}(1) \rightarrow \text{fib}(1) \rightarrow \text{fib}(0) \rightarrow \text{fib}(1)$

2. Prove the time complexity of the algorithm.

for the Equation the time complexity will be

$$T(n) = T(n-1) + T(n-2) + 1$$

For each value of n , the function makes two recursive calls with parameters $n-1$ & $n-2$. This branching continues until it reaches $n=0$, or $n=1$, at which the recursion stops. The number of function calls grows exponentially with the value of n because each function call results in two more functions $O(2^n)$.

3. We can improve this by storing the values of $\text{fib}(4)$, $\text{fib}(3)$ where these values can be reused instead of recalculation and calling all the below functions.

Problem 1

$$T(1) = 1$$

$$T(n) = T(n-1) + c$$

$$T(n) = T(n-2) + c$$

$$T(n) = T(n-k) + kc$$

$$\text{When } n-k = 1$$

$$K = n - 1$$

$$= n + 1$$

So time complexity will be $O(n)$

To improve the time complexity of the factorial calculation, we can use an iterative approach instead of recursion. Recursion can lead to stack overflow errors for large input values of n . This directly calculates the factorial of n by multiplying all integers from 1 to n and eliminates the need for recursive function calls.

Problem 2

$$T(n) = T(n-1) + T(n-2) + c$$

Assume $T(n-1) = T(n-2)$ as both are nearly close and similar.

$$T(n) = 2T(n-1) + c$$

$$\text{Assume } f(n) = O(2^n)$$

Substituting the above in original Eqⁿ

$$2T(n-1) + c \leq 2K \cdot 2^{n-1} + c$$

$$= K \cdot 2^n + c$$

\therefore the time complexity is $O(2^n)$

Improvement

Instead of comparing each element with its previous element to check for duplicates, use binary search to find duplicates efficiently in the sorted array.