STAT1050 STATISTICAL METHODS FOR TIME SERIES ANALYSIS

Tejesh Ramesh Bawa
001173097-8

Data Retrieval And Preprocessing

Data Description And Summary

This report will analyse the S&P 500 index from January 2000 to December 2023. The data was retrieved through the quantmod library utilizing Yahoo! Finance by using the "^GSPC". The focus of the analysis was on the adjusted closing prices, as it provides a more comprehensive valuation by incorporating the closing price, dividends, stock splits and new stock issuances. With that, it offers a more precise representation of the stock's value, as these factors can significantly affect the closing price.

Table 1 Data Summary

Minimum Adjusted Price	Maximum Adjusted Price	Mean Adjusted Price	Missing Values
67.5	4796.6	1973.6	0

Visualization And Logarithmic Transformation Of The S&P 5

Figure 1

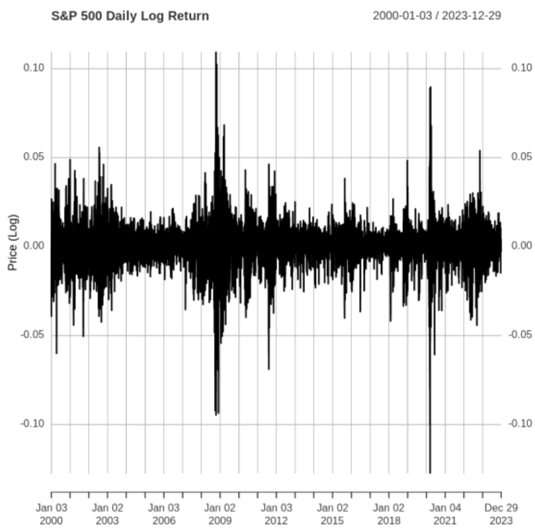


Figure 2

The performance of the index is illustrated through a plot of the adjusted closing price over time. Figure 1 demonstrates a price series with a strong upward trend with volatility during key events like the financial crisis of 2008 or Covid-19 pandemic. Notably, the series exhibits no seasonality, indicating the absence of recurring patterns with the average of the series fluctuating. The adjusted price was than computed to find the daily log return by eliminating the trend of exponential growth and stabilize the variance and mean which was plotted in **Error! Reference source not found.**. They were computed using $r_t = \ln \frac{P_t}{P_{t-1}}$ where P_t is the adjusted price at time t and P_{t-1} the adjusted price at $t - 1$.

Preliminary Analysis and Model Identification

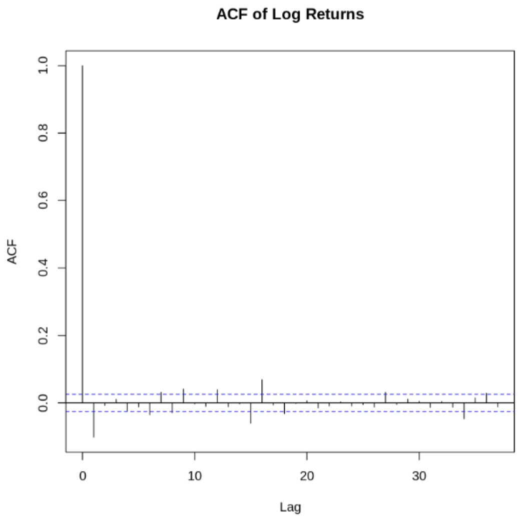
ADF And KPSS Test

To verify the stationarity of the daily log returns, two test were carried out, the ADF and KPSS Test. These tests will help to assess by evaluating distinct hypotheses regarding the stationarity of the time series. The ADF test specifically will investigate the presence of unit root within the series which help to identify whether the series is stationary or not. The null hypothesis (H_0) has unit root implying that the series is non-stationary, while the alternative hypothesis (H_1) has no unit root implying that the series is stationary. The computed p-value for the ADF test was 0.01 which is below the 0.05

significant threshold, whereby resulting in rejecting the null hypothesis and confirming that the series is stationary.

The KPSS test assesses the null hypothesis (H_0) level stationary while the alternative hypothesis (H_1) to be non-stationary. The outcome of the test was a p-value of 0.1 which surpassed the significance level of 0.05 whereby rejecting the alternative hypothesis and supporting the ADF test to validate that the series is indeed stationary.

ACF And PACF



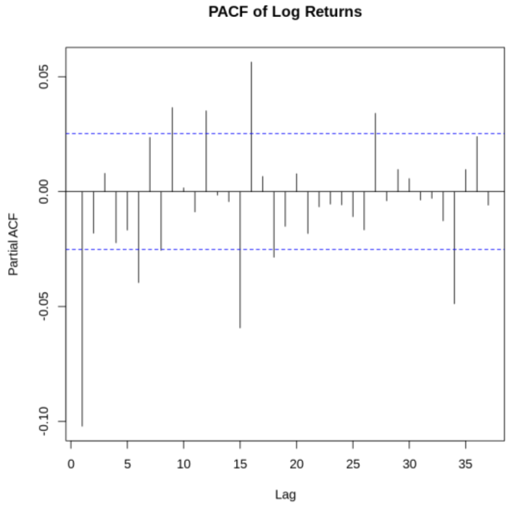


Figure 4

Figure 3

To assist with model identification, the analysis of the ACF and PACF plots were conducted. Figure 3 demonstrated a rapid decrease to approximately zero following the first lag, which indicates that there are minimal autocorrelations within the series. However, Figure 4 plot shows considerable spikes in the initial lags showing the existence of short-term dependencies. These results imply that the autoregressive term (p) should be incorporated into the model.

Model Identification

The preliminary ARIMA models were determined based on the outcomes of the ADF and KPSS test together with observing pattern from the ACF and PACF plots. Given that the series is stationary, no differencing will be required resulting in $d = 0$. The notable lags in the PACF imply that considering only 1 or 2 autoregressive terms ($p = 1 \ or \ 2$). As there was absence of definite patterns in the ACF plot, it suggests a minimal moving average component ($q = 0$ or 1) is suitable for this model. ARIMA Model (1,0,0),(1,0,1),(0,0,1),(2,0,0), and (2,0,1).

ARIMA Model Fitting And Comparison

Utilizing both the auto.arima function and manually specified models, various models were assessed to help determine the most suitable model.

Table 2

ARIMA Models	ARIMA (1,0,4)	ARIMA (1,0,0)	ARIMA (1,0,1)	ARIMA (0,0,1)	ARIMA (2,0,0)	ARIMA (2,0,1)
AIC	-35954.06	-35952.4	-35952.2	-35953.97	-35952.38	-35950.3
BIC	-35907.12	-35932.29	-35925.38	-35933.85	-35925.56	-35916.77

ARIMA(1,0,4) is the most efficient model, according to the auto.arima function. It consists of four moving average terms ($q = 4$) and one autoregressive term ($p = 1$). This result is different from the first analysis in the previous part, which indicated $q = 0$. The initial assumption was based on a visual analysis of the ACF plot, that showed a rapid drop to zero after the initial lag, which indicated little autocorrelation. However, as more complex dependencies spanning four lags were discovered by the auto.arima function, that has optimised the model's parameters. The noise adjustment that might not have been evident in the previous analysis are probably being addressed by the addition of $MA(4)$ terms.

The Equation fitted for ARIMA(1,0,4) is:
$y_t = \emptyset_1 y_{t-1} + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \theta_3 \epsilon_{t-3} + \theta_4 \epsilon_{t-4} + \mu$
$y_t = 0.4706 y_{t-1} - 0.5754 \epsilon_{t-1} + 0.0416 \epsilon_{t-2} + 0.0138 \epsilon_{t-3} - 0.0345 \epsilon_{t-4} + \mu$

In this context, the y_t denotes the logarithmic returns, et signifies the residual error at time t and the calculated mean by $\mu = 0.0002$. With the residual variance (σ^2) reduced to 0.0001515, this configuration produced the lowest AIC and BIC of -35,954.06 and -35,907.12 respectively. These results confirmed that the model has successfully compromised between complexity and fit.

A variety of manual ARIMA models were applied in order to evaluate the results derived from the auto.arima function. The single autoregressive component of ARIMA(1,0,0) model yielded higher AIC and BIC. This is because the short-term dependencies were effectively represented by this simpler model, however the noise patterns were not sufficiently addressed. Similarly, by adding one more moving average term ($MA(1)$) offered a little increase in fit. Nonetheless, it still fell short of the ARIMA(1,0,4) model's performance.

By adding a second autoregressive term such as the ARMIA(2,0,0) and ARIMA(2,0,1). Despite ARIMA(2,0,0) achieving a comparable AIC score did not improve significantly due to increase complexity. ARIMA(2,0,1) combining both moving average and autoregressive terms, however the resulting coefficients were not statistically significant and suggested a risk of overfitting without improving the overall quality of the model.

With its ability to capture short-term relationships and noise adjustments over four lags, the ARIMA(1,0,4) model performed better than any other option. the model is appropriate for examining the daily log return and that it adheres to the ARIMA modelling assumptions.

Residual Diagnostics And Assumption Validation

To evaluate the ARIMA models quality and verify whether that it adheres to the time series modelling as hypotheses. The residual diagnostics and assumption validation will be carried out. Three best models from the previous part ARIMA(1,0,4), ARIMA(1,0,0) and ARIMA(2,0,1) are compared. For this, the normality, homoscedasticity, and residual autocorrelation were the main criteria utilized to evaluate these models.

All the models had residual autocorrelation, according to the Ljung-Box Test ($p = 0.0006532$). But the ARIMA(1,0,4) model had demonstrated the lowest Ljung-Box value of $Q^* = 21.494$ and $df = 5$ which indicated that it might be better to adjust for noise. The residual and ACF plots, showed a lower degree compared to simpler models like ARIMA(1,0,0).

The K-S test and Q-Q plots were used to evaluate the residual normality. The ARIMA(1,0,4) model showed residuals that fell more in line with the normality, as seen by its Q-Q plot. This minor deviations in relation to the predicted hypothesized quantiles, thus rejecting the null hypothesis of normality. The residual histogram shows that ARIMA(1,0,4) more strongly resemble a normal distribution than the other models.

Through this investigation, all the models showed a consistent residual variance with no indication of heteroscedasticity. Interestingly, the ARIMA(1,0,4) model had the lowest AIC and BIC score which suggest that there needs to be a balance of complexity and good fit quality. Simple models like the ARIMA(1,0,0) were unable to detect the hidden noise patterns, but the addition ($q = 4$) was effective. Despite adding more additional parameters in ARIMA(2,0,1) model it might have indicated that it had an increased risk for overfitting without notably improving the residuals behaviour.

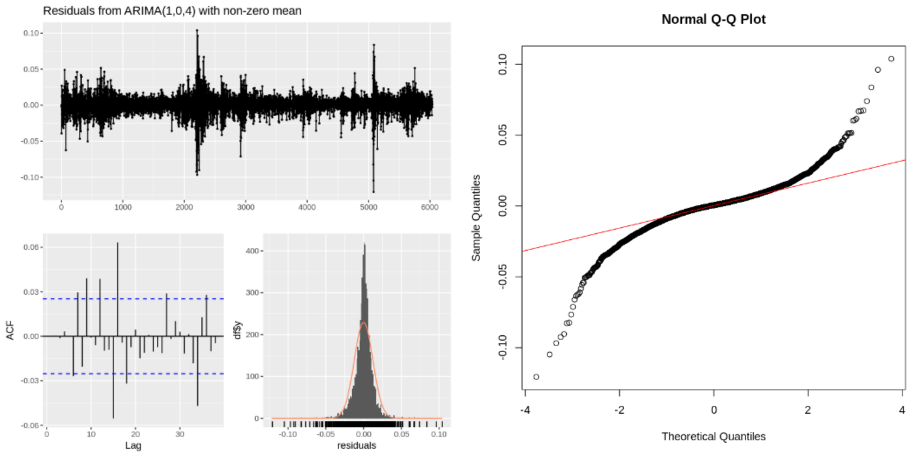


Figure 5

Model Evaluation And Performance Comparison

The test dataset was used to evaluate the ARIMA models (ARIMA(1,0,4) and ARIMA(2,0,1)) forecasting abilities by using Mean Error (ME), Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE). This will help highlight the benefits and drawbacks of each model.

Table 3

Model	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
ARIMA (1,0,4)	2.963423e-04	0.01364658	0.008895214	98.75527	103.7568	0.7422294	0.0007078051
ARIMA (1,0,1)	2.906892e-04	0.01364649	0.008894689	98.8951	103.7877	0.7421856	0.001133505

From table it can be seen that even though ARIMA(1,0,4) had strong predicting ability as seen. Even though there were some instances where ARIMA(1,0,1) was better than ARIMA(1,0,4), the MAPE was noticeably higher which suggested that it was difficult to precisely capture the percentage mistakes at times of high volatility. Additional information about the model's effectiveness was provided via the ACF1. ARIMA(1,0,1) had a higher residual autocorrelation (ACF1) score compared to ARIMA(1,0,4) which shows that it was better at handling patterns of noise.

Using one autoregressive term and four moving average term (ARIMA(1,0,4) model) is able to effectively capture the underlying noise at different lags. It was the preferred model due to it ability to balance the complexity together with a good quality fit. While the ARIMA(1,0,1) model with one autoregressive term and one moving average term had more straightforward approach. It also struggled with the underlying noise. Even though it could reduce the possibility of overfitting, it did not offer must advantage over the ARIMA(1,0,4) model.