COURSE MATERIAL

II Year B. Tech II- Semester

MECHANICAL ENGINEERING

STRENGTH OF MATERIALS

R18A0309

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

(Autonomous Institution-UGC, Govt. of India)
Secunderabad-500100, Telangana State, India.
www.mrcet.ac.in

CONTENTS

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

VISION

- To establish a pedestal for the integral innovation, team spirit, originality and competence in the students, expose them to face the global challenges and become technology leaders of Indian vision of modern society.

MISSION

- To become a model institution in the fields of Engineering, Technology and Management.
- To impart holistic education to the students to render them as industry ready engineers.
- To ensure synchronization of MRCET ideologies with challenging demands of International Pioneering Organizations.

QUALITY POLICY

- To implement best practices in Teaching and Learning process for both UG and PG courses meticulously.
- To provide state of art infrastructure and expertise to impart quality education.
- To groom the students to become intellectually creative and professionally competitive.
- To channelize the activities and tune them in heights of commitment and sincerity, the requisites to claim the never - ending ladder of **SUCCESS** year after year.

For more information: www.mrcet.ac.in

VISION

To become an innovative knowledge center in mechanical engineering through state-of-the-art teaching-learning and research practices, promoting creative thinking professionals.

MISSION

The Department of Mechanical Engineering is dedicated for transforming the students into highly competent Mechanical engineers to meet the needs of the industry, in a changing and challenging technical environment, by strongly focusing in the fundamentals of engineering sciences for achieving excellent results in their professional pursuits.

Quality Policy

- ✓ To pursuit global Standards of excellence in all our endeavors namely teaching, research and continuing education and to remain accountable in our core and support functions, through processes of self-evaluation and continuous improvement.
- ✓ To create a midst of excellence for imparting state of art education, industry-oriented training research in the field of technical education.

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Department of Mechanical Engineering

PROGRAM OUTCOMES

Engineering Graduates will be able to:

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and teamwork**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one’s own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. **Life-long learning**: Recognize the need for and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM SPECIFIC OUTCOMES (PSOs)

PSO1 Ability to analyze, design and develop Mechanical systems to solve the Engineering problems by integrating thermal, design and manufacturing Domains.

PSO2 Ability to succeed in competitive examinations or to pursue higher studies or research.

PSO3 Ability to apply the learned Mechanical Engineering knowledge for the Development of society and self.

Program Educational Objectives (PEOs)

The Program Educational Objectives of the program offered by the department are broadly listed below:

PEO1: PREPARATION

To provide sound foundation in mathematical, scientific and engineering fundamentals necessary to analyze, formulate and solve engineering problems.

PEO2: CORE COMPETANCE

To provide thorough knowledge in Mechanical Engineering subjects including theoretical knowledge and practical training for preparing physical models pertaining to Thermodynamics, Hydraulics, Heat and Mass Transfer, Dynamics of Machinery, Jet Propulsion, Automobile Engineering, Element Analysis, Production Technology, Mechatronics etc.

PEO3: INVENTION, INNOVATION AND CREATIVITY

To make the students to design, experiment, analyze, interpret in the core field with the help of other inter disciplinary concepts wherever applicable.

PEO4: CAREER DEVELOPMENT

To inculcate the habit of lifelong learning for career development through successful completion of advanced degrees, professional development courses, industrial training etc.

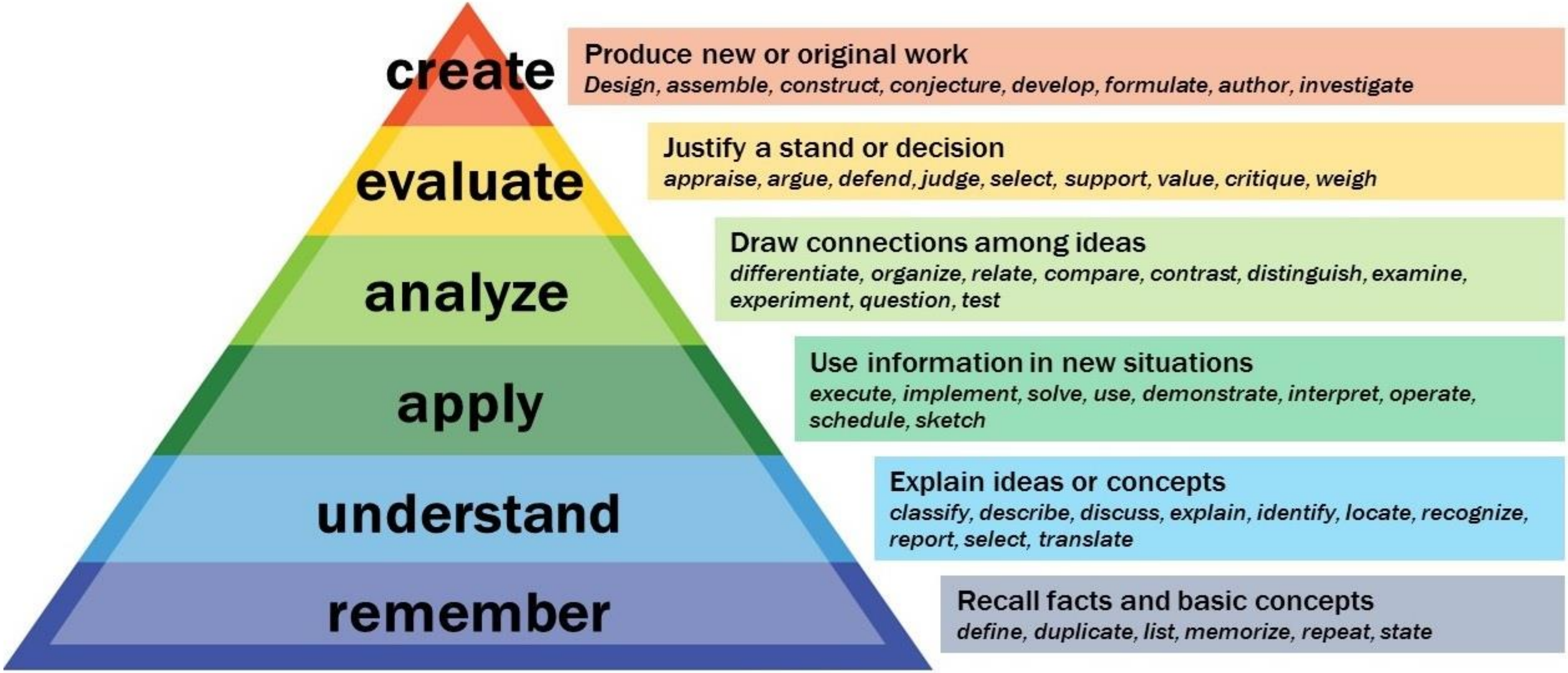
PEO5: PROFESSIONALISM

To impart technical knowledge, ethical values for professional development of the student to solve complex problems and to work in multi-disciplinary ambience, whose solutions lead to significant societal benefits.

Blooms Taxonomy

Bloom's Taxonomy is a classification of the different objectives and skills that educators set for their students (learning objectives). The terminology has been updated to include the following six levels of learning. These 6 levels can be used to structure the learning objectives, lessons, and assessments of a course.

1. **Remembering**: Retrieving, recognizing, and recalling relevant knowledge from long- term memory.
2. **Understanding**: Constructing meaning from oral, written, and graphic messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining.
3. **Applying**: Carrying out or using a procedure for executing or implementing.
4. **Analyzing**: Breaking material into constituent parts, determining how the parts relate to one another and to an overall structure or purpose through differentiating, organizing, and attributing.
5. **Evaluating**: Making judgments based on criteria and standard through checking and critiquing.
6. **Creating**: Putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning, or producing.


create
Produce new or original work
Design, assemble, construct, conjecture, develop, formulate, author, investigate
evaluate
Justify a stand or decision
appraise, argue, defend, judge, select, support, value, critique, weigh
analyze
Draw connections among ideas
differentiate, organize, relate, compare, contrast, distinguish, examine, experiment, question, test
apply
Use information in new situations
execute, implement, solve, use, demonstrate, interpret, operate, schedule, sketch
understand
Explain ideas or concepts
classify, describe, discuss, explain, identify, locate, recognize, report, select, translate
remember
Recall facts and basic concepts
define, duplicate, list, memorize, repeat, state

COURSE SYLLABUS

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

	L	P/D	C
II Year B. Tech ,ME-II Sem	3	0	3

(R18A0309) STRENGTH OF MATERIALS

Course Objectives:

- To understand the nature of stresses induced in material under different loads.
- To plot the variation of shear force and bending moments over the beams under different types of loads.
- To understand the behavior of beams subjected to shear loads.
- To understand the behavior of beams under complex loading.
- To analyze the cylindrical shells under circumferential and radial loading

UNIT-I

Simple Stresses & Strains : Elasticity and plasticity – Types of stresses & strains–Hooke's law – stress – strain diagram for mild steel – Working stress – Factor of safety – Lateral strain, Poisson's ratio & volumetric strain – Elastic moduli & the relationship between them – Bars of varying section – composite bars – Temperature stresses. Strain energy – Resilience – Gradual, sudden, impact and shock loadings.

UNIT-II

Shear Force and Bending Moment Diagrams: Definition of beam – Types of beams – Concept of shear force and bending moment – S.F and B.M diagrams for cantilever, simply supported and overhanging beams subjected to point loads, u.d.l., uniformly varying loads and combination of these loads – Point of contra flexure.

UNIT-III

Flexural Stresses: Theory of simple bending – Assumptions – Derivation of bending equation: M/I = f/y = E/R Neutral axis – Determination bending stresses – section modulus of rectangular and circular sections (Solid and Hollow), I, T, sections.

Shear Stresses: Derivation of formula – Shear stress distribution across various beams sections like rectangular, circular, triangular, I, T sections.

UNIT-IV

Deflection of Beams: Bending into a circular arc – slope, deflection and radius of curvature – Differential equation for the elastic line of a beam – Double integration and Macaulay's methods – Determination of slope and deflection for cantilever and simply supported beams subjected to point loads,- U.D.L uniformly varying load..

UNIT-V

Torsion of Circular Shafts: Theory of pure torsion, Derivation of torsion equations: $T/J=q/r=N\theta/L$- Assumptions made in theory of pure torsion-Torsional moment of resistance – Polar section modulus – Power transmitted by shafts.

Thin Cylinders: Thin seamless cylindrical shells – Derivation of formula for longitudinal and circumferential stresses – hoop, longitudinal and volumetric strains – changes in dia, and volume of thin cylinders.

TEXT BOOKS:

1.Strength of Materials by R.K. Bansal ,Laxmi Publications 2010.

2. Strength of materials by Sadhu Singh.Khanna Publications.

3. Strength of Materials by S.Timshenko

REFERENCE BOOKS:

1. Strength of Materials -By Jindal, Umesh Publications.
2. Strength of materials by Bhavikatti, Lakshmi publications.
3. Mechanics of Structures Vol-III, by S.B.Junnarkar.

Course Outcomes:

- Determine the simple stresses and strains when members are subjected to axial loads.
- Draw the shear force and bending moment diagrams for the beam subjected to different loading conditions.
- Evaluate stresses induced in different cross-sectional members subjected to shear loads.
- Evaluate the deflections in beams subjected to different loading conditions.
- Analyze the Shafts and thick cylindrical shells.

Lecturer Notes

UNIT 1

SIMPLE STRESSES & STRAINS

UNIT – I

SIMPLE STRESSES AND STRAINS

INTRODUCTION AND REVIEW

Preamble

Engineering science is usually subdivided into number of topics such as

1. Solid Mechanics
2. Fluid Mechanics
3. Heat Transfer
4. Properties of materials and soon Although there are close links between them in terms of the physical principles involved and methods of analysis employed.

The solid mechanics as a subject may be defined as a branch of applied mechanics that deals with behaviours of solid bodies subjected to various types of loadings. This is usually subdivided into further two streams i.e Mechanics of rigid bodies or simply Mechanics and Mechanics of deformable solids.

The mechanics of deformable solids which is branch of applied mechanics is known by several names i.e. strength of materials, mechanics of materials etc.

Mechanics of rigid bodies:

The mechanics of rigid bodies is primarily concerned with the static and dynamic behaviour under external forces of engineering components and systems which are treated as infinitely strong and undeformable Primarily we deal here with the forces and motions associated with particles and rigid bodies.

Mechanics of deformable solids:

Mechanics of solids:

The mechanics of deformable solids is more concerned with the internal forces and associated changes in the geometry of the components involved. Of particular importance are the properties of the materials used, the strength of which will determine whether the components fail by breaking in service, and the stiffness of which will determine whether the amount of deformation they suffer is acceptable. Therefore, the subject of mechanics of materials or strength of materials is central to the whole activity of engineering design. Usually the objectives in analysis here will be the determination of the stresses, strains, and deflections produced by loads. Theoretical analyses and experimental results have an equal roles in this field.

Analysis of stress and strain :

Concept of stress: Let us introduce the concept of stress as we know that the main problem of engineering mechanics of material is the investigation of the internal resistance of the body, i.e. the nature of forces set up within a body to balance the effect of the externally applied forces.

The externally applied forces are termed as loads. These externally applied forces may be due to any one of the reason.

(i) due to service conditions

(ii) due to environment in which the component works

(iii) through contact with other members

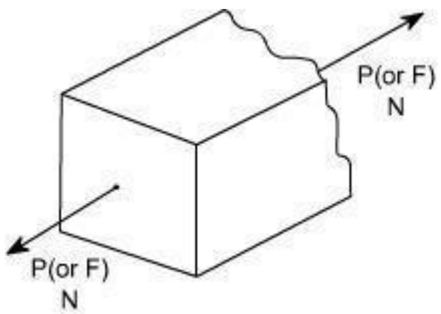
(iv) due to fluid pressures

(v) due to gravity or inertia forces.

As we know that in mechanics of deformable solids, externally applied forces acts on a body and body suffers a deformation. From equilibrium point of view, this action should be opposed or reacted by internal forces which are set up within the particles of material due to cohesion.

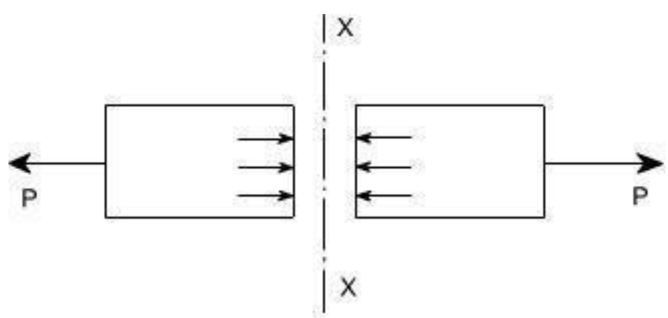
These internal forces give rise to a concept of stress. Therefore, let us define a stress Therefore, let us define a term stress

Stress:



Let us consider a rectangular bar of some cross – sectional area and subjected to some load or force (in Newtons)

Let us imagine that the same rectangular bar is assumed to be cut into two halves at section XX. The each portion of this rectangular bar is in equilibrium under the action of load P and the internal forces acting at the section XX has been shown



Now stress is defined as the force intensity or force per unit area. Here we use a symbol to represent the stress.

$$\sigma = \frac{P}{A}$$

Where A is the area of the X – section

Here we are using an assumption that the total force or total load carried by the rectangular bar is uniformly distributed over its cross – section.

But the stress distributions may be for from uniform, with local regions of high stress known as stress concentrations.

If the force carried by a component is not uniformly distributed over its cross – sectional area, A, we must consider a small area, ‘ A' which carries a small load P, of the total force ‘P', Then definition of stress is

$$\boxed{\sigma = \frac{\delta F}{\delta A}}$$

As a particular stress generally holds true only at a point, therefore it is defined mathematically as

$$\boxed{\sigma = \lim_{\delta A \to 0} \frac{\delta F}{\delta A}}$$

Units :

The basic units of stress in S.I units i.e. (International system) are N / m^2 (or Pa)

MPa = 10^6 Pa

GPa = 10^9 Pa

KPa = 10^3 Pa

Some times N / mm^2 units are also used, because this is an equivalent to MPa. While US customary unit is pound per square inch psi.

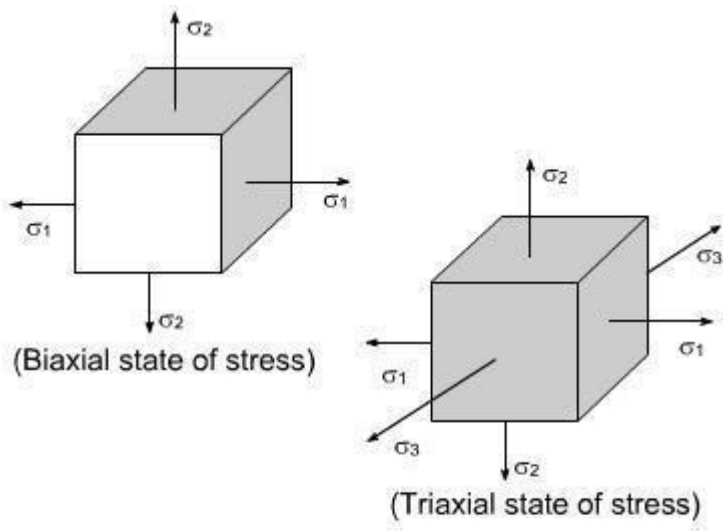
TYPES OF STRESSES :

only two basic stresses exists : (1) normal stress and (2) shear shear stress. Other stresses either are similar to these basic stresses or are a combination of these e.g. bending stress is a combination tensile, compressive and shear stresses. Torsional stress, as encountered in twisting of a shaft is a shearing stress.

Let us define the normal stresses and shear stresses in the following sections.

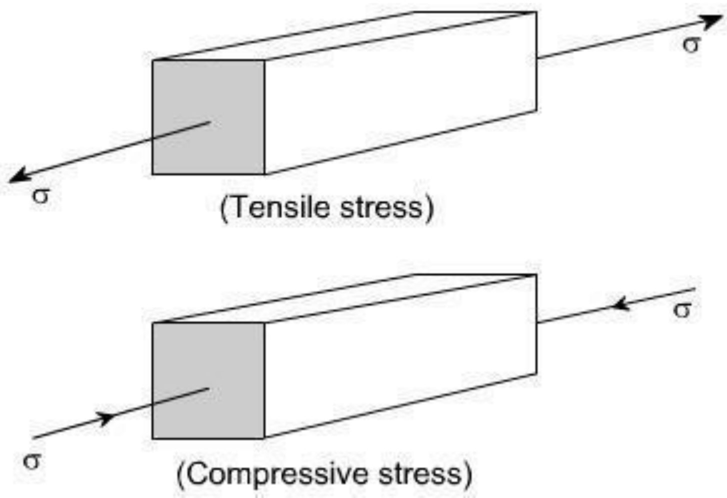
Normal stresses : We have defined stress as force per unit area. If the stresses are normal to the areas concerned, then these are termed as normal stresses. The normal stresses are generally denoted by a Greek letter ( )

This is also known as uniaxial state of stress, because the stresses acts only in one direction however, such a state rarely exists, therefore we have biaxial and triaxial state of stresses where either the two mutually perpendicular normal stresses acts or three mutually perpendicular normal stresses acts as shown in the figures below :

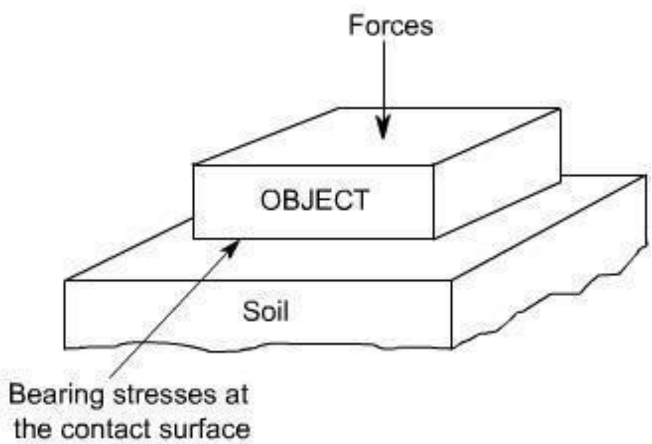


Tensile or compressive stresses :

The normal stresses can be either tensile or compressive whether the stresses acts out of the area or into the area

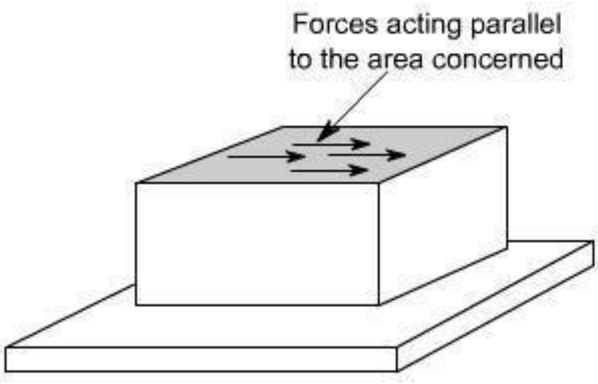


Bearing Stress : When one object presses against another, it is referred to a bearing stress (They are in fact the compressive stresses).



Shear stresses :

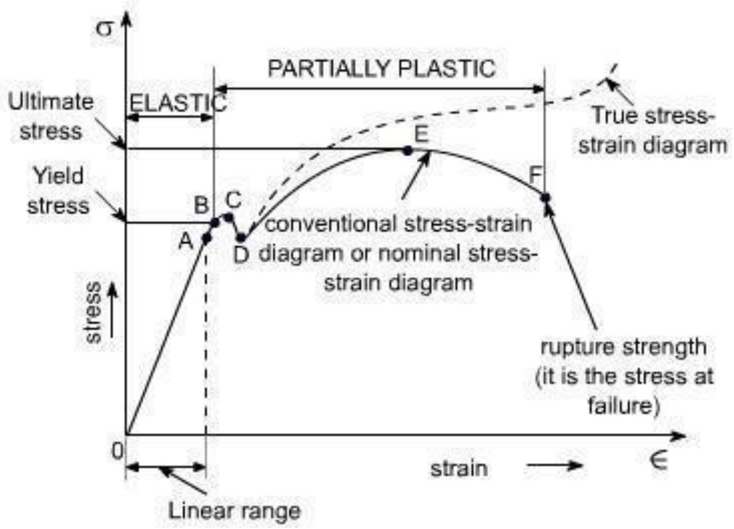
Let us consider now the situation, where the cross – sectional area of a block of material is subject to a distribution of forces which are parallel, rather than normal, to the area concerned. Such forces are associated with a shearing of the material, and are referred to as shear forces. The resulting force interistes are known as shear stresses.



The resulting force intensities are known as shear stresses, the mean shear stress being equal to

$$\tau = \frac{P}{A}$$

Where P is the total force and A the area over which it acts.



__Nominal stress – Strain OR Conventional Stress – Strain diagrams:__

Stresses are usually computed on the basis of the original area of the specimen; such stresses are often referred to as conventional or nominal stresses.

__True stress – Strain Diagram:__

Since when a material is subjected to a uniaxial load, some contraction or expansion always takes place. Thus, dividing the applied force by the corresponding actual area of the specimen at the same instant gives the so called true stress.

SALIENT POINTS OF THE GRAPH:

(A) So it is evident from the graph that the strain is proportional to strain or elongation isproportional to the load giving a st. line relationship. This law of proportionality is valid upto a point A.

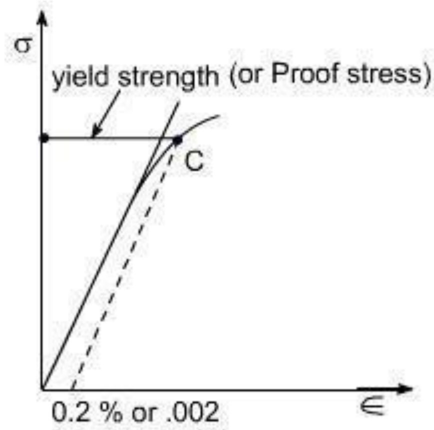
Or we can say that point A is some ultimate point when the linear nature of the graph ceases or there is a deviation from the linear nature. This point is known as **the limit of proportionality or the proportionality limit**.

(B) For a short period beyond the point A, the material may still be elastic in the sense that the deformations are completely recovered when the load is removed. The limiting point B is termed as **Elastic Limit**.

(C) and (D) - Beyond the elastic limit plastic deformation occurs and strains are not totally recoverable. There will be thus permanent deformation or permanent set when load is removed. These two points are termed as upper and lower yield points respectively. The stress at the yield point is called the yield strength.

A study a stress – strain diagrams shows that the yield point is so near the proportional limit that for most purpose the two may be taken as one. However, it is much easier to locate the former. For material which do not possess a well define yield points, In order to find the yield point or yield strength, an offset method is applied.

In this method a line is drawn parallel to the straight line portion of initial stress diagram by off setting this by an amount equal to 0.2% of the strain as shown as below and this happens especially for the low carbon steel.



(E) A further increase in the load will cause marked deformation in the whole volume of the metal. The maximum load which the specimen can with stand without failure is called the load at the ultimate strength.

The highest point 'E' of the diagram corresponds to the ultimate strength of a material.

σ_u = Stress which the specimen can with stand without failure & is known as Ultimate Strength or Tensile Strength.

σ_u is equal to load at E divided by the original cross-sectional area of the bar.

(F) Beyond point E, the bar begins to forms neck. The load falling from the maximum untilfracture occurs at F.

[Beyond point E, the cross-sectional area of the specimen begins to reduce rapidly over a relatively small length of bar and the bar is said to form a neck. This necking takes place whilst the load reduces, and fracture of the bar finally occurs at point F]

Note: Owing to large reduction in area produced by the necking process the actual stress at fracture is often greater than the above value. Since the designers are interested in maximum loads which can be carried by the complete cross section, hence the stress at fracture is seldom of any practical value.

Percentage Elongation:

The ductility of a material in tension can be characterized by its elongation and by the reduction in area at the cross section where fracture occurs.

It is the ratio of the extension in length of the specimen after fracture to its initial gauge length, expressed in percent.

$$\delta = \frac{(l_1 - l_g)}{l_1} \times 100$$

l_I = gauge length of specimen after fracture(or the distance between the gage marks at fracture)

l_g= gauge length before fracture(i.e. initial gauge length)

For 50 mm gage length, steel may here a % elongation of the order of 10% to 40%.

Ductile and Brittle Materials:

Based on this behaviour, the materials may be classified as ductile or brittle materials

Ductile Materials:

It we just examine the earlier tension curve one can notice that the extension of the materials over the plastic range is considerably in excess of that associated with elastic loading. The Capacity of materials to allow these large deformations or large extensions without failure is termed as ductility. The materials with high ductility are termed as ductile materials.

Brittle Materials:

A brittle material is one which exhibits a relatively small extensions or deformations to fracture, so that the partially plastic region of the tensile test graph is much reduced.

This type of graph is shown by the cast iron or steels with high carbon contents or concrete.

ELASTIC CONSTANTS

In considering the elastic behavior of an isotropic materials under, normal, shear and hydrostatic loading, we introduce a total of four elastic constants namely E, G, K, and μ .

It turns out that not all of these are independent to the others. In fact, given any two of them, the other two can be foundout . Let us define these elastic constants

(i) E = Young's Modulus of Rigidity

= Stress / strain

(ii) G = Shear Modulus or Modulus of rigidity

= Shear stress / Shear strain

(iii) μ = Possion's ratio

= lateral strain / longitudinal strain

(iv) K = Bulk Modulus of elasticity

= Volumetric stress / Volumetric strain

Where

Volumetric strain = sum of linear stress in x, y and z

direction. Volumetric stress = stress which cause the change

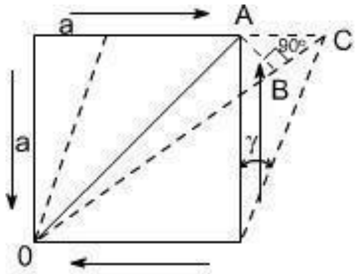
in volume. Let us find the relations between them

RELATION AMONG ELASTIC CONSTANTS

Relation between E, G and μ :

Let us establish a relation among the elastic constants E,G and . Consider a cube of material of side 'a' subjected to the action of the shear and complementary shear stresses as shown in the figure and producing the strained shape as shown in the figure below.

Assuming that the strains are small and the angle A C B may be taken as 45^0.



Therefore strain on the diagonal OA

= Change in length / original length

Since angle between OA and OB is very small hence OA OB therefore BC, is the change in the length of the diagonal OA

$$\text{Thus, strain on diagonal OA} = \frac{BC}{OA}$$

$$= \frac{AC\cos 45^0}{OA}$$

$$OA = \frac{a}{\sin 45^0} = a.\sqrt{2}$$

$$\text{hence} \qquad \text{strain} = \frac{AC}{a\sqrt{2}}.\frac{1}{\sqrt{2}}$$

$$= \frac{AC}{2a}$$

but $AC = a\gamma$

where γ = shear strain

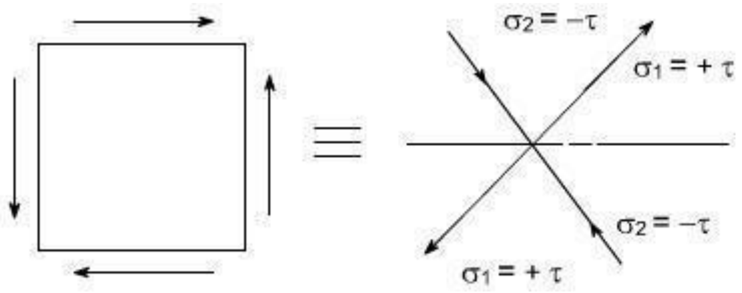
$$\text{Thus, the strain on diagonal} = \frac{a\gamma}{2a} = \frac{\gamma}{2}$$

From the definition

$$G = \frac{\tau}{\gamma} \text{ or } \gamma = \frac{\tau}{G}$$

$$\text{thus, the strain on diagonal} = \frac{\gamma}{2} = \frac{\tau}{2G}$$

Now this shear stress system is equivalent or can be replaced by a system of direct stresses at 45^0 as shown below. One set will be compressive, the other tensile, and both will be equal in value to the applied shear strain.



Thus, for the direct state of stress system which applies along the diagonals:

$$\text{strain on diagonal} = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$
$$= \frac{\tau}{E} - \mu\frac{(-\tau)}{E}$$
$$= \frac{\tau}{E}(1+\mu)$$

equating the two strains one may get

$$\frac{\tau}{2G} = \frac{\tau}{E}(1+\mu)$$

or $$\boxed{E = 2G(1+\mu)}$$

We have introduced a total of four elastic constants, i.e E, G, K and µ. It turns out that not all of these are independent of the others. Infact given any two of then, the other two can be found.

Again $E = 3K(1-2\gamma)$

$$\Rightarrow \frac{E}{3(1-2\gamma)} = K$$

if $\gamma = 0.5$ $K = \infty$

$$\epsilon_v = \frac{(1-2\gamma)}{E}(\epsilon_x + \epsilon_y + \epsilon_z) = 3\frac{\sigma}{E}(1-2\gamma)$$

(for $\epsilon_x = \epsilon_y = \epsilon_z$ hydrostatic state of stress)

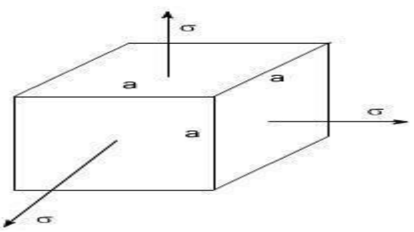
$\epsilon_v = 0$ if $\gamma = 0.5$

irrespective of the stresses i.e, the material is incompressible.

When µ = 0.5 Value of k is infinite, rather than a zero value of E and volumetric strain is zero, or in other words, the material is incompressible.

Relation between E, K and :

Consider a cube subjected to three equal stresses as shown in the figure below



The total strain in one direction or along one edge due to the application of hydrostatic stress or volumetric stress is given as

$$= \frac{\sigma}{E} - \gamma\frac{\sigma}{E} - \gamma\frac{\sigma}{E}$$
$$= \frac{\sigma}{E}(1-2\gamma)$$

volumetre strain = 3.linear strain

volumetre strain = $\epsilon_x + \epsilon_y + \epsilon_z$

or thus, $\epsilon_x = \epsilon_y = \epsilon_z$

$$\text{volumetric strain} = 3\frac{\sigma}{E}(1-2\gamma)$$

By definition

$$\text{Bulk Modulus of Elasticity (K)} = \frac{\text{Volumetric stress}(\sigma)}{\text{Volumetric strain}}$$

or

$$\text{Volumetric strain} = \frac{\sigma}{k}$$

Equating the two strains we get

$$\frac{\sigma}{k} = 3.\frac{\sigma}{E}(1-2\gamma)$$
$$\boxed{E = 3K(1-2\gamma)}$$

Relation between E, G and K :

$$\boxed{E = \frac{9\,GK}{(3K + G)}}$$

Relation between E, K and :

From the already derived relations, E can be eliminated

$$E = 2G(1 + \gamma)$$

$$E = 3K(1 - 2\gamma)$$

Thus,we get

$$3k(1 - 2\gamma) = 2G(1 + \gamma)$$

therefore

$$\gamma = \frac{(3K - 2G)}{2(G + 3K)}$$

or

$$\boxed{\gamma = 0.5(3K - 2G)(G + 3K)}$$

Engineering Brief about the elastic constants :

We have introduced a total of four elastic constants i.e E, G, K and ↓ It may be seen that not all of these are independent of the others. Infact given any two of them, the other two can be determined. Futher, it may be noted that

$$E = 3K(1 - 2\gamma)$$

or

$$K = \frac{E}{(1 - 2\gamma)}$$

$$\text{if } \gamma = 0.5;\ K = \infty$$

$$\text{Also } \epsilon_v = \frac{(1 - 2\gamma)}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$= \frac{(1 - 2\gamma)}{E}.3\sigma \text{ (for hydrostatic state of stress i.e } \sigma_x = \sigma_y = \sigma_z = \sigma \text{)}$$

hence if $\mu = 0.5$, the value of K becomes infinite, rather than a zero value of E and the volumetric strain is zero or in otherwords, the material becomes incompressible

Futher, it may be noted that under condition of simple tension and simple shear, all real materials tend to experience displacements in the directions of the applied forces and Under hydrostatic loading they tend to increase in volume. In otherwords the value of the elastic constants E, G and K cannot be negative

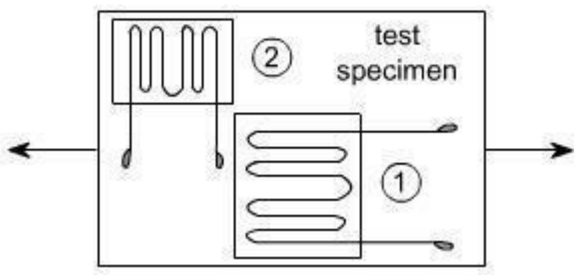
Therefore, the relations

$E = 2\ G\ (\ 1 + \mu)$

$E = 3\ K\ (\ 1 \quad)$

Yields $\boxed{-1 \le \nu \le 0.5}$

Determination of Poisson's ratio: Poisson's ratio can be determined easily by simultaneous use of two strain gauges on a test specimen subjected to uniaxial tensile or compressive load. One gage is mounted parallel to the longitudnal axis of the specimen and other is mounted perpendicular to the longitudnal axis as shown below:



Compression Test: Machines used for compression testing are basically similar to those used for tensile testing often the same machine can be used to perform both tests.

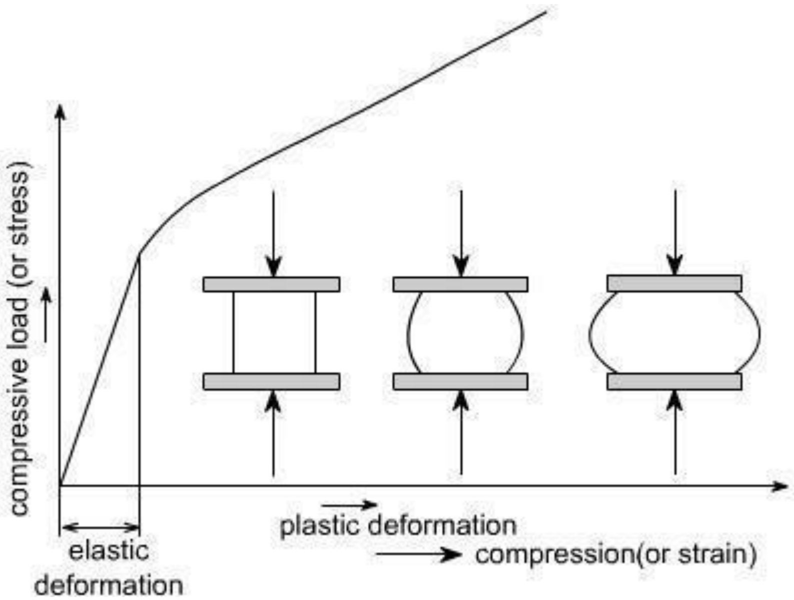
Shape of the specimen: The shape of the machine to be used for the different materials are as follows:

(i) For metals and certain plastics: The specimen may be in the from of a cylinder

(ii) For building materials: Such as concrete or stone the shape of the specimen may be in the from of a cube.

Shape of stress stain diagram

(a) Ductile materials: For ductile material such as mild steel, the load Vs compression diagram would be as follows



(1) The ductile materials such as steel, Aluminum, and copper have stress – strain diagrams similar to ones which we have for tensile test, there would be an elastic range which is then followed by a plastic region.

(2) The ductile materials (steel, Aluminum, copper) proportional limits in compression test are very much close to those in tension.

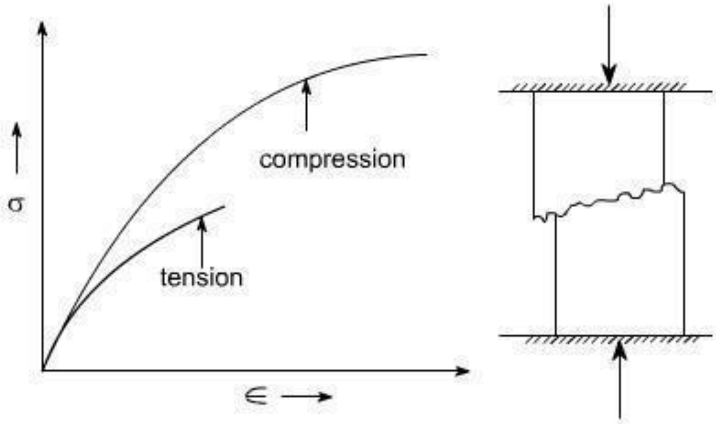
(3) In tension test, a specimen is being stretched, necking may occur, and ultimately fracture fakes place. On the other hand when a small specimen of the ductile material is compressed, it begins to bulge on sides and becomes barrel shaped as shown in the figure above. With increasing load, the specimen is flattened out, thus offering increased resistance to further shortening (which means that the stress – strains curve goes upward) this effect is indicated in the diagram.

Brittle materials (in compression test)

Brittle materials in compression typically have an initial linear region followed by a region in which the shortening increases at a higher rate than does the load. Thus, the compression stress – strain diagram has a shape that is similar to the shape of the tensile diagram.

However, brittle materials usually reach much higher ultimate stresses in compression than in tension.

For cast iron, the shape may be like this



Brittle materials in compression behave elastically up to certain load, and then fail suddenly by splitting or by craking in the way as shown in figure. The brittle fracture is performed by separation and is not accompanied by noticeable plastic deformation.

Practice Problems:

PROB 1: A standard mild steel tensile test specimen has a diameter of 16 mm and a gauge length of 80 mm such a specimen was tested to destruction, and the following results obtained.

Load at yield point = 87 kN

Extension at yield point = 173 x 16^{-6} m

Ultimate load = 124 kN

Total extension at fracture = 24 mm

Diameter of specimen at fracture = 9.8 mm

Cross - sectional area at fracture = 75.4 mm^2

Cross - sectional Area ‘A' = 200 mm^2

Compute the followings:

(i) Modulus of elasticity of steel

(ii) The ultimate tensile stream

(iii) The yield stress

(iv) The percentage elongation

(v) The Percentage reduction in Area.

PROB 2:

A light alloy specimen has a diameter of 16mm and a gauge Length of 80 mm. When tested in tension, the load extension graph proved linear up to a load of 6kN, at which point the extension was 0.034 mm. Determine the limits of proportionality stress and the modulus of elasticity of material.

Note: For a 16mm diameter specimen, the Cross – sectional area A = 200 mm^2

This is according to tables Determine the limit of proportion try stream & the modulus of elasticity for the material.

Ans: 30 MN /m^2 , 70.5 GN /m^2

solution:

$$\text{Limit of proportionaly stress} = \frac{6\ \text{kN}}{200 \times 10^{-6}}$$

$$= 30\ \text{MN/m}^2$$

$$\text{Young Modulus} \qquad E = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{strain} = \frac{.034}{80}$$

$$E = 30 \times 10^6 \Big/ \frac{.034}{80}$$

$$= 70.5\ \text{GN/m}^2$$

Strain Energy

Strain Energy of the member is defined as the internal work done in defoming the body by the action of externally applied forces. This energy in elastic bodies is known as **elastic strain energy** :

Strain Energy in uniaxial Loading

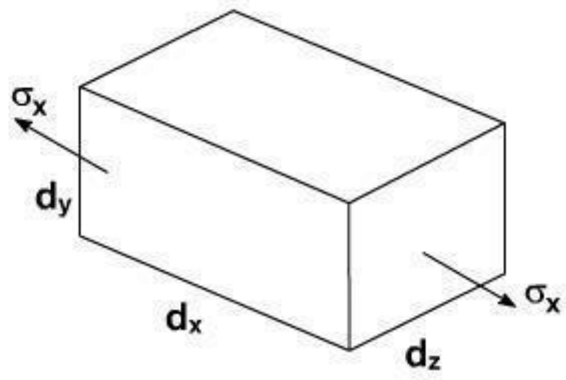


Fig .1

Let as consider an infinitesimal element of dimensions as shown in Fig .1. Let the element be subjected to normal stress σ_X.

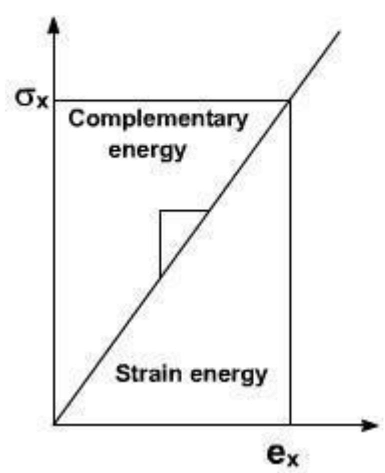
The forces acting on the face of this element is σ_X. dy. dz

where

dydz = Area of the element due to the application of forces, the element deforms to an amount = σ_X dx

$$= \frac{\text{Change in length}}{\text{Orginal in length}}$$

Assuming the element material to be as linearly elastic the stress is directly proportional to strain as shown in Fig . 2.



From Fig .2 the force that acts on the element increases linearly from zero until it attains its full value.

For a perfectly elastic body the above work done is the internal strain energy “du”.

$$\boxed{du = \frac{1}{2}\sigma_x dydz\, \epsilon_x\, dx} \qquad .....(2)$$

$$= \frac{1}{2}\sigma_x \epsilon_x\, dxdydz$$

$$\boxed{du = \frac{1}{2}\sigma_x\, \epsilon_x\, dv} \qquad ....(3)$$

where dv = dxdydz

= Volume of the element

By rearranging the above equation we can write

$$U_o = \boxed{\frac{du}{dv} = \frac{1}{2}\sigma_x\, \epsilon_x} \qquad .....(4)$$

The equation (4) represents the strain energy in elastic body per unit volume of the material its strain energy – density ‘u_o' .

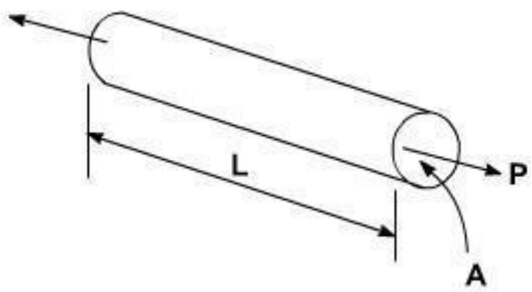
From Hook's Law for elastic bodies, it may be recalled that

$$\boxed{\sigma = E\epsilon}$$

$$\boxed{U_o = \frac{du}{dv} = \frac{\sigma_x^2}{2E} = \frac{E\,\epsilon_x^2}{2}} \qquad ......(5)$$

$$\boxed{U = \int_{Vol} \frac{\sigma_x^2}{2E}\, dv} \qquad ....(6)$$

In the case of a rod of uniform cross – section subjected at its ends an equal and opposite forces of magnitude P as shown in the Fig



$$U = \int_{Vol}^{L} \frac{\sigma_x^{\,2}}{2E}\, dv \qquad \sigma_x = \frac{P}{A}$$

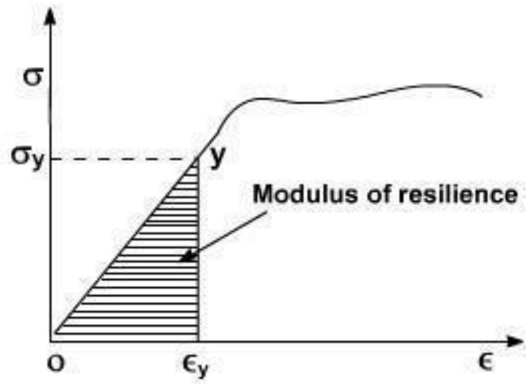
$$U = \int_{0}^{L} \frac{P^2}{2EA^2}\, A dx \qquad dv = Adx = \text{Element volume}$$

A = Area of the bar.

L = Length of the bar

$$\boxed{U = \frac{P^2 L}{2AE}} \qquad .....(7)$$

Modulus of resilience :



Suppose 'σ_x' in strain energy equation is put equal to σ_y i.e. the stress at proportional limit or yield point. The resulting strain energy gives an index of the materials ability to store or absorb energy without permanent deformation

So $$\boxed{U_y = \frac{\sigma_y^{\,2}}{2E}} \qquad .....(8)$$

The quantity resulting from the above equation is called the Modulus of resilience

The modulus of resilience is equal to the area under the straight line portion 'OY' of the stress – strain diagram as shown in Fig .4 and represents the energy per unit volume that the material can absorb without yielding. Hence this is used to differentiate materials for applications where energy must be absorbed by members.

Modulus of Toughness :

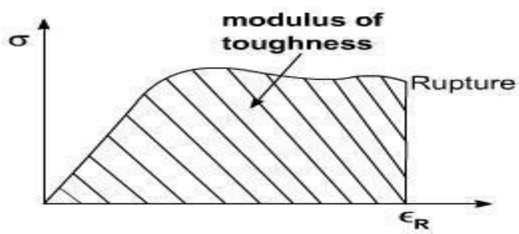


Fig .5

Suppose ‘ ' [strain] in strain energy expression is replaced by R strain at rupture, the resulting strain energy density is called modulus of toughness

$$U = \int_0^{\epsilon} E \epsilon_x \, dx = \frac{E \epsilon_R^2}{2} dv$$

$$\boxed{U = \frac{E \epsilon_R^2}{2}} \qquad \ldots\ldots(9)$$

From the stress – strain diagram, the area under the complete curve gives the measure of modules of toughness. It is the materials.

Ability to absorb energy upto fracture. It is clear that the toughness of a material is related to its ductility as well as to its ultimate strength and that the capacity of a structure to withstand an impact Load depends upon the toughness of the material used

Tutorial questions

1. Draw stress strain diagram for ductile materials and indicate all salient features on it. Explain the various mechanical properties can be estimated from that diagram.
2. Derive the relations between E,G,K
3. Derive the expression for the elongation for the circular tapered bar
4. Two parallel walls 6m apart are stayed together by a 25 mm diameter steel rod at 80^0C passing through washers and nuts at ends. If the rod cools down to 22^0C, calculate the pull induced in the rod, if
   (a) the walls do not yield and
   (b) the total yield at ends is 1.5 mm
   $E_{steel} = 2\times10^5 N/mm^2$, $\alpha_{steel} = 11\times10{-}6$ per^{0}C.
5. A)A metallic rod of 1 cm diameter, when tested under an axial pull of 10 kN was found to reduce its diameter by 0.0003 cm. The modulus of rigidity for the rod is 51 KN/mm^2. Find the Poisson's ratio, modulus of elasticity and Bulk Modulus.

   b) An aluminium bar 60 mm diameter when subjected to an axial tensile load 100 kN elongates 0.20 mm in a gage length 300 mm and the diameter is decreased by 0.012 mm. Calculate the modulus of elasticity and the Poisson's ratio of the material.
6. A specimen of diameter 13 mm and gauge length 50 mm was tested under tension. At 20 kN load, the extension was observed to be 0.0315 mm. Yielding occurred at a load of 35 kN and the ultimate load was 60 KN. The final gauge length at fracture was 70 mm. Calculate young's modulus, yield stress, ultimate strength and percentage elongation.

Assignment Questions

1. Determine the young's modulus and Possion's ratio of a metallic bar of length 25cm breadth 3cm depth 2cm when the beam is subjected to an axial compressive load 240KN. The decrease in length is given by 0.05cm and increase in breadth 0.002

2. Write the differences among Gradual, Sudden, Impact and Shock loadings with the help of expressions

3. A steel rod and two copper rods together support a load of 370 kN as shown in fig. The cross sectional area of steel road is 2500 mm^2 and of each copper road is 1600 mm^2. Find the stresses in the roads. Take E for steel is $2x10^5$ N/mm^2 and for copper is $1x10^5$ N/mm^2

4. A vertical tie, fixed rigidly at the top end consist of a steel rod 2.5 m long and 20 mm diameter encased throughout in a brass tube 20 mm internal diameter and 30 mm external diameter. The rod and the casing are fixed together at both ends. The compound rod is loaded in tension by a force of 10 kN. Calculate the maximum stress in steel and brass. Take $E_s=2x10^5 N/mm^2$ and $E_b=1x10^5 N/mm^2$

5. A steel tube 50mm in external diamerter and 3mm thick encloses centrally a solid copper bar of 35mm diameter. The bar and the tube are rigidly connected together at the ends at a temperature of 20 ^{0}C. Find the stress in each metal when heated to 170^0C. Also find the increase in length, if the original length of the assembly is 350mm. Take $\alpha_s=1.08 \times 10^{-5}$ per ^{0}C and $\alpha_c=1.7 \times 10^{-5}$ per ^{0}C . Take $E_s = 2X10^5$ N/mm^2 , $E_c = 1X10^5$ N/mm^2

UNIT-I

POWER POINT PRESENTAION

SIMPLE STRESSES & STRAINS:-

UNIT -I

MRCET CAMPUS
UGC Autonomous

DEPARTMENT OF MECHANICAL ENGINEERING

STRENGTH OF MATERIALS

- Predicting how geometric and physical properties of structure will influence its behaviour under service conditions.

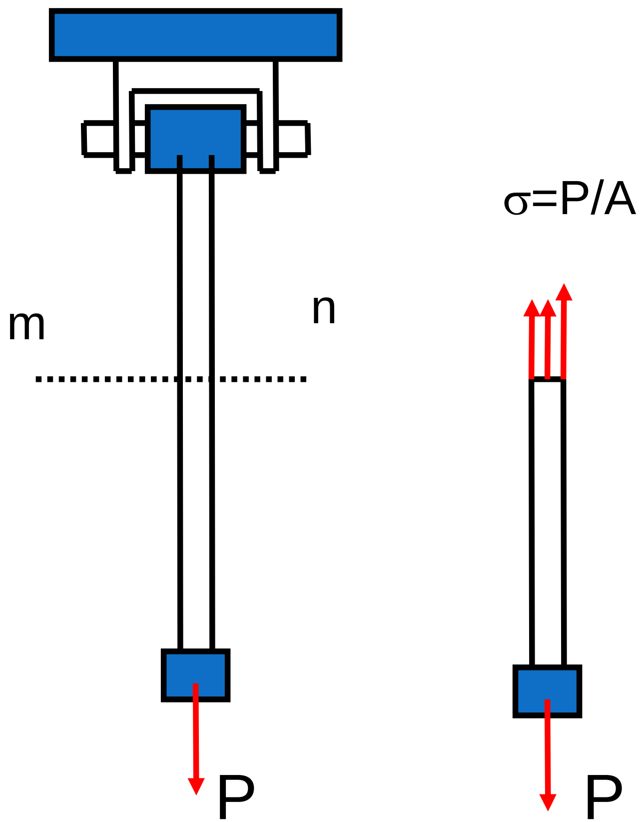
- Stresses can occur isolated or in combination.
- Is structure strong enough to withstand loads applied to it ?
- Is it stiff enough to avoid excessive deformations and deflections?
- Engineering Mechanics----> Statics----->

  deals with rigid bodies
- All materials are deformable and mechanics of solids takes this into account.

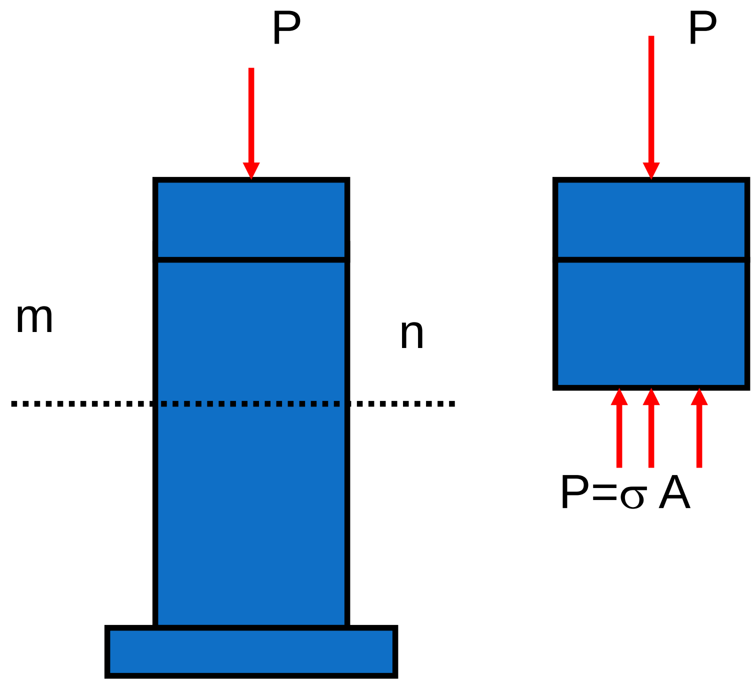
- Strength and stiffness of structures is function of size and shape, certain physical properties of material.

- **Properties of Material:-**
- Elasticity
- Plasticity
- Ductility
- Malleability
- Brittleness
- Toughness
- Hardness

INTERNAL FORCE:- STRESS

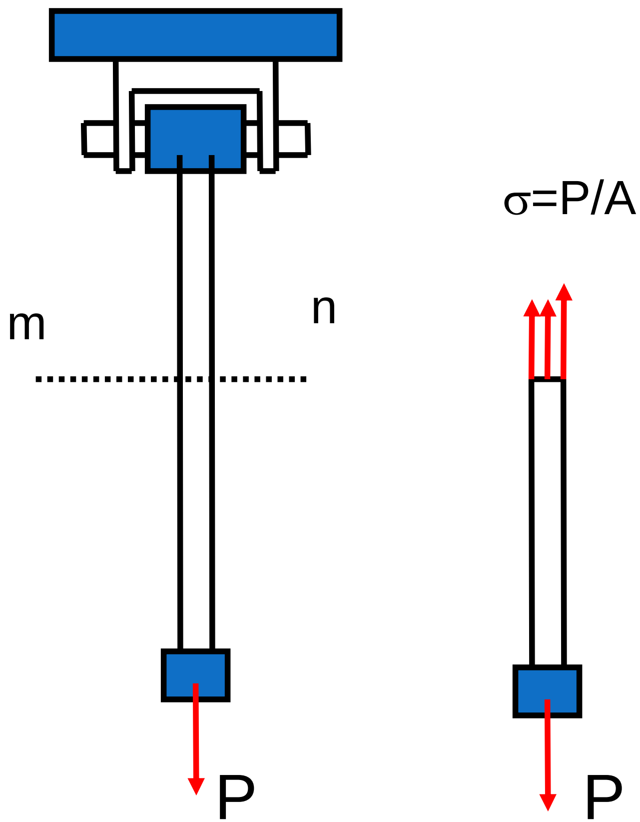


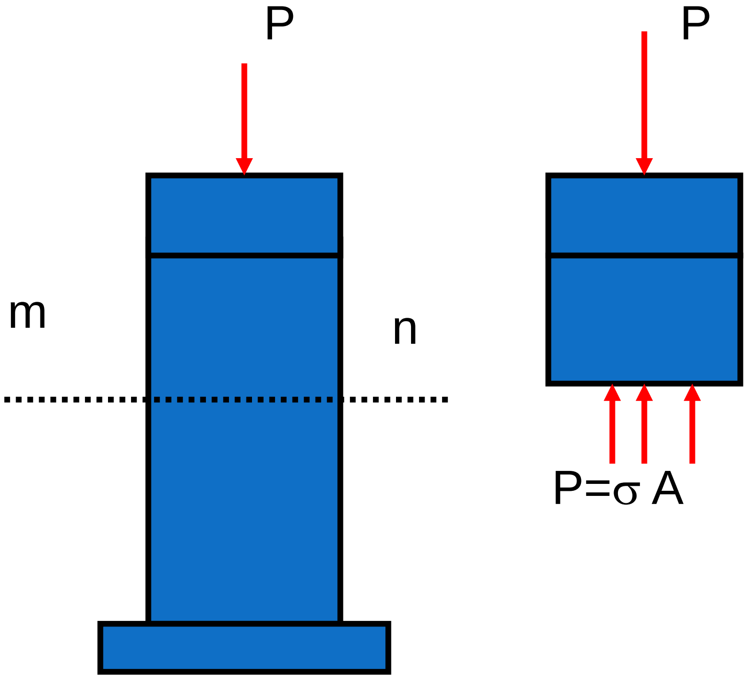
- Axial tension
- Stretches the bars & tends to pull it apart
- Rupture



- Axial Compression
- Shortens the bar
- Crushing
- Buckling

INTERNAL FORCE:- STRESS





- Axial tension
- Stretches the bars & tends to pull it apart
- Rupture

- Axial Compression
- Shortens the bar
- Crushing
- Buckling

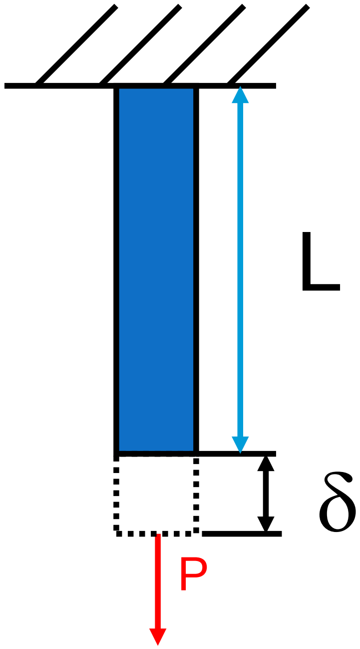
- Strain

- It is defined as deformation per unit length

- it is the ratio of change in length to original length

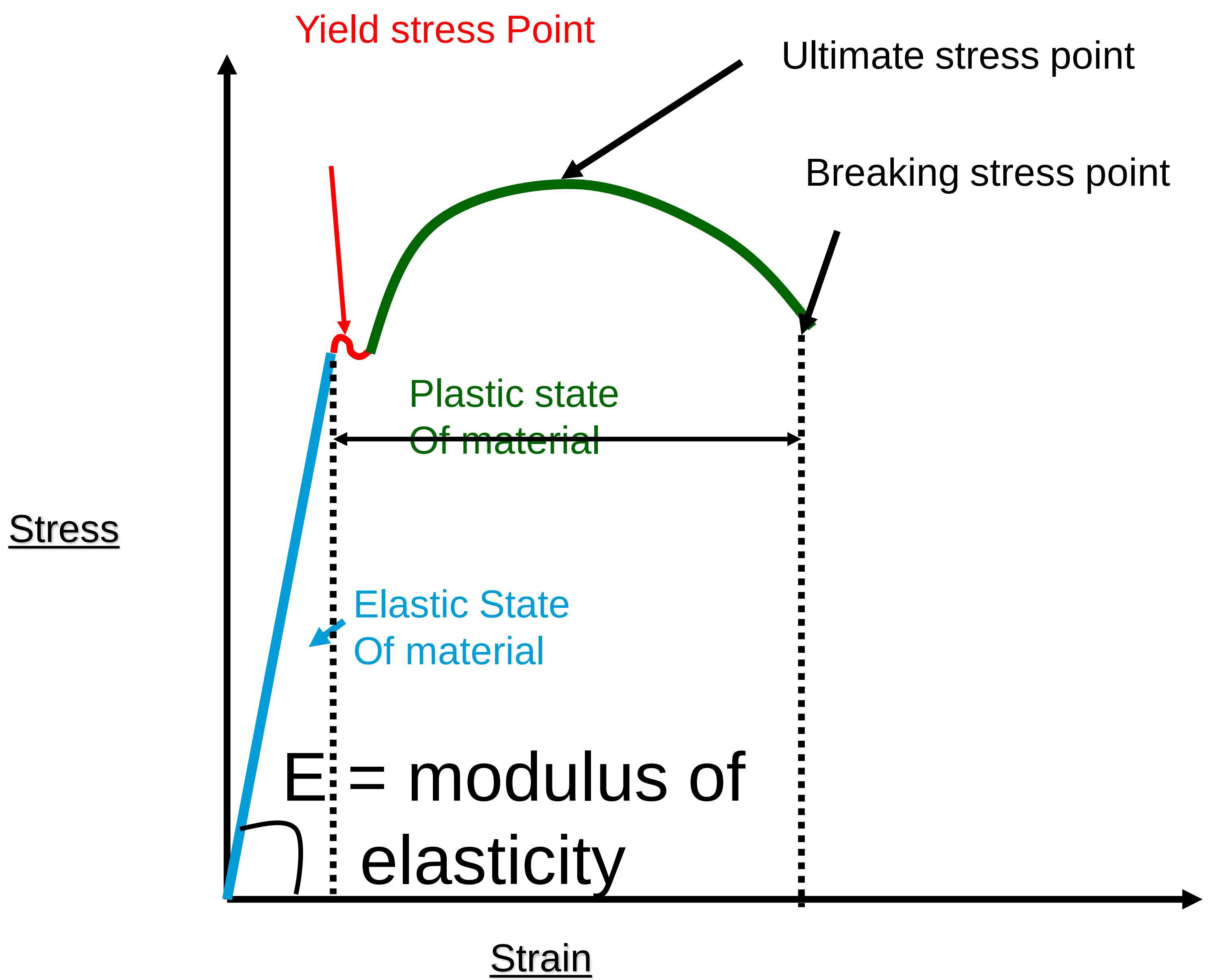
- Tensile strain (+ Ve) (ε) $= \dfrac{\text{increase in length}}{\text{Original length}} = \dfrac{\delta}{L}$

Compressive strain (- Ve) (ε) $= \dfrac{\text{decrease in length}}{\text{Original length}} = \dfrac{\delta}{L}$



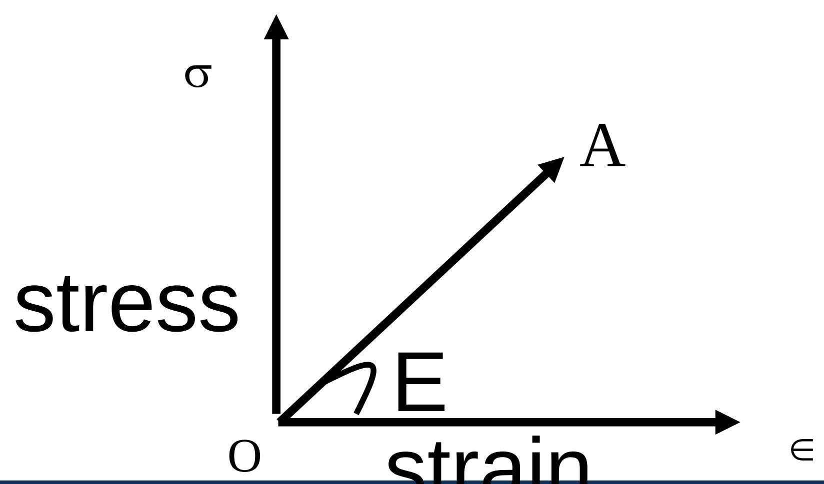
- Strain is dimensionless quantity.

Stress- Strain Curve for Mild Steel (Ductile Material)

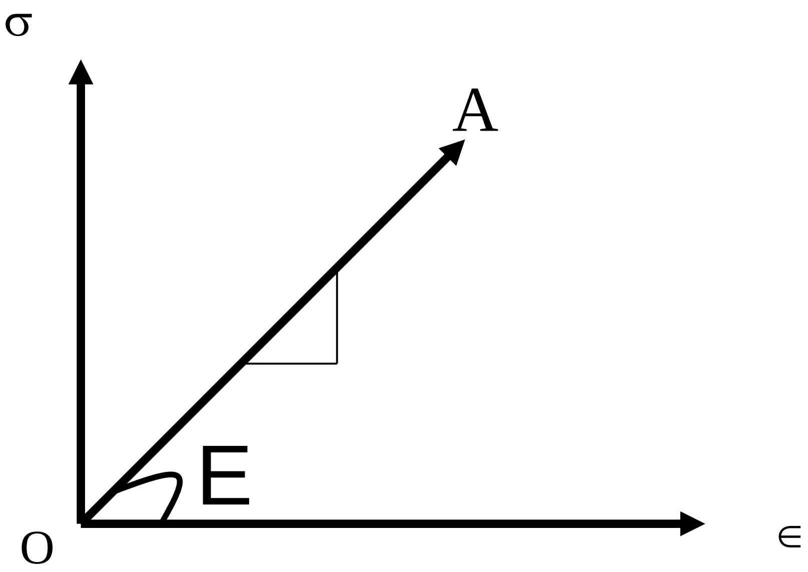


Modulus of Elasticity: $\sigma = E \in$

- Stress required to produce a strain of unity.
- i.e. the stress under which the bar would be stretched to twice its original length . If the material remains elastic throughout , such excessive strain.
- Represents slope of stress-strain line OA.



Value of E is same in Tension & Compression.



- Hooke's Law:-

Up to elastic limit, Stress is proportional to strain

$\sigma \ \alpha \ \in$

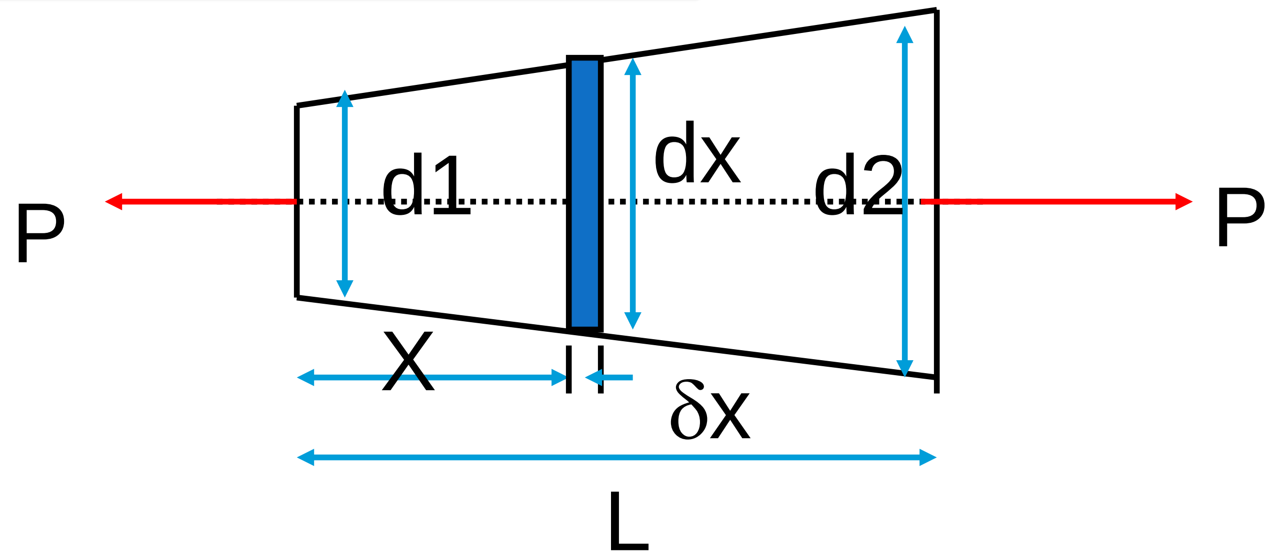
$\sigma = E \in$; where E=Young's modulus

$\sigma = P/A$ and $\in = \delta / L$

$P/A = E (\delta / L)$

$\delta = PL / AE$

Extension of Bar of Tapering cross Section from diameter d1 to d2:-



Bar of Tapering Section:

$$dx = d1 + [(d2 - d1) / L] * X$$

$$\delta\Delta = P\delta x / E[\pi /4\{d1 + [(d2 - d1) / L] * X\}^2]$$

$$\Delta = \int_0^L 4\,P\,dx / [E\,\pi\{d1+kx\}^2]$$

$$= -\,[4P/\,\pi\,E]\ x\quad 1/k\ \ [\,\{1/(d1+kx)\}]_0^L\ dx$$

$$=-\,[4PL/\,\pi\,E(d2-d1)]\ \{1/(d1+d2-d1) - 1/d1\}$$

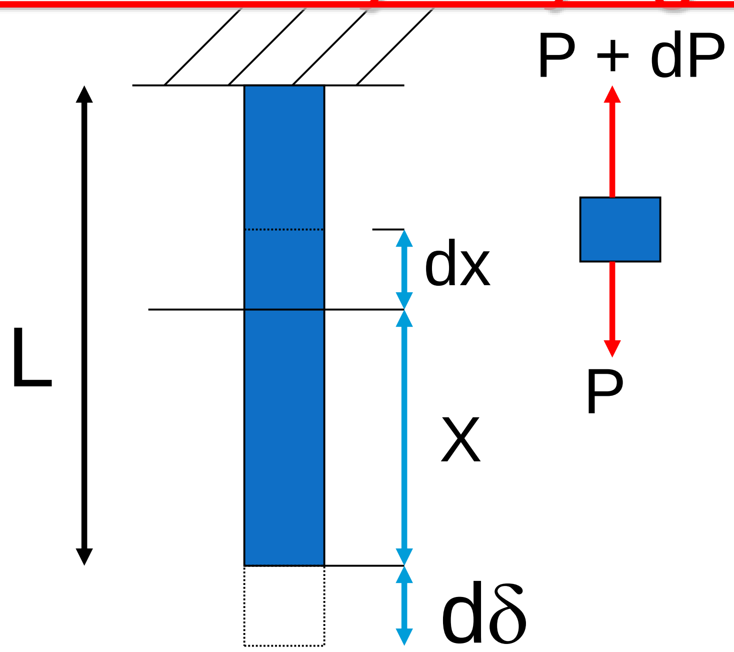
$$\Delta = 4PL/(\pi\,E\,d1\,d2)$$

Check :-

When d = d1=d2

Δ =PL/ [(π /4)* d^2E] = PL /AE (refer -24)

Extension of Uniform cross section bar subjected to uniformly varying tension due to self weight



$$P_X = \gamma A x$$

$$d\delta = P_X \, dx / A E;$$

$$\delta = \int_0^L P_X \, dx/AE = \int_0^L \gamma A x \, dx/AE$$

$$\delta = (\gamma / E) \int_0^L x \, dx = (\gamma L^2/2E)$$

0

If total weight of bar W= γ A L γ = W/AL

$\delta = WL/2AE$ (compare this results with slide-26)

POISSONS RATIO:- μ = lateral contraction per Unit axial elongation, (with in elastic limit)

$\mu = (\delta B/B)/(\delta L/L)$;

$= (\delta B/B)/(\varepsilon)$

So $\delta B = \varepsilon \mu B$;

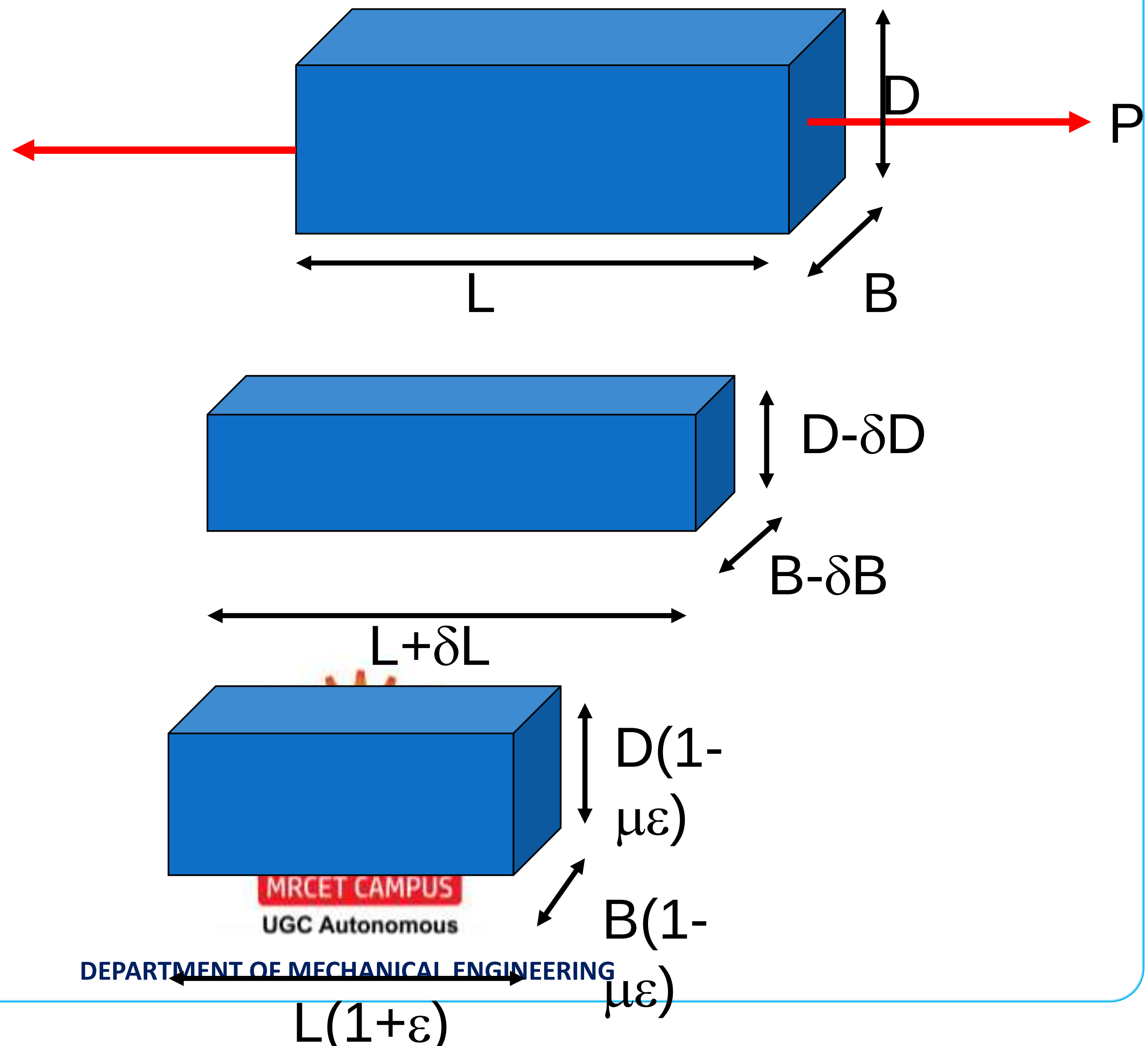
New breadth =

$B - \delta B = B - \varepsilon \mu B$

$= B(1 - \mu \varepsilon)$

Sim.,New depth=

$D(1-\mu \varepsilon)$



for isotropic materials $\mu = 1/4$ for steel $\mu = 0.3$

Volume of bar before deformation V= L * B*D

new length after deformation $L_1 = L + \delta L = L + \varepsilon L = L(1+\varepsilon)$

new breadth $B_1 = B - \delta B = B - \varepsilon \mu B = B(1 - \mu \varepsilon)$

new depth $D_1 = D - \delta D = D - \varepsilon \mu D = D(1 - \mu \varepsilon)$

new cross-sectional area = $A_1 = B(1-\mu \varepsilon) * D(1-\mu \varepsilon) = A(1-\mu \varepsilon)^2$

new volume $V1 = V - \delta V = L(1+\varepsilon) * A(1-\mu \varepsilon)^2$

$$\approx AL(1+\varepsilon - 2\mu \varepsilon)$$

Since ε is small

change in volume = $\delta V = V1 - V \quad = AL\varepsilon(1-2\mu)$

and unit volume change = $\delta V / V \quad = \{AL\varepsilon(1-2\mu)\}/AL$

$$\delta V / V \quad = \varepsilon(1-2\mu)$$

for isotropic materials $\mu = 1/4$ for steel $\mu = 0.3$

Volume of bar before deformation $V = L * B * D$

new length after deformation $L_1 = L + \delta L = L + \varepsilon L = L(1 + \varepsilon)$

new breadth $B_1 = B - \delta B = B - \varepsilon \mu B = B(1 - \mu \varepsilon)$

new depth $D_1 = D - \delta D = D - \varepsilon \mu D = D(1 - \mu \varepsilon)$

new cross-sectional area $= A_1 = B(1-\mu \varepsilon) * D(1-\mu \varepsilon) = A(1- \mu \varepsilon)^2$

new volume $V1 = V - \delta V = L(1+ \varepsilon) * A(1- \mu \varepsilon)^2$

$$\approx AL(1 + \varepsilon - 2 \mu \varepsilon)$$

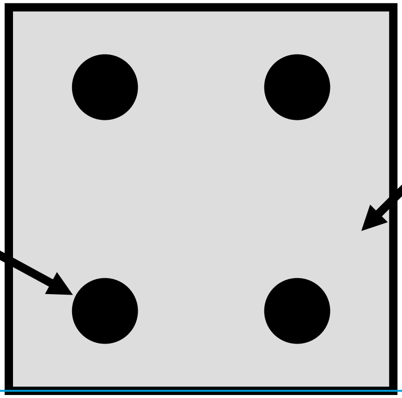
Since ε is small

change in volume $= \delta V = V1 - V = AL \varepsilon (1-2\mu)$

and unit volume change $= \delta V / V = \{AL \varepsilon (1-2\mu)\}/AL$

$$\delta V / V = \varepsilon (1-2\mu)$$

Composite Sections:



- as both the materials deforms axially by same value strain in both materials are same.

$$\varepsilon s = \varepsilon c = \varepsilon$$

$$\sigma s / Es = \sigma c / E \;(= \varepsilon = \delta L / L) \_\_\_\_\_\_ (1) \;\&\; (2)$$

- Load is shared between the two materials.

$$P_s + P_c = P \text{ i.e. } \sigma_s * A_s + \sigma_c * A_c = P \;\; \text{---}(3)$$

(unknowns are σs, σc and δL)

Temperature stresses:-

ELASTIC CONSTANTS:

Any direct stress produces a strain in its own direction and opposite strain in every direction at right angles to it.

Lateral strain /Longitudinal strain

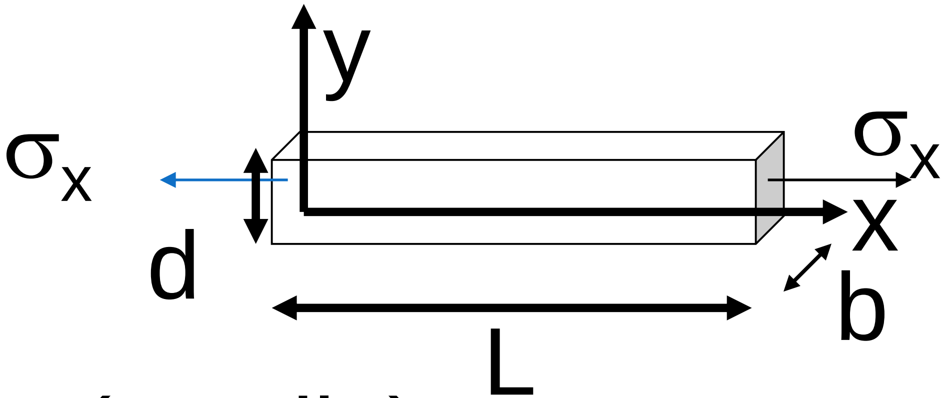
= Constant

= 1/m = μ = Poisson's ratio

Lateral strain = Poisson's ratio x Longitudinal strain

$\varepsilon_y = \mu\, \varepsilon_x$ -------------(1)

Single direct stress along longitudinal axis



$\varepsilon_x = \sigma_x/E$ (tensile)

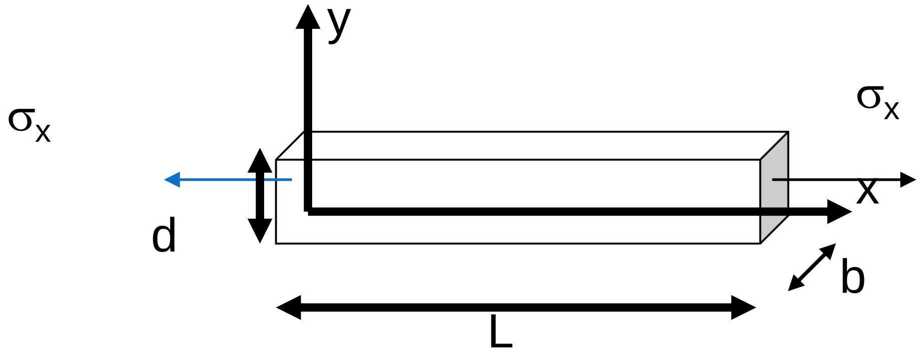
$\varepsilon_{y} = \mu\ \varepsilon_x = \mu\ [\sigma_x/E]$ (compressive)

Volume = L b d

$\delta V = bd\ \delta L - d\ L\delta b - L\ b\delta d$

$\delta V/V = \delta L/L - \delta b/b - \delta d/d$

$= \varepsilon_x - \varepsilon_y - \varepsilon_z = \varepsilon_x - \mu\ \varepsilon_x - \mu\varepsilon_x = \varepsilon_x - 2\mu\ \varepsilon_x = \varepsilon_x(1-2\ \mu)$



$= \varepsilon_x - \varepsilon_y - \varepsilon_z = \varepsilon_x - \mu\,\varepsilon_x - \mu\varepsilon_x = \varepsilon_x - 2\mu\,\varepsilon_x = \varepsilon_x(1-2\,\mu)$

$= [\sigma_x/E] \times (1-2\,\mu)$

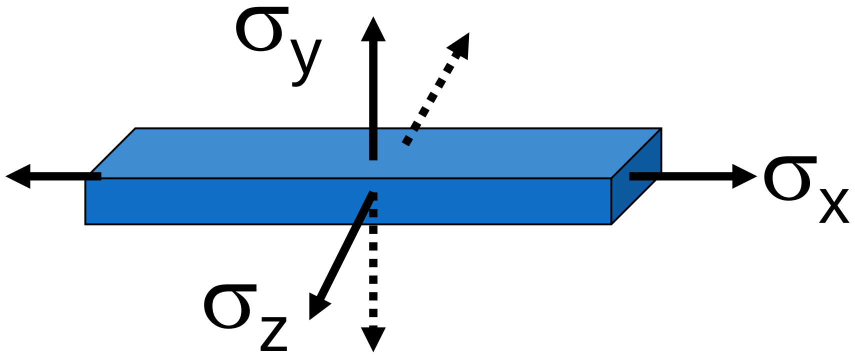
Volumetric strain= $\varepsilon_v = [\sigma_x/E] \times (1-2\,\mu)$ _

-----(2)

or $\varepsilon_v = [\sigma_x/E] \times (1-2/m)$

$\varepsilon_v = [\sigma_x/E] \times (1-2/m)$

Stress σ_x along the axis and σ_y and σ_z perpendicular to it.



$\varepsilon_x = \sigma_x/E - \sigma_y/mE - \sigma_z/mE$-----(i)

$\varepsilon_y = \sigma_y/E - \sigma_z/mE - \sigma_x/mE$-----(ii) --------(3)

$\varepsilon_z = \sigma_z/E - \sigma_x/mE - \sigma_y/mE$-----(iii)

Note:- If some of the stresses have opposite sign necessary changes in algebraic signs of the above expressions will have to be made.

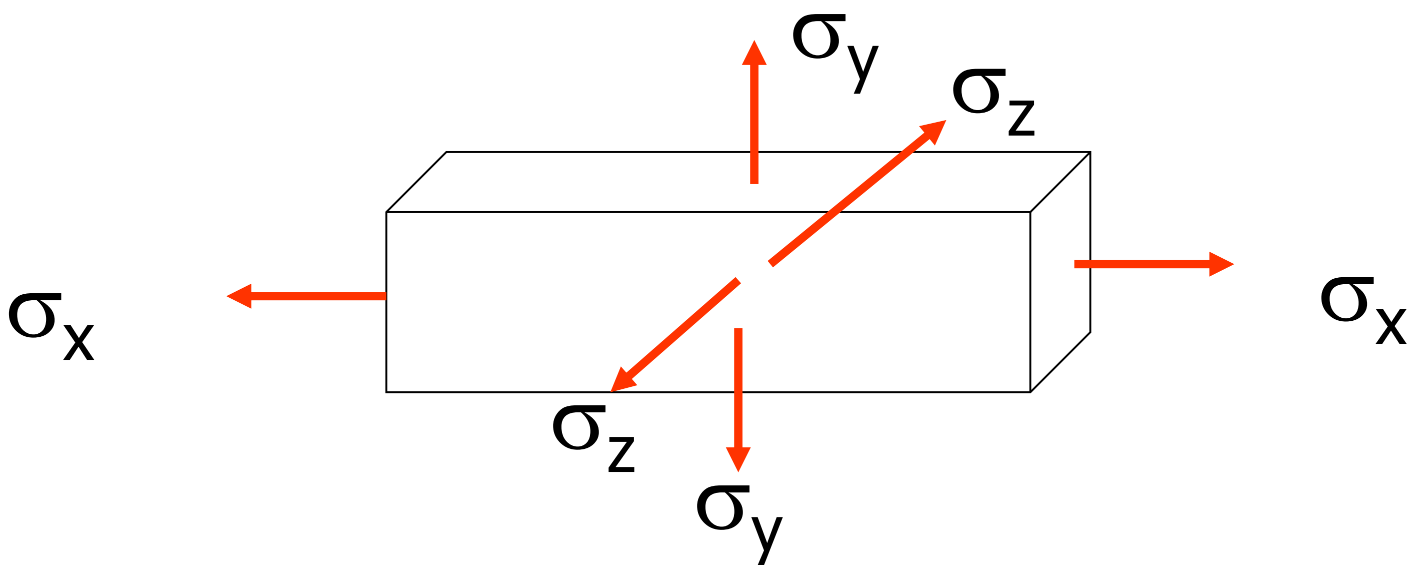
Upper limit of Poisson's Ratio:

adding (i),(ii) and (iii)

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = (1 - 2/m)(\sigma_x + \sigma_y + \sigma_z)/E_{-} \quad \text{--------(4)}$$

known as DILATATION

For small strains represents the change in volume /unit volume.



	ε_x	ε_y	ε_z
σ_x	σ_x/E	$-\mu\ \sigma_x/E$	$-\mu\ \sigma_x/E$
σ_y	$-\mu\ \sigma_y/E$	σ_y/E	$-\mu\ \sigma_y/E$
σ_z	$-\mu\ \sigma_z/E$	$-\mu\ \sigma_z/E$	σ_z/E

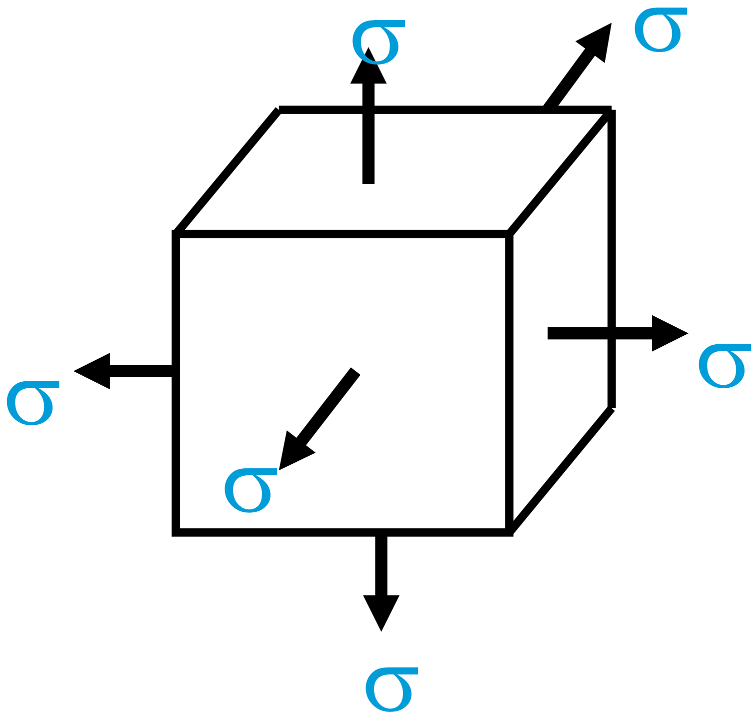
Sum all

BULK MODULUS (K):--

- When a body is subjected to the identical stress σ in three mutually perpendicular directions, the body undergoes uniform changes in three directions without the distortion of the shape.
- The ratio of change in volume to original volume has been defined as volumetric strain(ε_v)

- Then the bulk modulus, K is defined as $K = \sigma / \varepsilon_v$

BULK MODULUS (K):--



$$\boxed{K = \sigma / \varepsilon_v} \quad \text{-------------(6)}$$

Where, $\varepsilon_v = \Delta V/V$

$$= \frac{\text{Change in volume}}{\text{Original volume}}$$

$=$ Volumetric Strain

MODULUS OF RIGIDITY (N): OR

MODULUS OF TRANSVERSE ELASTICITY OR

SHEARING MODULUS

Up to the elastic limit,

shear stress (τ) $\propto$ shearing strain(ϕ)

$$\tau = N\,\phi$$

Expresses relation between shear stress and shear strain.

where

Modulus of Rigidity = N = τ / ϕ -------------(7)

ELASTIC CONSTANTS

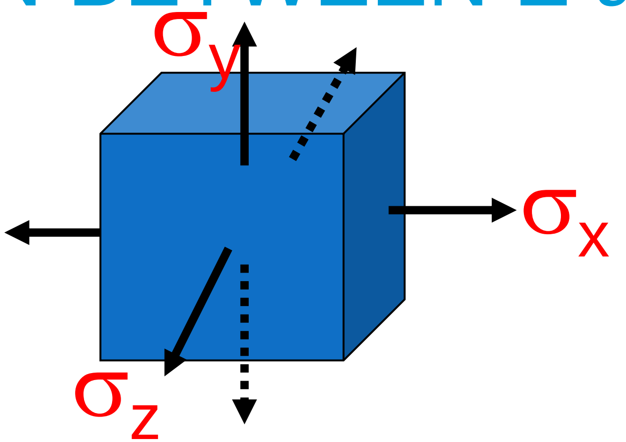
YOUNG'S MODULUS $E = \sigma / \varepsilon$ -------------(5)

BULK MODULUS $K = \sigma / \varepsilon_v$ -------------(6)

MODULUS OF RIGIDITY $N = \tau / \phi$ ------------(7)

RELATION BETWEEN ELASTIC CONSTANTS

(A) RELATION BETWEEN E and K



Let a cube having a side L be subjected to three mutually perpendicular stresses of intensity σ

By definition of bulk modulus

$K = \sigma / \varepsilon_v$

Now $\varepsilon_v = \delta_v / V = \sigma / K$ ----------------------------(i)

The total linear strain for each side

$\varepsilon = \sigma/E - \sigma/(mE) - \sigma/(mE)$

so $\delta L / L = \varepsilon = (\sigma/E) * (1-2/m)$--------------(ii)

now $V = L^3$

$\delta V = 3 L^2 \delta L$

$\delta V/V = 3 L^2 \delta L / L^3 = 3 \delta L/L$

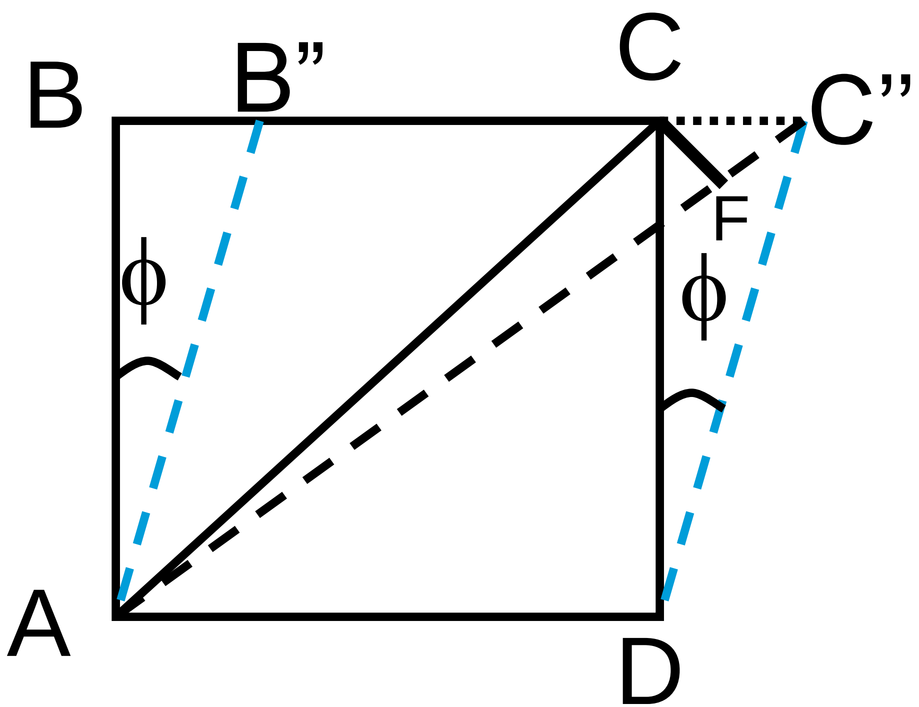
$= 3 (\sigma/E) * (1-2/m)$ ------------------(iii)

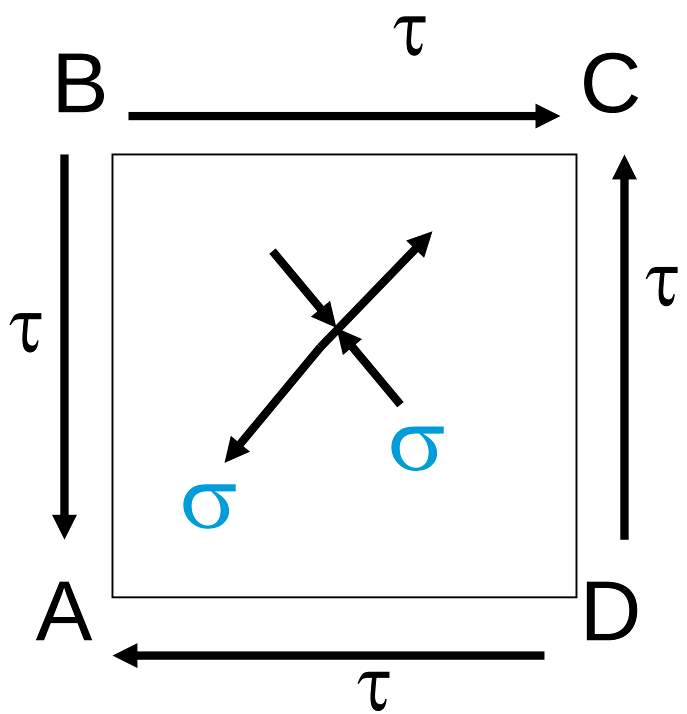
Equating (i) and (iii)

$$\sigma/K = 3(\sigma /E)(1-2 /m)$$

$$\boxed{E = 3 K(1-2 /m)} \quad \text{-----(9)}$$

(B) Relation between E and N





Linear strain of diagonal AC,

$\varepsilon = \phi/2 = \tau/2N$ ---------------------------(i)

State of simple shear produces tensile and compressive stresses along diagonal planes and

$\sigma = \tau$

Strain ε of diagonal AC, due to these two mutually perpendicular direct stresses

$\varepsilon = \sigma/E - (-\sigma/mE) = (\sigma/E)*(1+1/m)$ ---(ii)

But $\sigma = \tau$

so $\varepsilon = (\tau/E)*(1+1/m)$ -------------------(iii)

From equation (i) and (iii)

$\tau / 2N = (\tau / E)(1+1/m)$

OR

$E = 2N(1+1/m)$ --------(10)

But $E = 3K(1-2/m)$ -------(9)

Eliminating E from --(9) & --(10)

$\mu = 1/m = (3K - 2N) / (6K + 2N)$ -----(11)

Eliminating m from –(9) & --(10)

$\boxed{E = 9KN / (N+3K)}$ ---------(12)

(C) Relation between E ,K and N:--

$E = 2N(1+1/m)$ --------(10)

$E = 3K\ (1-2/m)$ ---------(9)

$E = 9KN / (N+3K)$ --------(12)

(D) Relation between μ,K and N:--

$\mu = 1/m = (3K-2N)/(6K+2N)$------(11)

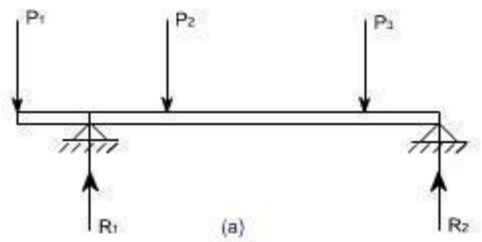
UNIT 2

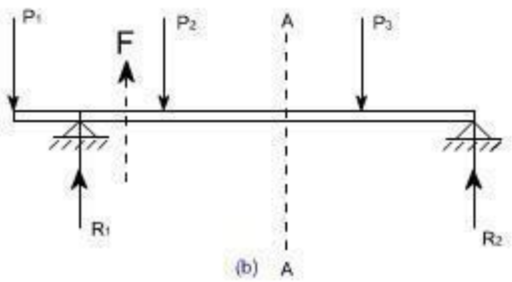
SHEAR FORCE & BENDING MOMENT DIAGRAMS

UNIT-II
SHEAR FORCE AND BENDING MOMENT DIAGRAMS

Concept of Shear Force and Bending moment in beams:

When the beam is loaded in some arbitrarily manner, the internal forces and moments are developed and the terms shear force and bending moments come into pictures which are helpful to analyze the beams further. Let us define these terms





Now let us consider the beam as shown in fig 1(a) which is supporting the loads P_1, P_2, P_3 and is simply supported at two points creating the reactions R_1 and R_2 respectively. Now let us assume that the beam is to divided into or imagined to be cut into two portions at a section AA. Now let us assume that the resultant of loads and reactions to the left of AA is ‘F' vertically upwards, and since the entire beam is to remain in equilibrium, thus the resultant of forces to the right of AA must also be F, acting downwards. This forces ‘F' is as a shear force. The shearing force at any x-section of a beam represents the tendency for the portion of the beam to one side of the section to slide or shear laterally relative to the other portion.

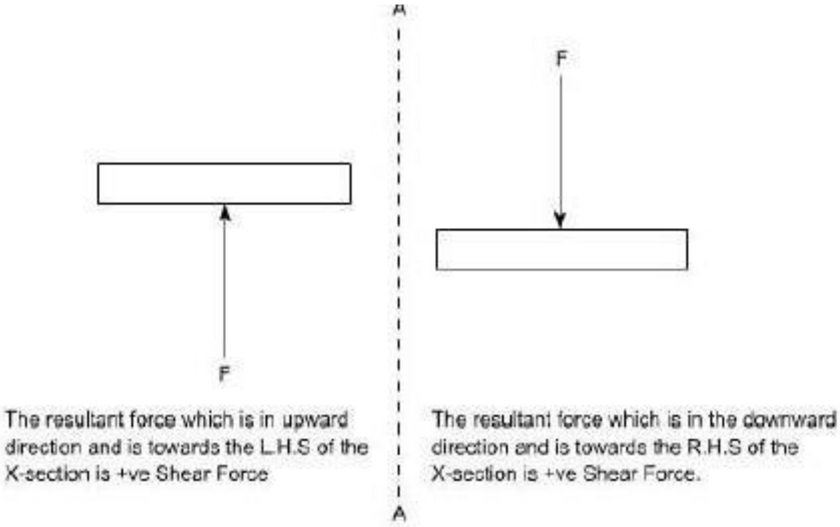
Therefore, now we are in a position to define the shear force ‘F' to as follows:

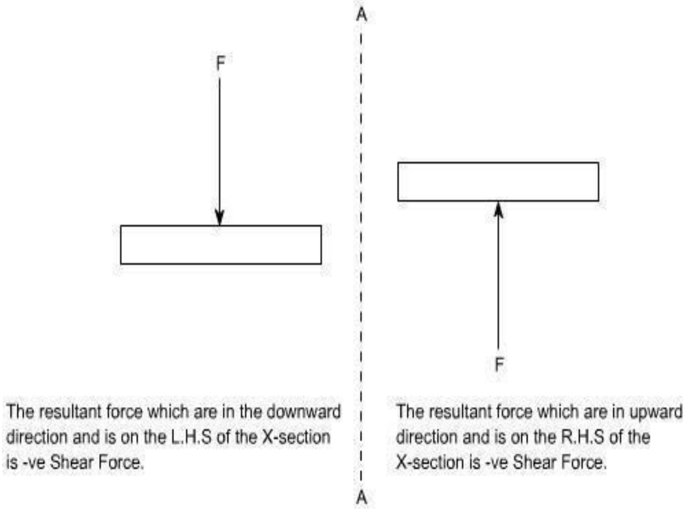
At any x-section of a beam, the shear force ‘F' is the algebraic sum of all the lateral components of the forces acting on either side of the x-section.

Sign Convention for Shear Force:

The usual sign conventions to be followed for the shear forces have been illustrated in figures 2 and 3.

Positive Shear Force





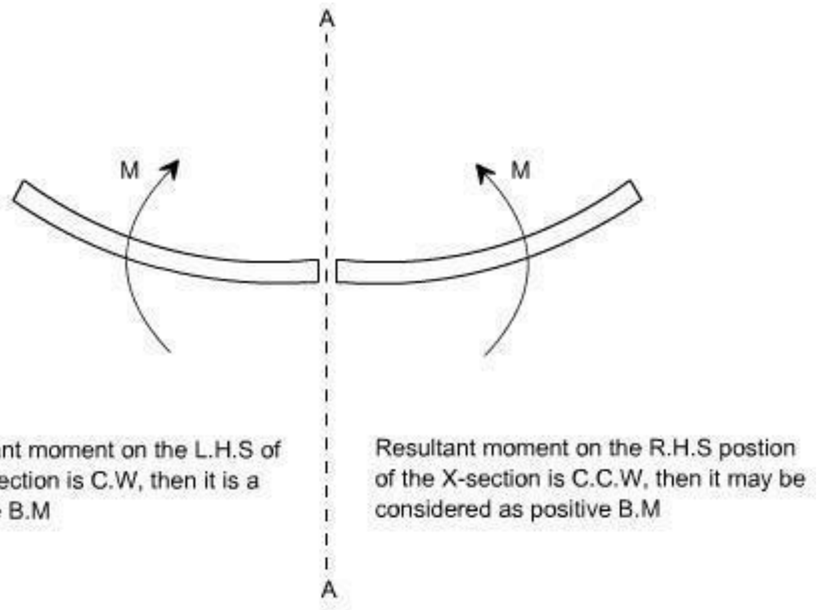
Negative Shear Force

Bending Moment:

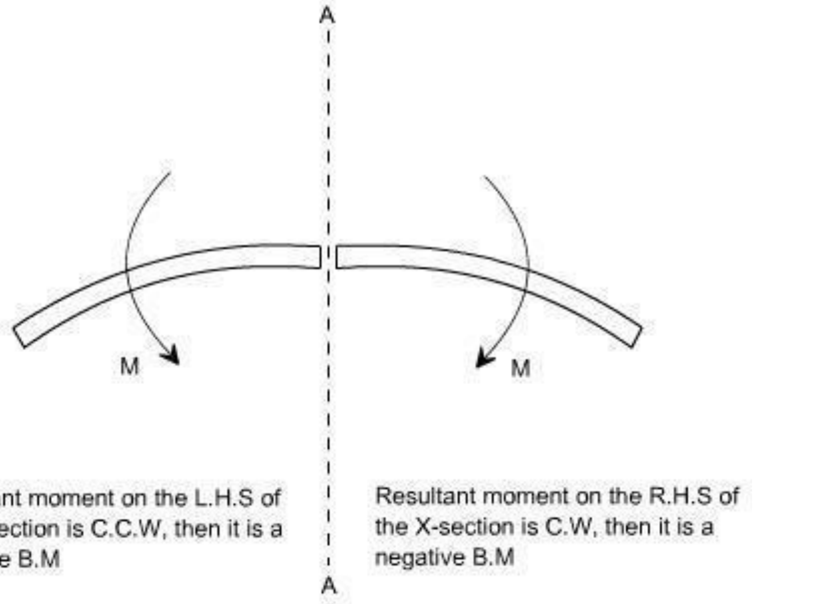
Let us again consider the beam which is simply supported at the two prints, carrying loads P_1, P_2 and P_3 and having the reactions R_1 and R_2 at the supports Fig 4. Now, let us imagine that the beam is cut into two potions at the x-section AA. In a similar manner, as done for the case of shear force, if we say that the resultant moment about the section AA of all the loads and reactions to the left of the x-section at AA is M in C.W direction, then moment of forces to the right of x-section AA must be 'M' in C.C.W. Then 'M' is called as the Bending moment and is abbreviated as B.M. Now one can define the bending moment to be simply as <u>the algebraic sum of the moments about an x-section of all the forces acting on either side of the section</u>

Sign Conventions for the Bending Moment:

For the bending moment, following sign conventions may be adopted as indicated in Fig 5 and Fig 6.



Positive Bending Moment



Negative Bending Moment

Some times, the terms ‘Sagging' and Hogging are generally used for the positive and negative bending moments respectively.

Bending Moment and Shear Force Diagrams:

The diagrams which illustrate the variations in B.M and S.F values along the length of the beam for any fixed loading conditions would be helpful to analyze the beam further.

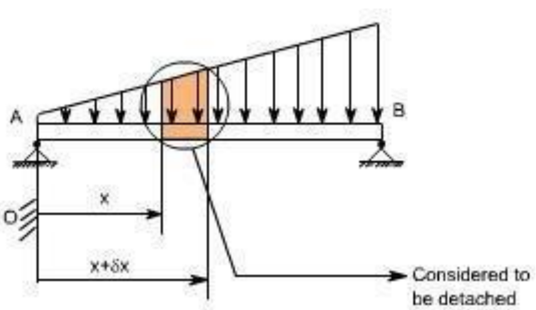
Thus, a shear force diagram is a graphical plot, which depicts how the internal shear force ‘F' varies along the length of beam. If x dentotes the length of the beam, then F is function x i.e. F(x).

Similarly a bending moment diagram is a graphical plot which depicts how the internal bending moment ‘M' varies along the length of the beam. Again M is a function x i.e. M(x).

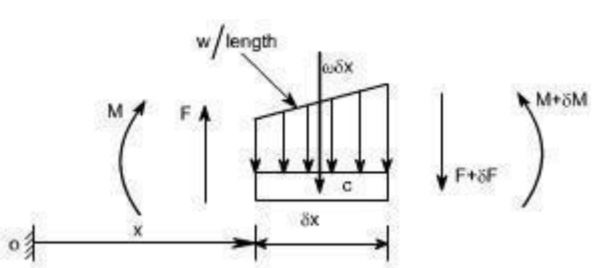
Basic Relationship Between The Rate of Loading, Shear Force and Bending Moment:

The construction of the shear force diagram and bending moment diagrams is greatly simplified if the relationship among load, shear force and bending moment is established.

Let us consider a simply supported beam AB carrying a uniformly distributed load w/length. Let us imagine to cut a short slice of length dx cut out from this loaded beam at distance ‘x' from the origin ‘0'.



Let us detach this portion of the beam and draw its free body diagram.



The forces acting on the free body diagram of the detached portion of this loaded beam are the following

- The shearing force F and F+ □F at the section x and x + □x respectively.

- The bending moment at the sections x and x + □x be M and M + dM respectively.

- Force due to external loading, if ‘w' is the mean rate of loading per unit length then the total loading on this slice of length □x is w. □x, which is approximately acting through the centre ‘c'. If the loading is assumed to be uniformly distributed then it would pass exactly through the centre ‘c'.

This small element must be in equilibrium under the action of these forces and couples.

Now let us take the moments at the point ‘c'. Such that

$$M + F.\frac{\delta x}{2} + (F + \delta F).\frac{\delta x}{2} = M + \delta M$$

$$\Rightarrow F.\frac{\delta x}{2} + (F + \delta F).\frac{\delta x}{2} = \delta M$$

$$\Rightarrow F.\frac{\delta x}{2} + F.\frac{\delta x}{2} + \delta F.\frac{\delta x}{2} = \delta M \text{ [Neglecting the product of } \delta F \text{ and } \delta x \text{ being small quantities]}$$

$$\Rightarrow F.\delta x = \delta M$$

$$\Rightarrow F = \frac{\delta M}{\delta x}$$

Under the limits $\delta x \rightarrow 0$

$$\boxed{F = \frac{dM}{dx}} \quad \ldots\ldots\ldots\ldots \quad (1)$$

Resolving the forces vertically we get

$$w.\delta x + (F + \delta F) = F$$

$$\Rightarrow w = -\frac{\delta F}{\delta x}$$

Under the limits $\delta x \rightarrow 0$

$$\Rightarrow w = -\frac{dF}{dx} \text{ or } -\frac{d}{dx}\left(\frac{dM}{dx}\right)$$

$$\boxed{w = -\frac{dF}{dx} = -\frac{d^2M}{dx^2}} \quad \ldots\ldots\ldots\ldots (2)$$

Conclusions: From the above relations,the following important conclusions may be drawn

- From Equation (1), the area of the shear force diagram between any two points, from the basic calculus is the bending moment diagram

$$M = \int F.dx$$

- The slope of bending moment diagram is the shear force,thus

$$F = \frac{dM}{dx}$$

Thus, if F=0; the slope of the bending moment diagram is zero and the bending moment is therefore constant.'

- The maximum or minimum Bending moment occurs where $\frac{dM}{dx} = 0.$

The slope of the shear force diagram is equal to the magnitude of the intensity of the distributed loading at any position along the beam. The –ve sign is as a consequence of our particular choice of sign conventions

Procedure for drawing shear force and bending moment diagram:

Preamble:

The advantage of plotting a variation of shear force F and bending moment M in a beam as a function of ‘x' measured from one end of the beam is that it becomes easier to determine the maximum absolute value of shear force and bending moment.

Further, the determination of value of M as a function of ‘x' becomes of paramount importance so as to determine the value of deflection of beam subjected to a given loading.

Construction of shear force and bending moment diagrams:

A shear force diagram can be constructed from the loading diagram of the beam. In order to draw this, first the reactions must be determined always. Then the vertical components of forces and reactions are successively summed from the left end of the beam to preserve the mathematical sign conventions adopted. The shear at a section is simply equal to the sum of all the vertical forces to the left of the section.

When the successive summation process is used, the shear force diagram should end up with the previously calculated shear (reaction at right end of the beam. No shear force acts through the beam just beyond the last vertical force or reaction. If the shear force diagram closes in this fashion, then it gives an important check on mathematical calculations.

The bending moment diagram is obtained by proceeding continuously along the length of beam from the left hand end and summing up the areas of shear force diagrams giving due regard to sign. The process of obtaining the moment diagram from the shear force diagram by summation is exactly the same as that for drawing shear force diagram from load diagram.

It may also be observed that a constant shear force produces a uniform change in the bending moment, resulting in straight line in the moment diagram. If no shear force exists along a certain portion of a beam, then it indicates that there is no change in moment takes place. It may also further observe that dm/dx= F therefore, from the fundamental theorem of calculus the maximum or minimum moment occurs where the shear is zero. In order to check the validity of the bending moment diagram, the terminal conditions for the moment must be satisfied. If the end is free or pinned, the computed sum must be equal to zero. If the end is built in, the moment computed by the summation must be equal to the one calculated initially for the reaction. These conditions must always be satisfied.

Illustrative problems:

In the following sections some illustrative problems have been discussed so as to illustrate the procedure for drawing the shear force and bending moment diagrams

1. A cantilever of length carries a concentrated load 'W' at its free end.

Draw shear force and bending moment.

Solution:

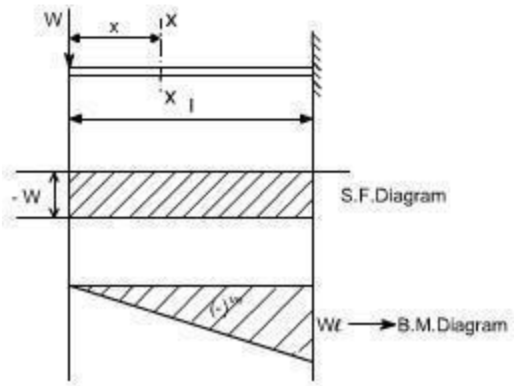
At a section a distance x from free end consider the forces to the left, then F = -W (for all values of x) -ve sign means the shear force to the left of the x-section are in downward direction and therefore negative

Taking moments about the section gives (obviously to the left of the section)

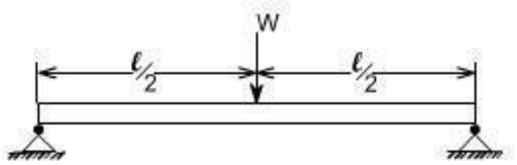
M = -Wx (-ve sign means that the moment on the left hand side of the portion is in the anticlockwise direction and is therefore taken as –ve according to the sign convention)

so that the maximum bending moment occurs at the fixed end i.e. M = -W l

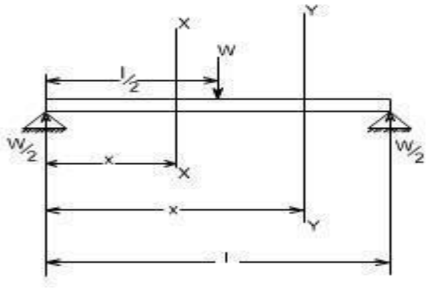
From equilibrium consideration, the fixing moment applied at the fixed end is Wl and the reaction is W. the shear force and bending moment are shown as,



2. Simply supported beam subjected to a central load (i.e. load acting at the mid-way)



By symmetry the reactions at the two supports would be W/2 and W/2. now consider any section X-X from the left end then, the beam is under the action of following forces.



So the shear force at any X-section would be = W/2 [Which is constant upto x < l/2]

If we consider another section Y-Y which is beyond l/2 then

$S.F_{Y\text{-}Y} = \frac{W}{2} - W = \frac{-W}{2}$ for all values greater = l/2

Hence S.F diagram can be plotted as,
.For B.M diagram:

If we just take the moments to the left of the cross-section,

$$B.M_{x\text{-}x} = \frac{W}{2}\, x \text{ for } x \text{ lies between } 0 \text{ and } l/2$$

$$B.M_{\text{at } x = \frac{l}{2}} = \frac{W}{2}\,\frac{l}{2} \text{ i.e } B.M \text{ at } x = 0$$

$$= \frac{Wl}{4}$$

$$B.M_{Y\text{-}Y} = \frac{W}{2}\, x - W\left(x - \frac{l}{2}\right)$$

Again

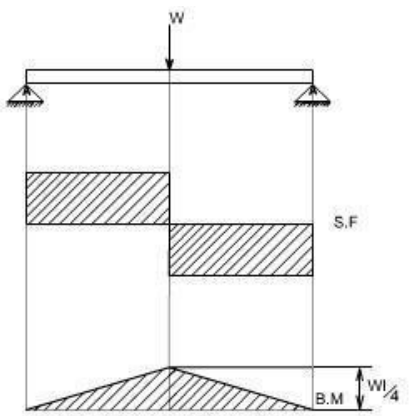
$$= \frac{W}{2}\, x - Wx + \frac{Wl}{2}$$

$$= -\frac{W}{2}\, x + \frac{Wl}{2}$$

$$B.M_{\text{at } x = l} = -\frac{Wl}{2} + \frac{Wl}{2}$$

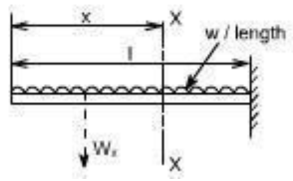
$$= 0$$

Which when plotted will give a straight relation i.e.



It may be observed that at the point of application of load there is an abrupt change in the shear force, at this point the B.M is maximum.

3. A cantilever beam subjected to U.d.L, draw S.F and B.M diagram.



Here the cantilever beam is subjected to a uniformly distributed load whose intensity is given w / length.

Consider any cross-section XX which is at a distance of x from the free end. If we just take the resultant of all the forces on the left of the X-section, then

$S.F_{XX} = -Wx$ for all values of ‘x'. **---------- (1)**

$S.F_{XX} = 0$

$S.F_{XX \text{ at } x=1} = -Wl$

So if we just plot the equation No. (1), then it will give a straight line relation. Bending Moment at X-X is obtained by treating the load to the left of X-X as a concentrated load of the same value acting through the centre of gravity.

Therefore, the bending moment at any cross-section X-X is

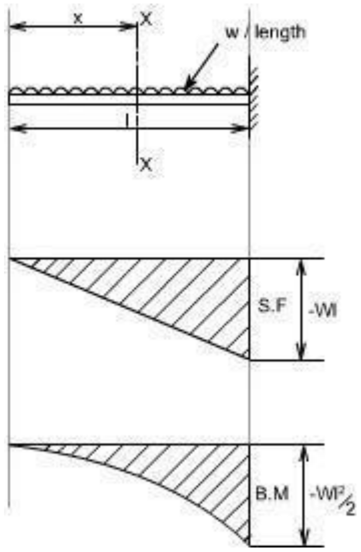
$$B.M_{X\text{-}X} = -Wx\,\frac{x}{2}$$
$$= -W\frac{x^2}{2}$$

The above equation is a quadratic in x, when B.M is plotted against x this will produces a parabolic variation.

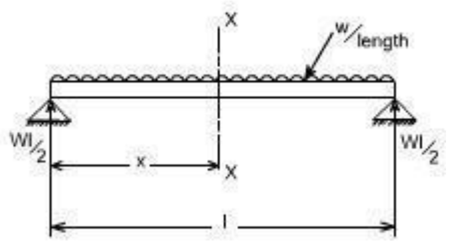
The extreme values of this would be at x = 0 and x = l

$$B.M_{\text{at } x=l} = -\frac{Wl^2}{2}$$
$$= \frac{Wl}{2} - Wx$$

Hence S.F and B.M diagram can be plotted as follows:



4. **Simply supported beam subjected to a uniformly distributed load [U.D.L].**



The total load carried by the span would be

= intensity of loading x length

= w x l

By symmetry the reactions at the end supports are each wl/2

If x is the distance of the section considered from the left hand end of the beam.

S.F at any X-section X-X is

$$= \frac{Wl}{2} - Wx$$

$$= W\left(\frac{l}{2} - x\right)$$

Giving a straight relation, having a slope equal to the rate of loading or intensity of the loading.

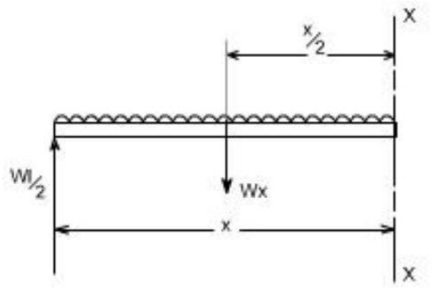
$$S.F_{at\ x=0} = \frac{wl}{2} - wx$$

so at

$$S.F_{at\ x=\frac{l}{2}} = 0 \text{ hence the S.F is zero at the centre}$$

$$S.F_{at\ x=l} = -\frac{Wl}{2}$$

The bending moment at the section x is found by treating the distributed load as acting at its centre of gravity, which at a distance of x/2 from the section



$$B.M_{X\text{-}X} = \frac{Wl}{2}x - Wx.\frac{x}{2}$$

so the

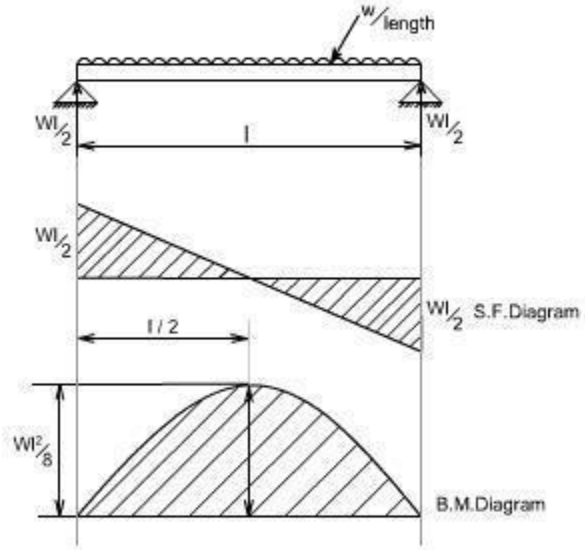
$$= W.\frac{x}{2}(l - 2) \quad ....(2)$$

$$B.M_{at\ x=0} = 0$$

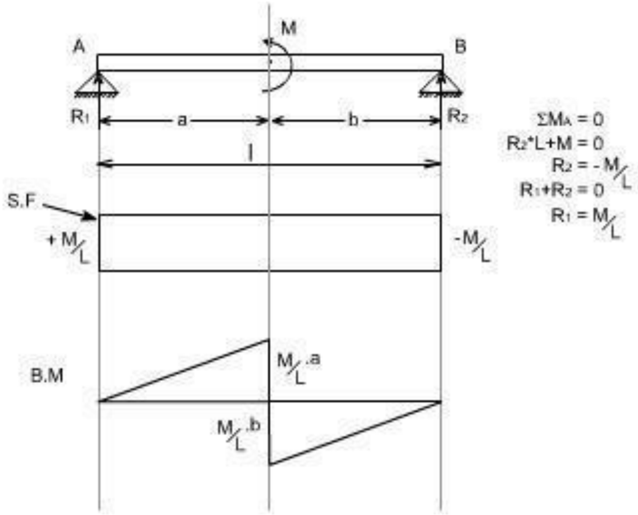
$$B.M_{at\ x=l} = 0$$

$$B.M\Big|_{at\ x=l} = -\frac{Wl^2}{8}$$

So the equation (2) when plotted against x gives rise to a parabolic curve and the shear force and bending moment can be drawn in the following way will appear as follows:

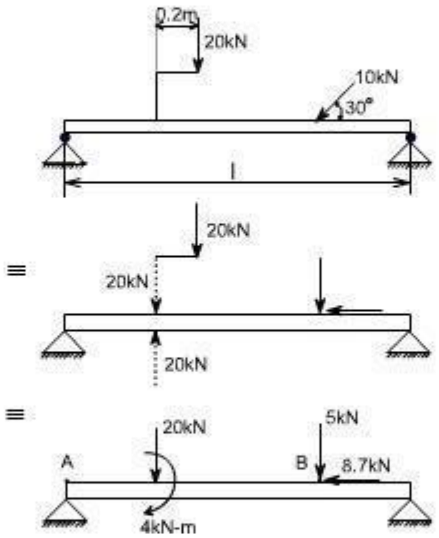


When the beam is subjected to couple, the shear force and Bending moment diagrams may be drawn exactly in the same fashion as discussed earlier.



5. Eccentric loads.

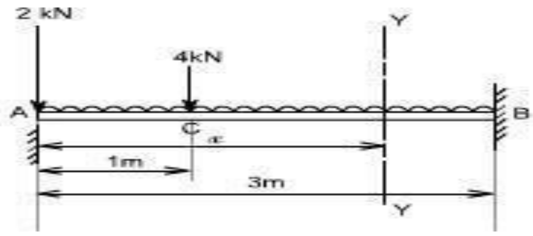
When the beam is subjected to an eccentric loads, the eccentric load are to be changed into a couple/ force as the case may be, In the illustrative example given below, the 20 kN load acting at a distance of 0.2m may be converted to an equivalent of 20 kN force and a couple of 2 kN.m. similarly a 10 kN force which is acting at an angle of 30^0 may be resolved into horizontal and vertical components.The rest of the procedure for drawing the shear force and Bending moment remains the same.



6. Loading changes or there is an abrupt change of loading:

When there is an aabrupt change of loading or loads changes, the problem may be tackled in a systematic way.consider a cantilever beam of 3 meters length. It carries a uniformly distributed load of 2 kN/m and a concentrated loads of 2kN at the free end and 4kN at 2 meters from fixed end.The shearing force and bending moment diagrams are required to be drawn and state the maximum values of the shearing force and bending moment.

Solution



Consider any cross section x-x, at a distance x from the free end

Shear Force at x-x = -2 -2x $\quad 0 < x < 1$

S.F at x = 0 i.e. at A = -2 kN

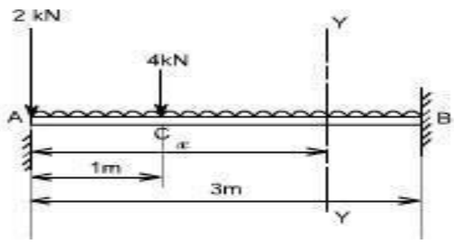
S.F at x = 1 = -2-2 = - 4kN

S.F at C (x = 1) = -2 -2x - 4 $\quad$ Concentrated load

= - 2 - 4 -2x1 kN

= - 8 kN

Again consider any cross-section YY, located at a distance x from the free end



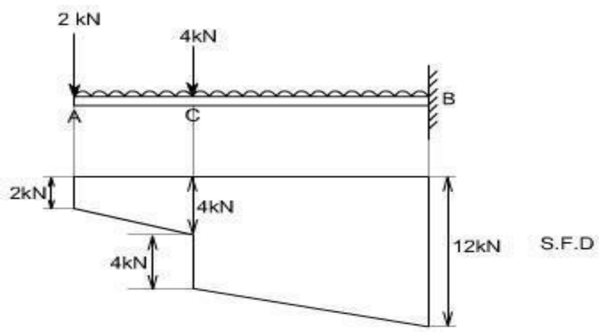
S.F at Y-Y = -2 - 2x - 4 $\quad 1 < x < 3$

This equation again gives S.F at point C equal to -8kN

S.F at x = 3 m = -2 -4 -2x3

= -12 kN

Hence the shear force diagram can be drawn as below:



For bending moment diagrams – Again write down the equations for the respective cross sections, as consider above

Bending Moment at xx = -2x - 2x.x/2 valid upto AC

B.M at x = 0 = 0

B.M at x =1m = -3 kN.m

For the portion CB, the bending moment equation can be written for the x-

section at Y-Y . B.M at YY = -2x - 2x.x/2 - 4(x -1)

This equation again gives,

B.M at point C = - 2.1 - 1 - 0 i.e. at x = 1

= -3 kN.m

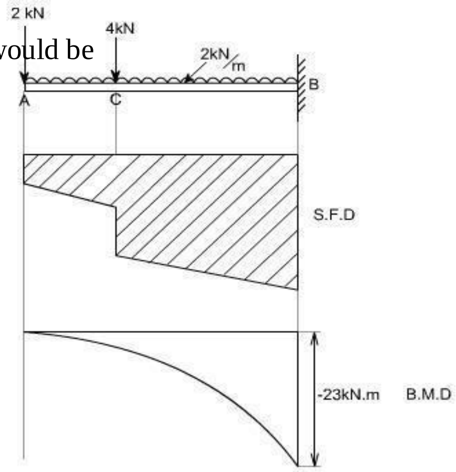
at point B i.e. at x = 3 m

= - 6 - 9 - 8

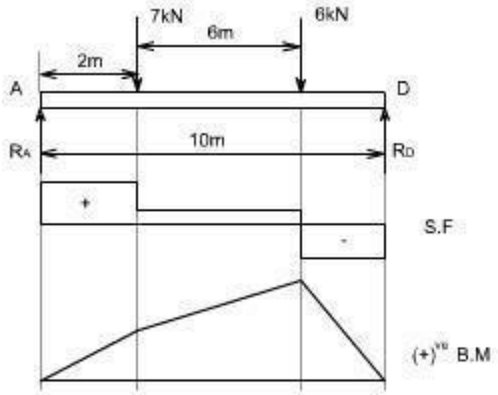
= - 23 kN-m

The variation of the bending moment diagrams would obviously be a

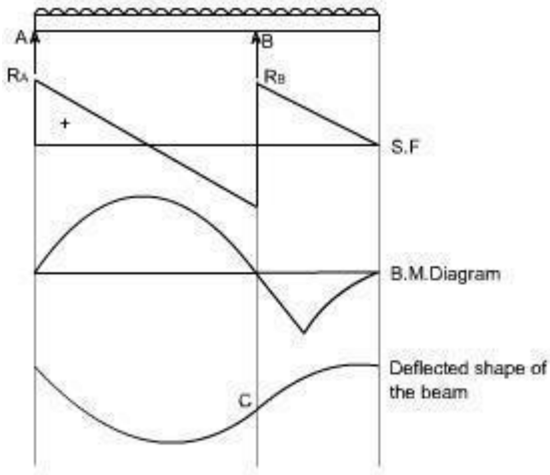
parabolic curve Hence the bending moment diagram would be



Point of Contraflexure:

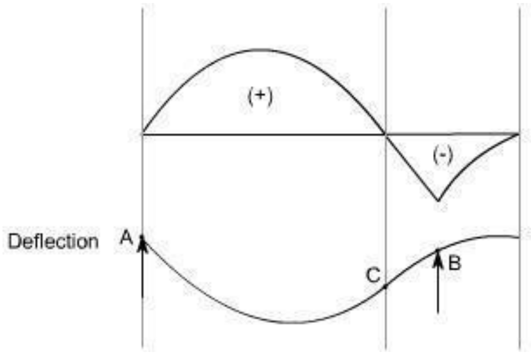


Consider the loaded beam a shown below along with the shear force and Bending moment diagrams for It may be observed that this case, the bending moment diagram is completely positive so that the curvature of the beam varies along its length, but it is always concave upwards or sagging.However if we consider a again a loaded beam as shown below along with the S.F and B.M diagrams, then



It may be noticed that for the beam loaded as in this case,

The bending moment diagram is partly positive and partly negative.If we plot the deflected shape of the beam just below the bending moment

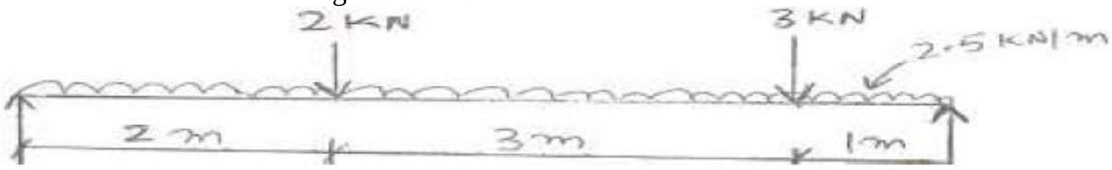


This diagram shows that L.H.S of the beam ‘sags' while the R.H.S of the beam ‘hogs'

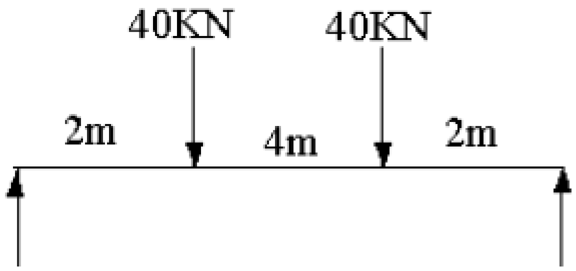
The point C on the beam where the curvature changes from sagging to hogging is a point of contraflexure.

Tutorial Questions

1. A cantilever of length 2.0 m carries a uniformly distributed load of 1 kN/m run over a length of 1.5 m from the free end. Draw the shear force and bending moment diagrams for the cantilever.
2. An overhanging beam ABC of length 7 m is simply supported at A and B over a span of 5 m and portion BC overhangs by 2 m. Draw the shearing force and bending moment diagrams abd determine the point of contra-flexure if it is subjected to uniformly distributed loads of 3 KN/m over the portion AB and a concentrated load of 8 kN at C.
3. A beam of span 10m is simply supported at two points 6m apart with equal over-hang on either side. Both the overhanging portions are loaded with a uniformly distributed load of 2 kN/m run and the beam also carries a concentrated load of 10 N at the midspan. Construct the SF and BM diagrams and locate the points of inflexion, if any.
4. Sketch the shear force and bending moment diagrams showing the salient values for the loaded beam shown in the figure below.



5. A Simply supported beam of span,9 m hL of 15 KN/m over 4 m from the left support and a concentrated load of 20KN at the center. Draw SF and BM diagrams
6. A Beam of length 12m is supported at left end and the other support is at a distance of 8m from the left support leaving a overhanging length of 4m on the right side.It carries a UDL of 10 KN/m over the entire length and a concentrated load of 8 KN at the right extreme end. Draw the shear force and bending moment diagrams and find the position of Contra flexure point
7. Draw the B. M. D and S. F.D



Assignment Questions

1. A cantilever beam of 2 m long carries a uniformly distributed load of 1.5kN/m over a length of 1.6 m from the free end. Draw shear force and bending moment diagrams for the beam
2. A simply supported beam 6 m long is carrying a uniformly distributed load of 5kN/m over a length of 3 m from the right end. Draw shear force and bending moment diagrams for the beam and also calculate the maximum bending moment on the beam
3. A simply supported beam of 16m long carries the point loads of 4KN, 5KN and 3KNat distances 3m, 7m and 10m respectively from the left support. Calculate the maximum shear force

   and bending moment. Draw the SFD and BMD.
4. A horizontal beam of 10m long is carrying a uniformly distributed load of 1kN/m. The beam is supported on two supports 6m apart. Find the position of supports, so that bending moment on the beam is small as possible. Also draw the SFD & BMD for the beam

5. A beam of length l carries a uniformly distributed load of w per unit length. The beam is supported on two supports at equal distances from the two ends. Determine the position of the supports, if the B.M, to which the beam is subjected to , is as small as possible. Draw the SFD & BMD for the beam.

6. A simply supported beam of length 10m, carries the uniformly distributed load and two point loads as shown in Fig.(2) Draw the S.F and B.M diagram for the beam and also calculate the Maximum bending moment

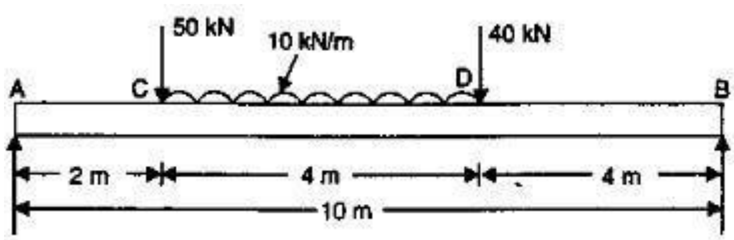


Fig.(2)

UNIT-II

POWER POINT PRESENTAION

SHEAR FORSE & BENDING MOMENT DIAGRAM

UNIT -IO

MRCET CAMPUS

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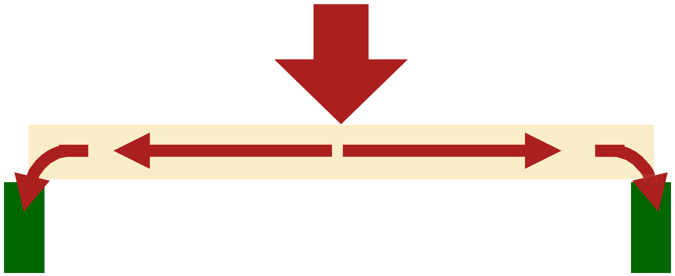
DEPARTMENT OF MECHANICAL ENGINEERING

BEAM

- A beam is a structural member (horizontal) that is design to support the applied load (vertical). It resists the applied loading by a combination of internal transverse **shear force** and bending **moment**.
- It is perhaps the most important and widely used structural members and can be classified according to its support conditions.

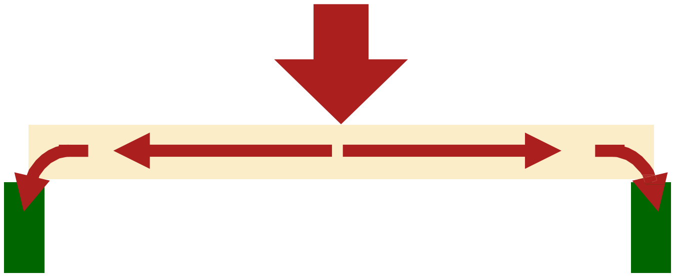
Beams



devices for transferring vertical loads horizontally

action of beams involves combination of **bending and shear**

Beams



devices for transferring vertical loads horizontally

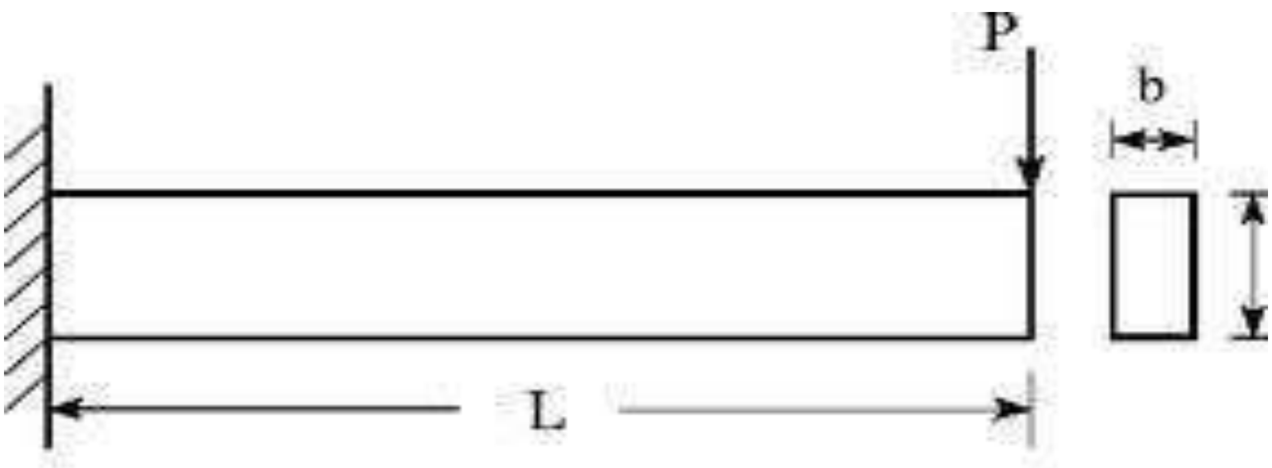
action of beams involves combination of bending and shear

TYPES OF BEAMS

- The following are the important types of beams:
- 1. Cantilever
- 2. simply supported
- 3. overhanging
- 4. Fixed beams
- 5. Continuous beam

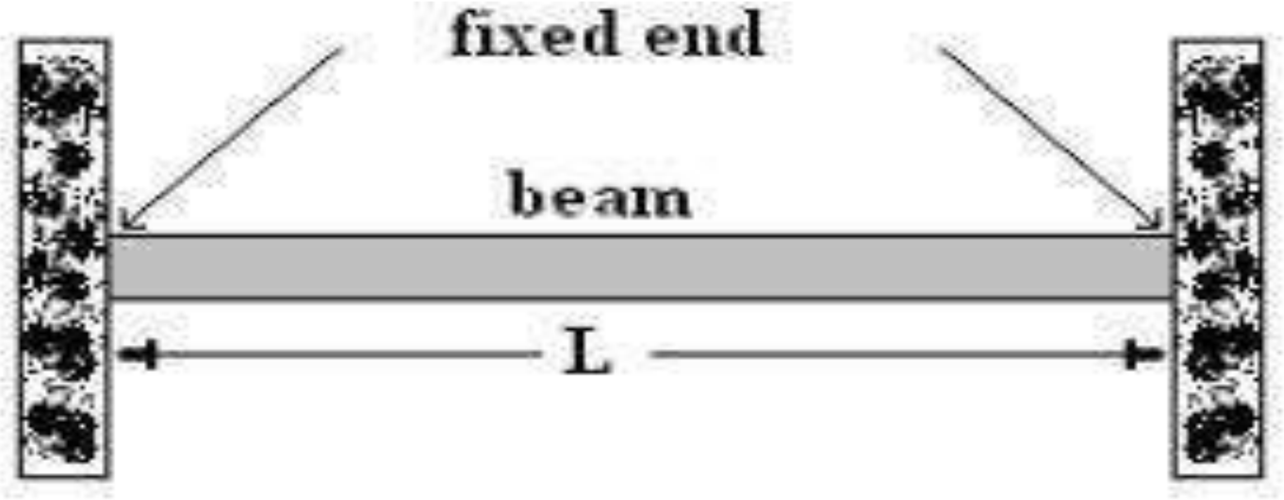
CANTILEVER BEAM

- A beam which is fixed at one end and free at the other end is known as cantilever beam.



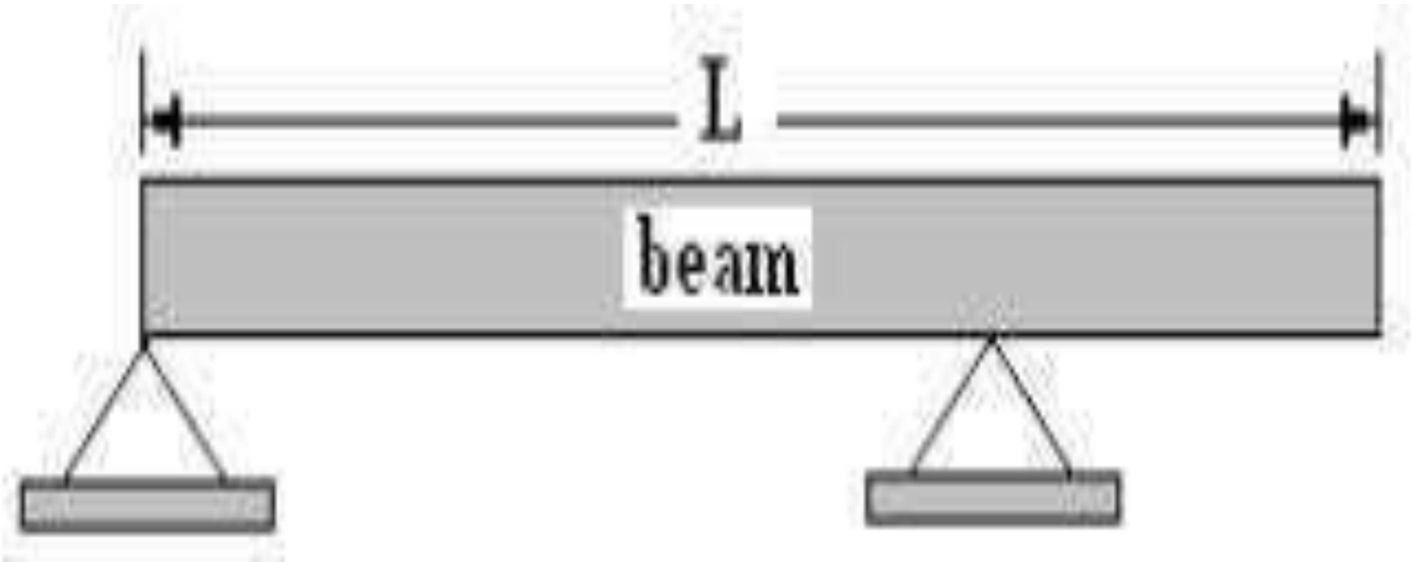
FIXED BEAMS

- A beam whose both ends are fixed and is restrained against rotation and vertical movement. Also known as built-in beam or encastred beam.



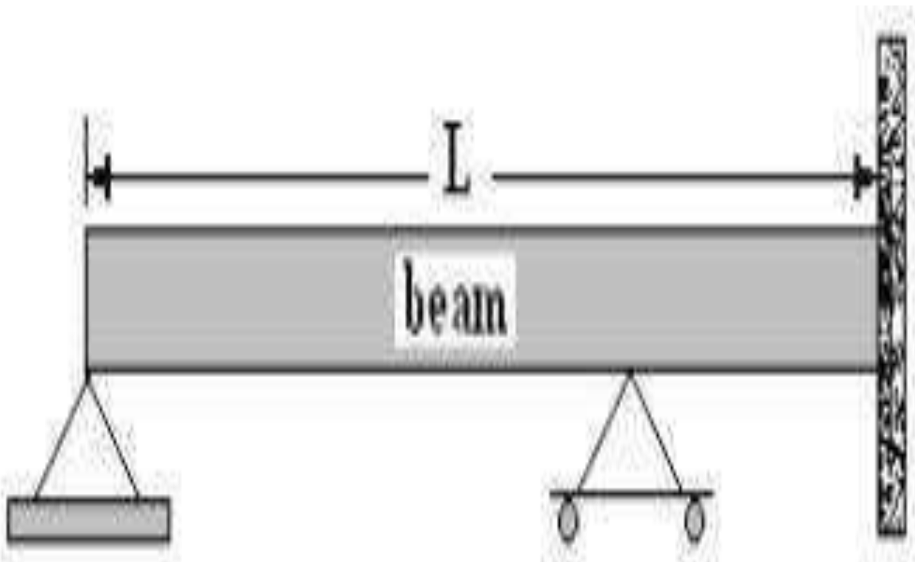
OVERHANGING BEAM

- If the end portion of a beam is extended outside the supports.



CONTINUOUS BEAMS

- A beam which is provided with more than two supports.



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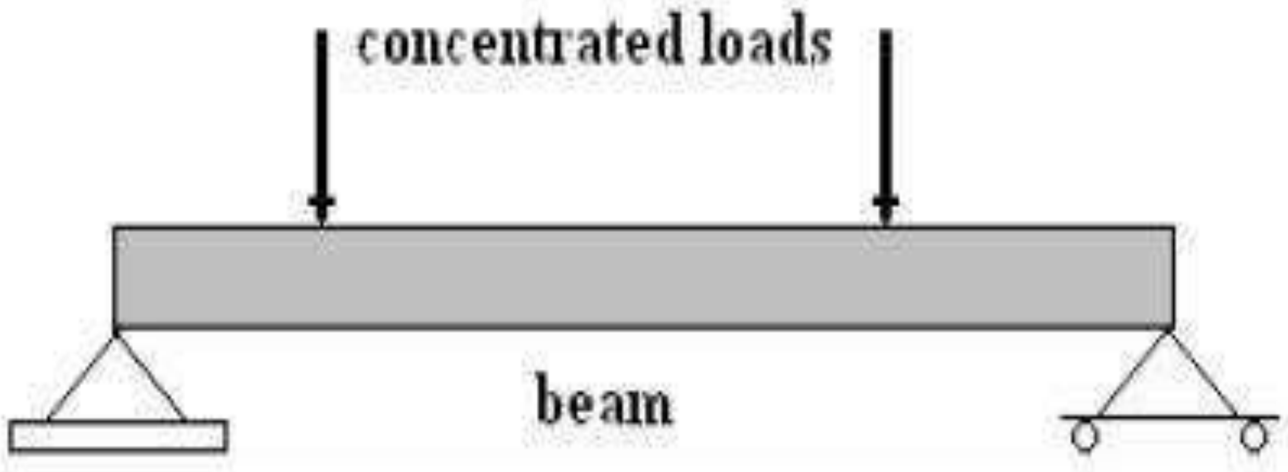
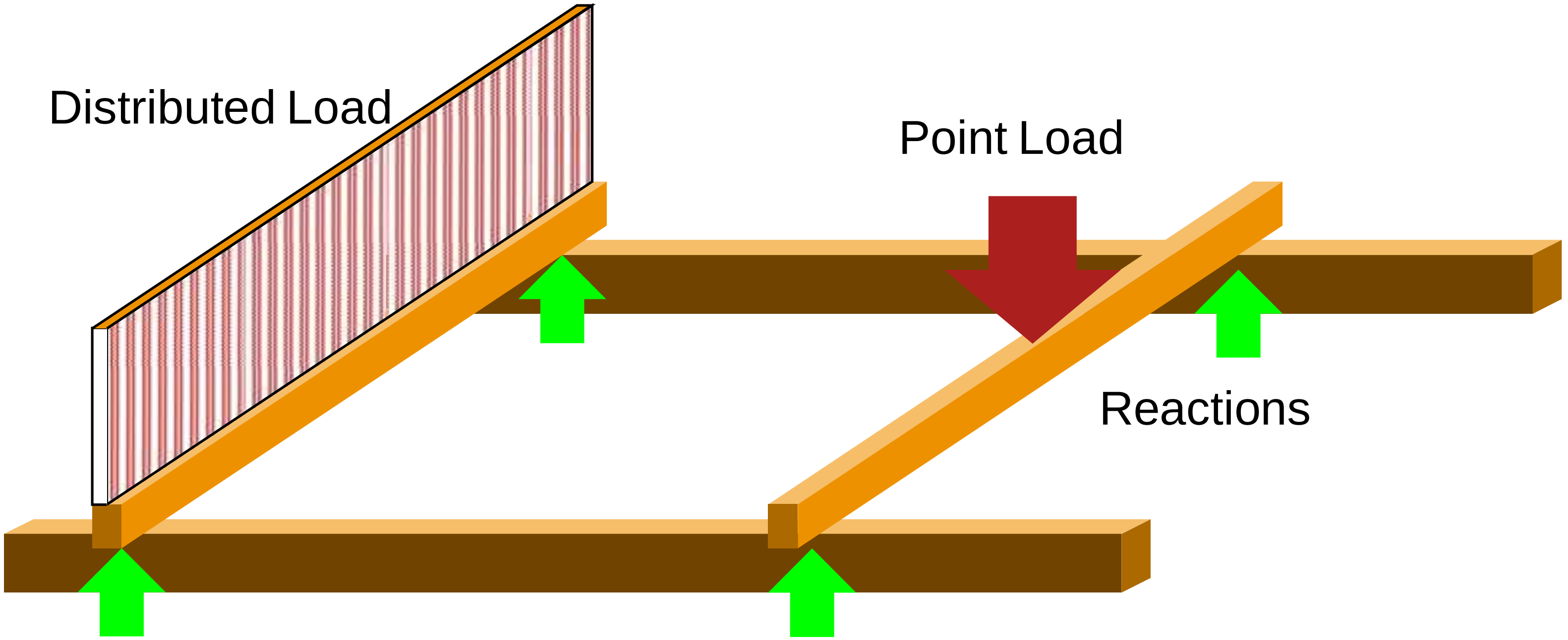
TYPES OF LOADS

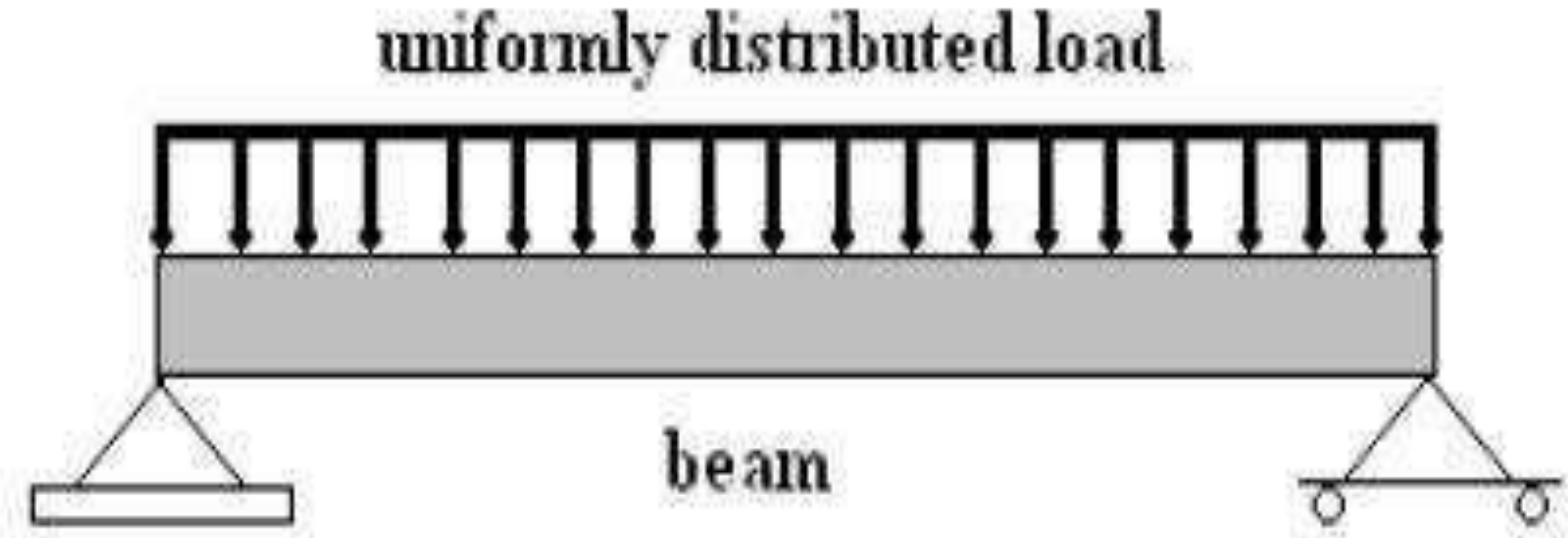
- Concentrated load assumed to act at a point and immediately introduce an oversimplification since all practical loading system must be applied over a finite area.

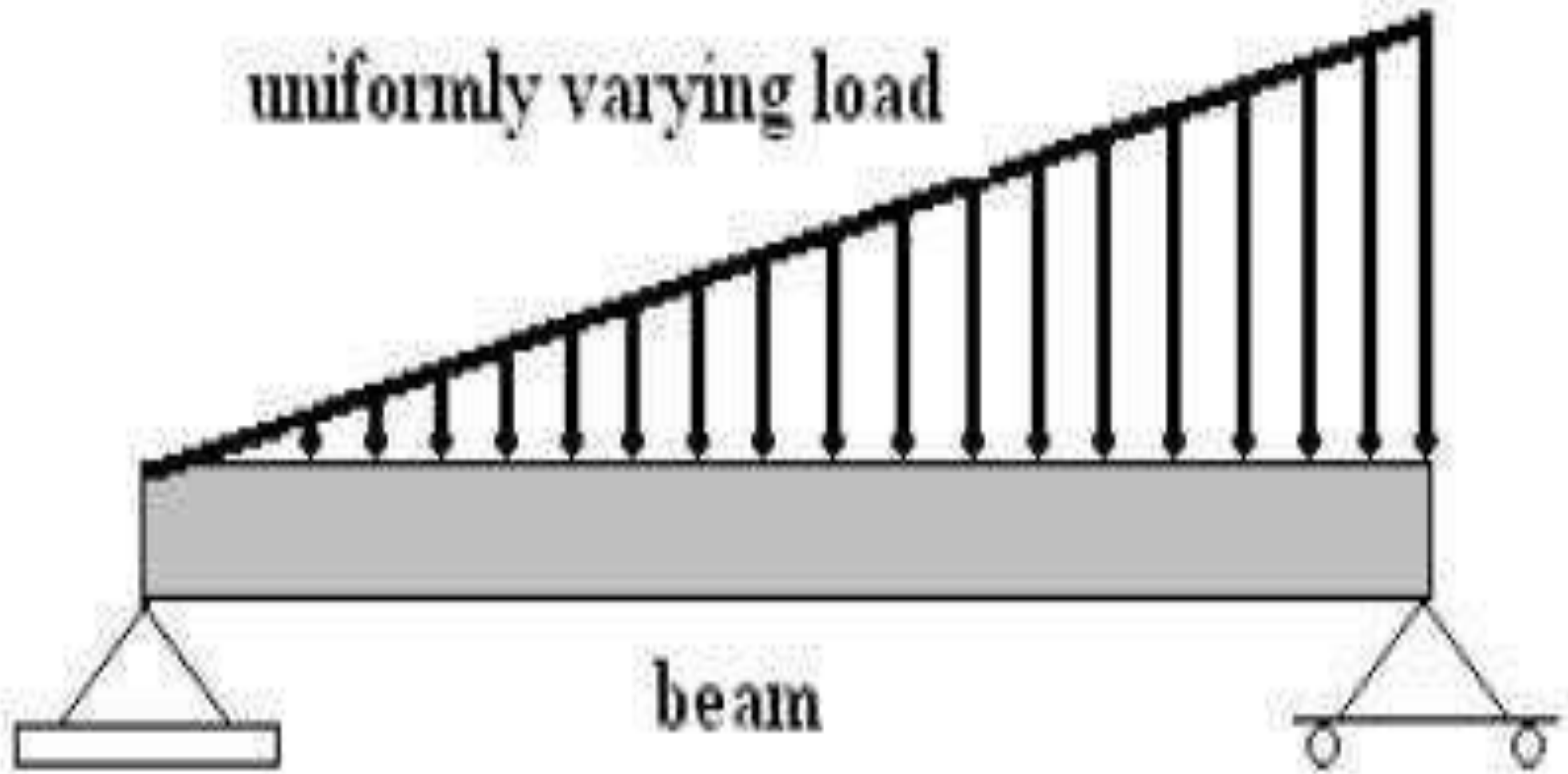


Loads on Beams

- POINT LOADS, FROM CONCENTRATED LOADS OR OTHER BEAMS
- Distributed loads, from anything continuous

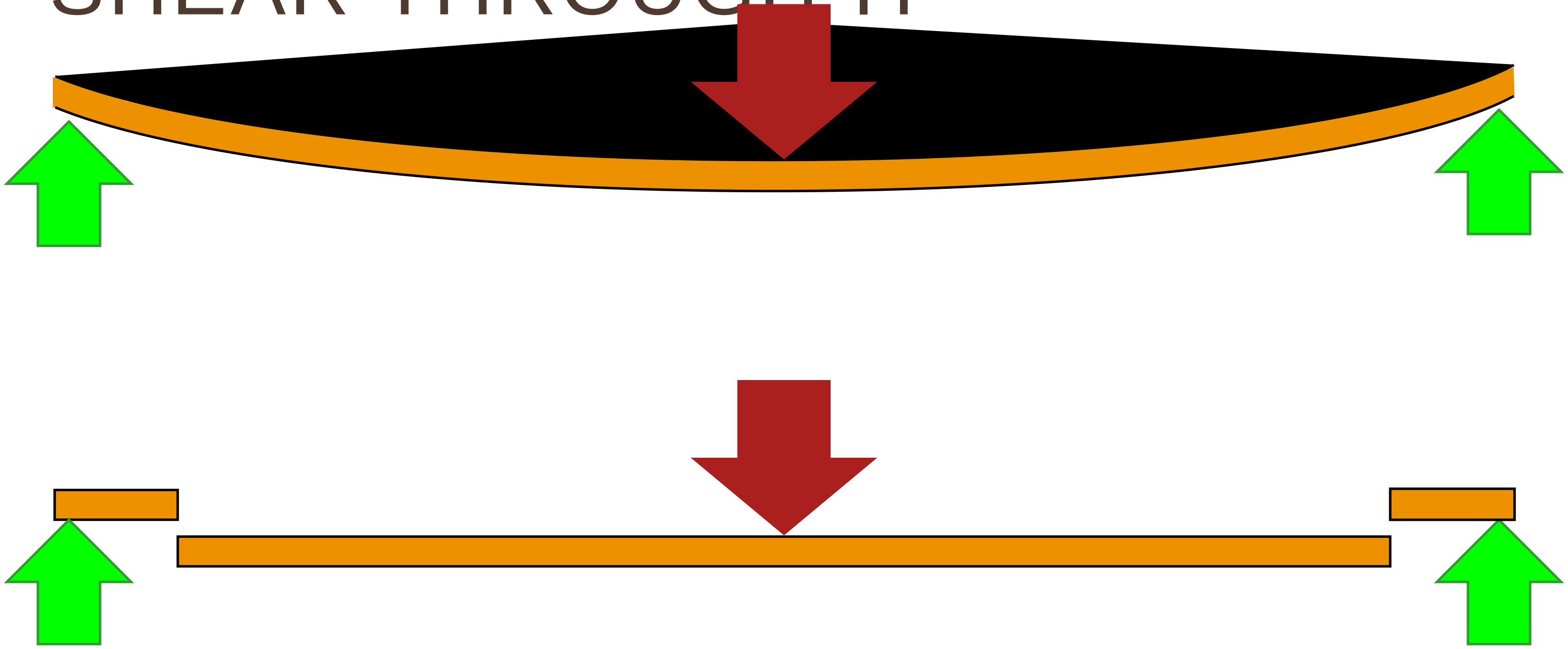



uniformly distributed load
beam


uniformly varying load
beam

What the Loads Do

- THE LOADS (& REACTIONS) BEND THE BEAM, AND TRY TO SHEAR THROUGH IT



for isotropic materials $\mu = 1/4$ for steel $\mu = 0.3$

Volume of bar before deformation $V = L * B * D$

new length after deformation $L_1 = L + \delta L = L + \varepsilon L = L(1+\varepsilon)$

new breadth $B_1 = B - \delta B = B - \varepsilon \mu B = B(1 - \mu \varepsilon)$

new depth $D_1 = D - \delta D = D - \varepsilon \mu D = D(1 - \mu \varepsilon)$

new cross-sectional area $= A_1 = B(1-\mu \varepsilon)^* D(1-\mu \varepsilon) = A(1-\mu \varepsilon)^2$

new volume $V1 = V - \delta V = L(1+\varepsilon) * A(1-\mu \varepsilon)^2$

$$\approx AL(1+\varepsilon - 2\mu\varepsilon)$$

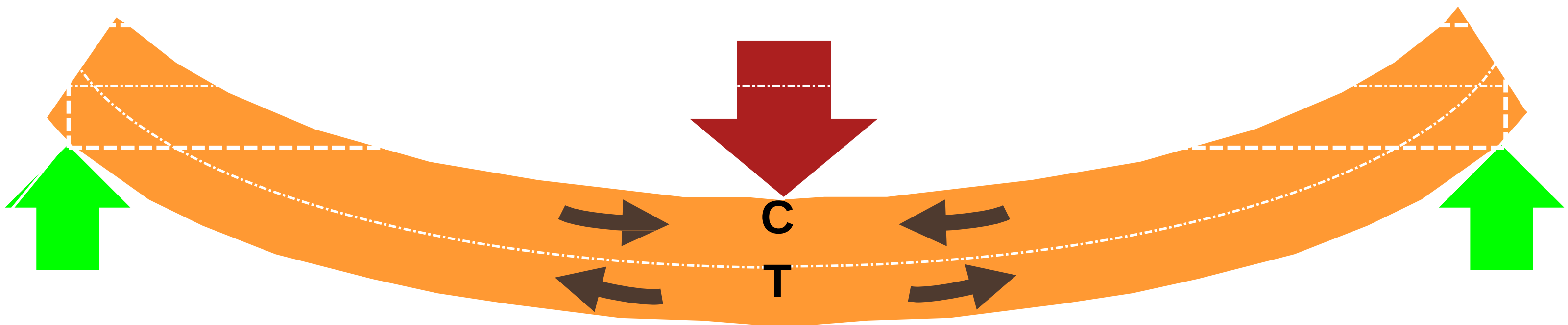
Since ε is small

change in volume $= \delta V = V1 - V = AL\varepsilon(1-2\mu)$

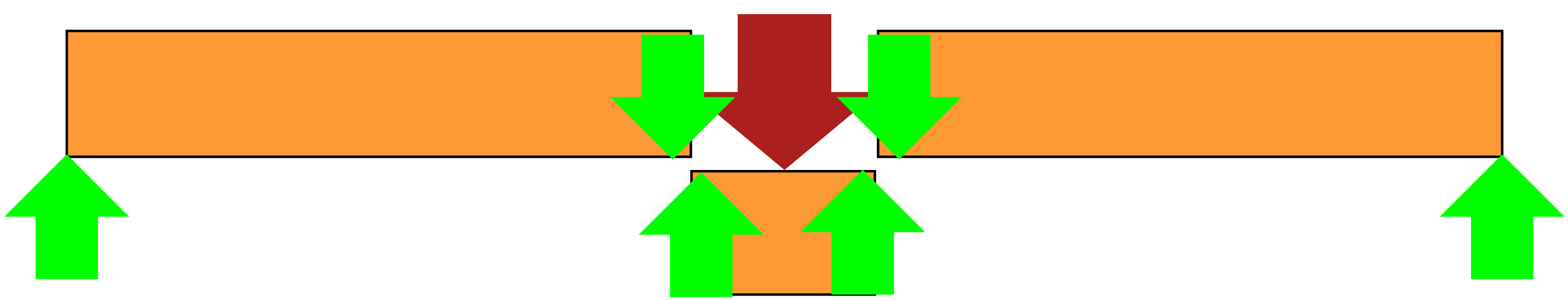
and unit volume change $= \delta V/V = \{AL\varepsilon(1-2\mu)\}/AL$

$$\delta V/V = \varepsilon(1-2\mu)$$

What the Loads Do



Bending



Shear

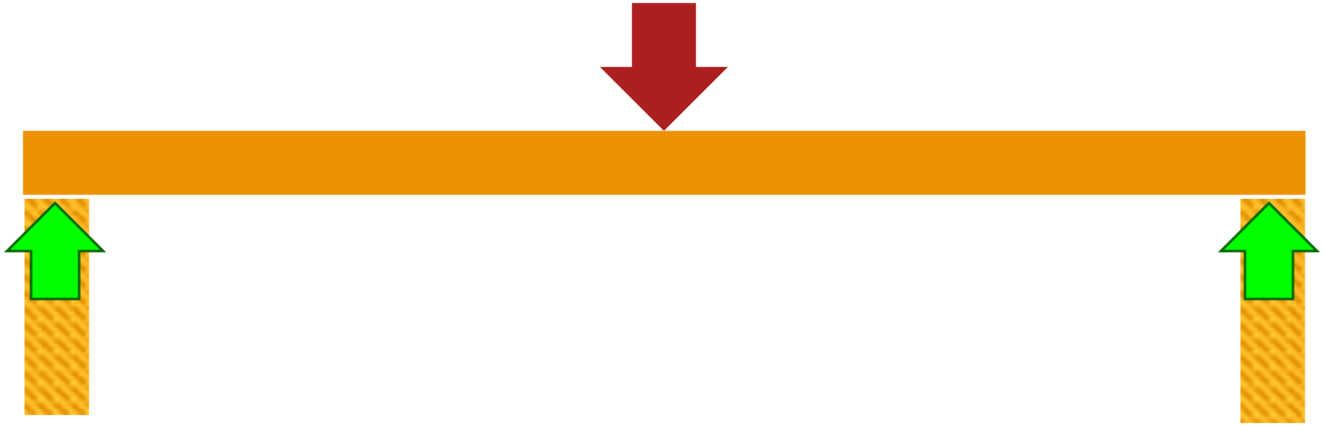
Designing Beams

- IN ARCHITECTURAL STRUCTURES, BENDING MOMENT MORE IMPORTANT
  - importance increases as span increases
- short span structures with heavy loads, shear dominant
  - e.g. pin connecting engine parts

beams in building designed for bending checked for shear

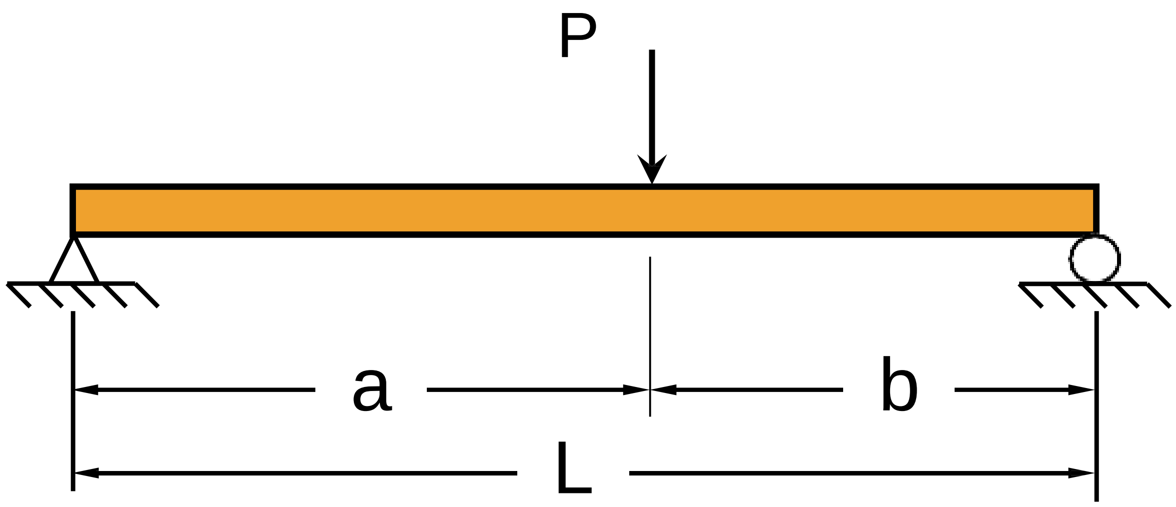
How we calculate the Effects

- FIRST, FIND ALL THE FORCES (LOADS AND REACTIONS)
- Make the beam into a free body (cut it out and artificially support it)
- Find the reactions, using the conditions of equilibrium



INTERNAL REACTIONS IN BEAMS

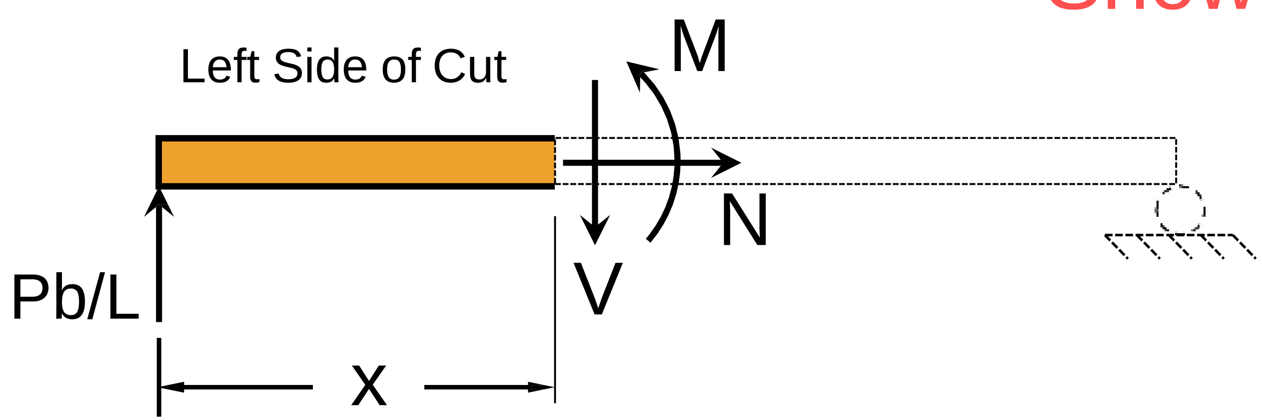
- At any cut in a beam, there are 3 possible internal reactions required for equilibrium:
- normal force,
- shear force,
- bending moment.



INTERNAL REACTIONS IN BEAMS

- At any cut in a beam, there are 3 possible internal reactions required for equilibrium:
- normal force,
- shear force,
  - bending moment.

Positive Directions Shown!!!



INTERNAL REACTIONS IN BEAMS

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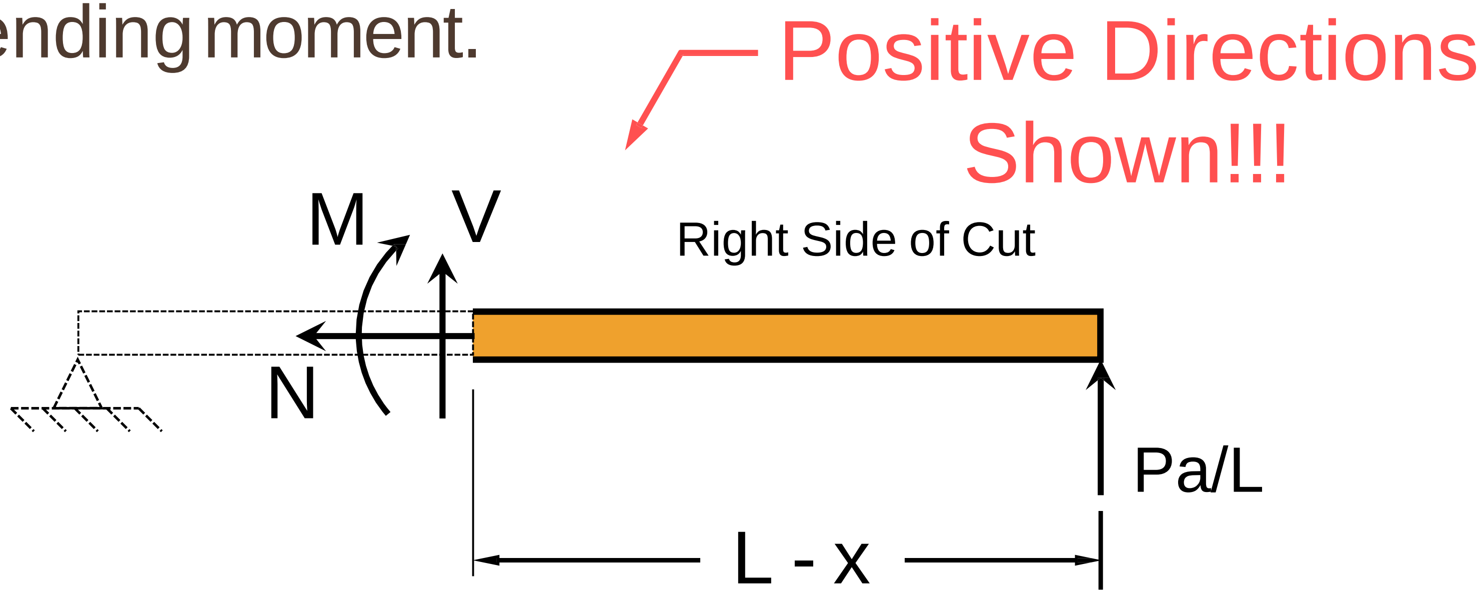
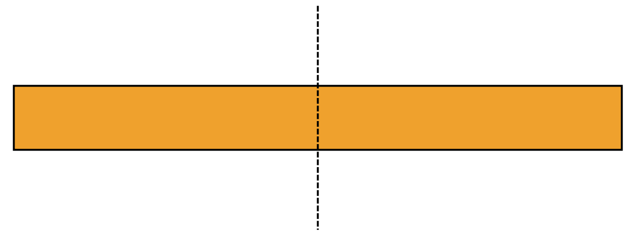


SHEAR FORCES, BENDING MOMENTS - SIGN CONVENTIONS



left section

right section

Shear forces:

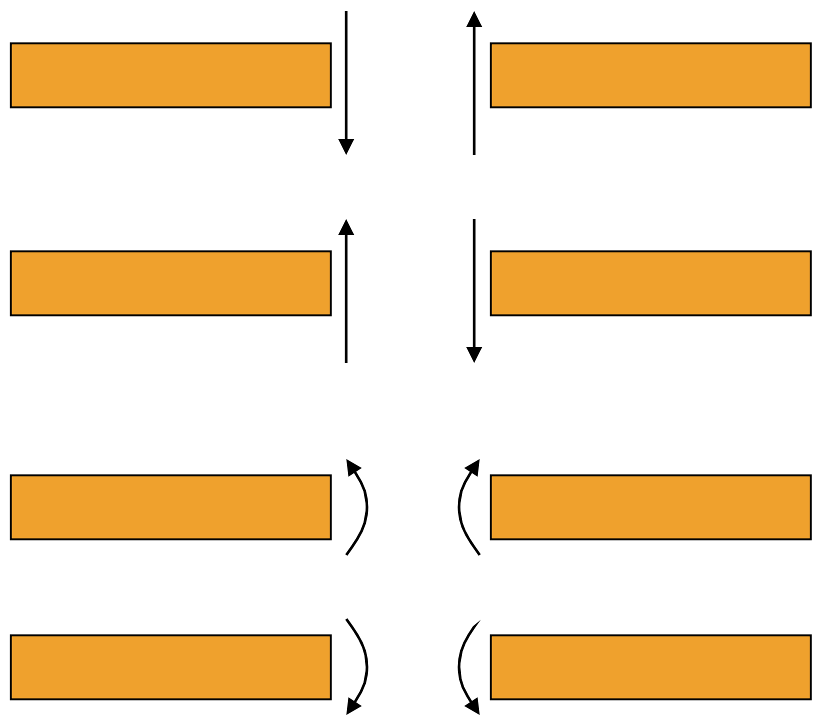
positive shear:

negative shear:

Bending moments:

Negative moment

positive moment

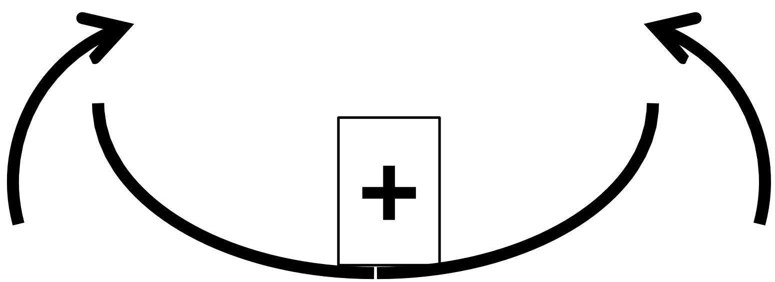


C.W

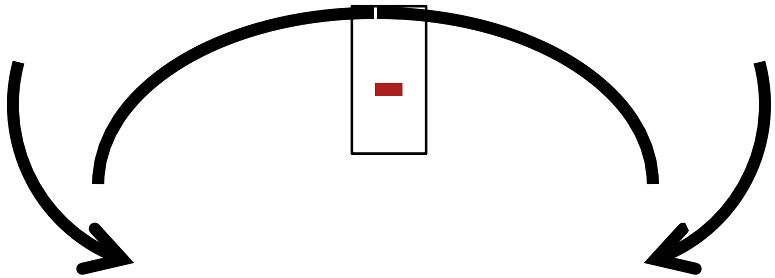
ACW

Sign Conventions
Bending Moment Diagrams (cont.)

Sagging bending moment is POSITIVE (happy)



Hogging bending moment is NEGATIVE (sad)

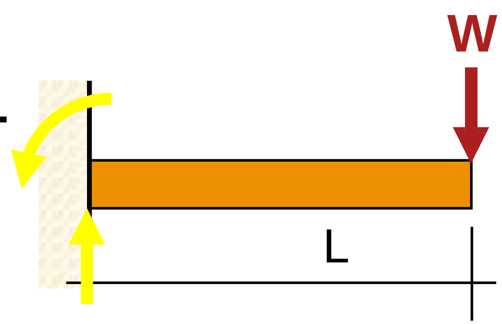


Cantilever Beam

Point Load at End

- CONSIDER CANTILEVER BEAM WITH POINT LOAD ON END

M_R = -WL



vertical reaction, R = -W
and moment reaction M_R = - WL

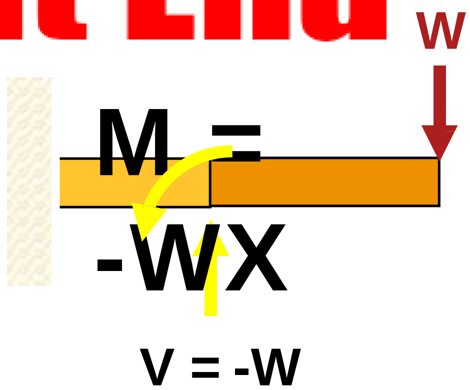
R =- W

- **Use the free body idea to isolate part of the beam**

- **Add in forces required for equilibrium**

Cantilever Beam

Point Load at End



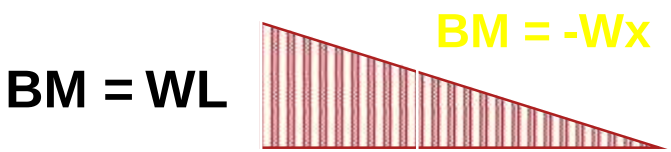
Take section anywhere at distance, x from end

Add in forces, V = -W and moment M = - Wx

Shear V =- W constant along length

Shear Force Diagram

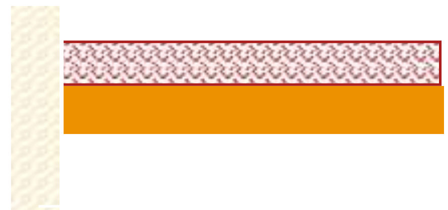
Bending Moment BM = -W.x
when x = L BM = -WL
when x = 0 BM = 0



Bending Moment Diagram

Cantilever Beam

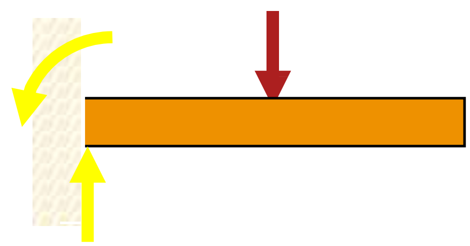
Uniformly Distributed Load



For maximum shear V and bending moment BM

Total Load W =

M_R = -WL/2
= -



R = W = wL

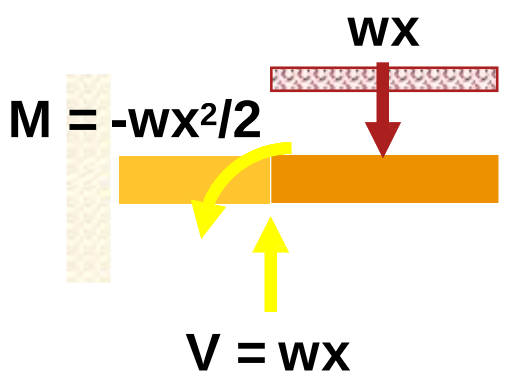
vertical reaction,	**R = W**	**= wL**
and moment reaction	M_R **= - WL/2**	**= - wL²/2**

Example 2 - Cantilever Beam Uniformly Distributed Load (cont.)

For distributed V and BM

Take section anywhere at distance, x from end

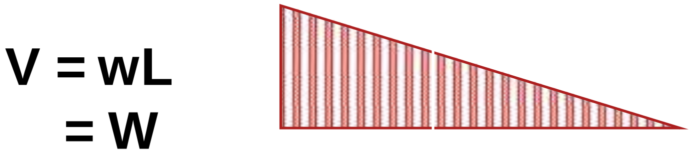
Add in forces, V = w.x and moment M = - wx.x/2



Shear $V = wx$

when x = L $V = W = wL$

when x = 0 $V = 0$



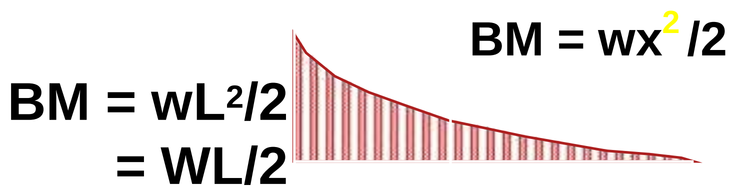
Shear Force Diagram

Bending Moment BM = $w.x^2/2$

when x = L BM = $wL^2/2 = WL/2$

when x = 0 BM = 0

(parabolic)



Bending Moment Diagram

Fig. 6.22 shows a cantilever of length L fixed at A and carrying a gradually varying load from zero at the free end to w per unit length at the fixed end.

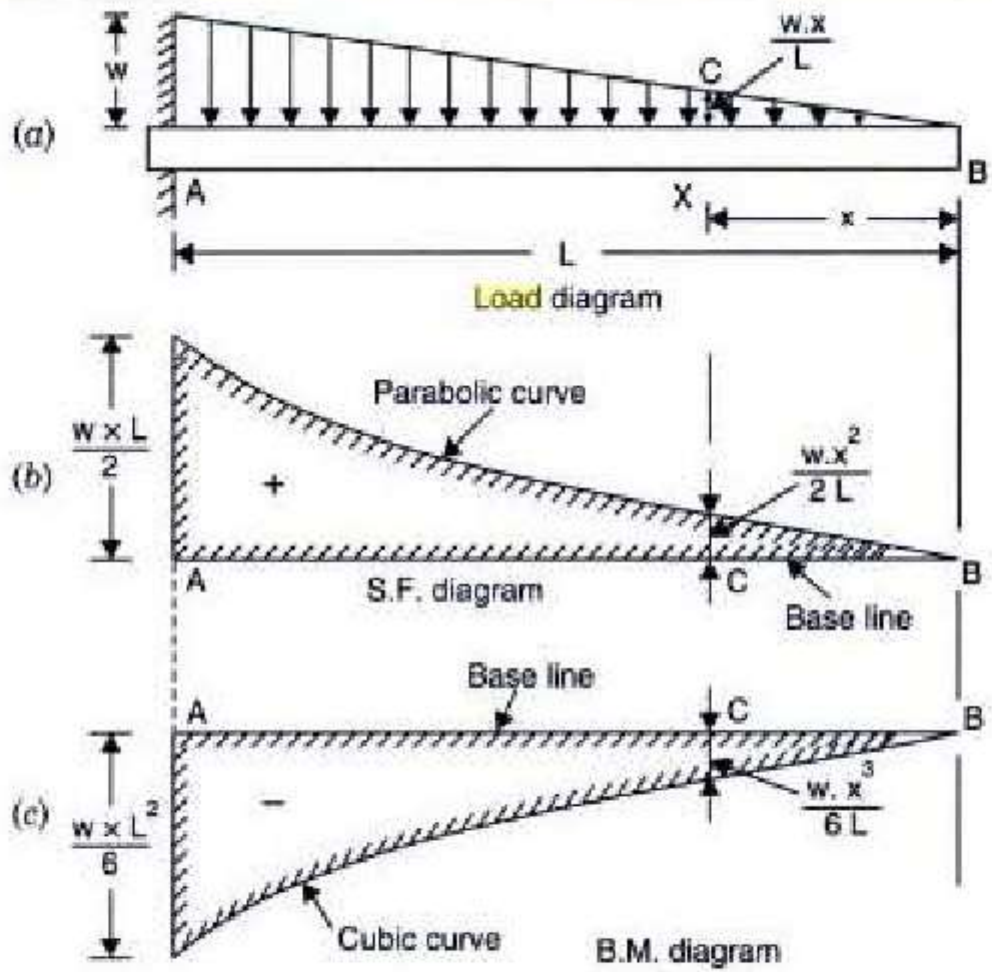


Fig. 6.22

Take a section X at a distance x from the free end B.

Let $\quad F_x$ = Shear force at the section X, and

ce.

UNIT 3

FLEXURAL & SHEAR STRESSES

UNIT-III
FLEXURAL AND SHEAR STRESSES

Members Subjected to Flexural Loads

Introduction:

In many engineering structures members are required to resist forces that are applied laterally or transversely to their axes. These type of members are termed as beam.

There are various ways to define the beams such as

Definition I: A beam is a laterally loaded member, whose cross-sectional dimensions are small as compared to its length.

Definition II: A beam is nothing simply a bar which is subjected to forces or couples that lie in a plane containing the longitudnal axis of the bar. The forces are understood to act perpendicular to the longitudnal axis of the bar.

Definition III: A bar working under bending is generally termed as a beam.

Materials for Beam:

The beams may be made from several usable engineering materials such commonly among them are as follows:

- Metal
- Wood
- Concrete
- Plastic

Examples of Beams:

Refer to the figures shown below that illustrates the beam

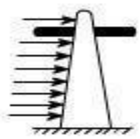


Fig 1

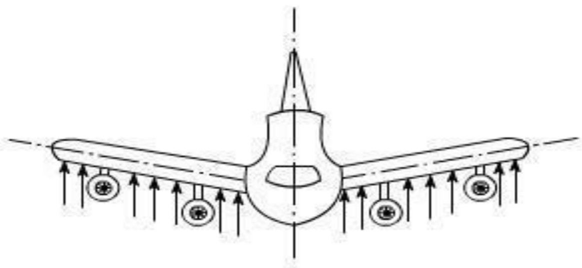


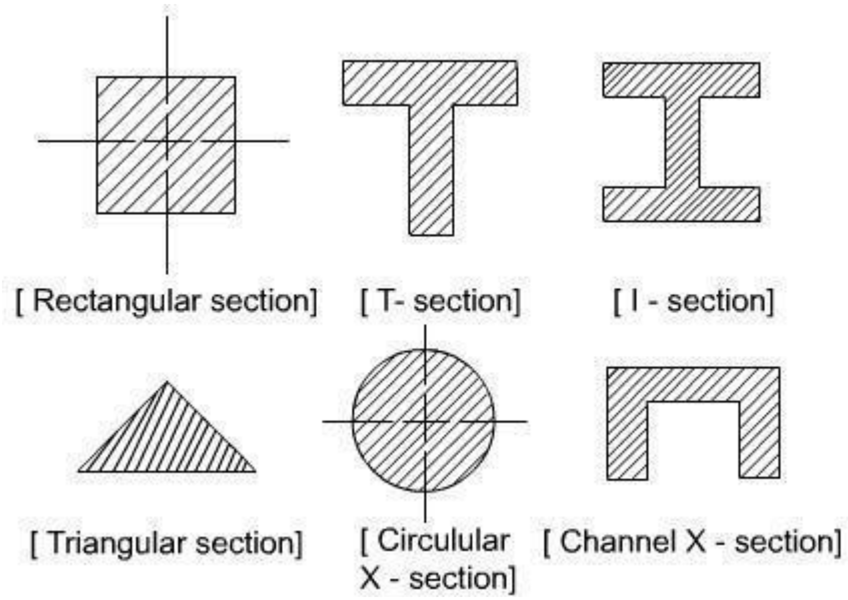
Fig 2

In the fig.1, an electric pole has been shown which is subject to forces occurring due to wind; hence it is an example of beam.

In the fig.2, the wings of an aeroplane may be regarded as a beam because here the aerodynamic action is responsible to provide lateral loading on the member.

Geometric forms of Beams:

The Area of X-section of the beam may take several forms some of them have been shown below:



Issues Regarding Beam:

Designer would be interested to know the answers to following issues while dealing with beams in practical engineering application

- At what load will it fail
- How much deflection occurs under the application of loads.

Classification of Beams:

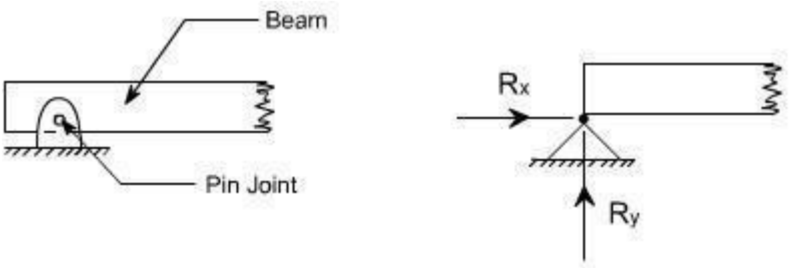
Beams are classified on the basis of their geometry and the manner in which they are supported.

Classification I: The classification based on the basis of geometry normally includes features such as the shape of the X-section and whether the beam is straight or curved.

Classification II: Beams are classified into several groups, depending primarily on the kind of supports used. But it must be clearly understood why do we need supports. The supports are required to provide constrainment to the movement of the beams or simply the supports resists the movements either in particular direction or in rotational direction or both. As a consequence of this, the reaction comes into picture whereas to resist rotational movements the moment comes into picture. On the basis of the support, the beams may be classified as follows:

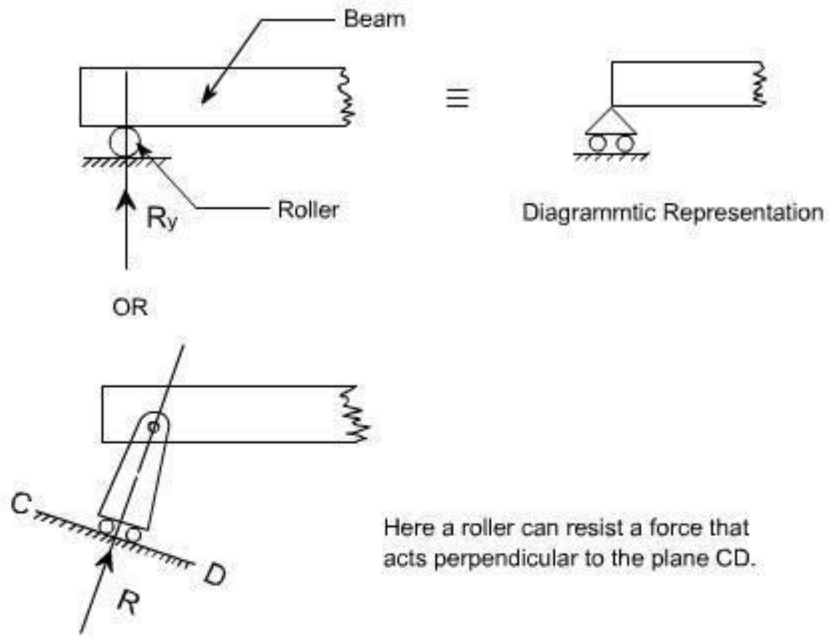
Cantilever Beam: A beam which is supported on the fixed support is termed as a cantilever beam: Now let us understand the meaning of a fixed support. Such a support is obtained by building a beam into a brick wall, casting it into concrete or welding the end of the beam. Such a support provides both the translational and rotational constrainment to the beam, therefore the reaction as well as the moments appears, as shown in the figure below

Simply Supported Beam: The beams are said to be simply supported if their supports creates only the translational constraints.



(a) Actual Representation (b) Diagrammatic Representation

Some times the translational movement may be allowed in one direction with the help of rollers and can be represented like this



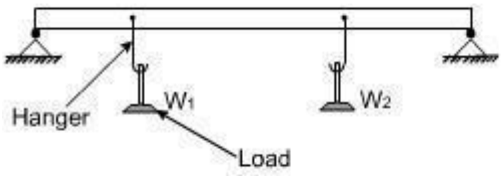
Statically Determinate or Statically Indeterminate Beams:

The beams can also be categorized as statically determinate or else it can be referred as statically indeterminate. If all the external forces and moments acting on it can be determined from the equilibrium conditions alone then. It would be referred as a statically determinate beam, whereas in the statically indeterminate beams one has to consider deformation i.e. deflections to solve the problem.

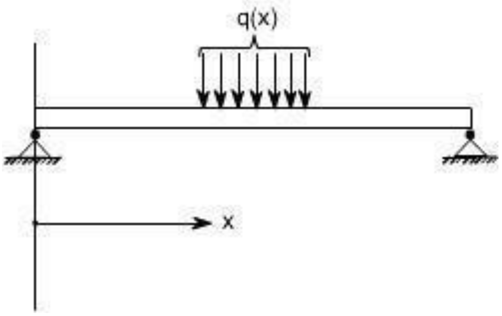
Types of loads acting on beams:

A beam is normally horizontal where as the external loads acting on the beams is generally in the vertical directions. In order to study the behaviors of beams under flexural loads. It becomes pertinent that one must be familiar with the various types of loads acting on the beams as well as their physical manifestations.

A. Concentrated Load: It is a kind of load which is considered to act at a point. By this we mean that the length of beam over which the force acts is so small in comparison to its total length that one can model the force as though applied at a point in two dimensional view of beam. Here in this case, force or load may be made to act on a beam by a hanger or though other means

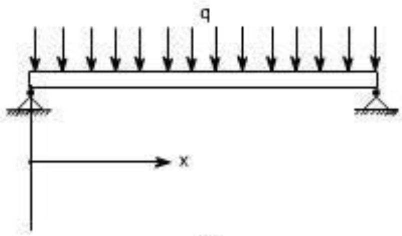


B. Distributed Load: The distributed load is a kind of load which is made to spread over a entire span of beam or over a particular portion of the beam in some specific manner

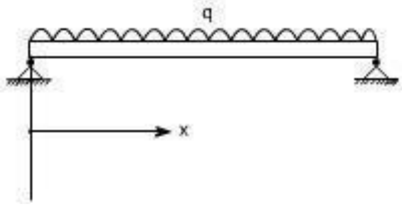


In the above figure, the rate of loading 'q' is a function of x i.e. span of the beam, hence this is a non uniformly distributed load.

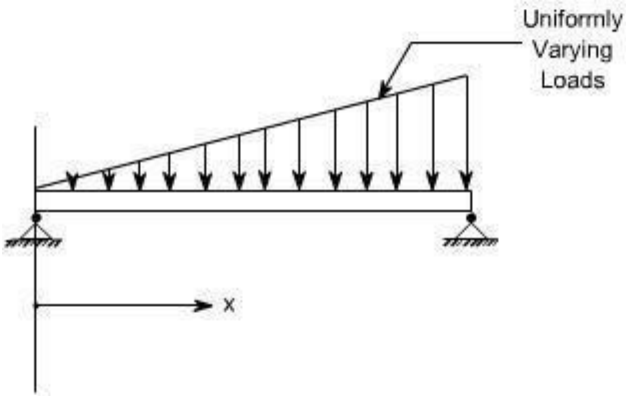
The rate of loading 'q' over the length of the beam may be uniform over the entire span of beam, then we cell this as a uniformly distributed load (U.D.L). The U.D.L may be represented in either of the way on the beams



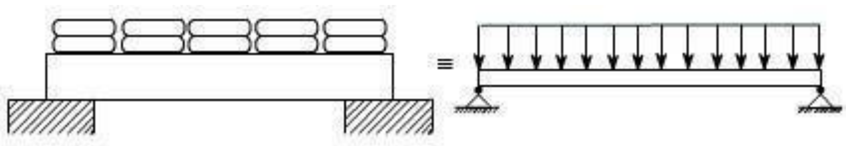
OR



some times the load acting on the beams may be the uniformly varying as in the case of dams or on inclind wall of a vessel containing liquid, then this may be represented on the beam as below:

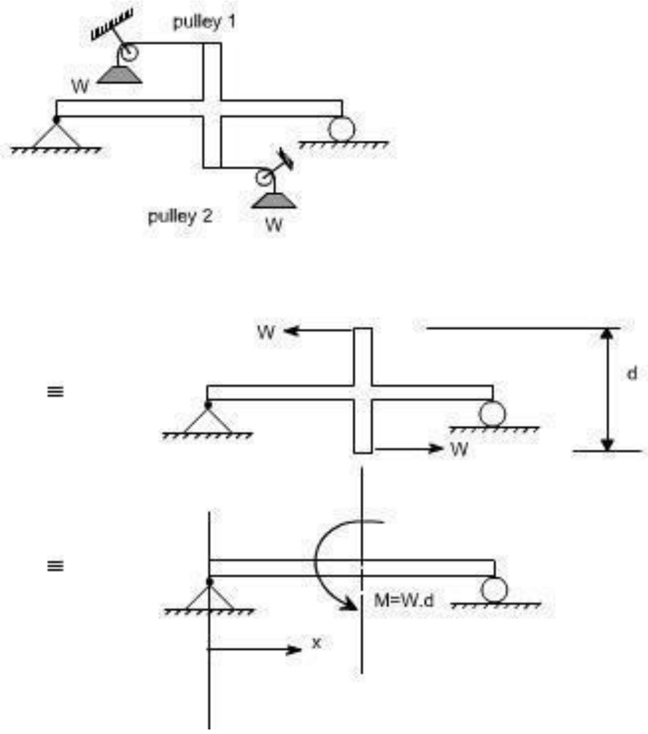


The U.D.L can be easily realized by making idealization of the ware house load, where the bags of grains are placed over a beam.



Concentrated Moment:

The beam may be subjected to a concentrated moment essentially at a point. One of the possible arrangement for applying the moment is being shown in the figure below:



Simple Bending Theory OR Theory of Flexure for Initially Straight Beams

(The normal stress due to bending are called flexure stresses) Preamble:

When a beam having an arbitrary cross section is subjected to a transverse loads the beam will bend. In addition to bending the other effects such as twisting and buckling may occur, and to investigate a problem that includes all the combined effects of bending, twisting and buckling could become a complicated one. Thus we are interested to investigate the bending effects alone, in order to do so, we have to put certain constraints on the geometry of the beam and the manner of loading.

Assumptions:

The constraints put on the geometry would form the **assumptions:**

1. Beam is initially **straight** , and has a **constant cross-section.**

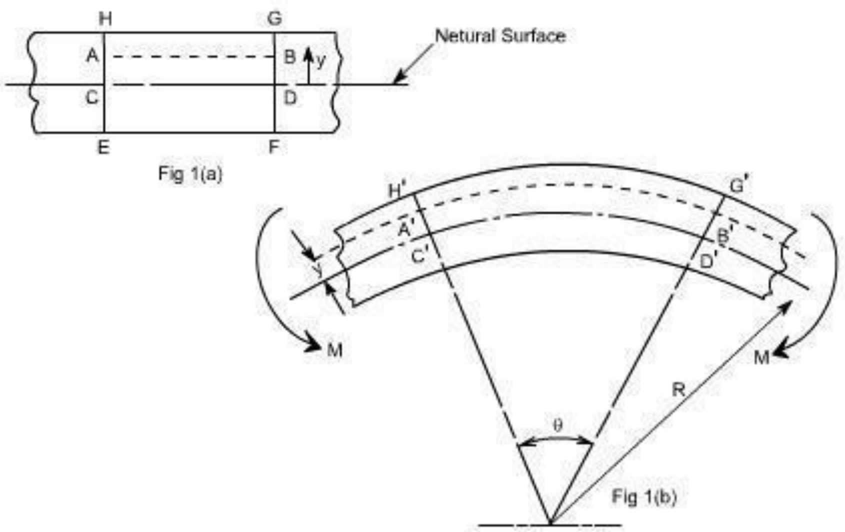
2. Beam is made of **homogeneous material** and the beam has a **longitudinal plane of symmetry.**

3. Resultant of the applied loads lies in the plane of symmetry.

4. The geometry of the overall member is such that bending not buckling is the primary cause of failure.

5. Elastic limit is nowhere exceeded and **'E'** is same in tension and compression.

6. Plane cross - sections remains plane before and after bending.



Let us consider a beam initially unstressed as shown in fig 1(a). Now the beam is subjected to a constant bending moment (i.e. 'Zero Shearing Force') along its length as would be obtained by applying equal couples at each end. The beam will bend to the radius R as shown in Fig 1(b)

As a result of this bending, the top fibers of the beam will be subjected to tension and the bottom to compression it is reasonable to suppose, therefore, **that some where between the two there are points at which the stress is zero. The locus of all such points is known as neutral axis** . The radius of curvature R is then measured to this axis. For symmetrical sections the N. A. is the axis of symmetry but what ever the section N. A. will always pass through the centre of the area or centroid.

The above restrictions have been taken so as to eliminate the possibility of '<u>twisting</u>' of the beam.

Concept of pure bending:

Loading restrictions:

As we are aware of the fact internal reactions developed on any cross-section of a beam may consists of a resultant normal force, a resultant shear force and a resultant couple. In order to ensure that the bending effects alone are investigated, we shall put a constraint on the loading such that the resultant normal and the resultant shear forces are zero on any cross-section perpendicular to the longitudinal axis of the member,

That means F = 0

since $\frac{dM}{dX} = F = 0$ or M = constant.

Thus, the zero shear force means that the bending moment is constant or the bending is same at every cross-section of the beam. Such a situation may be visualized or envisaged when the beam or some portion of the beam, as been loaded only by pure couples at its ends. It must be recalled that the couples are assumed to be loaded in the plane of symmetry.

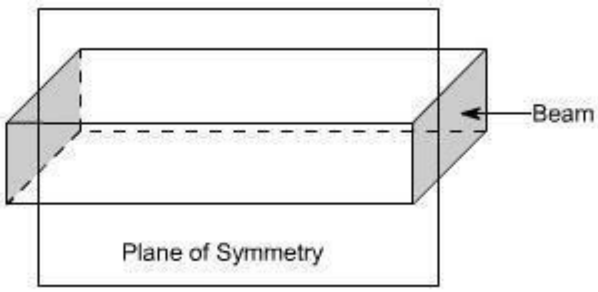


Fig (1)

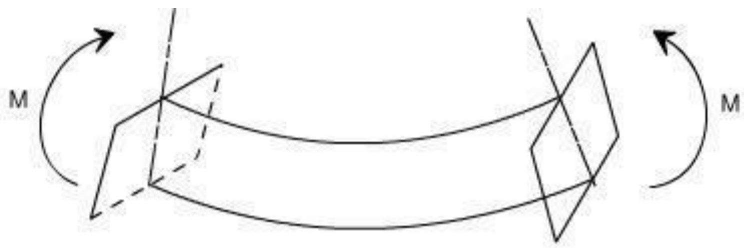
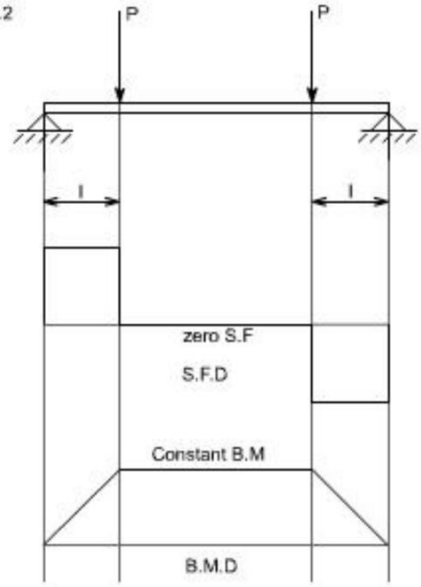


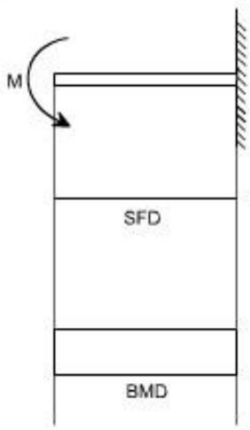
Fig (2)

When a member is loaded in such a fashion it is said to be in **pure bending.** The examples of pure bending have been indicated in EX 1and EX 2 as shown below :

EX .2



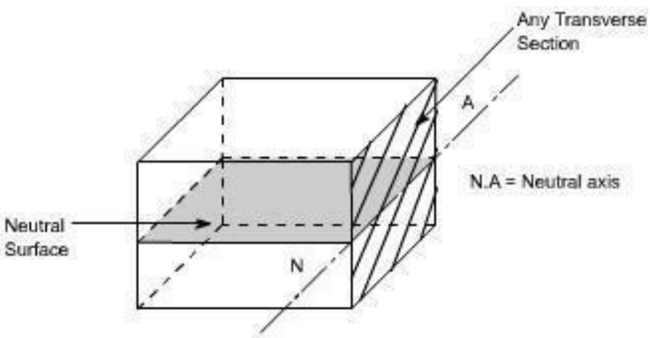
EX. 1



When a beam is subjected to pure bending are loaded by the couples at the ends, certain cross-section gets deformed and we shall have to make out the conclusion that,

1. Plane sections originally perpendicular to longitudinal axis of the beam remain plane and perpendicular to the longitudinal axis even after bending , i.e. the cross-section A'E', B'F' (refer Fig 1(a)) do not get warped or curved.

2. In the deformed section, the planes of this cross-section have a common intersection i.e. any time originally parallel to the longitudinal axis of the beam becomes an arc of circle.



We know that when a beam is under bending the fibres at the top will be lengthened while at the bottom will be shortened provided the bending moment M acts at the ends. In between these there are some fibres which remain unchanged in length that is they are not strained, that is they do not carry any stress. The plane containing such fibres is called neutral surface.

The line of intersection between the neutral surface and the transverse exploratory section is called the neutral axisNeutral axis **(N A)** .

Bending Stresses in Beams or Derivation of Elastic Flexural formula :

In order to compute the value of bending stresses developed in a loaded beam, let us consider the two cross-sections of a beam **HE** and **GF** , originally parallel as shown in fig 1(a).when the beam is to bend it is assumed that these sections remain parallel i.e. **H'E'** and **G'F'** , the final position of the sections, are still straight lines, they then subtend some angle .

Consider now fiber AB in the material, at adistance y from the N.A, when the beam bends this will stretch to A'B'

Therefore,

$$\text{strain in fibre AB} = \frac{\text{change in length}}{\text{orginal length}}$$

$$= \frac{A'B' - AB}{AB} \qquad \text{But } AB = CD \text{ and } CD = C'D'$$

refer to fig1(a) and fig1(b)

$$\therefore \text{strain} = \frac{A'B' - C'D'}{C'D'}$$

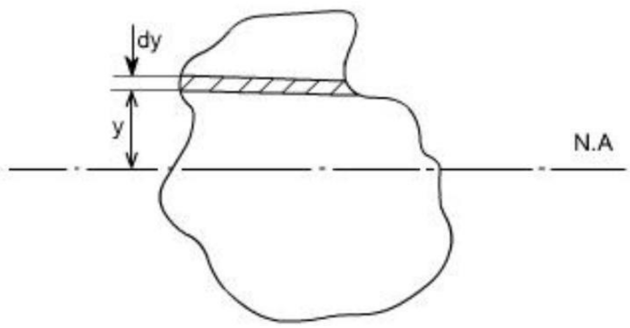
Since CD and C'D' are on the neutral axis and it is assumed that the Stress on the neutral axis zero. Therefore, there won't be any strain on the neutral axis

$$= \frac{(R+y)\theta - R\theta}{R\theta} = \frac{R\theta + y\theta - R\theta}{R\theta} = \frac{y}{R}$$

$$\text{However } \frac{\text{stress}}{\text{strain}} = E \qquad \text{where } E = \text{Young's Modulus of elasticity}$$

Therefore, equating the two strains as obtained from the two relations i.e,

$$\frac{\sigma}{E} = \frac{y}{R} \text{ or } \frac{\sigma}{y} = \frac{E}{R} \qquad \text{.............(1)}$$



Consider any arbitrary a cross-section of beam, as shown above now the strain on a fibre at a distance ‘y' from the N.A, is given by the expression

$$\sigma = \frac{E}{R} y$$

if the shaded strip is of area'dA'
then the force on the strip is

$$F = \sigma\ \delta A = \frac{E}{R} y\ \delta A$$

Moment about the neutral axis would be $= F.y = \frac{E}{R}\ y^2 \delta A$

The toatl moment for the whole
cross-section is therefore equal to

$$M = \sum \frac{E}{R} y^2\ \delta A = \frac{E}{R} \sum y^2 \delta A$$

Now the term $\sum y^2 \delta A$ is the property of the material and is called as a second moment of area of the cross-section and is denoted by a symbol I.

Therefore

$$M = \frac{E}{R} I \qquad ....(2)$$

combining equation 1 and 2 we get

$$\boxed{\frac{\sigma}{y} = \frac{M}{T} = \frac{E}{R}}$$

This equation is known as the Bending Theory Equation. The above proof has involved the assumption of pure bending without any shear force being present. Therefore this termed as the pure bending equation. This equation gives distribution of stresses which are normal to cross-section i.e. in x-direction.

<u>Section Modulus:</u>

From simple bending theory equation, the maximum stress obtained in any cross-section is given as

$$\sigma_{max}{}^{m} = \frac{M}{T} y_{max}{}^{m}$$

For any given allowable stress the maximum moment which can be accepted by a particular shape of cross-section is therefore

$$M = \frac{I}{y_{max}{}^{m}} \sigma_{max}{}^{m}$$

For ready comparison of the strength of various beam cross-section this relationship is some times written in the form

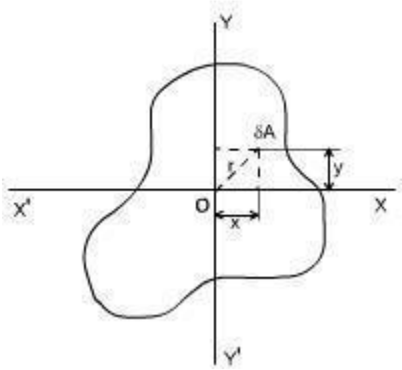
$$M = Z \sigma_{max}{}^{m} \text{ where } Z = \frac{I}{y_{max}{}^{m}}$$ **Is termed as section modulus**

The higher value of Z for a particular cross-section, the higher the bending moment which it can withstand for a given maximum stress.

Theorems to determine second moment of area: There are two theorems which are helpful to determine the value of second moment of area, which is required to be used while solving the simple bending theory equation.

Second Moment of Area :

Taking an analogy from the mass moment of inertia, the second moment of area is defined as the summation of areas times the distance squared from a fixed axis. (This property arised while we were driving bending theory equation). This is also known as the moment of inertia. An alternative name given to this is second moment of area, because the first moment being the sum of areas times their distance from a given axis and the second moment being the square of the distance or $\int y^2 \, dA$.



Consider any cross-section having small element of area d A then by the definition

I_X(Mass Moment of Inertia about x-axis) = $\int y^2 \, dA$ and I_y(Mass Moment of Inertia about y-axis) $= \int x^2 \, dA$

Now the moment of inertia about an axis through 'O' and perpendicular to the plane of figure is called the polar moment of inertia. (The polar moment of inertia is also the area moment of inertia).

i.e,

J = polar moment of inertia

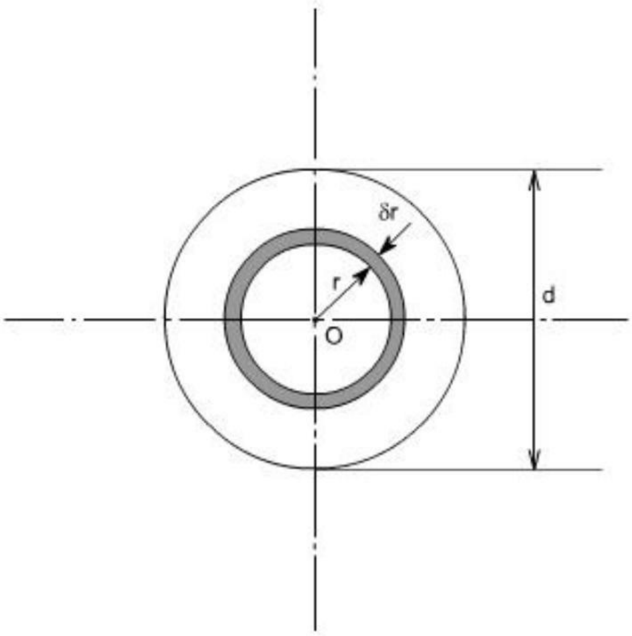
$$= \int r^2 dA$$
$$= \int (x^2 + y^2) dA$$
$$= \int x^2 dA + \int y^2 dA$$
$$= I_X + I_Y$$
$$\text{or } J = I_X + I_Y \qquad \text{.......... (1)}$$

The relation (1) is known as the **perpendicular axis theorem** and may be stated as follows:

The sum of the Moment of Inertia about any two axes in the plane is equal to the moment of inertia about an axis perpendicular to the plane, the three axes being concurrent, i.e, the three axes exist together.

CIRCULAR SECTION :

For a circular x-section, the polar moment of inertia may be computed in the following manner



Consider any circular strip of thickness □r located at a radius 'r'.

Than the area of the circular strip would be dA = 2□r. □r

$$J = \int r^2\, dA$$

Taking the limits of intergration from 0 to d/2

$$J = \int_0^{\frac{d}{2}} r^2\, 2\pi r\, \delta r$$

$$= 2\pi \int_0^{\frac{d}{2}} r^3\, \delta r$$

$$J = 2\pi \left[\frac{r^4}{4}\right]_0^{\frac{d}{2}} = \frac{\pi d^4}{32}$$

however,by perpendicular axistheorem

$$J = I_x + I_y$$

But for the circular cross-section ,the I_x and I_y are both equal being moment of inertia about a diameter

$$I_{dia} = \frac{1}{2} J$$

$$I_{dia} = \frac{\pi d^4}{64}$$

for a hollow circular sectionof diameter D and d, the values of J and I are defined as

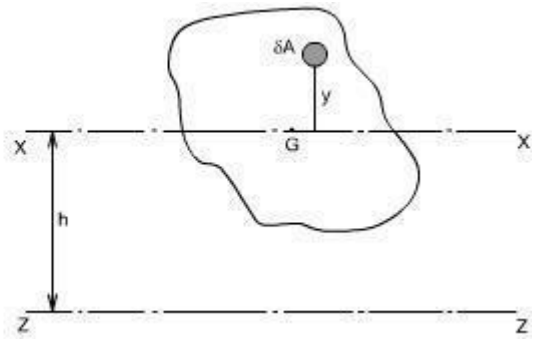
$$J = \frac{\pi\left(D^4 - d^4\right)}{32}$$

Thus

$$I = \frac{\pi\left(D^4 - d^4\right)}{64}$$

Parallel Axis Theorem:

The moment of inertia about any axis is equal to the moment of inertia about a parallel axis through the centroid plus the area times the square of the distance between the axes.



If ‘ZZ' is any axis in the plane of cross-section and ‘XX' is a parallel axis through the centroid G, of the cross-section, then

$I_z = \int (y + h)^2 \, dA$ by definition (moment of inertia about an axis ZZ)

$= \int (+ 2yh + h^2) dA$

$= \int y^2 dA + h^2 \int dA + 2h \int y dA$

Since $\int y dA = 0$

$= \int y^2 dA + h^2 \int dA$

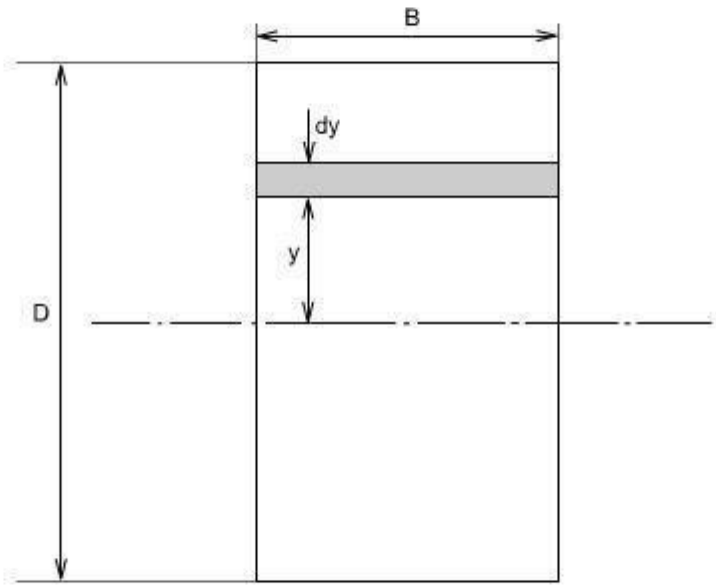
$= \int y^2 dA + h^2 A$

$I_z = I_x + Ah^2 \quad I_x = I_G$ (since cross-section axes also pass through G)

Where A = Total area of the section

Rectangular Section:

For a rectangular x-section of the beam, the second moment of area may be computed as below :



Consider the rectangular beam cross-section as shown above and an element of area **dA** , thickness **dy** , breadth **B** located at a distance **y** from the neutral axis, which by symmetry passes through the centre of section. The second moment of area **I** as defined earlier would be

$$I_{N.A} = \int y^2 \, dA$$

Thus, for the rectangular section the second moment of area about the neutral axis i.e., an axis through the centre is given by

$$I_{N.A} = \int_{\frac{-D}{2}}^{\frac{D}{2}} y^2 (B \, dy)$$

$$= B \int_{\frac{-D}{2}}^{\frac{D}{2}} y^2 \, dy$$

$$= B \left[\frac{y^3}{3} \right]_{\frac{D}{2}}^{\frac{-D}{2}}$$

$$= \frac{B}{3} \left[\frac{D^3}{8} - \left(\frac{-D^3}{8} \right) \right]$$

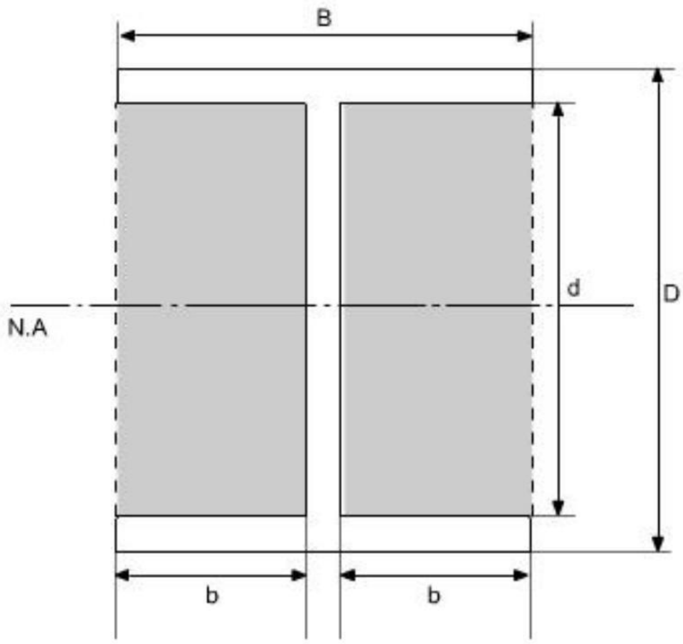
$$= \frac{B}{3} \left[\frac{D^3}{8} + \frac{D^3}{8} \right]$$

$$I_{N.A} = \frac{BD^3}{12}$$

Similarly, the second moment of area of the rectangular section about an axis through the lower edge of the section would be found using the same procedure but with integral limits of **0** to **D** .

Therefore $$I = B \left[\frac{y^3}{3} \right]_0^D = \frac{BD^3}{3}$$

These standards formulas prove very convenient in the determination of I_{NA} for build up sections which can be conveniently divided into rectangles. For instance if we just want to find out the Moment of Inertia of an I - section, then we can use the above relation.



$$I_{N.A} = I_{\text{of dotted rectangle}} - I_{\text{of shaded portion}}$$

$$\therefore\ I_{N.A} = \frac{BD^3}{12} - 2\left(\frac{bd^3}{12}\right)$$

$$I_{N.A} = \frac{BD^3}{12} - \frac{bd^3}{6}$$

Use of Flexure Formula:

Illustrative Problems:

An I - section girder, 200mm wide by 300 mm depth flange and web of thickness is 20 mm is used as simply supported beam for a span of 7 m. The girder carries a distributed load of 5 KN /m and a concentrated load of 20 KN at mid-span.

Determine the

(i). The second moment of area of the cross-section of the girder

(ii). The maximum stress set up.

Solution:

The second moment of area of the cross-section can be determained as follows :

For sections with symmetry about the neutral axis, use can be made of standard I value for a rectangle about an axis through centroid i.e. (bd 3)/12. The section can thus be divided into convenient rectangles for each of which the neutral axis passes through the centroid. Example in the case enclosing the girder by a rectangle

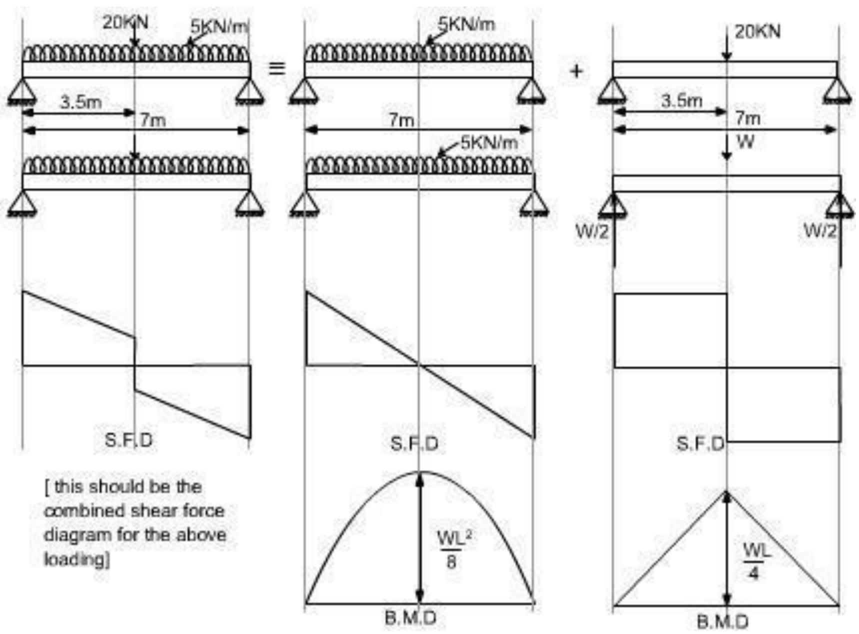
Computation of Bending Moment:

In this case the loading of the beam is of two types

(a) Uniformly distributed load

(b) Concentrated Load

In order to obtain the maximum bending moment the technique will be to consider each loading on the beam separately and get the bending moment due to it as if no other forces acting on the structure and then superimpose the two results.



Hence

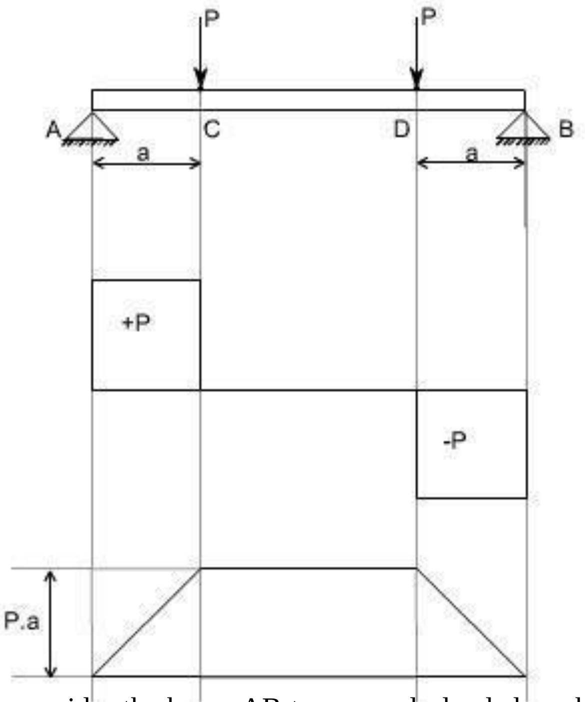
$$
\begin{aligned}
M_{max^m} &= \frac{wL}{4} + \frac{wL^2}{8} \\
&= \frac{20 \times 10^3 \times 7}{4} + \frac{5 \times 10^3 \times 7^2}{8} \\
&= (35.0 + 30.63)\,10^3 \\
&= 65.63 \text{ kNm}
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{max^m} &= \frac{M_{max^m}}{I}\, y_{max^m} \\
&= \frac{65.63 \times 10^3 \times 150 \times 10^3}{1.06 \times 10^{-4}}
\end{aligned}
$$

$$\sigma_{max^m} = 51.8 \text{ MN/m}^2$$

Shearing Stresses in Beams

All the theory which has been discussed earlier, while we discussed the bending stresses in beams was for the case of pure bending i.e. constant bending moment acts along the entire length of the beam.



Let us consider the beam AB transversely loaded as shown in the figure above. Together with shear force and bending moment diagrams we note that the middle potion CD of the beam is free from shear force and that its bending moment. M = P.a is uniform between the portion C and D. This condition is called the pure bending condition.

Since shear force and bending moment are related to each other F= dM/dX (eq) therefore if the shear force changes than there will be a change in the bending moment also, and then this won't be the pure bending.

Conclusions :

Hence one can conclude from the pure bending theory was that the shear force at each X-section is zero and the normal stresses due to bending are the only ones produced.

In the case of non-uniform bending of a beam where the bending moment varies from one X-section to another, there is a shearing force on each X-section and shearing stresses are also induced in the material. The deformation associated with those shearing stresses causes " warping " of the x-section so that the assumption which we assummed while deriving the relation $\frac{\sigma}{y}=\frac{M}{I}=\frac{E}{R}$ that the plane cross- section after bending remains plane is violated. Now due to warping the plane cross=section before bending do not remain plane after bending. This complicates the problem but more elaborate analysis shows that the normal stresses due to bending, as calculated from the equation $\frac{\sigma}{y}=\frac{M}{I}=\frac{E}{R}$.

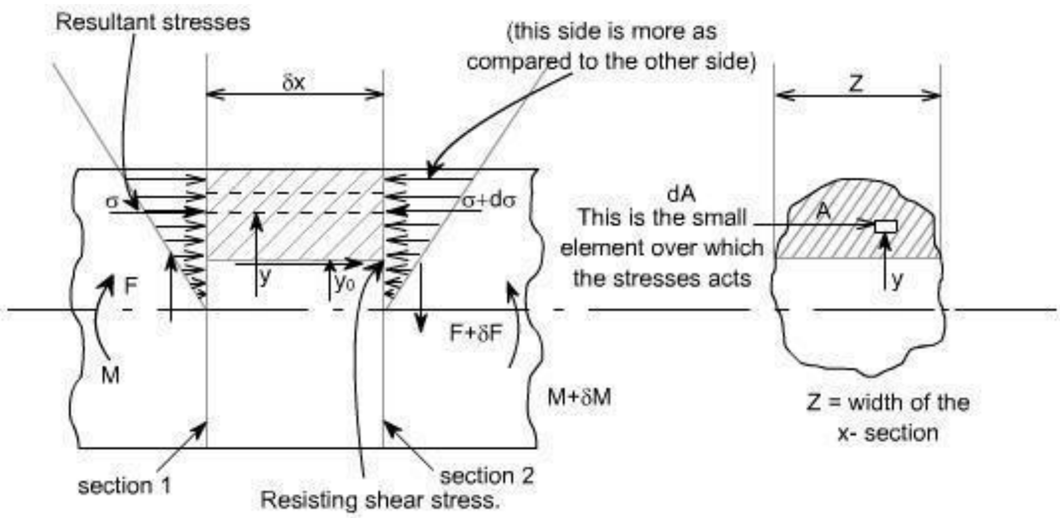
The above equation gives the distribution of stresses which are normal to the cross-section that is in x-direction or along the span of the beam are not greatly altered by the presence of these shearing stresses. Thus, it is justifiable to use the theory of pure bending in the case of non uniform bending and it is accepted practice to do so.

Let us study the shear stresses in the beams.

Concept of Shear Stresses in Beams :

By the earlier discussion we have seen that the bending moment represents the resultant of certain linear distribution of normal stresses $_{x|}$over the cross-section. Similarly, the shear force F $_X$ over any cross-section must be the resultant of a certain distribution of shear stresses.

Derivation of equation for shearing stress :



Assumptions :

1. Stress is uniform across the width (i.e. parallel to the neutral axis)

2. The presence of the shear stress does not affect the distribution of normal bending stresses.

It may be noted that the assumption no.2 cannot be rigidly true as the existence of shear stress will cause a distortion of transverse planes, which will no longer remain plane.

In the above figure let us consider the two transverse sections which are at a distance 'δx' apart. The shearing forces and bending moments being **F, F +δF** and **M, M + δM** respectively. Now due to the shear stress on transverse planes there will be a complementary shear stress on longitudinal planes parallel to the neutral axis.

i.e $\tau . z \delta x = \int d\sigma . dA$

from the bending theory equation

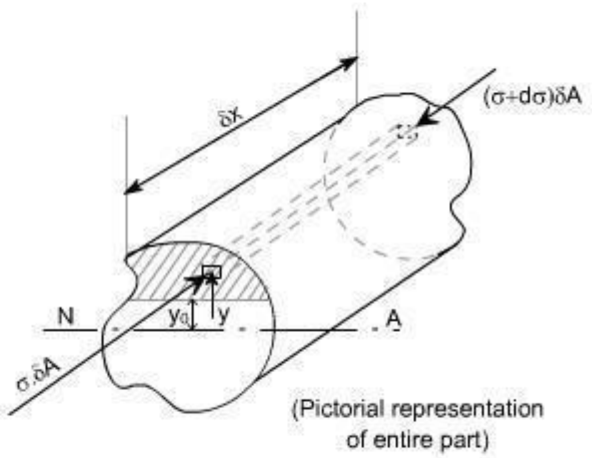
$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\sigma = \frac{M \cdot y}{I}$$

$$\sigma + d\sigma = \frac{(M + \delta M) \cdot y}{I}$$

Thus $\quad d\sigma = \frac{\delta M . y}{I}$

The figure shown below indicates the pictorial representation of the part.



$$d\sigma = \frac{\delta M . y}{I}$$

$$\tau . z\delta x = \int d\sigma . dA$$

$$= \int \frac{\delta M \ . \ y.\delta A}{I}$$

$$\tau . z\delta x = \frac{\delta M}{I} \int y.\delta A$$

But $\quad F = \frac{\delta M}{\delta x}$

i.e. $\quad \tau = \frac{F}{I.z}\int y.\delta A$

But from definition, $\int y.dA = A\bar{y}$

$\int y.dA$ is the first moment of area of the shaded portion

and $\bar{y}$ = centroid of the area'A'

Hence

$$\tau = \frac{F.A.\bar{y}}{I.z}$$

So substituting

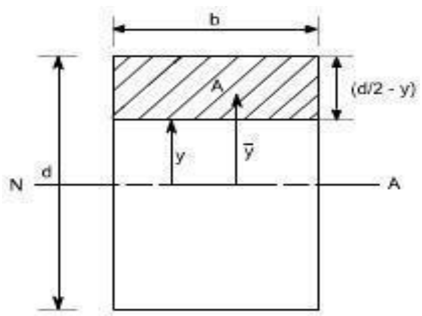
Where ‘z' is the actual width of the section at the position where ‘ ' is being calculated and I is the total moment of inertia about the neutral axis.

<u>Shearing stress distribution in typical cross-sections:</u>

Let us consider few examples to determine the sheer stress distribution in a given X- sections

Rectangular x-section:

Consider a rectangular x-section of dimension b and d



A is the area of the x-section cut off by a line parallel to the neutral axis. $\bar{y}$ is the distance of the centroid of A from the neutral axis

$$\tau = \frac{F.A.\bar{y}}{I.z}$$

for this case, $A = b\left(\frac{d}{2} - y\right)$

While $\bar{y} = \left[\frac{1}{2}\left(\frac{d}{2} - y\right) + y\right]$

i.e $\bar{y} = \frac{1}{2}\left(\frac{d}{2} + y\right)$ and $z = b$; $I = \frac{b.d^3}{12}$

substituting all these values, in the formula

$$\tau = \frac{F.A.\bar{y}}{I.z}$$

$$= \frac{F.b.\left(\frac{d}{2} - y\right).\frac{1}{2}.\left(\frac{d}{2} + y\right)}{b.\frac{b.d^3}{12}}$$

$$= \frac{\frac{F}{2}.\left\{\left(\frac{d}{2}\right)^2 - y^2\right\}}{\frac{b.d^3}{12}}$$

$$= \frac{6.F.\left\{\left(\frac{d}{2}\right)^2 - y^2\right\}}{b.d^3}$$

This shows that there is a parabolic distribution of shear stress with y.

The maximum value of shear stress would obviously beat the location y = 0.

Such that $\tau_{max} = \frac{6.F}{b.d^3}.\frac{d^2}{4}$

$$= \frac{3.F}{2.b.d}$$

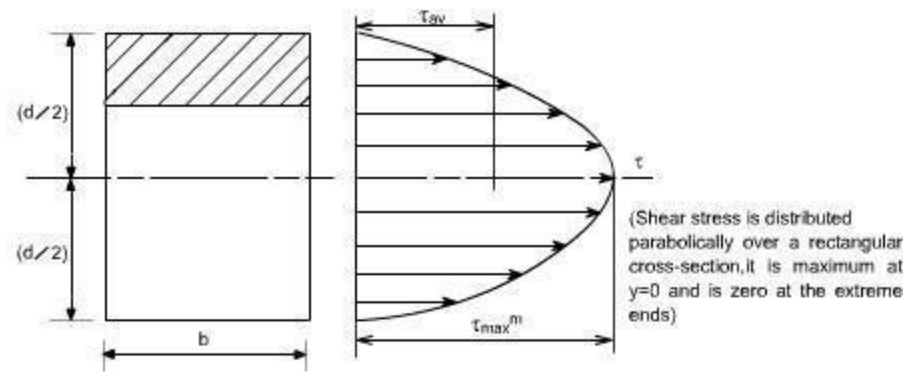
So $\boxed{\tau_{max} = \frac{3.F}{2.b.d}}$ The value of τ_{max} occurs at the neutral axis

The mean shear stress in the beam is defined as

τ_{mean} or $\tau_{avg} = F/A = F/b.d$

So $\tau_{max} = 1.5\,\tau_{mean} = 1.5\,\tau_{avg}$

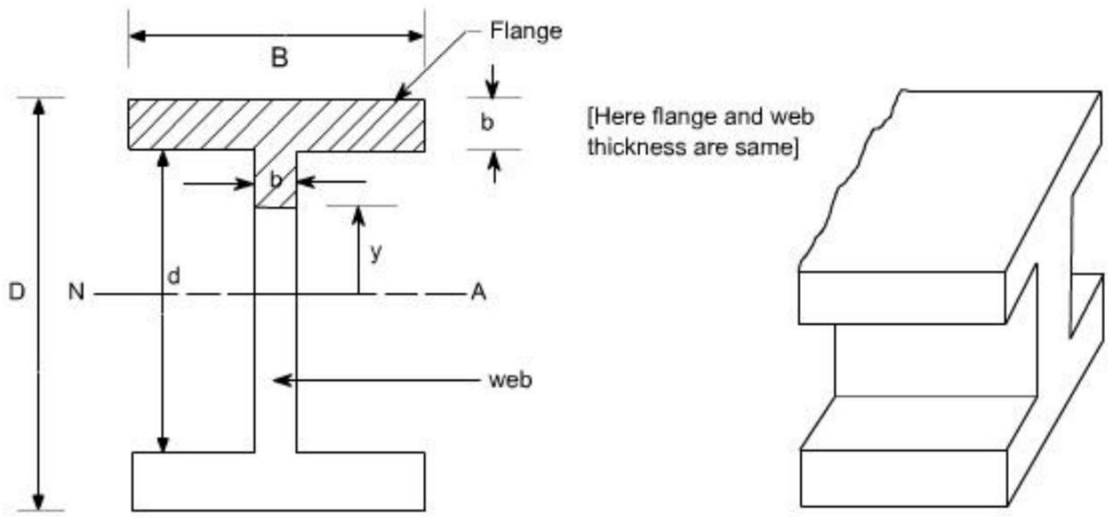
Therefore the shear stress distribution is shown as below.



It may be noted that the shear stress is distributed parabolically over a rectangular cross-section, it is maximum at y = 0 and is zero at the extreme ends.

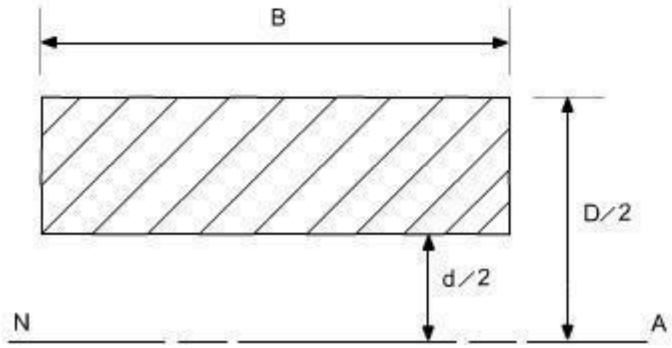
I - section :

Consider an I - section of the dimension shown below.



The shear stress distribution for any arbitrary shape is given $\tau = \frac{F A \bar{y}}{Z I}$

Let us evaluate the quantity $A\bar{y}$, the $A\bar{y}$ quantity for this case comprise the contribution due to flange area and web area



Flange area

$$\text{Area of the flange} = B\left(\frac{D-d}{2}\right)$$

Distance of the centroid of the flange from the N.A

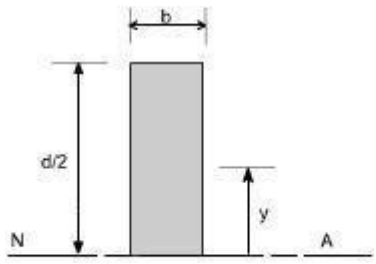
$$\bar{y} = \frac{1}{2}\left(\frac{D-d}{2}\right) + \frac{d}{2}$$

$$\bar{y} = \left(\frac{D+d}{4}\right)$$

Hence,

$$A\bar{y}\big|_{\text{Flange}} = B\left(\frac{D-d}{2}\right)\left(\frac{D-d}{4}\right)$$

Web Are



Area of the web

$$A = b\left(\frac{d}{2} - y\right)$$

Distance of the centroid from N.A

$$\bar{y} = \frac{1}{2}\left(\frac{d}{2} - y\right) + y$$

$$\bar{y} = \frac{1}{2}\left(\frac{d}{2} + y\right)$$

Therefore,

$$A\bar{y}\big|_{web} = b\left(\frac{d}{2} - y\right)\frac{1}{2}\left(\frac{d}{2} + y\right)$$

Hence,

$$A\bar{y}\big|_{Total} = B\left(\frac{D - d}{2}\right)\left(\frac{D + d}{4}\right) + b\left(\frac{d}{2} - y\right)\left(\frac{d}{2} + y\right)\frac{1}{2}$$

Thus,

$$A\bar{y}\big|_{Total} = B\left(\frac{D^2 - d^2}{8}\right) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)$$

Therefore shear stress,

$$\tau = \frac{F}{bI}\left[\frac{B\left(D^2 - d^2\right)}{8} + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

To get the maximum and minimum values of substitute in the above relation.

y = 0 at N. A. And y = d/2 at the tip.

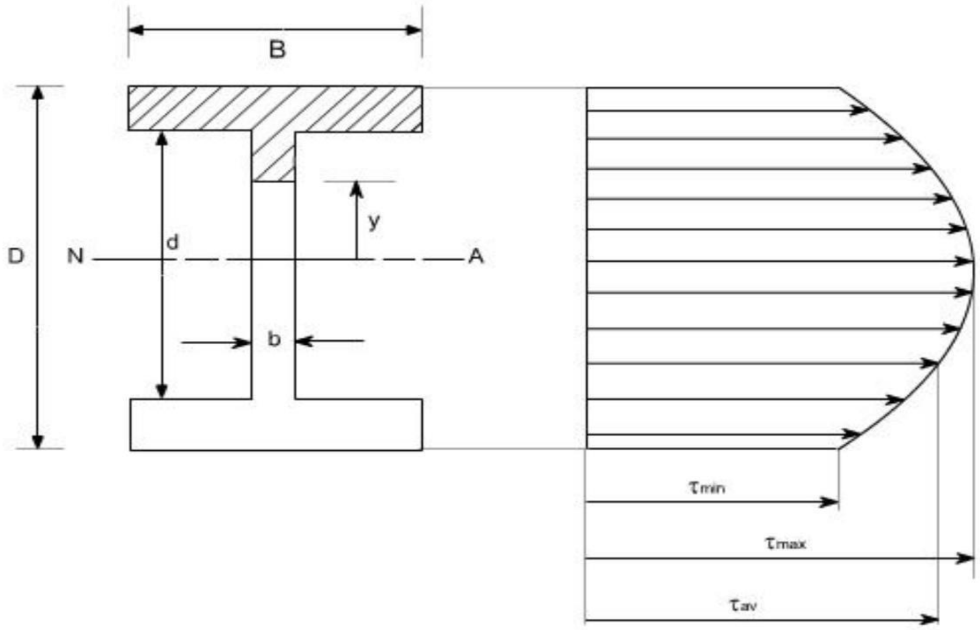
The maximum shear stress is at the neutral axis. i.e. for the condition y = 0 at N. A.

Hence, τ_{max} at $y = 0 = \dfrac{F}{8bI}\left[B\left(D^2 - d^2\right) + bd^2\right]$ ....(2)

The minimum stress occur at the top of the web, the term bd 2 goes off and shear stress is given by the following expression

τ_{min} at $y = d/2 = \dfrac{F}{8bI}\left[B\left(D^2 - d^2\right)\right]$ ....(3)

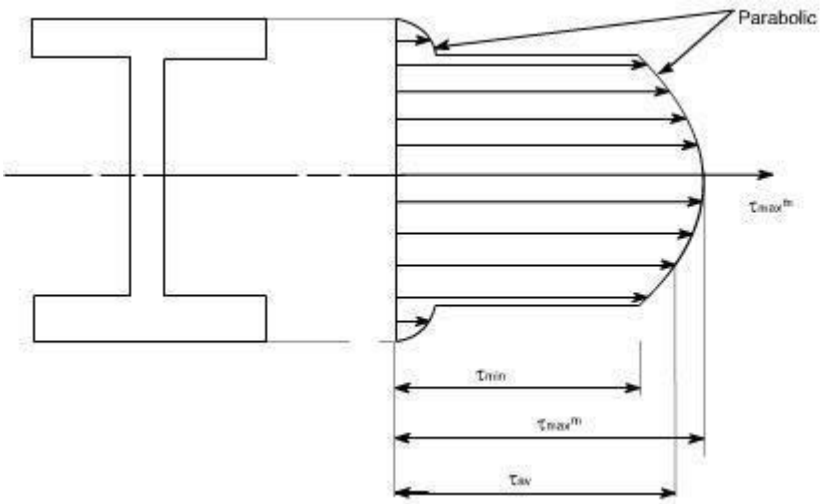
The distribution of shear stress may be drawn as below, which clearly indicates a parabolic distribution



$$\tau_{max^m} = \frac{F}{8bI}\left[B\left(D^2 - d^2\right) + bd^2\right]$$

Note: from the above distribution we can see that the shear stress at the flanges is not zero, but it has some value, this can be analyzed from equation (1). At the flange tip or flange or web interface y = d/2.Obviously than this will have some constant value and than onwards this will have parabolic distribution.

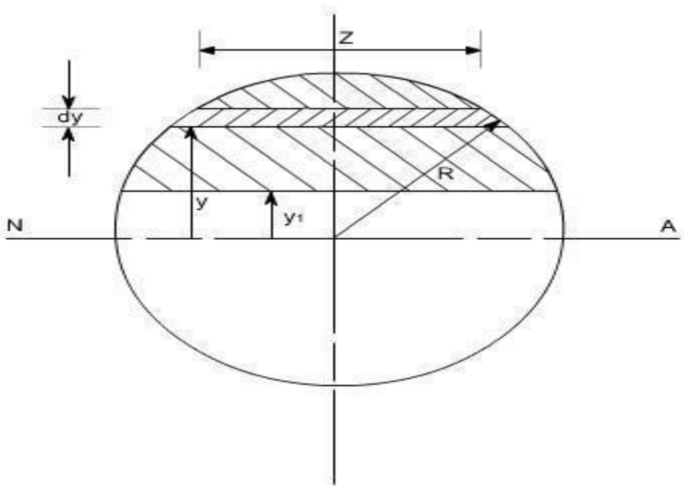
In practice it is usually found that most of shearing stress usually about 95% is carried by the web, and hence the shear stress in the flange is neglible however if we have the concrete analysis i.e. if we analyze the shearing stress in the flange i.e. writing down the expression for shear stress for flange and web separately, we will have this type of variation.



This distribution is known as the “top – hat” distribution. Clearly the web bears the most of the shear stress and bending theory we can say that the flange will bear most of the bending stress.

Shear stress distribution in beams of circular cross-section:

Let us find the shear stress distribution in beams of circular cross-section. In a beam of circular cross- section, the value of Z width depends on y.



Using the expression for the determination of shear stresses for any arbitrary shape or a arbitrary section.

$$\tau = \frac{FA\,\bar{y}}{ZI} = \frac{FA\int y\,dA}{ZI}$$

Where y dA is the area moment of the shaded portion or the first moment of area.

Here in this case 'dA' is to be found out using the Pythagoras theore

$$\left(\frac{Z}{2}\right)^2 + y^2 = R^2$$

$$\left(\frac{Z}{2}\right)^2 = R^2 - y^2 \text{ or } \frac{Z}{2} = \sqrt{R^2 - y^2}$$

$$Z = 2\sqrt{R^2 - y^2}$$

$$dA = Z\,dy = 2.\sqrt{R^2 - y^2}\,.dy$$

$I_{N.A}$ for a circular cross-section $= \frac{\pi R^4}{4}$

Hence,

$$\tau = \frac{FA\,\bar{y}}{ZI} = \frac{F}{\frac{\pi R^4}{4}\,2\sqrt{R^2 - y^2}}\int_{y_1}^{R} 2\,y\sqrt{R^2 - y^2}\,dy$$

Where R = radius of the circle.

[The limits have been taken from y_1 to R because we have to find moment of area the shaded portion]

$$= \frac{4F}{\pi R^4\sqrt{R^2 - y^2}}\int_{y_1}^{R} y\sqrt{R^2 - y^2}\,dy$$

The integration yields the final result to be

$$\tau = \frac{4F\left(R^2 - y_1^2\right)}{3\pi R^4}$$

Again this is a parabolic distribution of shear stress, having a maximum value when $y_1 = 0$

$$\tau_{max^m}|_{y_1 = 0} = \frac{4F}{3\pi R^2}$$

Obviously at the ends of the diameter the value of $y_1 = \pm R$ thus $\tau = 0$ so this again a parabolic distribution; maximum at the neutral axis

Also

$$\tau_{avg} \text{ or } \tau_{mean} = \frac{F}{A} = \frac{F}{\pi R^2}$$

Hence,

$$\boxed{\tau_{max^m} = \frac{4}{3}\,\tau_{avg}}$$

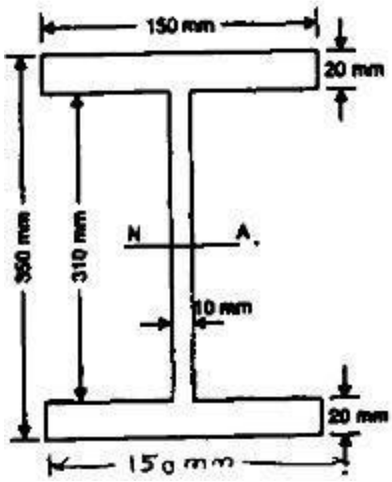
Tutorial Question

1. Derive the equation of bending moment and write down the assumptions for theory of simple bending.
2. A simply supported beam carries a U.D.L. of intensity 2.5 kN/metre over entire span of 5 meters. The cross-section of the beam is a T-section having the dimensions
   Top ange: 125 mm cm X 25 mm
   Web: 175 mm cm X25 mm
   Calculate the maximum shear stress for the section of the beam.
3. A cantilever beam of length 10 m has a cross section of 100 mm X 130 mm has a UDL of 10 KN/m over a length of 8 m from the left support and a concentrated load of 10 KN at the right end. Find the bending stress in the beam
4. A beam of T - section is having flange 120mm × 15mm and web 100mm × 15mm. It is subjected to a shear force of 24kN. Draw shear stress distribution across the depth marking values at salient points.
5. An I section is having overall depth as 550mm and overall width as 200mm. The thickness of the flanges is 25mm where as the thickness of the web is 20mm. If the section carries a shear force of 45kN, calculate the shear stress values at salient points and draw the sketch showing variation of shear stress.

Assignment Questions

1. An I section beam 350 x 150 mm as shown in Fig. has a web thickness of 10 mm and a flange thickness of 20 mm. If the shear force acting on the section is 40kN, find the maximum shear stress developed in the I section

2. A rectangular beam 300 mm deep is simply supported over a span of 4m. Determine the



uniformly distributed load per meter which the beam may carry, if the bending stress should not exceed 120 N/mm^2. Take $I = 8x10^6\ mm^4$.

3. An I-section beam 350mmX200mm has a web thickness of 12.5mm and a flange thickness of 25mm. It carries a shearing force of 200kN at a section. Sketch the shear stress distribution across the section.

4. A rolled steel joist 200mmx160mm wide has flange 22mm thick and web 12mm thick. Find the proportion, in which the flanges and web resist shear force.
5. A simply supported beam of 2m span carries a U.D.L. of 140 kN/m over the whole span. The cross section of the beam is T-section with a flange width of 120mm, web and flange thickness of 20mm and overall depth of 160mm. Determine the maximum shear stress in the beam and draw the shear stress distribution for the section.
6. A simply supported symmetric I-section has flanges of size 200 mmX 15 mm and its overall depth is 520 mm. Thickness of web is 10mm. It is strengthened with a plate of size 250 mm X 12mm on compression side. Find the moment of resistance of the section if permissible stress is 160 M Pa. How much uniformly distributed load it can carry if it is used as a cantilever of span 3.6m.

UNIT-III

POWER POINT PRESENTAION

FLEXURAL STRESSES & SHEAR STRESSES

UNIT III

DEPARTMENT OF MECHANICAL ENGINEERING

BEAM

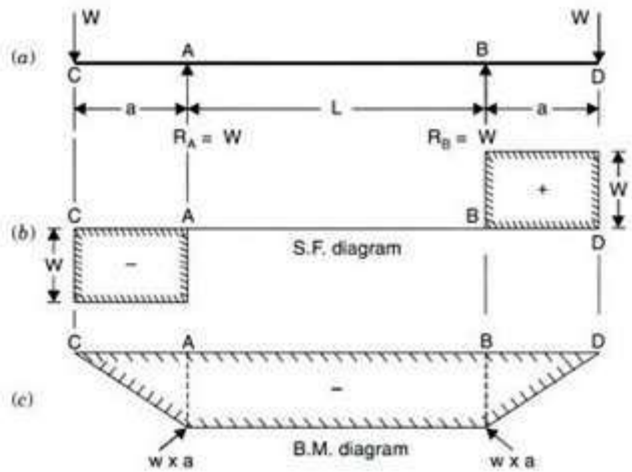
- A beam is a structural member (horizontal) that is design to support the applied load (vertical). It resists the applied loading by a combination of internal transverse **shear force** and bending **moment**.
- It is perhaps the most important and widely used structural members and can be classified according to its support conditions.

4.1 SIMPLE BENDING OR PURE BENDING

- When some external force acts on a beam, the shear force and bending moments are set up at all the sections of the beam
- Due to shear force and bending moment, the beam undergoes deformation. The material of the beam offers resistance to deformation
- Stresses introduced by bending moment are known as bending stresses
- Bending stresses are indirect normal stresses

4.1 SIMPLE BENDING OR PURE BENDING

- When a length of a beam is subjected to zero shear force and constant bending moment, then that length of beam is subjected to pure bending or simple pending.
- The stress set up in that length of the beam due



4.1 SIMPLE BENDING OR PURE BENDING

- Consider a simply supported beam with over hanging portions of equal lengths. Suppose the beam is subjected to equal loads of intensity W at either ends of the over hanging portion
- In the portion of beam of length l, the beam is subjected to constant bending moment of intensity w x a and shear force in this portion is zero
- Hence the portion AB is said to be subjected to pure bending or simple bending

4.2 ASSUMPTIONS FOR THE THEORY OF PURE BENDING

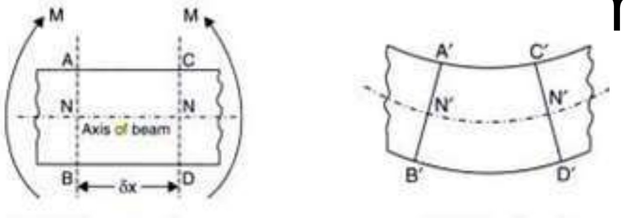
- The material of the beam is isotropic and homogeneous. Ie of same density and elastic properties throughout
- The beam is initially straight and unstressed and all the longitudinal filaments bend into a circular arc with a common centre of curvature
- The elastic limit is nowhere exceeded during the bending
- Young's modulus for the material is the same in tension and compression

4.2 ASSUMPTIONS FOR THE THEORY OF PURE BENDING

- The transverse sections which were plane before bending remain plane after bending also
- Radius of curvature is large compared to the dimensions of the cross section of the beam
- There is no resultant force perpendicular to any cross section
- All the layers of the beam are free to elongate and contract, independently of the layer, above or below it.

4.3 THEORY OF SIMPLE BENDING

- Consider a beam subjected to simple bending. Consider an infinitesimal element of length dx which is a part of this beam. Consider two transverse sections AB and CD which are normal to the axis of the beam and



- Due to the bending action the element ABCD is deformed to A'B'C'D' (concave curve).
- The layers of the beam are not of the same length before bending and after bending .

4.3 THEORY OF SIMPLE BENDING

- The layer AC is shortened to A'C'. Hence it is subjected to compressive stress
- The layer BD is elongated to B'D'. Hence it is subjected to tensile stresses.
- Hence the amount of shortening decrease from the top layer towards bottom and the amount of elongation decreases from the bottom layer towards top
- Therefore there is a layer in between which neither elongates nor shortens. This layer is called neutral layer .

4.3 THEORY OF SIMPLE BENDING

- The filaments/ fibers of the material are subjected to neither compression nor to tension
- The line of intersection of the neutral layer with transverse section is called neutral axis (N-N).
- Hence the theory of pure bending states that the amount by which a layer in a beam subjected to pure bending, increases or decreases in length, depends upon the position of the layer w.r.t neutral axis N-N.

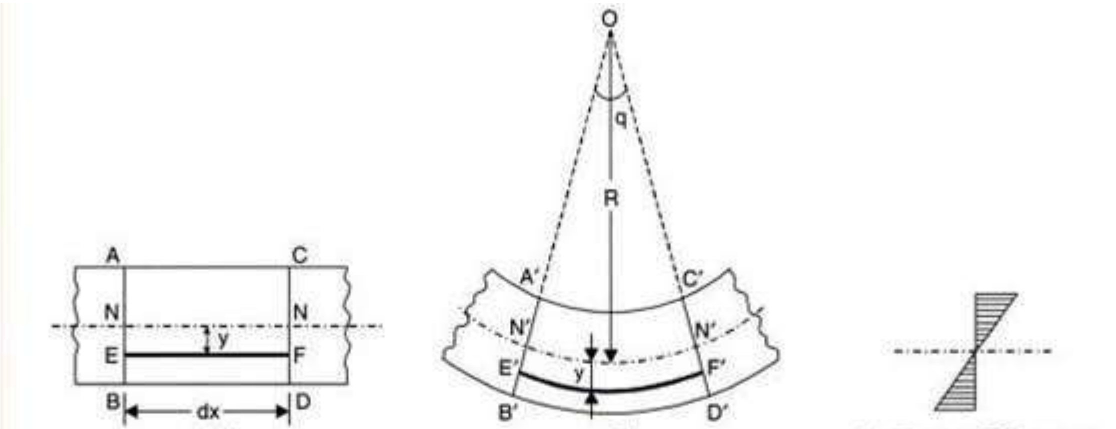
4.4 EXPRESSION FOR BENDING STRESS

- Consider a beam subjected to simple bending. Consider an infinitesimal element of length dx which is a part of this beam. Consider two transverse sections AB and CD which are normal to the axis of the beam. Due to the bending action the element ABCD is deformed to A'B'C'D' (concave curve).
- The lines B'A' and D'C' when extended meet at point O (which is the centre of curvature for the circular arc formed).
- Let R be the radius of curvature of the neutral axis.

4.4.1 STRAIN VARIATION ALONG THE DEPTH OF BEAM

- Consider a layer EF at a distance y from the neutral axis. After bending this layer will be deformed to E'F'.
- Strain developed= (E'F'-EF)/EF

EF=NN=dx=R x θ



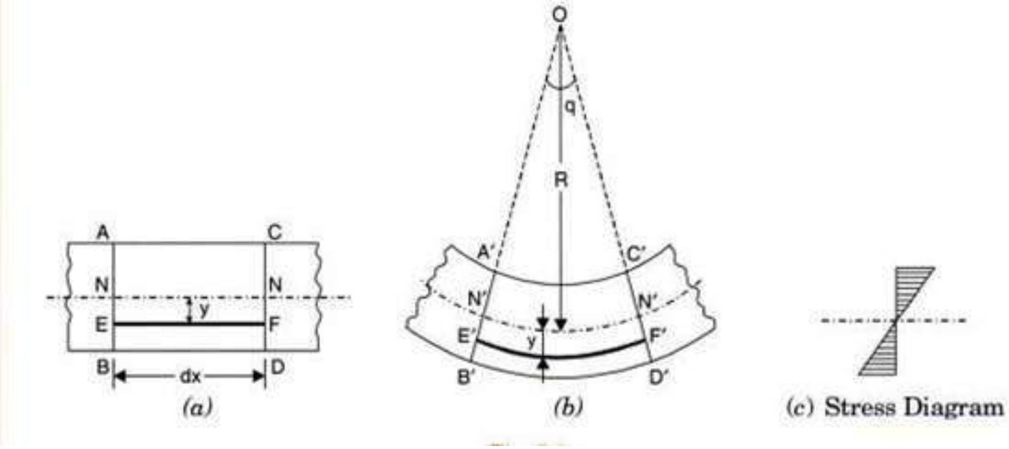
(a) (b) (c) Stress Diagram

4.4.1 STRAIN VARIATION ALONG THE DEPTH OF BEAM

- Strain developed eb= { (R + y) x θ - R x θ)}/ R x θ=y/R
- STRESS VARIATION WITH DEPTH OF BEAM
- σ/E= y/R or σ= Ey/R or σ/y = E/R
- Hence σ varies linearly with y (distance from neutral axis)
- Therefore stress in any layer is directly proportional to the distance of the layer from the neutral layer

4.5 NEUTRAL AXIS

- For a beam subjected to a pure bending moment, the stresses generated on the neutral layer is zero.
- Neutral axis is the line of intersection of neutral layer with the transverse section
- Consider the cross section of a beam subjected to pure bending. The stress at a distance y from
- =E/R



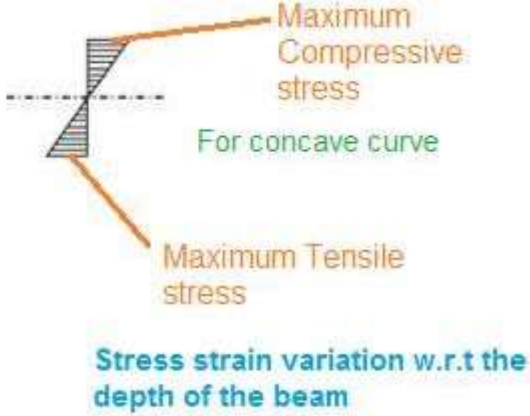
4.5 NEUTRAL AXIS

- $\sigma = E \times y/R$;
- The force acting perpendicular to this section, $dF = E \times y/R \times dA$, where dA is the cross sectional area of the strip/layer considered.
- Pure bending theory is based on an assumption that "There is no resultant force perpendicular to any cross section". Hence F=0;
- Hence, $E/R \times \int y dA = 0$

=> $\int y dA$= Moment of area of the entire cross section w.r.t the neutral axis=0

4.5 NEUTRAL AXIS

- Moment of area of any surface w.r.t the centroidal axis is zero. Hence neutral axis and centroidal axis for a beam subjected to simple bending are the same.
- Neutral axis coincides with centrodial axis or the centroidal axis gives the position of neutral axis



4.6 MOMENT OF RESISTANCE

- Due to the tensile and compressive stresses, forces are exerted on the layers of a beam subjected to simple bending
- These forces will have moment about the neutral axis. The total moment of these forces about the neutral axis is known as moment of resistance of that section
- We have seen that force on a layer of cross sectional area dA at a distance y from the neutral axis,

$dF = (E \times y \times dA)/R$

Moment of force dF about the neutral axis= dF x y= (E x y x dA)/R x y= E/R x ($y^2 dA$)

4.6 MOMENT OF RESISTANCE

- Hence the total moment of force about the neutral axis= Bending moment applied= $\int E/R \times (y^2 dA)$= $E/R \times I_{xx}$; Ixx is the moment of area about the neutral axis/centroidal axis.

Hence $M=E/R \times I_{xx}$ Or $M/I_{xx}=E/R$

- Hence $M/I_{xx}=E/R = \sigma_b/y$; σ_b is also known as flexural stress (Fb)
- Hence $\boxed{\mathbf{M/I_{xx}=E/R=F_b/y}}$
- The above equation is known as bending equation
- This can be remembered using the sentence "Elizabeth Rani May I Follow You"

4.7 CONDITION OF SIMPLE BENDING & FLEXURAL RIGIDITY

- Bending equation is applicable to a beam subjected to pure/simple bending. Ie the bending moment acting on the beam is constant and the shear stress is zero
- However in practical applications, the bending moment varies from section to section and the shear force is not zero
- But in the section where bending moment is maximum, shear force (derivative of bending moment) is zero
- Hence the bending equation is valid for the section where bending moment is maximum

4.7 CONDITION OF SIMPLE BENDING & FLEXURAL RIGIDITY

- Or in other words, the condition of simple bending may be satisfied at a section where bending moment is maximum.
- Therefore beams and structures are designed using bending equation considering the section of maximum bending moment
- Flexural rigidity/Flexural resistance of a beam:-
- For pure bending of uniform sections, beam will deflect into circular arcs and for this reason the term circular bending is often used.

4.7 CONDITION OF SIMPLE BENDING & FLEXURAL RIGIDITY

- The radius of curvature to which any beam is bent by an applied moment M is given by R=EI/M
- Hence for a given bending moment, the radius of curvature is directly related to "EI"
- Since radius of curvature is a direct indication of the degree of flexibility of the beam (larger the value of R, less flexible the beam is, more rigid the beam is), EI is known as flexural rigidity of flexural stiffness of the beam.
- The relative stiffnesses of beam sections can then easily be compared by their EI value

4.8 SECTIONAL MODULUS (Z)

- Section modulus is defined as the ratio of moment of area about the centroidal axis/neutral axis of a beam subjected to bending to the distance of outermost layer/fibre/filament from the centroidal axis
- **$Z = I_{xx}/y_{max}$**
- From the bending equation, $M/I_{xx} = \sigma_{bmax}/y_{max}$

Hence $I_{xx}/y_{max} = M/\sigma_{bmax}$ $M = \sigma bmax \times Z$

- Higher the Z value for a section, the higher the BM which it can withstand for a given maximum stress

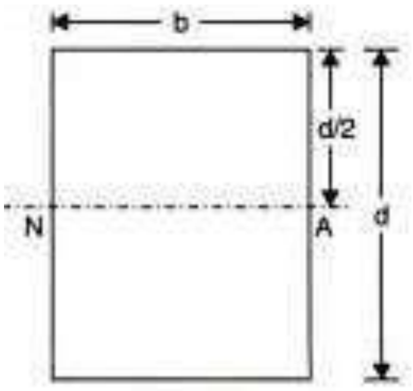
VARIOUS SHAPES OR BEAM SECTIONS

- 1) For a Rectangular section

  $Z = I_{xx}/y_{max}$

$I_{xx} = I_{NA} = bd^3/12 \quad y_{max} = d/2$

$Z = bd^2/6$

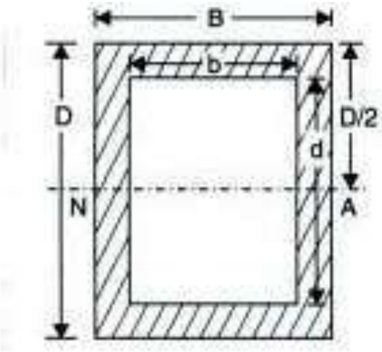


- 2) For a Rectangular hollow sect

  $I_{xx} = 1/12 \times (BD^3/12 - \quad bd^3/12)$ i

  $Y_{max} = D/2$

$Z = (BD^3 - bd^3)/6D$



SHEAR STRESSES IN BEAMS

Introduction:

In the earlier chapter, the variation of bending stress across a beam section was studied. The bending stress is due to bending moment at the section.

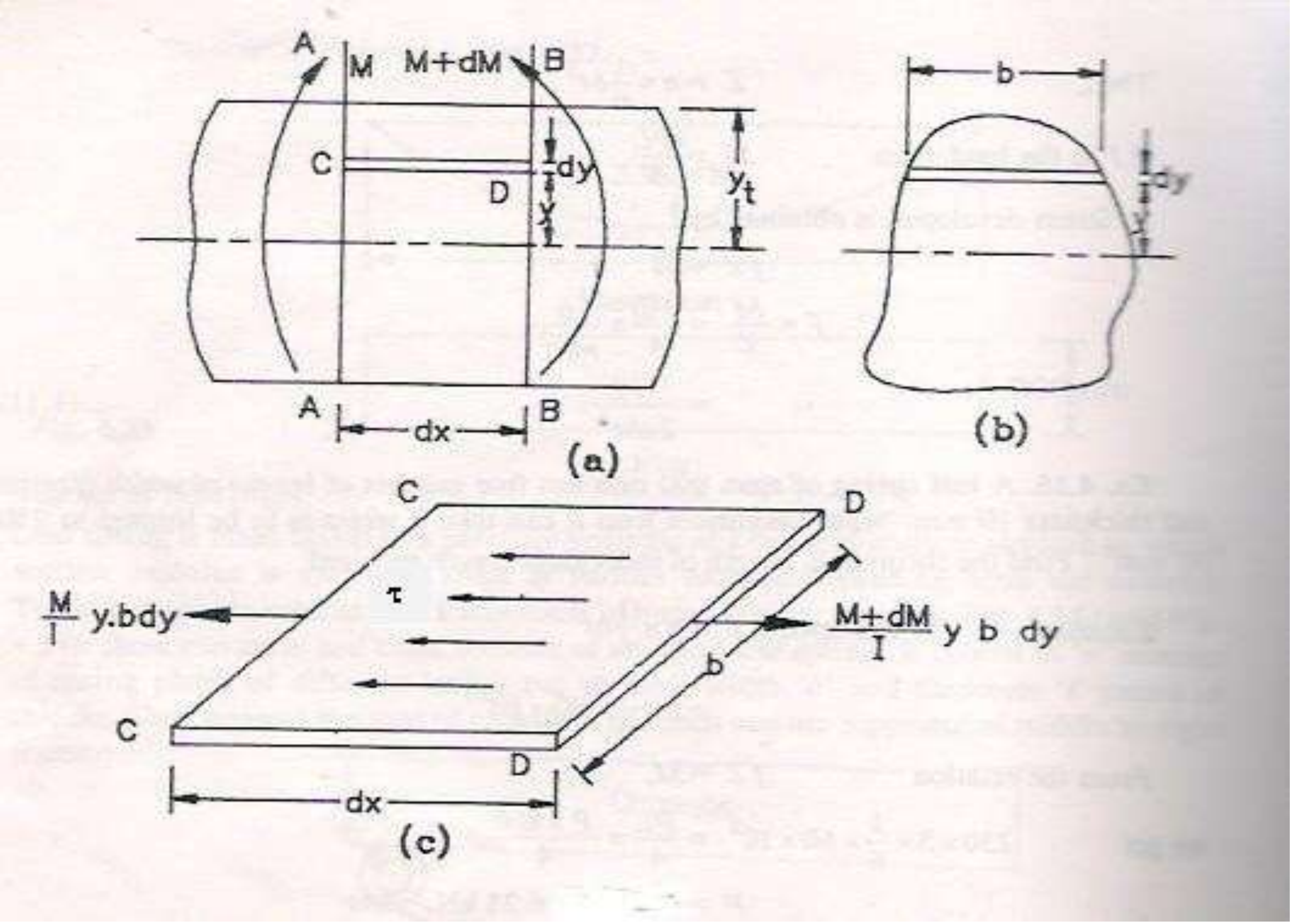
The bending stress act longitudinally and its intensity is directly proportional to its distance from neutral axis.

A typical beam section is subjected to shear force in addition to bending moment. The variation of shearing stress, which is due to the presence of shear force, is studied in this chapter.

The stresses induced by shear force at a section in a beam may be analyzed as follows:

Consider an elemental length of a beam between the sections AA and BB separated by a distance dx, as shown in the following figure. Let the moments acting at AA and BB be M and M+ dM respectively.

Let CD be a fibre of thickness dy at a distance y from the neutral axis. Then bending stress at left side of the fibre CD = $\frac{M\,y}{I}$



A
M
M+dM
B
C
dy
D
y
y_t
A
B
dx
(a)
b
dy
y
(b)
C
D
τ
$\frac{M}{I}$ y.bdy
$\frac{M+dM}{I}$ y b dy
b
C
D
dx
(c)

UNIT 4

DEFLECTION OF BEAMS

UNIT IV

Deflection of Beams

Introduction

The deformation of a beam is usually expressed in terms of its deflection from its original unloaded position. The deflection is measured from the original neutral surface of the beam to the neutral surface of the deformed beam. The configuration assumed by the deformed neutral surface is known as the elastic curve of the beam

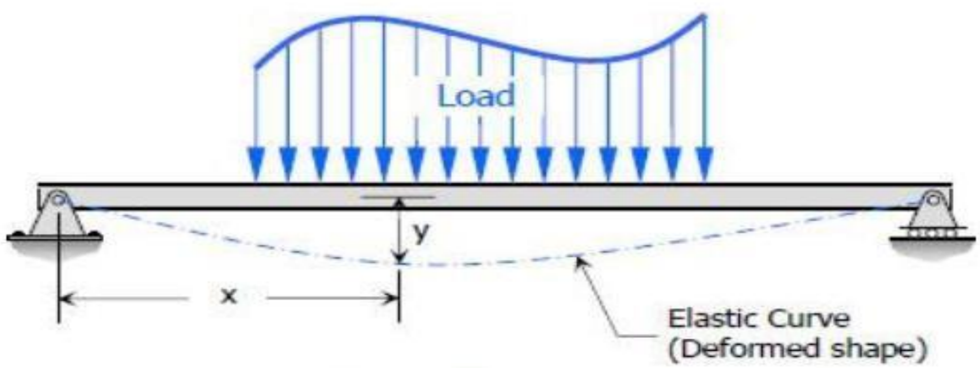


Figure: Elastic curve

A. Methods of Determining Beam Deflections

Methods for the determination of beam deflections include:

1. Double-Integration Method
2. Macaulay's Method
3. Moment-Area Method
4. Conjugate-beam Method
5. Strain - Energy method (Castigliano's Theorem)
6. Virtual work method

Of these methods, the first four shall be discussed in this course.

The stress, strain, dimension, curvature, elasticity, are all related, under certain assumption, by the theory of simple bending. This theory relates to beam flexure resulting from couples applied to the beam without consideration of the shearing forces.

B. Superposition Principle

The superposition principle is one of the most important tools for solving beam loading problems allowing simplification of very complicated design problems.

For beams subjected to several loads of different types the resulting shear force, bending moment, slope and deflection can be found at any location by summing the effects due to each load acting separately to the other loads.

C. Nomenclature

e = strain

E = Young's Modulus = σ /e (N/m^2)

y = distance of surface from neutral surface (m).

R = Radius of neutral axis (m).

I = Moment of Inertia (m^4 - more normally cm^4)

Z = section modulus = I/y_{max}(m^3 - more normally cm^3)

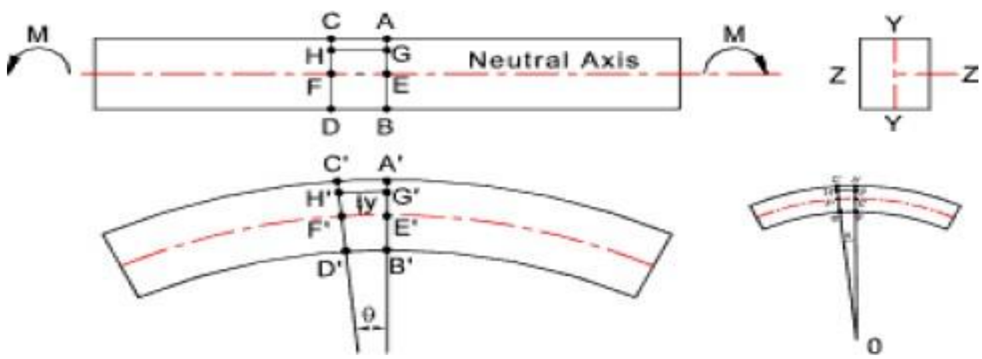
F = Force (N) x = Distance along beam

δ = deflection (m)

θ = Slope (radians) σ = stress (N/m^2)

D. Review of Simple Bending

A straight bar of homogeneous material is subject to only a moment at one end and an equal and opposite moment at the other end...



Assumptions

The beam is symmetrical about Y-Y. The traverse plane sections remain plane and normal to the longitudinal fibres after bending (Beroulli's assumption). The fixed relationship between stress and strain (Young's Modulus)for the beam material is the same for tension and compression (σ= E.e)

Consider two section very close together (AB and CD).

After bending the sections will be at A'B' and C'D' and are no longer parallel. AC will have extended to A'C' and BD will have compressed to B'D' .

The line EF will be located such that it will not change in length. This surface is called neutral surface and its intersection with Z_Z is called the neutral axis.

The development lines of A'B' and C'D' intersect at a point 0 at an angle of θ radians and the radius of E'F' = R

Let y be the distance(E'G') of any layer H'G' originally parallel to EF..Then

H'G'/E'F' =(R+y)θ /R θ = (R+y)/R

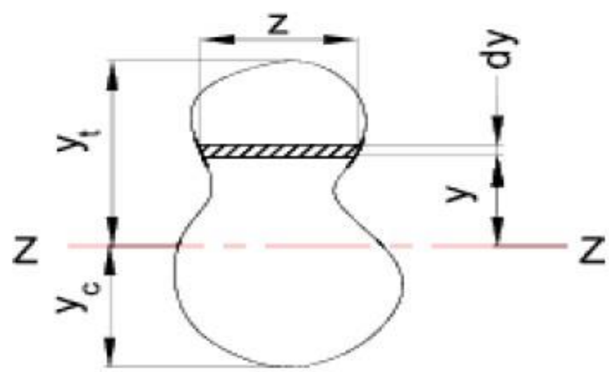
And the strain e at layer H'G' =

e = (H'G'- HG) / HG = (H'G'- HG) / EF = [(R+y)θ - R θ] /R θ = y /R

The accepted relationship between stress and strain is σ= E.e Therefore
σ = E.e = E. y /R σ / E = y / R

Therefore, for the illustrated example, the tensile stress is directly related to the distance above the neutral axis. The compressive stress is also directly related to the distance below the neutral axis. Assuming E is the same for compression and tension the relationship is the same.

As the beam is in static equilibrium and is only subject to moments (no vertical shear forces) the forces across the section (AB) are entirely longitudinal and the total compressive forces must balance the total tensile forces. The internal couple resulting from the sum of (σ.dA .y) over the whole section must equal the externally applied moment.



$$\sum(\sigma.\delta A\ ) = 0 \quad \text{therefore} \quad \sum(\sigma.z\ \delta y) = 0$$

$$\text{As} \quad \sigma = .\frac{yE}{R} \quad \text{therefore} \quad \frac{E}{R}\sum(y.\ \delta A\ ) = 0 \quad \text{and} \quad \frac{E}{R}\sum(y.\ z\delta y) = 0$$

This can only be correct if Σ(yδa) or Σ(y.z.δy) is the moment of area of the section about the neutral axis. This can only be zero if the axis passes through the centre of gravity (centroid) of the section.

The internal couple resulting from the sum of (σ.dA .y) over the whole section must equal the externally applied moment. Therefore the couple of the force resulting from the stress on each area when totalled over the whole area will equal the applied moment

The force on each area element = $\sigma . \delta A = \sigma . z . \delta y$

The resulting moment = $y.\sigma . \delta A = \sigma . z . y . \delta y$

The total moment $M = \sum(y.\sigma.\delta A)$ and $\sum(\sigma.z.y\ \delta y)$

Using $\frac{E}{R} y = \sigma$

$M = \frac{E}{R}\sum(y^2 . \delta A)$ and $\frac{E}{R}\sum(z.y^2 \delta y)$

$\sum(y^2 . \delta A)$ is the Moment of Inertia of the section(I)

From the above the following important simple beam bending relationship results

It is clear from above that a simple beam subject to bending generates a maximum stress at the surface furthest

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

away from the neutral axis. For sections symmetrical about Z-Z the maximum compressive and tensile stress is equal.

$$\sigma_{max} = y_{max} . M / I$$

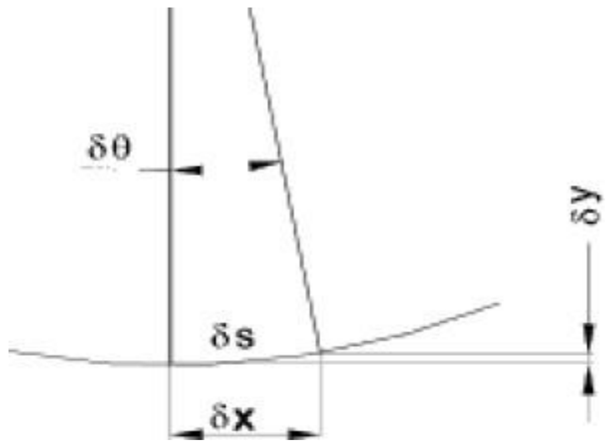
The factor I / y_{max} is given the name section Modulus (Z) and therefore

$$\sigma_{max} = M / Z$$

Values of Z are provided in the tables showing the properties of standard steel sections.

Differential Equation for the Elastic Curve

Below is shown the arc of the neutral axis of a beam subject to bending



For small angle dy/dx = tan θ = θ

The curvature of a beam is identified as dθ /ds = 1/R

In the figure δθ is small and δx; is practically = δs; i.e ds /dx =1

$$\frac{1}{R} = \frac{d\theta}{ds} = \frac{d\theta}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

From this simple approximation the following relationships are derived.

$$\frac{1}{R} = \frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$\text{Slope} = \theta = \frac{dy}{dx} = \int\left(\frac{d^2y}{dx^2}\right)dx = \int \frac{M}{EI}\,dx$$

The deflection between limits is obtained by further integration

$$\text{Deflection} = x = \int \theta\, dx = \int\left(\frac{dy}{dx}\right)dx = \int\int \frac{M}{EI}\,dx$$

It has been proved earlier that dM/dx = -S and dS/dx = w = - d^2M/dx^2 Where S = the shear force M is the moment and w is the distributed load /unit length of beam. Therefore

$$S = \frac{dy}{dx}\left(EI\frac{d^2y}{dx^2}\right) = EI\frac{d^3y}{dx^3} \text{ and } -w = EI\frac{d^4y}{dx^4}$$

If w is constant or a integrable function of x then this relationship can be used to arrive at general expressions for S, M, dy/dx, or y by progressive integrations with a constant of integration being added at each stage. The properties of the supports or fixings may be used to determine the constants. (x= 0 - simply supported, dx/dy = 0 fixed end etc)

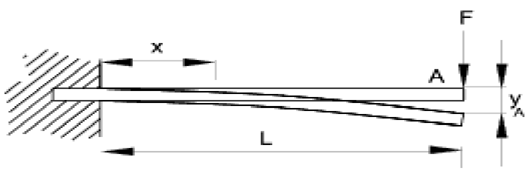
In a similar manner if an expression for the bending moment is known then the slope and deflection

can be obtained at any point x by single and double integration of the relationship and applying

suitable constants of integration of $\frac{d^2y}{dx^2} = \frac{M}{EI}$

Evaluation of deflection by double-integration method

A. Example 1- Cantilever beam
Consider a cantilever beam (uniform section) with a single concentrated load at the end. At the fixed end x = 0, dy = 0 , dy/dx = 0



From the equilibrium balance ..At the support there is a resisting moment -FL and a vertical upward

force F. At any point x along the beam there is a moment $F(x - L) = M_x = EI\, d^2y/dx^2$

$$EI\frac{d^2y}{dx^2} = -F\,(L-x) \quad \text{Integrating}$$

$$EI\frac{dy}{dx} = -F\left(Lx - \frac{x^2}{2}\right) + C_1 \quad \text{.....}(C_1 = 0 \text{ because } dy/dx = 0 \text{ at } x = 0)$$

Integrating again

$$EI\, y = -F\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) + C_2 \quad \text{.....}(C = 0 \text{ because } y = 0 \text{ at } x = 0)$$

$$\text{At end A } \left(\frac{dy}{dx}\right)_A = -\frac{F}{EI}\left(L^2 - \frac{L^2}{2}\right) = -\frac{FL^2}{2EI} \quad \text{and} \quad y_A = -\frac{F}{EI}\left(\frac{L^3}{2} - \frac{L^3}{6}\right) = -\frac{FL^3}{3EI}$$

Macaulay's Method / Singularity Functions

The basic equation governing the slope and deflection of beams is

$\frac{d^2y}{dx^2} = \frac{M}{EI}$, where M is a function of x. This is derived from the Euler-Bernoulli beam theory, based on the simplifying assumptions.

The method of integration of the above equation provides a convenient and effective way of determining the slope and deflection at any point of a beam, as long as the bending moment can be represented by a single analytical function M(x). However, when the loading of the beam is such that two different functions are needed to represent the bending moment over the entire length of the beam four constants of integration are required, and an equal number of equations, expressing continuity conditions at point of concentrated load, as well as boundary conditions at the supports A and B, must be used to determine these constants. If three or more functions were needed to represent the bending moment, additional constants and a corresponding number of additional equations would be required, resulting in rather lengthy computations. In this section these computations will be simplified through the use of the singularity functions.
This is the Macaulay's method.
For general case of loadings, M(x), can be expressed in the form:

$$M(x) = M_1(x) + P_1\langle x - a_1\rangle + P_2\langle x - a_2\rangle + P_3\langle x - a_3\rangle + \ldots$$

where the quantity $P_i\langle x - a_i\rangle$ represents the bending moment at the section 'x' due to point load P_i located at distance a_i from the end. The quantity $\langle x - a_i\rangle$ is a Macaulay bracket defined as

$$\langle x - a_i\rangle = \begin{cases} 0 & \text{if } x < a_i \\ x - a_i & \text{if } x > a_i \end{cases}$$

Ordinarily, when integrating $P_i(x - a_i)$ we get,

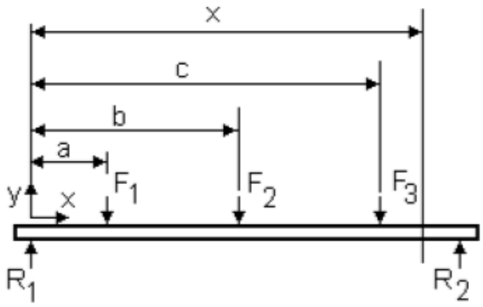
$$\int P(x-a)\ dx = P\left[\frac{x^2}{2} - ax\right] + C$$

However, when integrating expressions containing Macaulay brackets, we have to do this way:

$$\int P\langle x-a\rangle\ dx = P\frac{\langle x-a\rangle^2}{2} + C_m$$

Using these integration rules makes the calculation of the deflection of Euler-Bernoulli beams simple in situations where there are multiple point loads and point moments.

The steps for finding deflections by Macaulay's method are shown by the following example of a simply supported beam:



1. Write down the bending moment equation placing x on the extreme right hand end of the beam so that it contains all the loads. Write all terms containing x in angle brackets.

$$EI\frac{d^2y}{dx^2} = M = R_1\langle x\rangle - F_1\langle x-a\rangle - F_2\langle x-b\rangle - F_3\langle x-c\rangle$$

2. Integrate once treating the whole brackets as the variables.

$$EI\frac{dy}{dx} = R_1\frac{\langle x\rangle^2}{2} - F_1\frac{\langle x-a\rangle^2}{2} - F_2\frac{\langle x-b\rangle^2}{2} - F_3\frac{\langle x-c\rangle^2}{2} + C_1$$

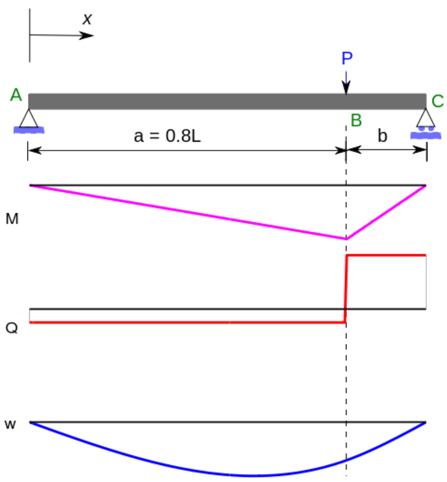
3. Integrate again using the same rules.

$$EI\,y = R_1\frac{\langle x\rangle^3}{6} - F_1\frac{\langle x-a\rangle^3}{6} - F_2\frac{\langle x-b\rangle^3}{6} - F_3\frac{\langle x-c\rangle^3}{6} + C_1x + C_2$$

4. Use boundary conditions to solve C_1 and C_2.

5. Solve slope and deflection by putting in appropriate value of x. IGNORE any brackets containing negative values.

Example 1: Simply Supported Beam with Eccentric Point Load



Consider a simply supported beam with a single eccentric concentrated load as shown in the figure.

The notations used in this example

(a) bending moment = M

(b) shear force = Q

(c) deflection = w (instead of y)

The first step is to find M. The reactions at the supports A and C are determined from the balance of forces and moments as

$$R_A + R_C = P, \quad LR_C = Pa$$

Therefore $R_A = Pb/L$ and the bending moment at a point D between A and B $(0 < x < a)$ is given by

$$M = R_A x = Pbx/L$$

Using the moment-curvature relation and the Euler-Bernoulli expression for the bending moment, we have

$$EI\frac{d^2w}{dx^2} = \frac{Pbx}{L}$$

Integrating the above equation we get, for $0 < x < a$,

$$EI\frac{dw}{dx} = \frac{Pbx^2}{2L} + C_1 \qquad \text{(i)}$$

$$EIw = \frac{Pbx^3}{6L} + C_1x + C_2 \qquad \text{(ii)}$$

At $x = a_-$

$$EI\frac{dw}{dx}(a_-) = \frac{Pba^2}{2L} + C_1 \qquad \text{(iii)}$$

$$EIw(a_-) = \frac{Pba^3}{6L} + C_1a + C_2 \qquad \text{(iv)}$$

For a point D in the region BC $(a < x < L)$, the bending moment is

$$M = R_A x - P(x - a) = Pbx/L - P(x - a)$$

In Macaulay's approach we use the Macaulay bracket form of the above expression to represent the fact that a point load has been applied at location B, i.e.,

$$M = \frac{Pbx}{L} - P\langle x-a\rangle$$

Therefore the Euler-Bernoulli beam equation for this region has the form

$$EI\frac{d^2w}{dx^2} = \frac{Pbx}{L} - P\langle x-a\rangle$$

Integrating the above equation, we get for $a < x < L$

$$EI\frac{dw}{dx} = \frac{Pbx^2}{2L} - P\frac{\langle x-a\rangle^2}{2} + D_1 \qquad \text{(v)}$$

$$EIw = \frac{Pbx^3}{6L} - P\frac{\langle x-a\rangle^3}{6} + D_1x + D_2 \qquad \text{(vi)}$$

At $x = a_+$

$$EI\frac{dw}{dx}(a_+) = \frac{Pba^2}{2L} + D_1 \qquad \text{(vii)}$$

$$EIw(a_+) = \frac{Pba^3}{6L} + D_1a + D_2 \qquad \text{(viii)}$$

Comparing equations (iii) & (vii) and (iv) & (viii) we notice that due to continuity at point B, $D_1 = C_1$ and $D_2 = C_2$. The above observation implies that for the two regions considered, though the equation for bending moment and hence for the curvature are different, the constants of integration got during successive integration of the equation for curvature for the two regions are the same.

The above argument holds true for any number/type of discontinuities in the equations for curvature, provided that in each case the equation retains the term for the subsequent region in the form $\langle x-a\rangle^n, \langle x-b\rangle^n, \langle x-c\rangle^n$ etc. It should be remembered that for any x, giving the quantities within the brackets, as in the above case, -ve should be neglected, and the calculations should be made considering only the quantities which give +ve sign for the terms within the brackets.

Reverting to the problem, we have

$$EI\frac{d^2w}{dx^2} = \frac{Pbx}{L} - P\langle x-a\rangle$$

It is obvious that the first term only is to be considered for $x < a$ and both the terms for $x > a$ and the solution is

$$EI\frac{dw}{dx} = \left[\frac{Pbx^2}{2L} + C_1\right] - \frac{P\langle x-a\rangle^2}{2}$$

$$EIw = \left[\frac{Pbx^3}{6L} + C_1x + C_2\right] - \frac{P\langle x-a\rangle^3}{6}$$

Note that the constants are placed immediately after the first term to indicate that they go with the first term when x < a and with both the terms when x > a. The Macaulay brackets help as a reminder that the quantity on the right is zero when considering points with x < a.

Boundary conditions:

As $w = 0$ at $x = 0$, $C2 = 0$. Also, as $w = 0$ at $x = L$,

$$\left[\frac{PbL^2}{6} + C_1 L\right] - \frac{P(L-a)^3}{6} = 0$$

or,

$$C_1 = -\frac{Pb}{6L}(L^2 - b^2)\ .$$

Hence,

$$EI\frac{dw}{dx} = \left[\frac{Pbx^2}{2L} - \frac{Pb}{6L}(L^2 - b^2)\right] - \frac{P\langle x-a\rangle^2}{2}$$

$$EIw = \left[\frac{Pbx^3}{6L} - \frac{Pbx}{6L}(L^2 - b^2)\right] - \frac{P\langle x-a\rangle^3}{6}$$

Maximum Deflection:

For w to be maximum, $dw/dx = 0$. Assuming that this happens for $x < a$ we have

$$\frac{Pbx^2}{2L} - \frac{Pb}{6L}(L^2 - b^2) = 0$$

or

$$x = \pm\frac{(L^2 - b^2)^{1/2}}{\sqrt{3}}$$

Clearly $x < 0$ cannot be a solution. Therefore, the maximum deflection is given by

$$EIw_{\max} = \frac{1}{3}\left[\frac{Pb(L^2 - b^2)^{3/2}}{6\sqrt{3}L}\right] - \frac{Pb(L^2 - b^2)^{3/2}}{6\sqrt{3}L}$$

or,

$$w_{\max} = -\frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}EIL}\ .$$

Deflection at load application point

At $x = a$, i.e., at point B, the deflection is

$$EIw_B = \frac{Pba^3}{6L} - \frac{Pba}{6L}(L^2 - b^2) = \frac{Pba}{6L}(a^2 + b^2 - L^2)$$

or

$$w_B = -\frac{Pa^2b^2}{3LEI}$$

Deflection at the mid-point

It is instructive to examine the ratio of $w_{\max}/w(L/2)$. At $x = L/2$

$$EIw(L/2) = \frac{PbL^2}{48} - \frac{Pb}{12}(L^2 - b^2) = -\frac{Pb}{12}\left[\frac{3L^2}{4} - b^2\right]$$

Therefore,

$$\frac{w_{\max}}{w(L/2)} = \frac{4(L^2 - b^2)^{3/2}}{3\sqrt{3}L\left[\frac{3L^2}{4} - b^2\right]} = \frac{4(1 - \frac{b^2}{L^2})^{3/2}}{3\sqrt{3}\left[\frac{3}{4} - \frac{b^2}{L^2}\right]} = \frac{16(1 - k^2)^{3/2}}{3\sqrt{3}\,(3 - 4k^2)}$$

where k = b/L and for a < b we get 0 < k < 0.5. Even when the load is as near as 0.05L from the support, the error in estimating the deflection is only 2.6%. Hence in most of the cases the estimation of maximum deflection may be made fairly accurately with reasonable margin of error by working out deflection at the centre.

Special case of symmetrically applied load

When $a = b = L/2$, for w to be maximum

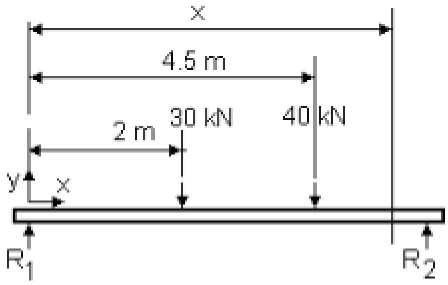
$$x = \frac{[L^2 - (L/2)^2]^{1/2}}{\sqrt{3}} = \frac{L}{2}$$

and the maximum deflection is

$$w_{\max} = -\frac{P(L/2)b[L^2 - (L/2)^2]^{3/2}}{9\sqrt{3}EIL} = -\frac{PL^3}{48EI} = w(L/2)\ .$$

Example 2: Simply Supported Beam with Two Point Loads

The beam shown is 7 m long with an EI value of 200 MN/m^2. Determine the slope and deflection at the middle of the span.



P.T.O.

SOLUTION

First solve the reactions by taking moments about the right end.

30 x 5 + 40 x 2.5 = 7 R_1 hence R_1 = 35.71 kN

R_2 = 70 - 35.71 = 34.29 kN

Next write out the bending equation.

$$EI\frac{d^2y}{dx^2} = M = 35710[x] - 30000[x-2] - 40000[x-4.5]$$

Integrate once treating the square bracket as the variable.

$$EI\frac{dy}{dx} = 35710\frac{[x]^2}{2} - 30000\frac{[x-2]^2}{2} - 40000\frac{[x-4.5]^2}{2} + A.....(1)$$

Integrate again

$$EIy = 35710\frac{[x]^3}{6} - 30000\frac{[x-2]^3}{6} - 40000\frac{[x-4.5]^3}{6} + Ax + B......(2)$$

BOUNDARY CONDITIONS

x = 0, y = 0 and x = 7 y = 0

Using equation 2 and putting x = 0 and y = 0 we get

$$EI(0) = 35710\frac{[0]^3}{6} - 30000\frac{[0-2]^3}{6} - 40000\frac{[0-4.5]^3}{6} + A(0) + B$$

Ignore any bracket containing a negative value.

$0 = 0 - 0 - 0 + 0 + B$ hence $B = 0$

Using equation 2 again but this time x=7 and y = 0

$$EI(0) = 35710\frac{[7]^3}{6} - 30000\frac{[7-2]^3}{6} - 40000\frac{[7-4.5]^3}{6} + A(7) + 0$$

Evaluate A and $A = -187400$

Moment-Area Method

The moment-area theorem is a method to derive the slope, rotation and deflection of beams and frames. This theorem was developed by Mohr and later stated namely by Charles E. Greene in 1873. This method is advantageous when we solve problems involving beams, especially for those subjected to a series of concentrated loadings or having segments with different moments of inertia. If we draw the moment diagram for the beam and then divided it by the flexural rigidity(EI), the 'M/EI diagram' results by the following:

$$\frac{d^2y}{dx^2} = \frac{d\theta}{dx} = \frac{M}{EI} \Rightarrow \theta(x) = \int \frac{M}{EI} dx$$

A. Mohr's Theorems

Theorem 1: The change in slope between any two points on the elastic curve equals the area of the $\frac{M}{EI}$ diagram between these two points.

$$\theta_{AB} = \int_A^B \frac{M}{EI} dx$$

where,

- M = bending moment expression as a function of x
- EI = flexural rigidity
- θ_{AB} = change in slope between points A and B
- A, B = points on the elastic curve

Theorem 2: The vertical deviation of a point A on an elastic curve with respect to the tangent which is extended from another point B equals the moment of the area under the M/EI diagram between those two points (A and B). This moment is computed about point A where the deviation from B to A is to be determined.

$$t_{A/B} = \int_A^B \left(\frac{M}{EI}\right) x dx$$

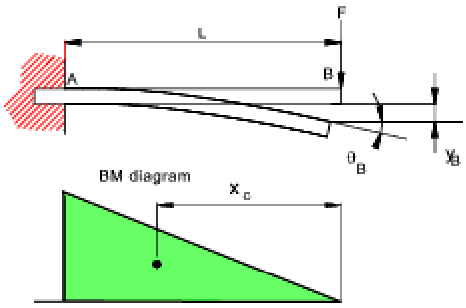
where,

- M = bending moment expression as a function of x
- EI = flexural rigidity
- $t_{A/B}$ = deviation of tangent at point B with respect to the tangent at point A

- A, B = points on the elastic curve

Two simple examples are provide below to illustrate these theorems

Example 1) Determine the deflection and slope of a cantilever as shown..



The bending moment at A = M_A = FL

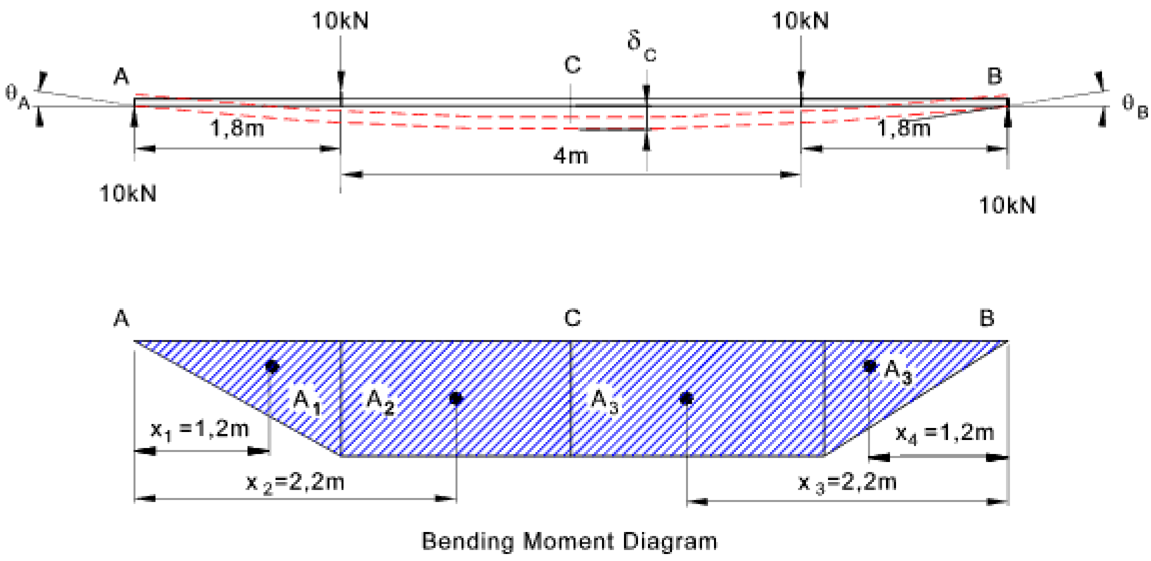
The area of the bending moment diagram $A_M = F.L^2 /2$

The distance to the centroid of the BM diagram from B = x_c = 2L/3 The deflection of B = $y_b = A_M.x_c /EI = F.L^3 /3EI$

The slope at B relative to the tan at A = $\theta_b = A_M /EI = FL^2 /2EI$

Example 2) Determine the central deflection and end slopes of the simply supported beam as shown..

E = 210 GPa ......I = 834 cm^4...... EI = 1,7514. $10^6 Nm^2$



A_1 = 10.1, 8.1, 8/2 =
16.2kNm A_2 = 10.1,8.2 =
36kNm
A_2 = 10.1,8.2 = 36kNm
A_1 = 10.1,8.1,8/2 = 16,2kNm
x_1 = Centroid of A_1 = (2/3).1,8 =
1,2 x_2 = Centroid of A_2 = 1,8 + 1
= 2,8 x_3 = Centroid of A_3 = 1,8 +
1 = 2,8 x_4 = Centroid of A_4 =
(2/3).1,8 = 1,2

The slope at A is given by the area of the moment diagram between A and C divided by EI.

$\theta_A = (A_1 + A_2)$ /EI = (16,2+36).10^{3} / (1,7514. 10^{6})
= 0,029rads = 1,7 degrees

The deflection at the centre (C) is equal to the deviation of the point A above a line that is tangent to C.
Moments must therefore be taken about the deviation line at A.

$\delta_C = (A_M.x_M)$ /EI = $(A_1 x_1 + A_2 x_2)$ / EI = 120,24.10^{3}/ (1,7514. 10^{6})
= 0.0686m = 68.6mm

Tutorial Questions

1. A cantilever 3m long has moment of inertia 800 Cm^4 for 1m length from the free end, 1600 cm^4 for the next 1m length 2400 Cm^4 for the last 1m. length. At the free end a load of 1 KN acts on the cantilever. Determine the slope and deflections at the free end of the cantilever E= 210 GN/ m2
2. A simply supported beam of span 6m carries two point loads of 60KN and 50KN at 1m and 3m respectively from the left end. Find the position and magnitude of max. deflection. Take E= as 200 GPa and I =8500cm^4. Also determine the value of deflection at the same point if one more load of 60KN is placed over the left support.
3. A beam AB of 8 m span is simply supported at the ends. It carries a point load of 10 kN at a distance of 1 m from the end A and a uniformly distributed load of 5 kN/m for a length of 2 m from the end B. If I = 10 _ 106 m4, Using Macaulay`s Method, Determine:
   (a) Deection at the mid-span,
   (b) Maximum deection, and
   (c) Slope at the end A.
4. A simply supported beam of span 6m carries two point loads of 60KN and 50KN at 1m and 3m respectively from the left end. Find the position and magnitude of max. deflection. Take E= as 200 GPa and I =8500cm^4. Also determine the value of deflection at the same point if one more load of 60KN is placed over the left support.
5. A simply supported beam of 8m carries a partial u d l of intensity 5KN/m and length 2m, sarting from 2m from the left end. Find slope at left support and central deflection. Take E= 200Gpa and I=$8\times10^8$$mm^4$

Assignment Questions

1. A simply supported beam of 8m carries a partial u d l of intensity 5KN/m and length 2m, sarting from 2m from the left end. Find slope at left support and central deflection. Take E= 200Gpa and I=8×10^8mm^4
2. A simply supported beam span 14m, carrying concentrated loads of 12KN and 8KN at two points 3mts and 4.5m from the two ends respectively. Moment of Inertia I for the beam is 160 x10^3 mm^4 and E = 210KN/mm^2. Calculate deflection of the beam at points under the two loads by macaulay's method
3. A Cantilever beam AB 6 mts long is subjected to u.d.l of w KN/m spread over the entire length. Assume rectangular cross-section with depth equal to twice the breadth. Determine the minimum dimension of the beam so that the vertical deflection at free end does not exceed 1.5 cm and the maximum stress due to bending does not exceed 10 KN/cm^2. E = 2 X 10^7 N/ cm^2.
4. A beam section is 10m long and is simply supported at ends. It carries concentrated loads of100kN and 60kN at a distance of 2m and 5m respectively from the left end. Calculate the deflection under the each load find also the maximum deflection. Take I = 18 X 10^8mm^4 and E = 200kN/mm^2.
5. A simply supported beam of span 6m carries two point loads of 60KN and 50KN at 1m and 3m respectively from the left end. Find the position and magnitude of max. deflection. Take E= as 200 GPa and I =8500cm^4. Also determine the value of deflection at the same point if one more load of 60KN is placed over the left support.

UNIT-IV

POWER POINT PRESENTAION

DEFLECTION OF BEAMS

UNIT IV

MRCET CAMPUS
UGC Autonomous

DEPARTMENT OF MECHANICAL ENGINEERING

BEAM

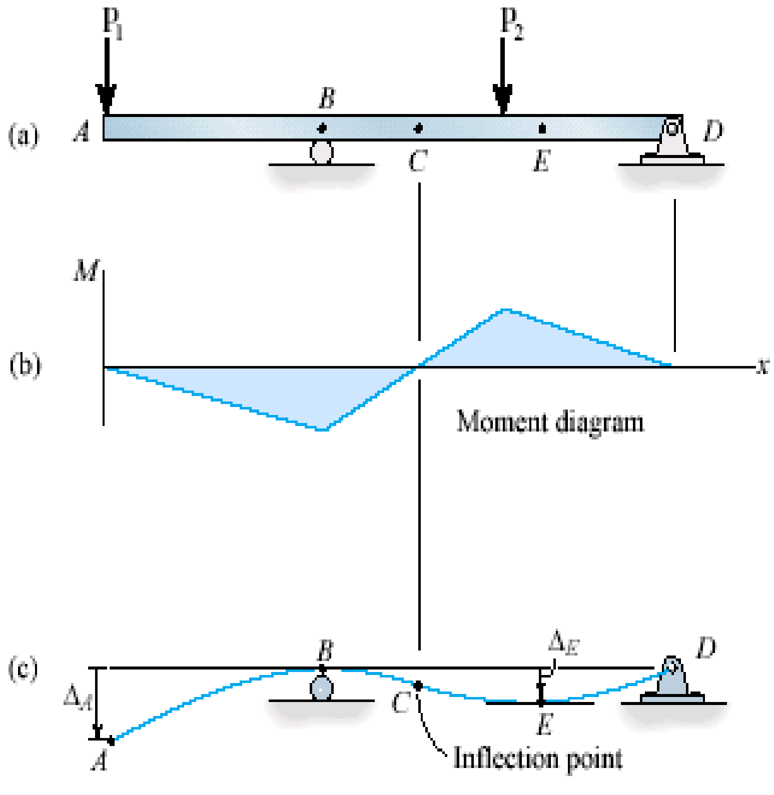
- A beam is a structural member (horizontal) that is design to support the applied load (vertical). It resists the applied loading by a combination of internal transverse **shear force** and bending **moment**.
- It is perhaps the most important and widely used structural members and can be classified according to its support conditions.

INTRODUCTION

- Calculation of deflections is an important part of structural analysis
- Excessive beam deflection can be seen as a mode of failure.
  - Extensive glass breakage in tall buildings can be attributed to excessive deflections
  - Large deflections in buildings are unsightly (and unnerving) and can cause cracks in ceilings and walls.
  - Deflections are limited to prevent undesirable vibrations

BEAM DEFLECTION

- Bending changes the initially straight longitudinal axis of the beam into a curve that is called the **Deflection Curve** or **Elastic Curve**



Elastic curve

BEAM DEFLECTION

- Consider a cantilever beam with a concentrated load acting upward at the free end.
- Under the action of this load the axis of the beam deforms into a curve
- The deflection Δ is the displacement in the y direction on any point on the axis of the beam

BEAM DEFLECTION

- Because the y axis is positive upward, the deflections are also positive when upward.
  - Traditional symbols for displacement in the x, y, and z directions are u, v, and w respectively.

BEAM DEFLECTION

- To determine the deflection curve:
  - Draw shear and moment diagram for the beam
  - Directly under the moment diagram draw a line for the beam and label all supports
  - At the supports displacement is zero
  - Where the moment is negative, the deflection curve is concave downward.
  - Where the moment is positive the deflection curve is concave upward
  - Where the two curve meet is the Inflection Point

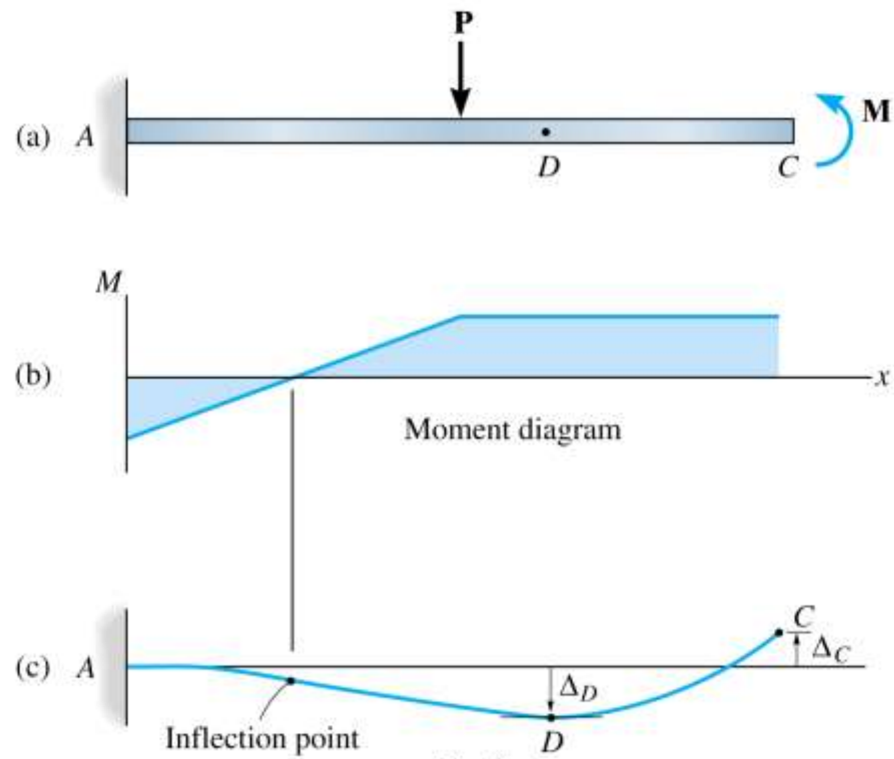
BEAM DEFLECTION

Positive internal moment concave upwards

(a)

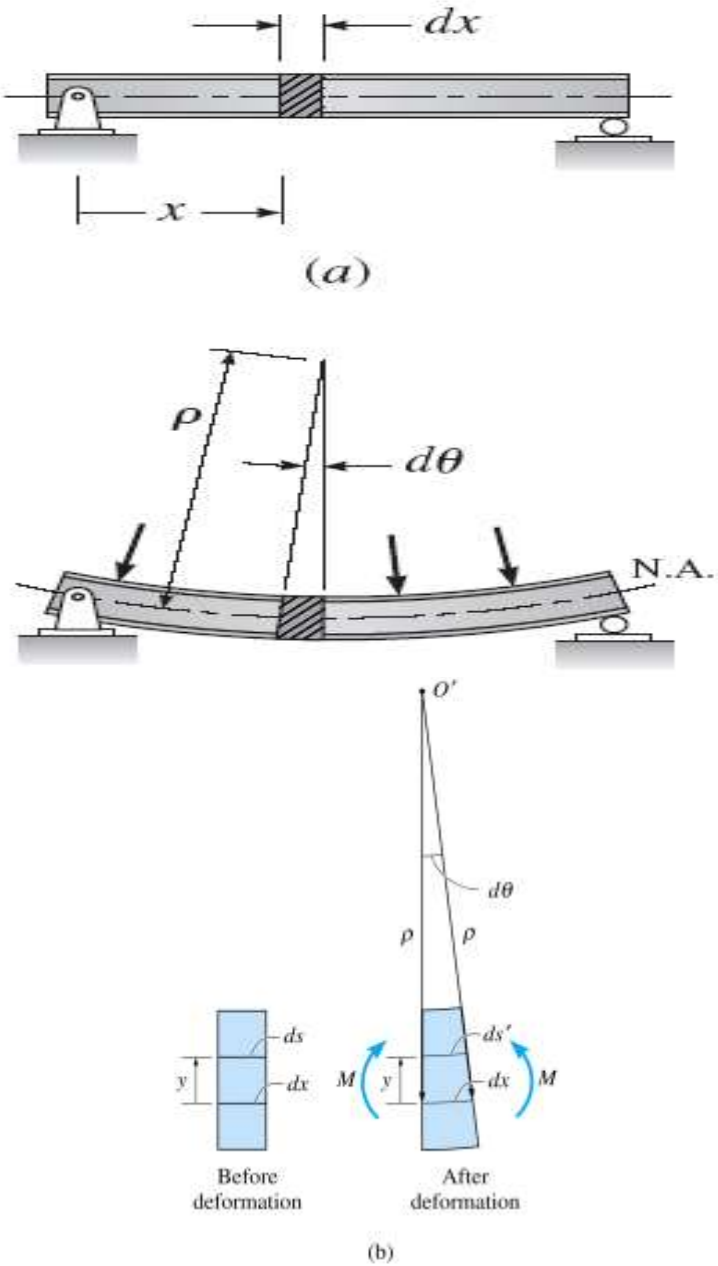
Negative internal moment concave downwards

(b)



ELASTIC-BEAM THEORY

- Consider a differential element of a beam subjected to pure bending.
- The radius of curvature ρ is measured from the center of curvature to the neutral axis
- Since the NA is unstretched, the $dx = \rho d\theta$



ELASTIC-BEAM THEORY

- The fibers below the NA are lengthened
- The unit strain in these fibers is:

$$\varepsilon = \frac{\left(ds' - ds\right)}{ds} = \frac{\left(\rho - y\right)d\theta - \rho d\theta}{\rho d\theta} \qquad \text{or}$$

$$\frac{1}{\rho} = -\frac{\varepsilon}{y}$$

ELASTIC-BEAM THEORY

- Below the NA the strain is positive and above the NA the strain is negative for positive bending moments.
- Applying Hooke's law and the Flexure formula, we obtain:

- The Moment curvature equation

$$\frac{1}{\rho} = \frac{M}{EI}$$

ELASTIC-BEAM THEORY

- The product *EI* is referred to as the flexural rigidity.
- Since $dx = \rho d\theta$, then

$$d\theta = \frac{M}{EI} dx \quad (Slope)$$

- In most calculus books

$$\frac{1}{\rho} = \frac{d^2v/dx^2}{\left[1+(dv/dx)^2\right]^{\frac{3}{2}}}$$

$$\frac{M}{EI} = \frac{d^2v/dx^2}{\left[1+(dv/dx)^2\right]^{\frac{3}{2}}} \qquad (exact \quad solution)$$

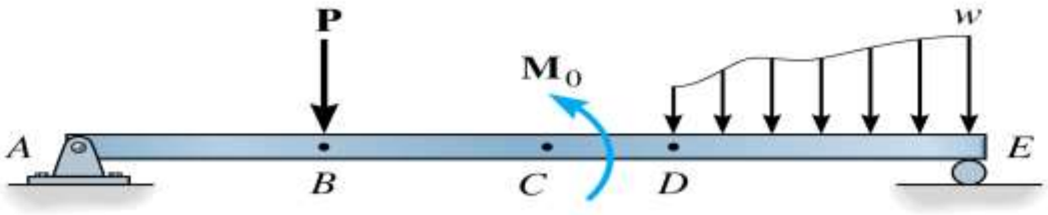
$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

THE DOUBLE INTEGRATION METHOD

- Once M is expressed as a function of position x, then successive integrations of the previous equations will yield the beams slope and the equation of the elastic curve, respectively.
- Wherever there is a discontinuity in the loading on a beam or where there is a support, there will be a discontinuity.

Consider a beam with several applied loads.

  – The beam has four intervals, AB, BC, CD, DE
  – Four separate functions for Shear and Moment



THE DOUBLE INTEGRATION METHOD

Relate Moments to Deflections

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

$$\theta(x) = \frac{dv}{dx} = \int \frac{M(x)}{EI(x)}dx$$

$$v(x) = \iint \frac{M(x)}{EI(x)}dx^2$$

Integration Constants

Use Boundary Conditions to Evaluate Integration Constants

MOMENT-AREA THEOREMS

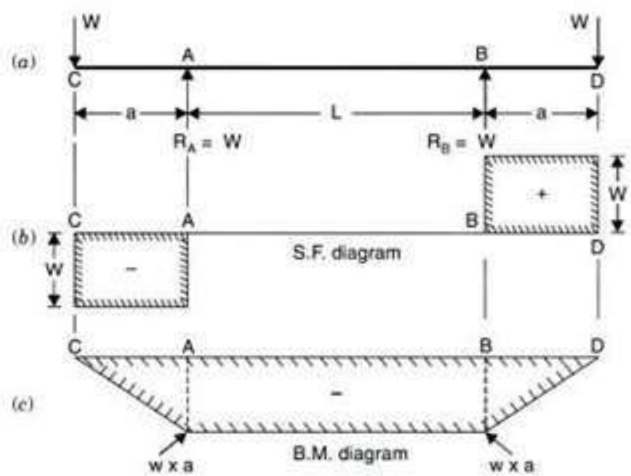
- The moment-area theorems procedure can be summarized as:
- If A and B are two points on the deflection curve of a beam, EI is constant and B is a point of zero slope, then the Mohr's theorems state that:
- (1) Slope at A = 1/EI x area of B.M. diagram between A and B
- (2) Deflection at A relative to B = 1/EI x first moment of area of B.M diagram between A and B about A.

4.1 SIMPLE BENDING OR PURE BENDING

- When some external force acts on a beam, the shear force and bending moments are set up at all the sections of the beam
- Due to shear force and bending moment, the beam undergoes deformation. The material of the beam offers resistance to deformation
- Stresses introduced by bending moment are known as bending stresses
- Bending stresses are indirect normal stresses

4.1 SIMPLE BENDING OR PURE BENDING

- When a length of a beam is subjected to zero shear force and constant bending moment, then that length of beam is subjected to pure bending or simple pending.
- The stress set up in that length of the beam due



4.1 SIMPLE BENDING OR PURE BENDING

- Consider a simply supported beam with over hanging portions of equal lengths. Suppose the beam is subjected to equal loads of intensity W at either ends of the over hanging portion
- In the portion of beam of length l, the beam is subjected to constant bending moment of intensity w x a and shear force in this portion is zero
- Hence the portion AB is said to be subjected to pure bending or simple bending

4.2 ASSUMPTIONS FOR THE THEORY OF PURE BENDING

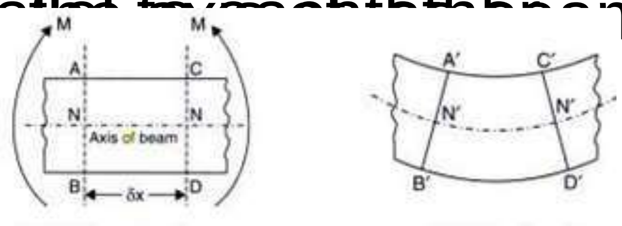
- The material of the beam is isotropic and homogeneous. Ie of same density and elastic properties throughout
- The beam is initially straight and unstressed and all the longitudinal filaments bend into a circular arc with a common centre of curvature
- The elastic limit is nowhere exceeded during the bending
- Young's modulus for the material is the same in tension and compression

4.2 ASSUMPTIONS FOR THE THEORY OF PURE BENDING

- The transverse sections which were plane before bending remain plane after bending also
- Radius of curvature is large compared to the dimensions of the cross section of the beam
- There is no resultant force perpendicular to any cross section
- All the layers of the beam are free to elongate and contract, independently of the layer, above or below it.

4.3 THEORY OF SIMPLE BENDING

- Consider a beam subjected to simple bending. Consider an infinitesimal element of length dx which is a part of this beam. Consider two transverse sections AB and CD which are normal to the axis of the beam and



- Due to the bending action the element ABCD is deformed to A’B’C’D’ (concave curve).
- The layers of the beam are not of the same length before bending and after bending .

4.3 THEORY OF SIMPLE BENDING

- The layer AC is shortened to A'C'. Hence it is subjected to compressive stress
- The layer BD is elongated to B'D'. Hence it is subjected to tensile stresses.
- Hence the amount of shortening decrease from the top layer towards bottom and the amount of elongation decreases from the bottom layer towards top
- Therefore there is a layer in between which neither elongates nor shortens. This layer is called neutral layer .

4.3 THEORY OF SIMPLE BENDING

- The filaments/ fibers of the material are subjected to neither compression nor to tension
- The line of intersection of the neutral layer with transverse section is called neutral axis (N-N).
- Hence the theory of pure bending states that the amount by which a layer in a beam subjected to pure bending, increases or decreases in length, depends upon the position of the layer w.r.t neutral axis N-N.

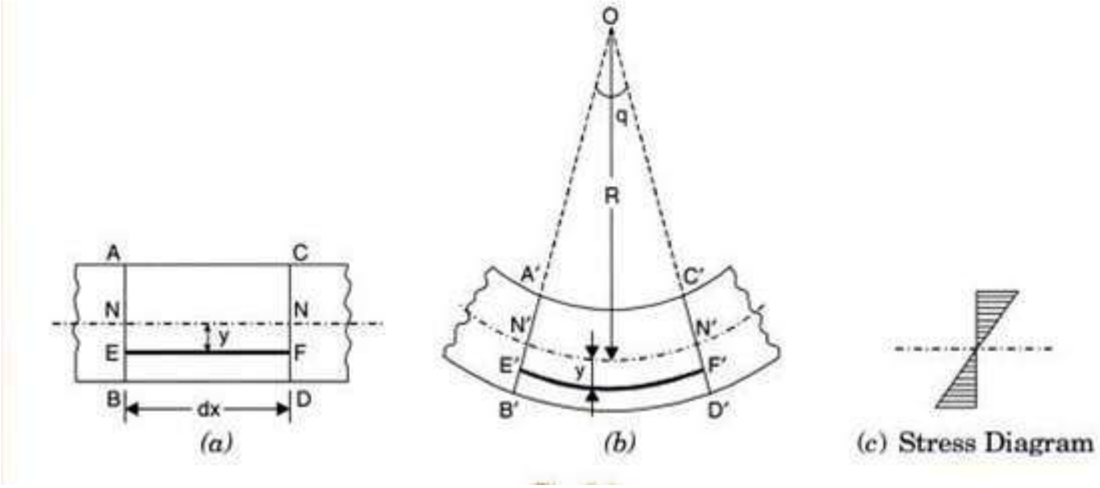
4.4 EXPRESSION FOR BENDING STRESS

- Consider a beam subjected to simple bending. Consider an infinitesimal element of length dx which is a part of this beam. Consider two transverse sections AB and CD which are normal to the axis of the beam. Due to the bending action the element ABCD is deformed to A'B'C'D' (concave curve).
- The lines B'A' and D'C' when extended meet at point O (which is the centre of curvature for the circular arc formed).
- Let R be the radius of curvature of the neutral axis.

4.4.1 STRAIN VARIATION ALONG THE DEPTH OF BEAM

- Consider a layer EF at a distance y from the neutral axis. After bending this layer will be deformed to E'F'.
- Strain developed= (E'F'-EF)/EF

EF=NN=dx=R x θ

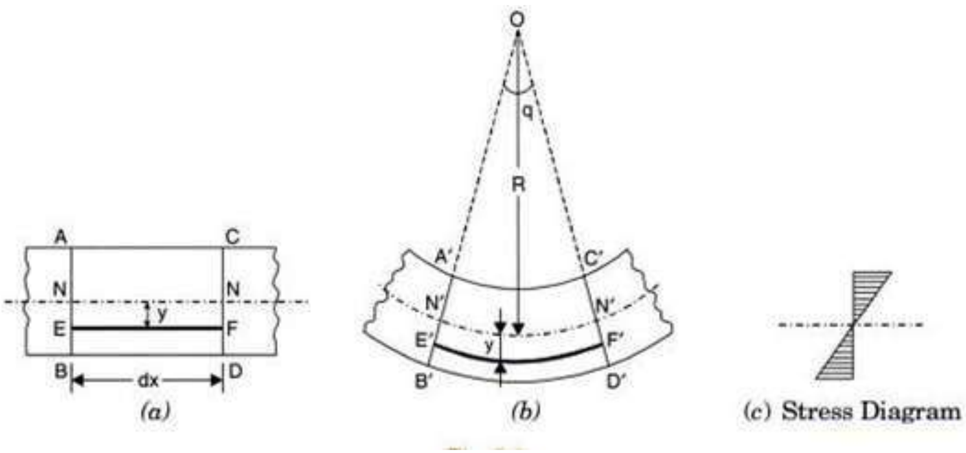


4.4.1 STRAIN VARIATION ALONG THE DEPTH OF BEAM

- Strain developed eb= { (R + y) x θ - R x θ)}/ R x θ=y/R
- STRESS VARIATION WITH DEPTH OF BEAM
- σ/E= y/R or σ= Ey/R or σ/y = E/R
- Hence σ varies linearly with y (distance from neutral axis)
- Therefore stress in any layer is directly proportional to the distance of the layer from the neutral layer

4.5 NEUTRAL AXIS

- For a beam subjected to a pure bending moment, the stresses generated on the neutral layer is zero.
- Neutral axis is the line of intersection of neutral layer with the transverse section
- Consider the cross section of a beam subjected to pure bending. The stress at a distance y from
- =E/R



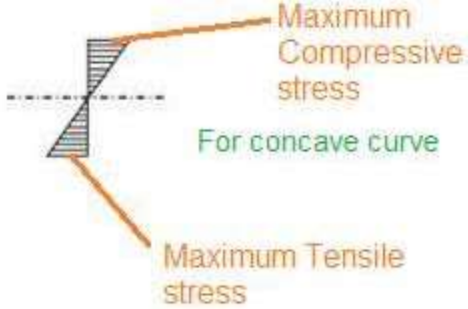
4.5 NEUTRAL AXIS

- $\sigma = E \times y/R$;
- The force acting perpendicular to this section, $dF = E \times y/R \times dA$, where dA is the cross sectional area of the strip/layer considered.
- Pure bending theory is based on an assumption that "There is no resultant force perpendicular to any cross section". Hence $F=0$;
- Hence, $E/R \times \int y\,dA = 0$

=> $\int y\,dA$= Moment of area of the entire cross section w.r.t the neutral axis=0

4.5 NEUTRAL AXIS

- Moment of area of any surface w.r.t the centroidal axis is zero. Hence neutral axis and centroidal axis for a beam subjected to simple bending are the same.
- Neutral axis coincides with centrodial axis or the centroidal axis gives the position of neutral axis



Stress strain variation w.r.t the depth of the beam

4.6 MOMENT OF RESISTANCE

- Due to the tensile and compressive stresses, forces are exerted on the layers of a beam subjected to simple bending
- These forces will have moment about the neutral axis. The total moment of these forces about the neutral axis is known as moment of resistance of that section
- We have seen that force on a layer of cross sectional area dA at a distance y from the neutral axis,

$dF = (E \times y \times dA)/R$

Moment of force dF about the neutral axis= $dF \times y = (E \times y \times dA)/R \times y = E/R \times (y^2 dA)$

4.6 MOMENT OF RESISTANCE

- Hence the total moment of force about the neutral axis= Bending moment applied= $\int E/R \times (y^2 dA)$= $E/R \times I_{xx}$; Ixx is the moment of area about the neutral axis/centroidal axis.

Hence $M=E/R \times I_{xx}$ Or $M/I_{xx}=E/R$

- Hence $M/I_{xx}=E/R = \sigma_b/y$; σ_b is also known as flexural stress (Fb)
- Hence $\boxed{\mathbf{M/I_{xx}=E/R=F_b/y}}$
- The above equation is known as bending equation
- This can be remembered using the sentence "Elizabeth Rani May I Follow You"

4.7 CONDITION OF SIMPLE BENDING & FLEXURAL RIGIDITY

- Bending equation is applicable to a beam subjected to pure/simple bending. Ie the bending moment acting on the beam is constant and the shear stress is zero
- However in practical applications, the bending moment varies from section to section and the shear force is not zero
- But in the section where bending moment is maximum, shear force (derivative of bending moment) is zero
- Hence the bending equation is valid for the section where bending moment is maximum

4.7 CONDITION OF SIMPLE BENDING & FLEXURAL RIGIDITY

- Or in other words, the condition of simple bending may be satisfied at a section where bending moment is maximum.
- Therefore beams and structures are designed using bending equation considering the section of maximum bending moment
- Flexural rigidity/Flexural resistance of a beam:-
- For pure bending of uniform sections, beam will deflect into circular arcs and for this reason the term circular bending is often used.

4.7 CONDITION OF SIMPLE BENDING & FLEXURAL RIGIDITY

- The radius of curvature to which any beam is bent by an applied moment M is given by R=EI/M
- Hence for a given bending moment, the radius of curvature is directly related to “EI”
- Since radius of curvature is a direct indication of the degree of flexibility of the beam (larger the value of R, less flexible the beam is, more rigid the beam is), EI is known as flexural rigidity of flexural stiffness of the beam.
- The relative stiffnesses of beam sections can then easily be compared by their EI value

4.8 SECTIONAL MODULUS (Z)

- Section modulus is defined as the ratio of moment of area about the centroidal axis/neutral axis of a beam subjected to bending to the distance of outermost layer/fibre/filament from the centroidal axis
- **$Z = I_{xx}/y_{max}$**
- From the bending equation, $M/I_{xx} = \sigma_{bmax}/y_{max}$

Hence $I_{xx}/y_{max} = M/\sigma_{bmax}$ $M = \sigma bmax \times Z$

- Higher the Z value for a section, the higher the BM which it can withstand for a given maximum stress

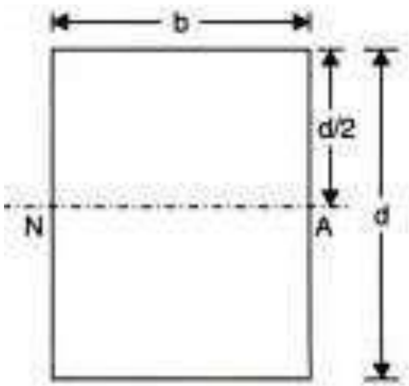
VARIOUS SHAPES OR BEAM SECTIONS

- 1) For a Rectangular section

  $Z = I_{xx}/y_{max}$

$I_{xx} = I_{NA} = bd^3/12 \quad y_{max} = d/2$

$Z = bd^2/6$

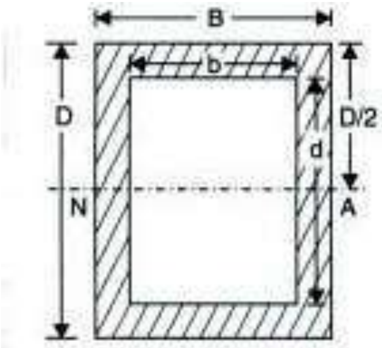


- 2) For a Rectangular hollow sect

  $I_{xx} = 1/12 \times (BD^3/12 - \quad bd^3/12)$ i

  $Y_{max} = D/2$

$Z = (BD^3 - bd^3)/6D$



UNIT 5

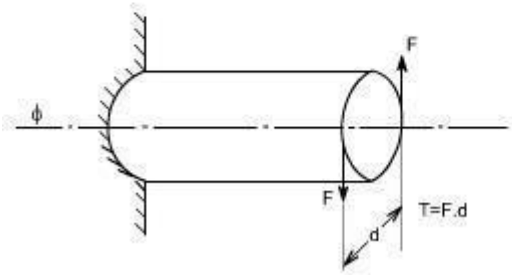
TORSION OF CIRCULAR SHAFTS &THIN CYLINDERS

TORSION OF CIRCULAR SHAFTS AND THIN CYLINDERS

Members Subjected to Torsional Loads Torsion

of circular shafts

Definition of Torsion: Consider a shaft rigidly clamped at one end and twisted at the other end by a torque T = F.d applied in a plane perpendicular to the axis of the bar such a shaft is said to be in torsion.



Effects of Torsion: The effects of a torsional load applied to a bar are

(i) To impart an angular displacement of one end cross – section with respect to the other end.

(ii) To setup shear stresses on any cross section of the bar perpendicular to its axis.

GENERATION OF SHEAR STRESSES

The physical understanding of the phenomena of setting up of shear stresses in a shaft subjected to a torsion may be understood from the figure 1-3.

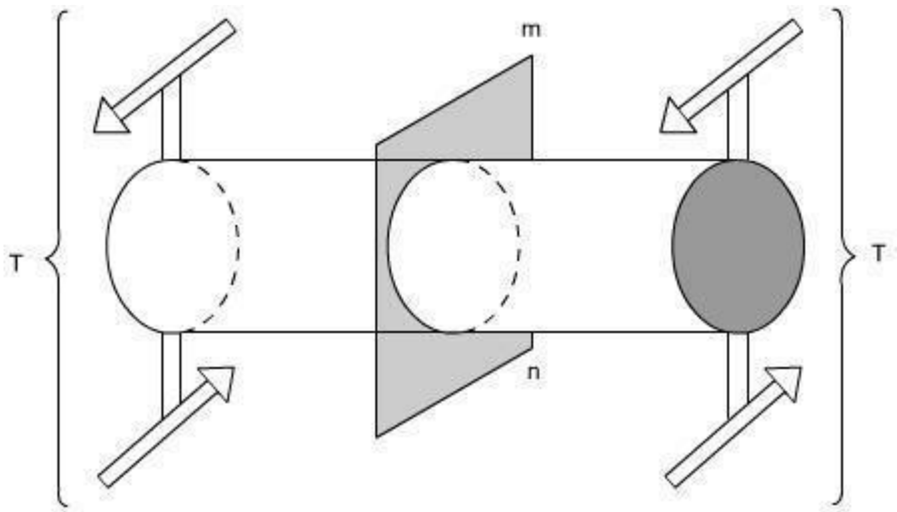


Fig 1: Here the cylindrical member or a shaft is in static equilibrium where T is the resultant external torque acting on the member. Let the member be imagined to be cut by some imaginary plane 'mn'.

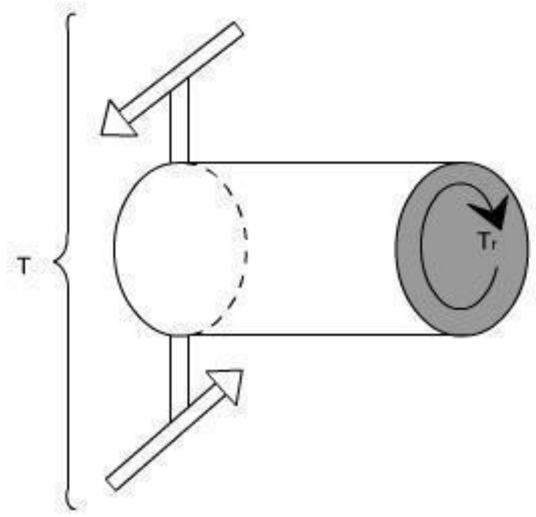


Fig 2: When the plane 'mn' cuts remove the portion on R.H.S. and we get a fig 2. Now since the entire member is in equilibrium, therefore, each portion must be in equilibrium. Thus, the member is in equilibrium under the action of resultant external torque T and developed resisting Torque T_r.

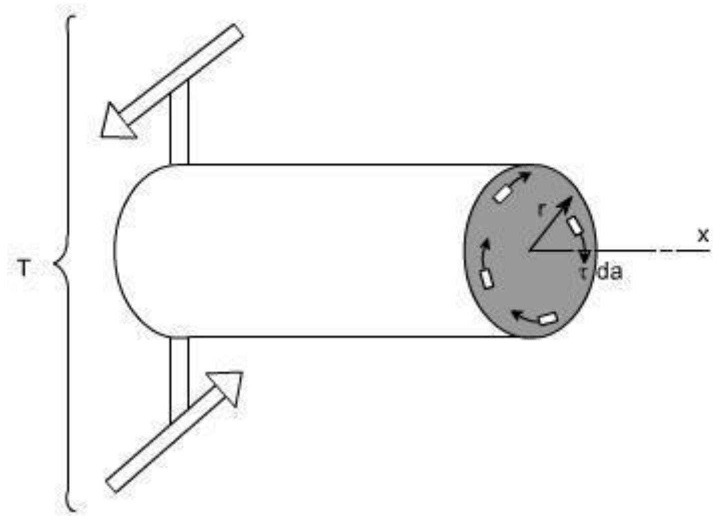


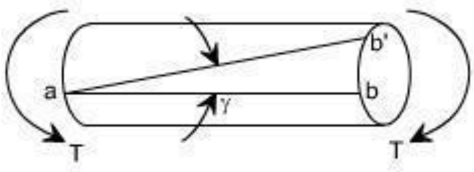
Fig 3: The Figure shows that how the resisting torque T_r is developed. The resisting torque T_r is produced by virtue of an infinites mal shear forces acting on the plane perpendicular to the axis of the shaft. Obviously such shear forces would be developed by virtue of sheer stresses.

Therefore we can say that when a particular member (say shaft in this case) is subjected to a torque, the result would be that on any element there will be shear stresses acting. While on other faces the complementary sheer forces come into picture. Thus, we can say that when a member is subjected to torque, an element of this member will be subjected to a state of pure shear.

Shaft: The shafts are the machine elements which are used to transmit power in machines.

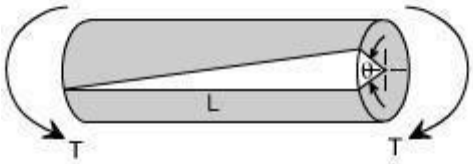
Twisting Moment: The twisting moment for any section along the bar / shaft is defined to be the algebraic sum of the moments of the applied couples that lie to one side of the section under consideration. The choice of the side in any case is of course arbitrary.

Shearing Strain: If a generator a – b is marked on the surface of the unloaded bar, then after the twisting moment 'T' has been applied this line moves to ab'. The angle '□' measured in radians, Between the final and original positions of the generators is defined as the shearing strain at the surface of the bar or shaft. The same definition will hold at any interior point of the bar.



Modulus of Elasticity in shear: The ratio of the shear stress to the shear strain is called the modulus of elasticity in shear OR Modulus of Rigidity and in represented by the symbol $G = \frac{\tau}{r}$

Angle of Twist: If a shaft of length L is subjected to a constant twisting moment T along its length, than the angle through which one end of the bar will twist relative to the other is known is the angle of twist.

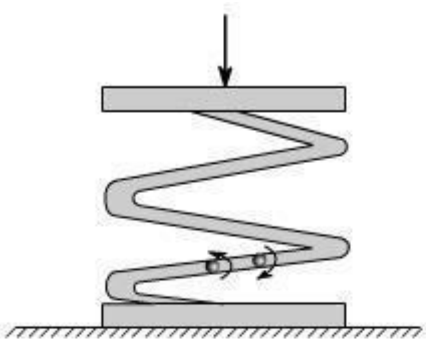


- Despite the differences in the forms of loading, we see that there are number of similarities between bending and torsion, including for example, a linear variation of stresses and strain with position.

  In torsion the members are subjected to moments (couples) in planes normal to their axes.

- For the purpose of desiging a circular shaft to withstand a given torque, we must develop an equation giving the relation between twisting moment, maximum shear stress produced, and a quantity representing the size and shape of the cross-sectional area of the shaft.

Not all torsion problems, involve rotating machinery, however, for example some types of vehicle suspension system employ torsional springs. Indeed, even coil springs are really curved members in torsion as shown in figure.



- Many torque carrying engineering members are cylindrical in shape. Examples aredrive shafts, bolts and screw drivers.

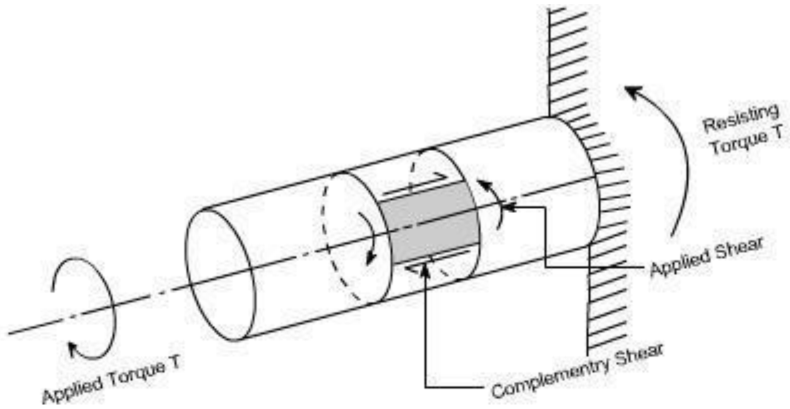
Simple Torsion Theory or Development of Torsion Formula : Here we are basically interested to derive an equation between the relevant parameters

Relationship in Torsion: $\frac{T}{J} = \frac{\tau}{r} = \frac{G.\theta}{l}$

1 st Term: It refers to applied loading ad a property of section, which in the instance is the polar second moment of area.

2 nd Term: This refers to stress, and the stress increases as the distance from the axis increases.

3 rd Term: it refers to the deformation and contains the terms modulus of rigidity & combined term which is equivalent to strain for the purpose of designing a circular shaft to with stand a given torque we must develop an equation giving the relation between Twisting moments max m shear stain produced and a quantity representing the size and shape of the cross – sectional area of the shaft.



Refer to the figure shown above where a uniform circular shaft is subjected to a torque it can be shown that every section of the shaft is subjected to a state of pure shear, the moment of resistance developed by the shear stresses being every where equal to the magnitude, and opposite in sense, to the applied torque. For the purpose of deriving a simple theory to describe the behavior of shafts subjected to torque it is necessary make the following base assumptions.

Assumption:

(i) The materiel is homogenous i.e of uniform elastic properties exists throughout the material.

(ii) The material is elastic, follows Hook's law, with shear stress proportional to shear strain.

(iii) The stress does not exceed the elastic limit.

(iv) The circular section remains circular

(v) Cross section remain plane.

(vi) Cross section rotate as if rigid i.e. every diameter rotates through the same angle.

Consider now the solid circular shaft of radius R subjected to a torque T at one end, the other end being fixed Under the action of this torque a radial line at the free end of the shaft twists through an angle , point A moves to B, and AB subtends an angle ‘ ' at the fixed end. This is then the angle of distortion of the shaft i.e the shear strain.

Since angle in radius = arc / Radius

arc AB = Rθ

= L [since L and | also constitute the arc AB]

Thus, | = Rθ / L (1)

From the definition of Modulus of rigidity or Modulus of elasticity in shear

$$G = \frac{\text{shear stress}(\tau)}{\text{shear strain}(\gamma)}$$

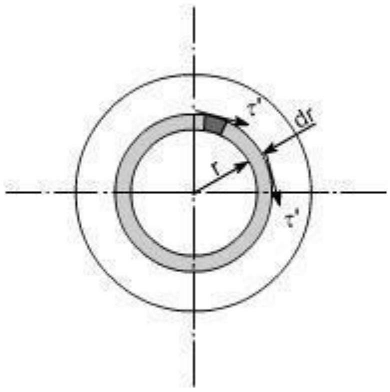
where γ is the shear stress set up at radius R.

Then $\frac{\tau}{G} = \gamma$

Equating the equations (1) and (2) we get $\frac{R\theta}{L} = \frac{\tau}{G}$

$\frac{\tau}{R} = \frac{G\theta}{L}\left(= \frac{\tau'}{r}\right)$ where τ' is the shear stress at any radius r.

Stresses: Let us consider a small strip of radius r and thickness dr which is subjected to shear stress.



The force set up on each element

= stress x area

The total torque T on the section, will be the sum of all the contributions. $T = \int_0^R 2\pi\tau' r^2\, dr$

Since □' is a function of r, because it varies with radius so writing down□□' in terms of r from the equation (1).

i.e $\tau' = \dfrac{G\theta . r}{L}$

we get $T = \int_0^R 2\pi \dfrac{G\theta}{L} . r^3\, dr$

$$T = \frac{2\pi G\theta}{L} \int_0^R r^3\, dr$$

$$= \frac{2\pi G\theta}{L} . \left[\frac{R^4}{4}\right]_0^R$$

$$= \frac{G\theta}{L} . \frac{2\pi R^4}{4}$$

$$= \frac{G\theta}{L} . \frac{\pi R^4}{2}$$

$$= \frac{G\theta}{L} . \left[\frac{\pi d^4}{32}\right] \text{ now substituting } R = d/2$$

$$= \frac{G\theta}{L} . J$$

since $\dfrac{\pi d^4}{32} = J$ the polar moment of inertia

or $\dfrac{T}{J} = \dfrac{G\theta}{L}$ ......(2)

if we combine the equation no.(1) and (2) we get $\boxed{\dfrac{T}{J} = \dfrac{\tau'}{r} = \dfrac{G.\theta}{L}}$

Where

T = applied external Torque, which is constant over Length L;

J = Polar moment of Inertia

$= \dfrac{\pi d^4}{32}$ for solid shaft

$= \dfrac{\pi(D^4 - d^4)}{32}$ for a hollow shaft. [D = Outside diameter ; d = inside diameter]

G = Modules of rigidity (or Modulus of elasticity in shear)

θ = It is the angle of twist in radians on a length L.

Tensional Stiffness: The tensional stiffness k is defined as the torque per radius twist

Power Transmitted by a shaft : If T is the applied Torque and ωis the angular velocity of the shaft, then the power transmitted by the shaft is

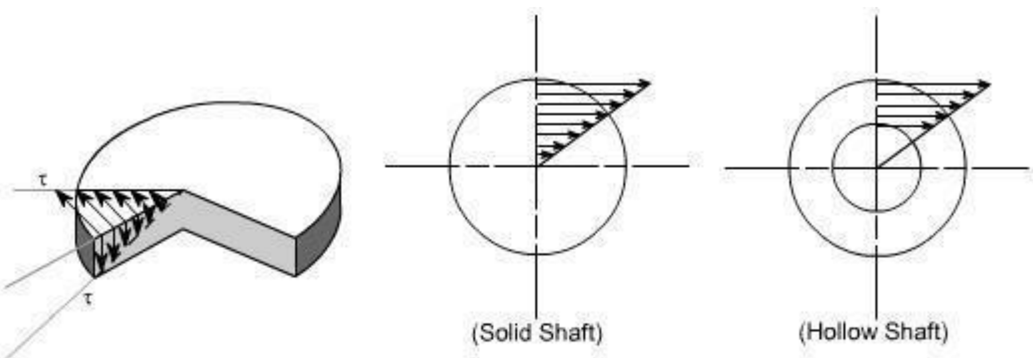
$$P = T.\omega = \frac{2\pi NT}{60} = \frac{2\pi NT}{60.10^3} kw$$
$$\text{where } N = \text{rpm}$$

Distribution of shear stresses in circular Shafts subjected to torsion :

The simple torsion equation is written as

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G.\theta}{l}$$
$$\text{or}$$
$$\tau = \frac{G\theta.r}{L}$$

This states that the shearing stress varies directly as the distance ‘r' from the axis of the shaft and the following is the stress distribution in the plane of cross section and also the complementary shearing stresses in an axial plane.



Hence the maximum strear stress occurs on the outer surface of the shaft where r = R

The value of maximum shearing stress in the solid circular shaft can be determined as

$$\frac{\tau}{r} = \frac{T}{J}$$
$$\tau_{max}\Big|_{r=d/2} = \frac{T.R}{J} = \frac{T}{\frac{\pi d^4}{32}} \cdot d/2$$
$$\text{where d=diameter of solid shaft}$$
$$\text{or } \tau_{max^m} = \frac{16T}{\pi d^3}$$

Power Transmitted by a shaft:

In practical application, the diameter of the shaft must sometimes be calculated from the power which it is required to transmit.

Given the power required to be transmitted, speed in rpm ‘N' Torque T, the formula connecting

These quantities can be derived as follows

$$P = T.\omega$$
$$= \frac{T.2\pi N}{60} \text{watts}$$
$$= \frac{2\pi NT}{60 \times 10^3} (kw)$$

Torsional stiffness: The torsional stiffness k is defined as the torque per radian twist .

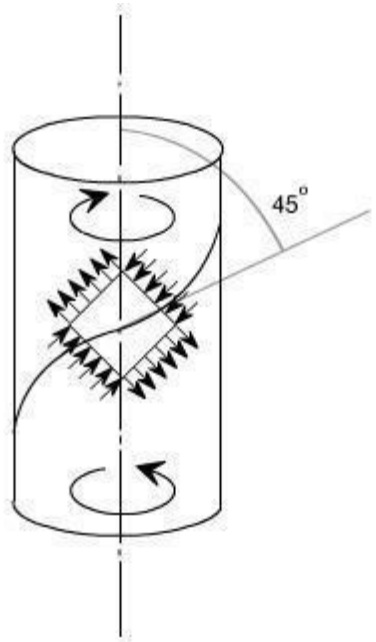
$$k = \frac{T}{\theta}$$
$$\text{i.e} \ = \frac{GJ}{L}$$
$$\text{or} \ k = \frac{G.J}{L}$$

For a ductile material, the plastic flow begins first in the outer surface. For a material which is weaker in shear longitudinally than transversely – for instance a wooden shaft, with the fibres parallel to axis the first cracks will be produced by the shearing stresses acting in the axial section and they will upper on the surface of the shaft in the longitudinal direction.

In the case of a material which is weaker in tension than in shear. For instance a, circular shaft of cast iron or a cylindrical piece of chalk a crack along a helix inclined at 45^0 to the axis of shaft often occurs.

Explanation: This is because of the fact that the state of pure shear is equivalent to a state of stress tension in one direction and equal compression in perpendicular direction.

A rectangular element cut from the outer layer of a twisted shaft with sides at 45^0 to the axis will be subjected to such stresses, the tensile stresses shown will produce a helical crack mentioned.



TORSION OF HOLLOW SHAFTS:

From the torsion of solid shafts of circular x – section , it is seen that only the material at the outer surface of the shaft can be stressed to the limit assigned as an allowable working stresses. All of the material within the shaft will work at a lower stress and is not being used to full capacity. Thus, in these cases where the weight reduction is important, it is advantageous to use hollow shafts. In discussing the torsion of hollow shafts the same assumptions will be made as in the case of a solid shaft. The general torsion equation as we have applied in the case of torsion of solid shaft will hold good

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G.\theta}{l}$$

For the hollow shaft

$$J = \frac{\pi(D_0{}^4 - d_i{}^4)}{32} \quad \text{where } D_0 = \text{Outside diameter}$$

$$d_i = \text{Inside diameter}$$

$$\text{Let } d_i = \frac{1}{2}.D_0$$

$$\tau_{max^m}\Big|_{solid} = \frac{16T}{\pi D_0{}^3} \qquad (1)$$

$$\tau_{max^m}\Big|_{hollow} = \frac{T.D_0/2}{\frac{\pi}{32}(D_0{}^4 - d_i{}^4)}$$

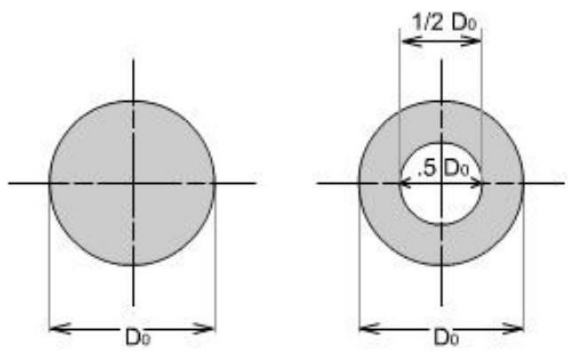
$$= \frac{16T.D_0}{\pi D_0{}^4\left[1 - (d_i/D_0)^4\right]}$$

$$= \frac{16T}{\pi D_0{}^3\left[1 - (1/2)^4\right]} = 1.066.\frac{16T}{\pi D_0{}^3} \qquad (2)$$

Hence by examining the equation (1) and (2) it may be seen that the $\tau_{max}{}^m$ in the case of hollow shaft is 6.6% larger then in the case of a solid shaft having the same outside diameter.

Reduction in weight:

Considering a solid and hollow shafts of the same length 'l' and density '□' with d_i = 1/2 D_o



Weight of hollow shaft

$$= \left[\frac{\pi D_0^{\,2}}{4} - \frac{\pi (D_0/2)^2}{4}\right] l \times \rho$$

$$= \left[\frac{\pi D_0^{\,2}}{4} - \frac{\pi D_0^{\,2}}{16}\right] l \times \rho$$

$$= \frac{\pi D_0^{\,2}}{4}[1 - 1/4] l \times \rho$$

$$= 0.75\frac{\pi D_0^{\,2}}{4} l \times \rho$$

$$\text{Weight of solid shaft} = \frac{\pi D_0^{\,2}}{4} . l . \rho$$

$$\text{Reduction in weight} = (1 - 0.75)\frac{\pi D_0^{\,2}}{4} l \times \rho$$

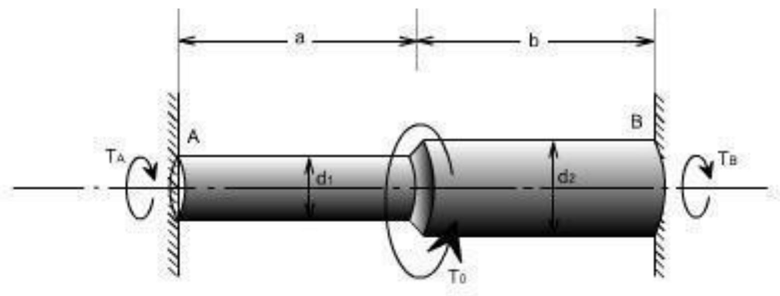
$$= 0.25\frac{\pi D_0^{\,2}}{4} l \times \rho$$

Hence the reduction in weight would be just 25%.

<u>Illustrative Examples :</u>

<u>Problem 1</u>

A stepped solid circular shaft is built in at its ends and subjected to an externally applied torque. T_0 at the shoulder as shown in the figure. Determine the angle of rotation $_0$ of the shoulder section where T_0 is applied ?



Solution: This is a statically indeterminate system because the shaft is built in at both ends. All that we can find from the statics is that the sum of two reactive torque T_A and T_B at the built – in ends of the shafts must be equal to the applied torque T_0

Thus $T_A + T_B = T_0$------ (1)

[from static principles]

Where T_A ,T_B are the reactive torque at the built in ends A and B. wheeras T_0 is the applied torque

From consideration of consistent deformation, we see that the angle of twist in each portion of the shaft must be same.

using the relation for angle of twist

$$\frac{T}{J} = \frac{G.\theta}{l}$$

$$\text{or } \theta_A = \frac{T_A a}{J_A G}$$

$$\theta_B = \frac{T_B a}{J_B G}$$

$$\Rightarrow \frac{T_A a}{J_A G} = \frac{T_B b}{J_B G} = \theta_0 \quad \text{or } \frac{T_A}{T_B} = \frac{J_A}{J_B}.\frac{b}{a} \quad (2)$$

N.B: Assuming modulus of rigidity G to be same for the two portions

So the defines the ratio of T_A and T_B

So by solving (1) & (2) we get

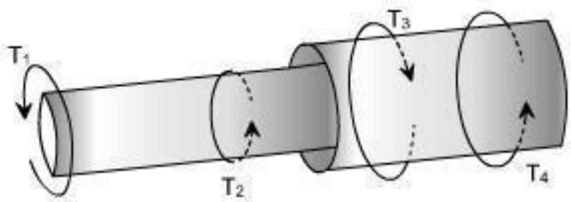
$$T_A = \frac{T_0}{1 + \frac{J_B a}{J_A b}}$$

$$T_b = \frac{T_0}{1 + \frac{J_a b}{J_b a}}$$

Using either of these values in (2) we have the angle of rotation θ_0 at the junction

$$\theta_0 = \frac{T_0.a.b}{[J_A.b + J_B.a]G}$$

Non Uniform Torsion: The pure torsion refers to a torsion of a prismatic bar subjected to torques acting only at the ends. While the non uniform torsion differs from pure torsion in a sense that the bar / shaft need not to be prismatic and the applied torques may vary along the length.



Here the shaft is made up of two different segments of different diameters and having torques applied at several cross sections. Each region of the bar between the applied loads between changes in cross section is in pure torsion, hence the formula's derived earlier may be applied. Then form the internal torque, maximum shear stress and angle of rotation for each region can be calculated from the relation

$$\frac{T}{J} = \frac{\tau}{r} \text{ and } \frac{T}{J} = \frac{G\theta}{L}$$

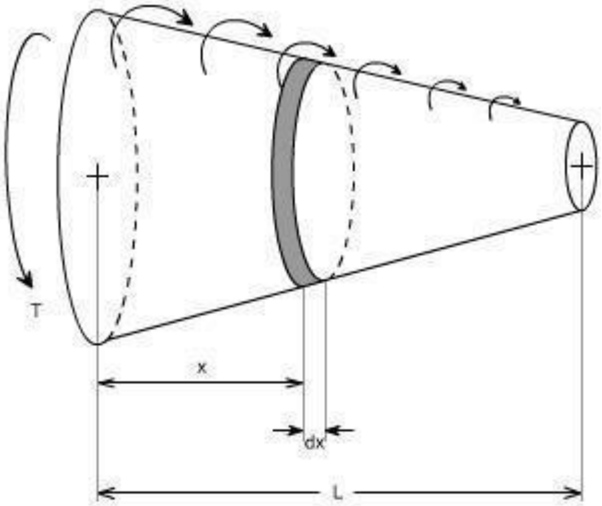
The total angle to twist of one end of the bar with respect to the other is obtained by summation using the formula

$$\theta = \sum_{i=1}^{n} \frac{T_i L_i}{G_i J_i}$$

i = index for no. of parts

n = total number of parts

If either the torque or the cross section changes continuously along the axis of the bar, then the (summation can be replaced by an integral sign (∫). i.e We will have to consider a differential element.



After considering the differential element, we can write $d\theta = \frac{T_x dx}{GI_x}$

Substituting the expressions for T_X and J_X at a distance x from the end of the bar, and then integrating between the limits 0 to L, find the value of angle of twist may be determined.

$$\theta = \int_0^L d\theta = \int_0^L \frac{T_x dx}{GI_x}$$

Members Subjected to Axisymmetric Loads

Pressurized thin walled cylinder:

Preamble : Pressure vessels are exceedingly important in industry. Normally two types of pressure vessel are used in common practice such as cylindrical pressure vessel and spherical pressure vessel.

In the analysis of this walled cylinders subjected to internal pressures it is assumed that the radial plans remains radial and the wall thickness dose not change due to internal pressure. Although the internal pressure acting on the wall causes a local compressive stresses (equal to pressure) but its value is negligibly small as compared to other stresses & hence the sate of stress of an element of a thin walled pressure is considered a biaxial one.

Further in the analysis of them walled cylinders, the weight of the fluid is considered neglible.

Let us consider a long cylinder of circular cross - section with an internal radius of R $_2$ and a constant wall thickness‘t' as showing fig.

This cylinder is subjected to a difference of hydrostatic pressure of ‘p' between its inner and outer surfaces. In many cases, ‘p' between gage pressure within the cylinder, taking outside pressure to be ambient.

By thin walled cylinder we mean that the thickness‘t' is very much smaller than the radius R_i and we may quantify this by stating than the ratio t / R_i of thickness of radius should be less than 0.1.

The small piece of the cylinder wall is shown in isolation, and stresses in respective direction have also been shown.

Type of failure:

Such a component fails in since when subjected to an excessively high internal pressure. While it might fail by bursting along a path following the circumference of the cylinder. Under normal circumstance it fails by circumstances it fails by bursting along a path parallel to the axis. This suggests that the hoop stress is significantly higher than the axial stress.

In order to derive the expressions for various stresses we make following

Applications :

Liquid storage tanks and containers, water pipes, boilers, submarine hulls, and certain air plane components are common examples of thin walled cylinders and spheres, roof domes.

ANALYSIS : In order to analyse the thin walled cylinders, let us make the following assumptions :

- There are no shear stresses acting in the wall.
- The longitudinal and hoop stresses do not vary through the wall.
- Radial stresses $_r$ which acts normal to the curved plane of the isolated element are neglibly small as compared to other two stresses especially when $\left[\frac{t}{R_i} < \frac{1}{20}\right]$

The state of tress for an element of a thin walled pressure vessel is considered to be biaxial, although the internal pressure acting normal to the wall causes a local compressive stress equal to the internal pressure, Actually a state of tri-axial stress exists on the inside of the vessel. However, for then walled pressure vessel the third stress is much smaller than the other two stresses and for this reason in can be neglected.

Thin Cylinders Subjected to Internal Pressure:

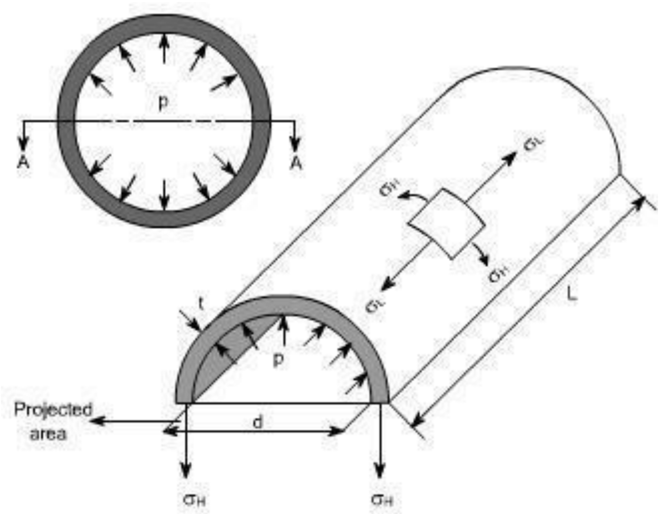
When a thin – walled cylinder is subjected to internal pressure, three mutually perpendicular principal stresses will be set up in the cylinder materials, namely

- Circumferential or hoop stress
- The radial stress
- Longitudinal stress

now let us define these stresses and determine the expressions for them

Hoop or circumferential stress:

This is the stress which is set up in resisting the bursting effect of the applied pressure and can be most conveniently treated by considering the equilibrium of the cylinder.



In the figure we have shown a one half of the cylinder. This cylinder is subjected to an internal pressure p.

i.e. p = internal pressure

d = inside diametre

L = Length of the cylinder

t = thickness of the wall

Total force on one half of the cylinder owing to the internal pressure 'p'

= p x Projected Area

= p x d x L

= p .d. L------- (1)

The total resisting force owing to hoop stresses σ_H set up in the cylinder walls

= **2 .** σ_H **.L.t** ---------(2)

Because σ_H .L.t. is the force in the one wall of the half cylinder.

the equations (1) & (2) we get

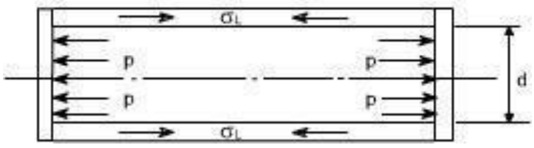
2 . σ_H **. L . t = p . d . L**

σ_H = **(p . d) / 2t**

Circumferential or hoop Stress (σ_H) = (p.d)/ 2t

Longitudinal Stress:

Consider now again the same figure and the vessel could be considered to have closed ends and contains a fluid under a gage pressure p.Then the walls of the cylinder will have a longitudinal stress as well as a ciccumferential stress.



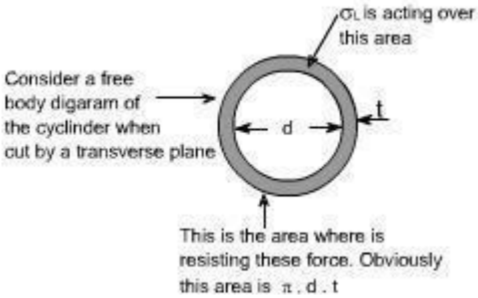
Total force on the end of the cylinder owing to internal pressure

= pressure x area

= p x Π /4 x d^2

Area of metal resisting this force = □d.t. (approximately)

because □d is the circumference and this is multiplied by the wall thickness



Hence the longitudnal stresses

$$\text{Set up} = \frac{\text{force}}{\text{area}} = \frac{[p \times \pi d^2/4]}{\pi dt}$$

$$= \frac{pd}{4t} \quad \text{or} \quad \sigma_L = \frac{pd}{4t}$$

or alternatively from equilibrium conditions

$$\sigma_L.(\pi dt) = p.\frac{\pi d^2}{4}$$

$$\text{Thus} \boxed{\sigma_L = \frac{pd}{4t}}$$

Change in Dimensions :

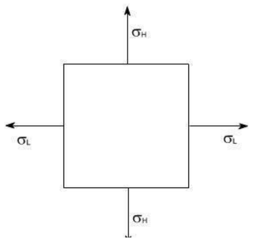
The change in length of the cylinder may be determined from the longitudinal strain.

Since whenever the cylinder will elongate in axial direction or longitudinal direction, this will also get decreased in diametre or the lateral strain will also take place. Therefore we will have to also take into consideration the lateral strain.as we know that the poisson's ratio (ν) is

$$\nu = \frac{-\text{lateral strain}}{\text{longitudnal strain}}$$

where the -ve sign emphasized that the change is negative

Let E = Young's modulus of elasticity



$$\text{Resultant Strain in longitudnal direction} = \frac{\sigma_L}{E} - \nu\frac{\sigma_H}{E} = \frac{1}{E}(\sigma_L - \nu\sigma_H)$$

recalling

$$\sigma_L = \frac{pd}{4t} \qquad \sigma_H = \frac{pd}{2t}$$

$$\epsilon_1 \text{ (longitudnal strain)} = \frac{pd}{4Et}[1-2\nu]$$

or

$$\text{Change in Length} = \text{Longitudal strain x original Length}$$
$$= \epsilon_1 . L$$

$$\text{Similarly the hoop Strain } \epsilon_2 = \frac{1}{E}(\sigma_H - \nu\sigma_L) = \frac{1}{E}\left[\frac{pd}{2t} - \nu\frac{pd}{4t}\right]$$

$$\epsilon_2 = \frac{pd}{4Et}[2-\nu]$$

Infact ϵ_2 is the hoop strain if we just go by the definition then

$$\epsilon_2 = \frac{\text{Change in diametre}}{\text{Original diametre}} = \frac{\delta d}{d}$$

where d = original diameter.

if we are interested to find out the change in diametre then

Change in diametre $= \epsilon_2$ .Original diametre

i.e $\delta d = \epsilon_2$ .d substituting the value of ϵ_2 we get

$$\delta d = \frac{p.d}{4.t.E}[2-\nu].d$$
$$= \frac{p.d^2}{4.t.E}[2-\nu]$$

$$\text{i.e } \boxed{\delta d = \frac{p.d^2}{4.t.E}[2-\nu]}$$

Volumetric Strain or Change in the Internal Volume:

When the thin cylinder is subjected to the internal pressure as we have already calculated that there is a change in the cylinder dimensions i.e, longitudinal strain and hoop strains come into picture. As a result of which there will be change in capacity of the cylinder or there is a change in the volume of the cylinder hence it becomes imperative to determine the change in volume or the volumetric strain.

The capacity of a cylinder is defined as

V = Area X Length

= $d^2/4$ x L

Let there be a change in dimensions occurs, when the thin cylinder is subjected to an internal pressure.

(i) The diameter **d** changes to **d +** **d**

(ii) The length **L** changes to **L +** **L**

Therefore, the change in volume = Final volume Original volume

Therefore to find but the increase in capacity or volume, multiply the volumetric strain by original volume.

Hence

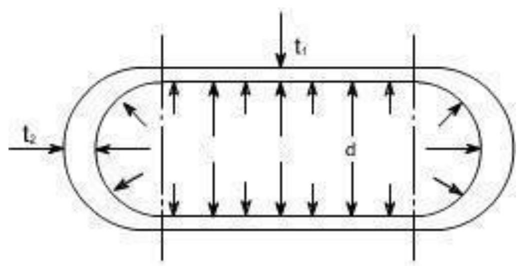
Change in Capacity / Volume or

$$\boxed{\text{Increase in volume} = \frac{pd}{4tE}[5 - 4\nu]V}$$

Cylindrical Vessel with Hemispherical Ends:

Let us now consider the vessel with hemispherical ends. The wall thickness of the cylindrical and hemispherical portion is different. While the internal diameter of both the portions is assumed to be equal

Let the cylindrical vassal is subjected to an internal pressure p.



For the Cylindrical Portion

hoop or circumferential stress = σ_{HC} 'c' here synifies the cylindrical portion.

$$= \frac{pd}{2t_1}$$

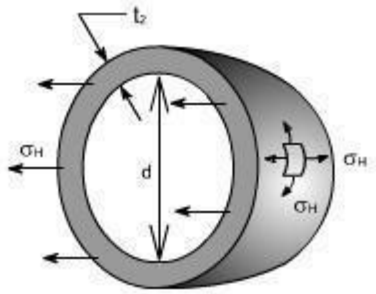
longitudnal stress = σ_{LC}

$$= \frac{pd}{4t_1}$$

hoop or circumferential strain $\epsilon_2 = \frac{\sigma_{HC}}{E} - \nu\frac{\sigma_{LC}}{E} = \frac{pd}{4t_1E}[2-\nu]$

or $\boxed{\epsilon_2 = \frac{pd}{4t_1E}[2-\nu]}$

For The Hemispherical Ends:



Because of the symmetry of the sphere the stresses set up owing to internal pressure will be two mutually perpendicular hoops or circumferential stresses of equal values. Again the radial stresses are neglected in comparison to the hoop stresses as with this cylinder having thickness to diametre less than1:20.

Consider the equilibrium of the half – sphere

Force on half-sphere owing to internal pressure = pressure x projected Area

$= p.\ \Pi d^2/4$

Resisting force = $\sigma_H \cdot \pi.d.t_2$

$$\therefore \quad p.\frac{\pi.d^2}{4} = \sigma_H \cdot \pi d.t_2$$

$$\Rightarrow \sigma_H \text{ (for sphere)} = \frac{pd}{4t_2}$$

similarly the hoop strain $= \frac{1}{E}\left[\sigma_H - \nu.\sigma_H\right] = \frac{\sigma_H}{E}[1-\nu] = \frac{pd}{4t_2E}[1-\nu]$ or $\boxed{\epsilon_{2s} = \frac{\mathbf{pd}}{\mathbf{4t_2E}}[\mathbf{1-\nu}]}$

Fig – shown the (by way of dotted lines) the tendency, for the cylindrical portion and the spherical ends to expand by a different amount under the action of internal pressure. So owing to difference in stress, the two portions (i.e. cylindrical and spherical ends) expand by a different amount. This incompatibly of deformations causes a local bending and sheering stresses in the neighborhood of the joint. Since there must be physical continuity between the ends and the cylindrical portion, for this reason, properly curved ends must be used for pressure vessels.

Thus equating the two strains in order that there shall be no distortion of the junction

$$\frac{pd}{4t_1E}[2-\nu] = \frac{pd}{4t_2E}[1-\nu] \quad \text{or} \quad \frac{t_2}{t_1} = \frac{1-\nu}{2-\nu}$$

But for general steel works $\nu = 0.3$, therefore, the thickness ratios becomes

t_2 / t_1 = **0.7/1.7** or

$\boxed{t_1 = 2.4\, t_2}$

i.e. the thickness of the cylinder walls must be approximately 2.4 times that of the hemispheroid ends for no distortion of the junction to occur.

SUMMARY OF THE RESULTS : Let us summarise the derived results

(A) The stresses set up in the walls of a thin cylinder owing to an internal pressure p are :

(i) Circumferential or loop stress

σ_H **= pd/2t**

(ii) Longitudinal or axial stress

σ_L **= pd/4t**

Where d is the internal diametre and t is the wall thickness of the cylinder.

(B) Change of internal volume of cylinder under pressure

$$= \frac{pd}{4tE}[5-4\nu]V$$

(C) Fro thin spheres circumferential or loop stress

$$\sigma_H = \frac{pd}{4t}$$

Tutorial Questions

1. Derive an expression for the shear stress produced in a circular shaft which is subjected to torsion. What are the assumptions made in the above derivation ?
2. a)Derive the formula for the hoop stress in a thin cylindrical shell subjected to an internal pressure.
   b) A gas cylinder of thickness 25 mm and has an internal diameter of 1500 mm. The tensile stress in the gas cylinder material is not to exceed 100 N/mm2. Calculate the allowable internal pressure of the gas inside the cylinder.
3. A thin cylindrical shell is 3m long and 1m in internal diameter. It is subjected to internal pressure of 1.2 MPa. If the thickness of the sheet is 12mm, find the circumferential stress, longitudinal stress, changes in diameter, length and volume. Take E=200 GPa and μ= 0.3.
4. A Hollow shaft is to transmit 400 KW power at 120 rpm. If the shear stress is not exceed 60 N/mm^2 and internal diameter is 0.65 of external diameter. Find the internal and external diameters assuming maximum torque is 1.5 times the mean
5. A hollow shaft of diameter ratio 3/8 is to transmit 395 kW at 120 rpm. The maximum torque being 24% greater than the mean, the shear stress is not to exceed 65 MPa and the twist in a length of 6 m is not to exceed 3 degrees. Calculate its external and internal diameters which would satisfy both the above said conditions. Take $G=9.2\times10^4$ MPa.

Assignment Questions

1. A cylindrical vessel 2m long and 500mm in diameter with 10mm thick plates is subjected to an internal pressure of 3MPa.Calculate the change in volume of the vessel .Take E=200GPa and poissons ratio=0.3 for the vessel material.
2. A shaft is to be transmitted 100KW at 240 rpm. If the allowable shear stresses of the material is 60MPa. The shaft is not to twist more than 1^0 in a length of 3.5 mts. Find the diameter of the shaft based on strength and stiffness criteria. The modulus of rigidity of the material (N) is 80 X $10^3 N/mm^2$.
3. A cylindrical vessel 3m long and 500mm in diameter with 10mm thick plates is subjected to an internal pressure of 3MPa.Calculate the change in volume of the vessel .Take E=210GPa and Poisson's ratio=0.3 for the vessel material
4. A thin cylindrical shell is 3m long and 1m in internal diameter. It is subjected to internal pressure of 1.2 MPa. If the thickness of the sheet is 12mm, find the circumferential stress, longitudinal stress, changes in diameter, length and volume. Take E=200 GPa and μ= 0.3.
5. A thin cylindrical shell is 3m long and 1m in internal diameter. It is subjected to internal pressure of 1.2 MPa. If the thickness of the sheet is 12mm, find the circumferential stress, longitudinal stress, changes in diameter, length and volume. Take E=200 GPa and μ= 0.3.
6. A hallow shaft of outside diameter 80 mm and inside diameter 50 mm is made of aluminium having shear modulus G = 27GPa. When the shaft is subjected to a torque T = 4.8 kN-m, what is the maximum shear strain and maximum normal strain in the bar?

UNIT-V

POWER POINT PRESENTAION

TORSION OF CIRCULAR SHAFTS & THIN CYLINDRES

Unit v

MRCET CAMPUS
UGC Autonomous

DEPARTMENT OF MECHANICAL ENGINEERING

TORSION OF CIRCULAR SHAT

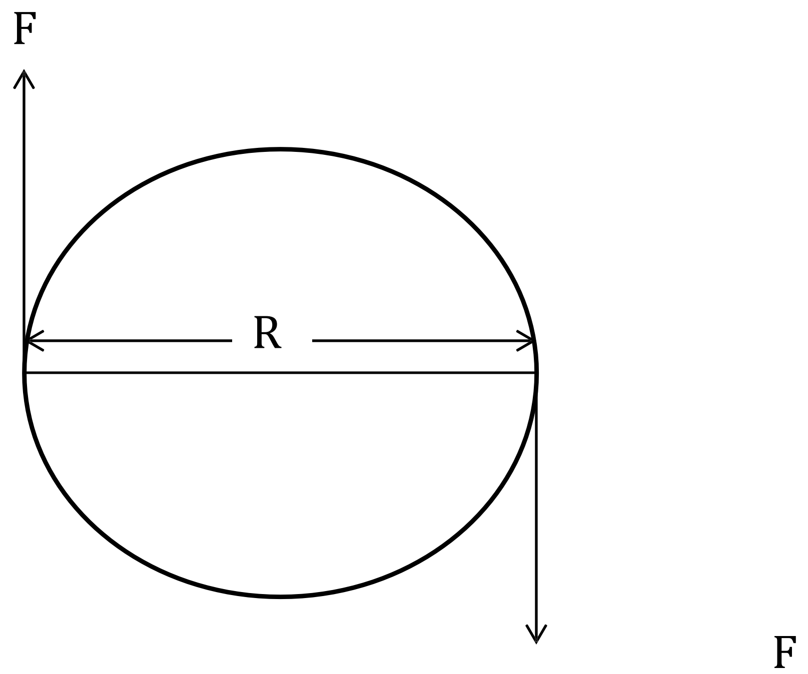
TORSION OF CIRCULAR SHAFT

- <u>TORQUE OR TURNING MOMENT OR TWISTING MOMENT</u> :-

➢ In factories and workshops, shafts is used to transmit energy from one end to other end.

➢ To transmit the energy, a turning force is applied either to the rim of a pulley, keyed to the shafts, or to any other suitable point at some distance from the axis of the shaft.

➢ The moment of couple acting on the shaft is called torque or turning moment or twisting moment.



Torque = turning force x diameter of shaft

T = F x 2R

where :

T=Torque F=Turning force

S=Radius of the shaft Unit of Torque(T) is N.mm or kN.mm

SHEAR STRESS IN SHAFT:(τ)

- When a shaft is subjected to equals and opposite end couples, whose axes coincide with the axis of the shaft, the shaft is said to be in pure torsion and at any point in the section of the shaft stress will be induced.

- That stress is called shear stress in shaft.

STRENGTH OF SHAFTS

Maximum torque or power the shaft can transmit from one pulley to another, is called strength of shaft.

(a) For solid circular shafts:

Maximum torque (T)is given by :

$$T = \frac{\pi}{16} \times \tau \times D^3$$

where, D = dia. of the shaft

τ=shear stress in the shaft

(B) for hollow circular shaft

maximum torque (t) is given by.

$$T = \frac{\pi}{16} \times \tau \times \frac{D^4 - d^4}{D}$$

Where, d= outer dia of shaft
d=inner dia of shaft.

ASSUMPTION IN THE THEORY OF TORSION:

- The following assumptions are made while finding out shear stress in a circular shaft subjected to torsion.

1) The material of shaft is uniform throughout the length.
2) The twist along the shaft is uniform.
3) The shaft is of uniform circular section throughout the length.
4) Cross section of the shaft, which are plane before twist remain plain after twist.
5) All radii which are straight before twist remain straight after twist.

POLLAR MOMENT OF INERTIA :

- The moment of inertia of a plane area, with respect to an axis perpendicular to the plane of the figure is called polar moment of inertia.
- As per the perpendicular axis theorem.

$$I_{ZZ} = I_{XX} + I_{YY} = J$$

$$= \frac{\pi}{64} \times D^4 + \frac{\pi}{64} \times D^4$$

$$J = \frac{\pi}{32} \times D^4$$

TORSION RIGIDITY

- Let twisting moment Produce a twist radians in length L.

$T = C.\theta$

- for given shaft the twist is therefore proportional to the twisting moment T.
- In a beam the bending moment produce deflection, in the same manner a torque produces a twist in shaft .
- The quantity CJ stands for the torque required to produce a twist of 1 radian per unit of the shaft.
- The quantity CJ corresponding to a similar EI, in expression for deflection of beams, EI is known as flexure rigidity.

THEORY OF TORSION AND TORSION EQUATION

- Consider a shaft fixed at one end subjected to torque at the other end.

Let T= Torque

l= length of the shaft

R=Radius of the shaft

- As a result of torque every cross-section of the shaft will be subjected to shear stress.
- Line CA on the surface of the shaft will be deform to CA’ and OA to OA’, as shown in figure.
- Let, ∠ACA’=shear strain
- ∠AOA’=angle of twist

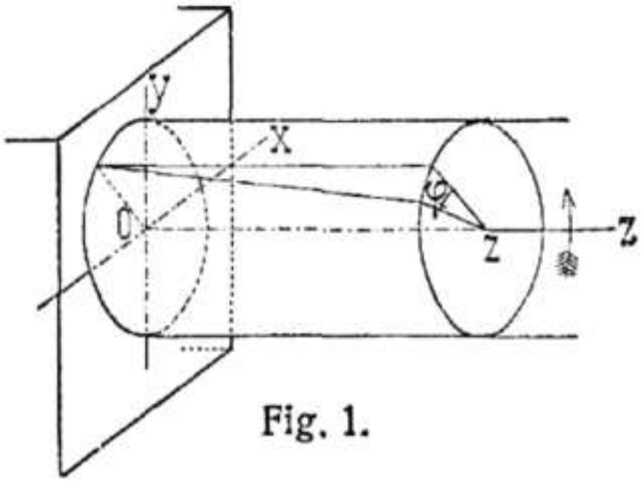


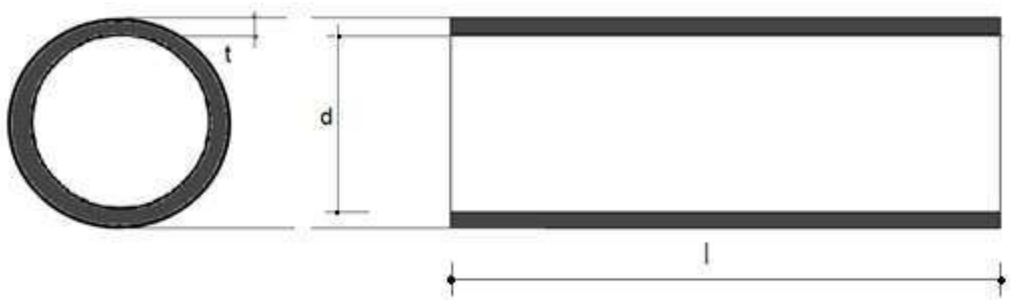
Fig. 1.

TORSION RIGIDITY

- Let twisting moment Produce a twist radians in length L.
- $\frac{T}{J} = \frac{C.\theta}{L}$
- for given shaft the twist is therefore proportional to the twisting moment T.
- In a beam the bending moment produce deflection, in the same manner a torque produces a twist in shaft .
- The quantity CJ stands for the torque required to produce a twist of 1 radian per unit of the shaft.
- The quantity CJ corresponding to a similar EI, in expression for deflection of beams, EI is known as flexure rigidity.

Thin Cylinders

- Shape
  - Use of Shape (Tanks, Boilers, pipelines, Vault)



  - Thin and Thick

- Pressure
  - Internal and External atmospheric

- Stress
  - Failure
    - Hoop type failure
    - Longitudinal failure

  - Strain change in
    - diameter
    - length
    - volume

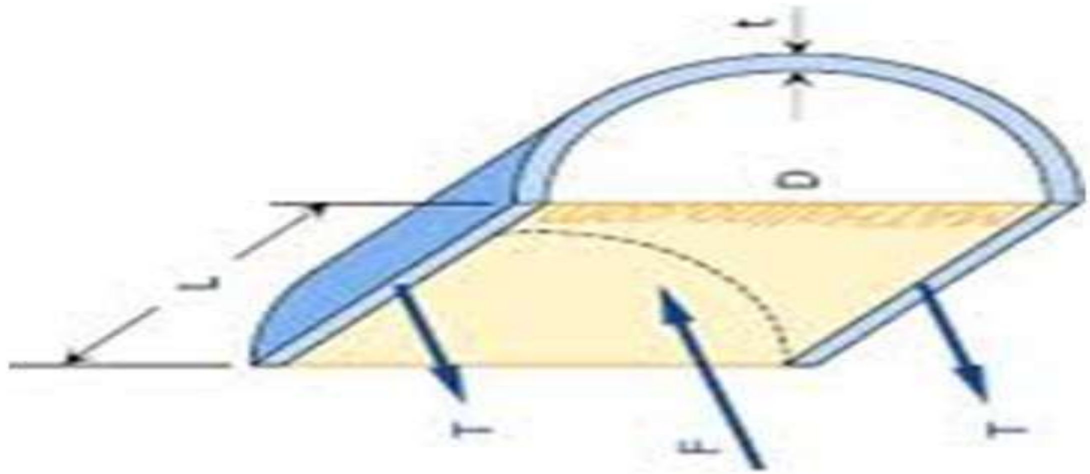
INTRODUCTION:

- In many engineering applications, cylinders transporting or storing of liquids, gases or fluids. Eg: Pipes, Boilers, storage tanks etc.

- These cylinders are subjected to fluid pressures.

- When a cylinder is subjected to a internal pressure, at any point on the cylinder wall, three types of stresses are induced on three mutually perpendicular planes.

- They are ,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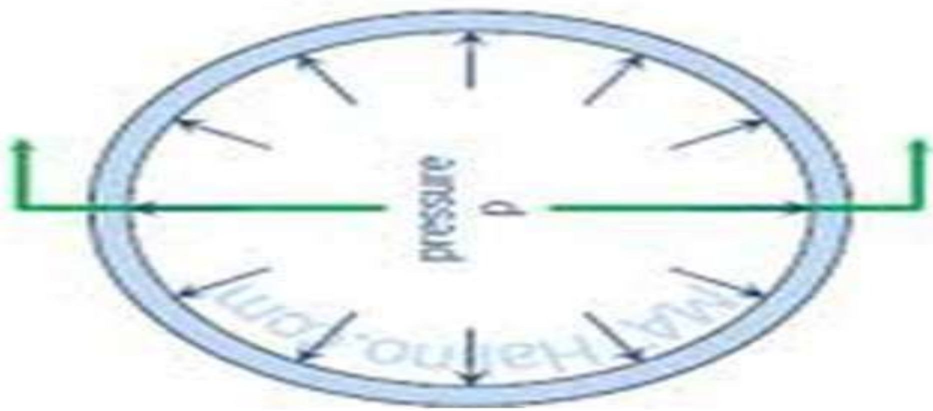
pressure
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pressure
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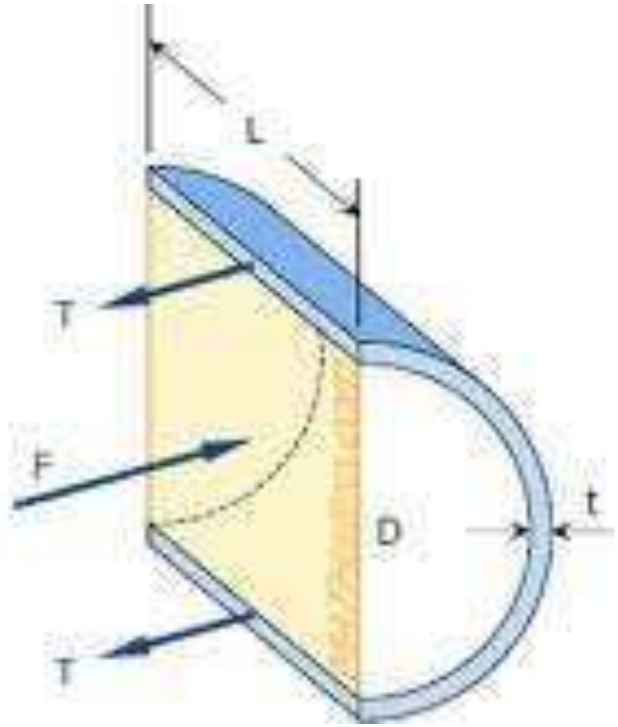
L
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Three types of stresses :

1.Hoop or Circumferential Stress (σ_C) or (σ_1) – This is directed along the tangent to the circumference and tensile in nature. Thus, there will be increase in diameter.

2.2. Longitudinal Stress (σ_L) or (σ_2) – This stress is directed along the length of the cylinder. This is also tensile in nature and tends to increase the length.

3.3. Radial pressure (p_r) – It is compressive in nature. <u>Its magnitude is equal to fluid pressure on the inside wall and zero on the outer wall if it is open to atmosphere</u>

- A cylinder or spherical shell is considered to be thin when the metal thickness is small compared to internal diameter.

i.e., when the wall thickness, ‘t’ is equal to or less than ‘d/20’, where ‘d’ is the internal diameter of the cylinder or shell, we consider the cylinder or shell to be thin, otherwise thick

- Magnitude of radial pressure is very small compared to other two stresses in case of thin cylinders and hence neglected

R15

Code No: R15A0305

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

II B. Tech I Semester Supplementary Examinations, May 2018

Strength of Materials

(ME)

Roll No										

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B

Part A is compulsory which carriers 25 marks and Answer all questions.

Part B Consists of 5 SECTIONS (One SECTION for each UNIT). Answer FIVE Questions, Choosing ONE Question from each SECTION and each Question carries 10 marks.

******

PART – A **(25 Marks)**

1. (a) Distinguish clearly the properties elasticity and plasticity[2M]
   (b) Define poisson's ratio. Mention its significance in material selection.[3M]
   (c) List different types of beams.[2M]
   (d) What is meant by the term point of contraflexure. Explain.[3M]
   (e) Find out section modulus for a rectangular beam having width 'b' and depth 'd'[2M]
   (f) Draw shear stress distribution across the rectangular cross section of the beam if the beam is carrying a concentrated load P at mid point.[3M]
   (g) Explain moment area theorm-I applicable for beams.[2M]
   (h) What is the condition to be satisfied for a perfect truss? Explain.[3M]
   (i) A solid shaft is to be transmitted 25 kw by running at 800 rpm. Find the torque induced in the shaft material.[2M]
   (j) Develop the relationship between circumferential and longitudinal stress in case thin cylinder subjected to internal fluid pressure.[3M]

PART – B **(50 Marks)**

SECTION – I

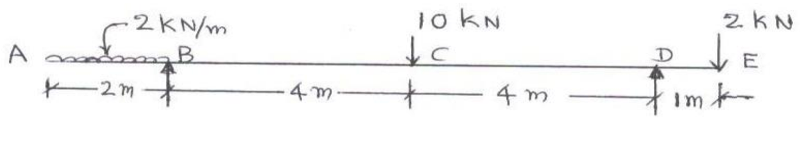
2. Derive the relationship between Elatic Moduli E, G and K from fundamentals of solid mechanics. [10M]

(OR)

3. Derive the formula for elongation of uniformly tapered circular cross section bar under axial load. Also deduce the relation for strain energy stored in the bar. [10M]

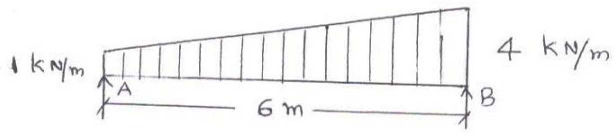
SECTION – II

4. Sketch the S.F. &B.M. diagrams for an Overhanging beam ABCDE shown. Mark all the salient points with respective values. [10M]



(OR)

5. Draw SF& BM diagrams for the simply supported beam marking all the salient values. [10M]



SECTION – III

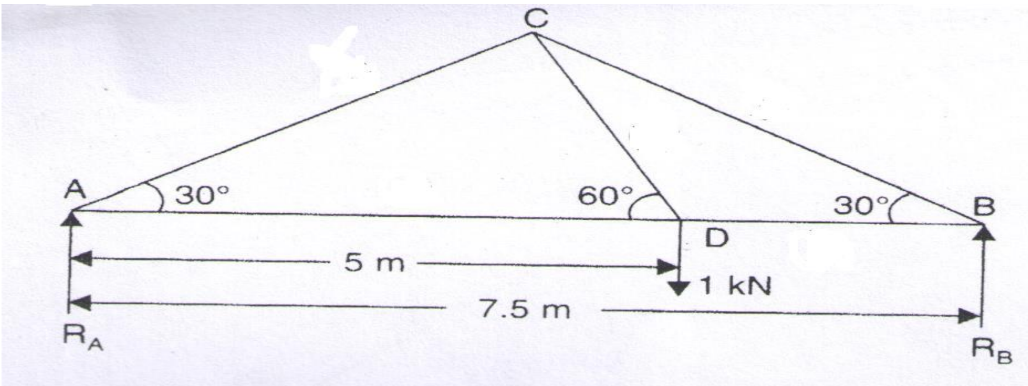
6. An I – beam of 200mm depth is simply supported over an effective span of 8m. Find what max. intensity of udl it can carry over entire length if the allowable bending stresses in tension and compression are 30 and 45 N/mm^2 respectively. Take I_{NA} = 5935.5×10^4 mm^4. Distance of bottom fibre from NA is 87.38mm. [10M]

(OR)

7. A simply supported beam having span 4 m is subjected to a UDL of 30 kN/m over whole span. The cross-section of beam is T section. The dimensions of flange are 120mmx10mm and that of web are 200mmx15mm. Draw shear stress distribution across the depth of cross-section marking the values at salient points. [10M]

SECTION – IV

8. A truss of span 7.5 m carries a point load of 1 kN at joint 'D' as shown in figure. Find the reactions and forces in the members of the truss. [10M]



(OR)

9. A simply supported beam of 8m carries a partial u d l of intensity 5KN/m and length 2m, sarting from 2m from the left end. Find slope at left support and central deflection. Take E= 200Gpa and I=8×10^{8}mm^4 [10M]

SECTION – V

10. A solid circular bar of steel (G=80GPa) with length L= 3.5 m and diameter d=120 mm is subjected to pure torsion by a torque T. How much strain energy is stored in the bar when the maximum shear stress is 60 MPa? [10M]

(OR)

11. Derive torsion equation with assumptions. [10M]

****

R15

Code No: R15A0305

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

II B.Tech I Semester supplementary Examinations, November 2018

Strength of Materials

(ME)

Roll No										

Time: 3 hours Max. Marks: 75

Note: This question paper contains two parts A and B
Part A is compulsory which carriers 25 marks and Answer all questions.
Part B Consists of 5 SECTIONS (One SECTION for each UNIT). Answer FIVE Questions, Choosing ONE Question from each SECTION and each Question carries 10 marks.

PART-A (25 Marks)

1). a What is composite bar, how will you find the stress in composite bar due to external loading. **[2M]**

b A rod 200 cm long and of diameter 3.0 cm is subjected to axial pull of 30KN if the young's modulus of the material is 2×10^5N/mm^2 then determine i) stress ii) Elongation iii) strain **[3M]**

c Define and explain the following terms **[2M]**
i) Bending stress ii) Section modulus

d Define and explain the following terms **[3M]**
i) Shear force ii) Bending moment iii) Bending moment diagram

e Derive an expression to find out section modulus for Hallow rectangular section **[2M]**

f Write torsional equation and explain the terms. **[3M]**

g What are the different types of Frames, Explain with the help of figures. **[2M]**

h Explain the procedure to do the analysis of frames by using method of joints and method of sections. **[3M]**

i Explain about polar section modulus. **[2M]**

j Derive the equation to find out volumetric strain in thin cylinder subjected to internal pressure. **[3M]**

PART-B (50 MARKS)

SECTION-I

2 Prove that the total extension (dL) of uniformly taper rod of diameter D_1 and D_2, when the rod is subjected to axial load **P** with the help of diagram. $dL = \frac{4PL}{\pi E D_1 D_2}$ **[10M]**

OR

3 a) Derive the equation to show the relation between E,K & G. **[(7+3)M]**

b) A steel bar 320 mm long and 40 mm wide 30 mm thickness is subjected to a pull of 250KN in the direction of its length. Determine the change in volume.

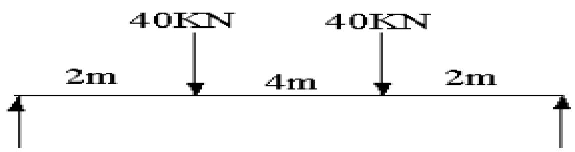
SECTION-II

4 a) Derive the relation between shear force, bending moment and loading for beam carrying U.d.l **[(3+7)M]**

b) A simply supported beam of length 6m is loaded with gradually varying load of 0KN/m from left support 750KN/m to right support. Draw the shear force and bending moment diagrams for the beam.

OR

5 Draw the B. M. D and S. F.D **[10M]**



SECTION-III

6 A T-section beam having flange 2cm x 10cm, web 10cm x 2cm is simply supported over a span of 6m. It carries a U.D.L of 3kN/m including its own weight over its entire span, together with a load of 2.5kN at mid span. Find the maximum tensile and compressive stresses occurring in beam section. **[10M]**

OR

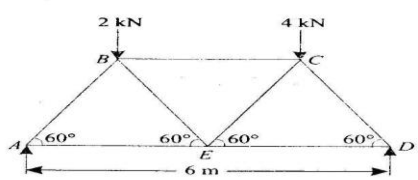
7 Define simple bending and what are the assumption made in simple bending theory and derive the bending moment eqation. **[10M]**

SECTION-IV

8 A beam section is 10m long and is simply supported at ends. It carries concentrated loads of100kN and 60kN at a distance of 2m and 5m respectively from the left end. Calculate the deflection under the each load find also the maximum deflection. Take I = 18 X 10^8mm^4 and E = 200kN/mm^2. **[10M]**

OR

9 Figure shows a Warren girder, each member having 3 m length supported freely at its end Points. The girder is loaded at B and C as shown. Find the forces in all members of the girder by using Method of joints. **[10M]**



SECTION-V

10 A shaft is to be transmitted 200KW at 300rpm. The max. shear stress should not exceed 30 MPa and twist should not be more than 1° in a shaft length of 2.5 mts. If the modulus of rigidity of the material is 10^5 MPa, Find the required diameter of the shaft to transmit above given power. **[10M]**

OR

11 A thin cylindrical shell is 3m long and 1m in internal diameter. It is subjected to internal pressure of 1.2 MPa. If the thickness of the sheet is 12mm, find the circumferential stress, longitudinal stress, changes in diameter, length and volume. Take E=200 GPa and μ= 0.3. **[10M]**

******

Code No: R15A0305 **R15**

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

II B.Tech I Semester supplementary Examinations, May 2019

Strength of Materials

(ME)

Roll No										

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B
Part A is compulsory which carriers 25 marks and Answer all questions.
Part B Consists of 5 SECTIONS (One SECTION for each UNIT). Answer FIVE Questions, Choosing ONE Question from each SECTION and each Question carries 10 marks.

***

PART-A (25 Marks)

1). a Define the following terms **[2M]**
i) Possions ratio ii) Strain energy iii) resilience iv) Proof Resilience

b Determine the change in length breadth and thickness of steel bar which is 5m long, 40 mm wide 30 mm thick and is subjected to axial pull of 35KN in the direction of its length. ($E=2\times10^5 N/mm^2$ Poisson's ratio=0.32) **[3M]**

c What are the types of beams and types of loads? **[2M]**

d A cantilever beam of length 4m carries point loads 1KN, 2KN and 3KN at 1m, 2 m and 4m respectively from fixed end. Draw the shear force and bending moment diagrams for the beam. **[3M]**

e Derive an expression to find out the section modulus for 'I' section **[2M]**

f A rectangular beam of 100mm wide is subjected to maximum shear force 100KN. Find the depth of beam if the shear stress is $6N/mm^2$ **[3M]**

g Explain the procedure to do the analysis of frames by using tension coefficient method. **[2M]**

h Explain with neat sketches, what is Beam, Frame, Truss? **[3M]**

i Explain the torsional moment of resistance of the shafts. **[2M]**

j Derive the equation to find out hoop stress and longitudinal stress in thin cylinder subjected to internal pressure. **[3M]**

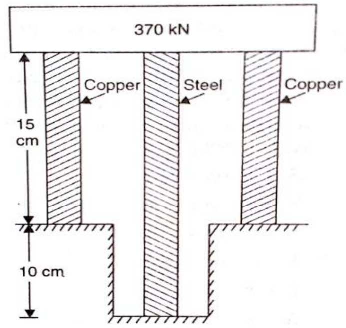
PART-B (50 MARKS)

SECTION-I

2 a) Determine the young's modulus and Possion's ratio of a metallic bar of length 25cm breadth 3cm depth 2cm when the beam is subjected to an axial compressive load 240KN. The decrease in length is given by 0.05cm and increase in breadth 0.002 **[(6+4)M]**
b) Write the differences among Gradual, Sudden, Impact and Shock loadings with the help of expressions

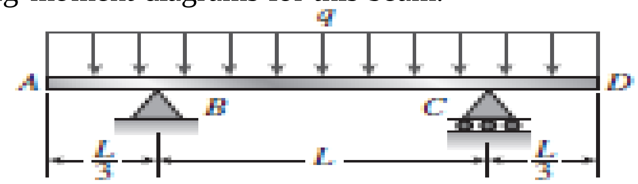
OR

3 A steel rod and two copper rods together support a load of 370 kN as shown in fig. The cross sectional area of steel road is 2500 mm^2 and of each copper road is 1600 mm^2. Find the stresses in the roads. Take E for steel is $2x10^5$ N/mm^2 and for copper is $1x10^5$ N/mm^2 **[10M]**



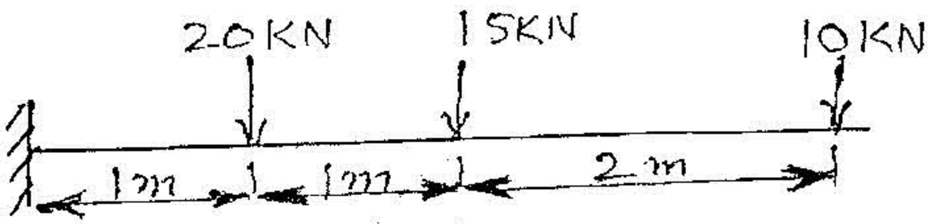
SECTION-II

4 Beam ABCD is simply supported at B and C and has overhangs at each end. The beam length between A and B is ‘L’ and each overhang has length L/ 3. A uniform load of intensity **q** acts on entire length of the beam. Draw the shear-force and bending-moment diagrams for this beam. **[10M]**



OR

5 Draw SF and BM diagrams for the cantilever shown in Fig **[10M]**



SECTION-III

6 Derive and Prove the following relation $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$, Where M is moment applied on the beam, I is moment of Inertia, σ is bending stress, y is the distance between neutral axis and extreme fiber, E is young’s modules, R is Radius if curvature. **[10M]**

OR

7 A rectangular beam of 100mm wide and 150mm deep is subjected to Shear force of 30KN, Determine ratio of Maximum shear stress to Average shear stress. Derive the equation which is used to find out the shear stress. **[10M]**

SECTION-IV

8 A simply supported beam span 14m, carrying concentrated loads of 12KN and 8KN at two points 3mts and 4.5m from the two ends respectively. Moment of Inertia I for the beam is 160 x10^3 mm^4 and E = 210KN/mm^2. Calculate deflection of the beam at points under the two loads by macaulay’s method. **[10M]**

OR

9 A Cantilever beam AB 6 mts long is subjected to u.d.l of w KN/m spread over the entire length. Assume rectangular cross-section with depth equal to twice the breadth. Determine the minimum dimension of the beam so that the vertical deflection at free end does not exceed 1.5 cm and the maximum stress due to bending does not exceed 10 KN/cm^2. E = 2 X 10^7 N/ cm^2. **[10M]**

SECTION-V

10 A shaft is to be transmitted 100KW at 240 rpm. If the allowable shear stresses of the material is 60MPa. The shaft is not to twist more than 1^0 in a length of 3.5 mts. Find the diameter of the shaft based on strength and stiffness criteria. The modulus of rigidity of the material (N) is 80 X 10^3N/mm^2. **[10M]**

OR

11 A cylindrical vessel 3m long and 500mm in diameter with 10mm thick plates is subjected to an internal pressure of 3MPa.Calculate the change in volume of the vessel .Take E=210GPa and Poisson's ratio=0.3 for the vessel material **[10M]**

******

Code No: R17A0305 **R17**

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

II B.Tech I Semester Supplementary Examinations, May 2019

Strength of Materials

(ME)

Roll No										

Time: 3 hours **Max. Marks: 70**

Note: This question paper Consists of 5 Sections. Answer **FIVE** Questions, Choosing ONE Question from each SECTION and each Question carries 14 marks.

****

SECTION-I

1 a) Draw stress-strain curve for a mild steel rod subjected to tension and explain about the salient points on it. **[7M]**

b) A vertical tie, fixed rigidly at the top end consist of a steel rod 2.5 m long and 20 mm diameter encased throughout in a brass tube 20 mm internal diameter and 30 mm external diameter. The rod and the casing are fixed together at both ends. The compound rod is loaded in tension by a force of 10 kN. Calculate the maximum stress in steel and brass. Take E_s=2x10^5N/mm^2 and E_b=1x10^5N/mm^2. **[7M]**

OR

2 A steel tube 50mm in external diamerter and 3mm thick encloses centrally a solid copper bar of 35mm diameter. The bar and the tube are rigidly connected together at the ends at a temperature of 20 ^{0}C. Find the stress in each metal when heated to 170^{0}C. Also find the increase in length, if the original length of the assembly is 350mm. Take α_s=1.08 x 10^{-5} per ^{0}C and α_c=1.7 x 10 $^{-5}$ per ^{0}C . Take E_s =2X10^5 N/mm^2 , E_c =1X10^5 N/mm^2. **[14M]**

SECTION-II

3 A 30m long horizontal beam carries a uniformly distributed load of 1 kN/ m on the whole length along with a point load of 3 kN at the right end. The beam is freely supported at the left end. Determine the position of the second support so that the maximum bending moment on the beam is as small as possible. Also draw the shear force and bending moment diagrams indicating main values. **[14M]**

OR

4 A Beam A B C, 5m long has one support at the end A and other support at B, 8m from A. It carries a point load of 4kN at the middle point of AB and a point load of 3kN at C Draw SFD and BMD. **[14M]**

SECTION-III

5 a) A simply supported symmetric I-section has flanges of size 200 mmX 15 mm and its overall depth is 520 mm. Thickness of web is 10mm. It is strengthened with a plate of size 250 mm X 12mm on compression side. Find the moment of resistance of the section if permissible stress is 160 M Pa. How much uniformly distributed load it can carry if it is used as a cantilever of span 3.6m. **[7M]**

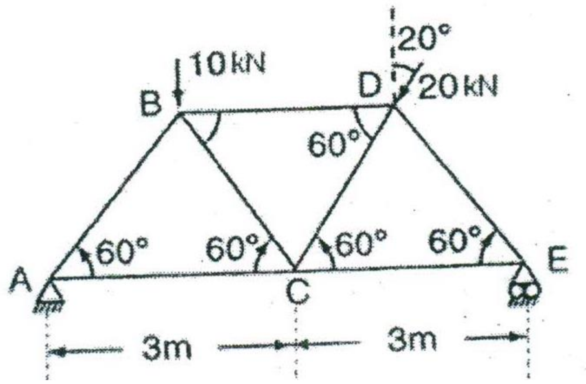
b) A simply supported beam of 2m span carries a U.D.L. of 140 kN/m over the whole span. The cross section of the beam is T-section with a flange width of 120mm, web and flange thickness of 20mm and overall depth of 160mm. Determine the maximum shear stress in the beam and draw the shear stress distribution for the section. **[7M]**

OR

6 A steel beam of I – section, 200mm deep and 160mm wide has 16 mm thick flanges and 10mm thick web. The beam is subjected to a shear force of 200 KN. Determine the shear stress distribution over the beam section if the web of the beam is kept horizontal. **[14M]**

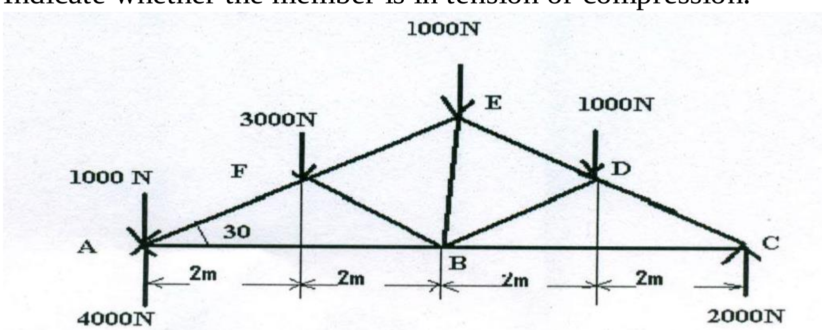
SECTION-IV

7 Find the forces in all the members of the truss as shown in the figure using method of joints. **[14M]**



OR

8 a) Determine the force in member EB of the roof truss shown in the figure. Indicate whether the member is in tension or compression. **[14M]**



SECTION-V

9 **a)** A solid shaft of 200mm diameter gas the same cross sectional area as a hollow shaft of the same material with inside diameter of 150mm. Find the ratio of powers transmitted by both the shafts at the same angular velocity. **[10M]**

b) Derive the expression for circumferential stress for a thin cylinder. . **[4M]**

OR

10 A shell 3.25m long and 1m diameter is subjected to an internal pressure of 1.2 N/mm^2. If the thickness to the shell is 10mm, find the circumferential and longitudinal stresses. Find also the maximum shear stress and changes in dimensions of the shell. Take E = 200 kN/mm^2, poissons ratio=0.3. **[14M]**

****

R17

Code No: R17A0305

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

II B. Tech I Semester Regular Examinations, November 2018

Strength of Materials

(ME)

Roll No										

Time: 3 hours **Max. Marks: 70**

Note: This question paper Consists of 5 Sections. Answer **FIVE** Questions, Choosing ONE Question from each SECTION and each Question carries 14 marks.

*****

SECTION-I

1 Derive the relation between E, G, K. **[14M]**

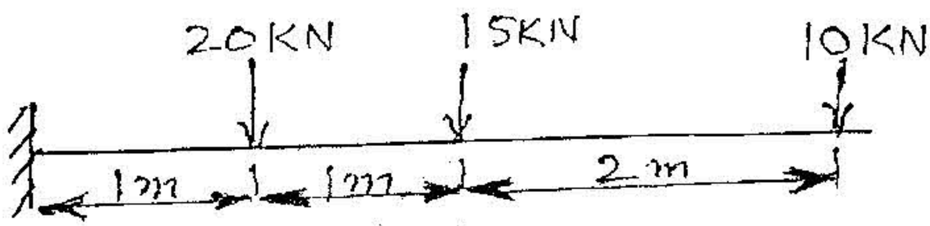
OR

2 a) What is proof resilience and modulous of resilience? (7M) **[14M]**

b)A steel tube of 30 mm external diameter and 25 mm internal diameter encloses a gun metal rod of 20 mm diameter to which it is rigidly joined at each end. The temperature of the whole assembly is raised to 140^0C and the nuts on the rod are then screwed lightly home on the ends of the tube. Find the intensity of stress in the rod when the common temperature has fallen to 30^0C. The value of E for steel and gun metal are $2.1x10^5$N/mm^2 and $1x10^5$N/mm^2 respectively. The linear co efficient of expansion for steel and gun metal are 12 x 10^{-6} per ^{0}C and 20 x 10^{-6} per ^{0}C. (7 M)

SECTION-II

3 Draw SF and BM diagrams for the cantilever shown in Fig **[14M]**



OR

4 A horizontal beam AB of length 4m in hinged at A and supported on rollers at B. the beam carries inclined loads of 100N, 200N and 300N incised towards the roller support at 60^0; 45^0 and 30^0 Respectively to the horizontal, at 1m, 2m and 3m respectively from A. draw the SF and BM diagrams. **[14M]**

SECTION-III

5 a) Explain theory of simple bending, and the assumptions made. Draw stress distribution diagram for a beam with rectangular section.(7M) **[14M]**

b) A timber beam of rectangle section is simply supported at the ends and carries a point load at the center of the beam. The maximum bending stress is 12 N/mm^2 and maximum shearing stress is 1 N/mm^2. Find the ratio of the span to the depth.

OR

6 A simply supported beam carries a U.D.L. of intensity 2.5 kN/m over entire span of 5 meters. The cross-section of the beam is a T-section having the dimensions **[14M]**

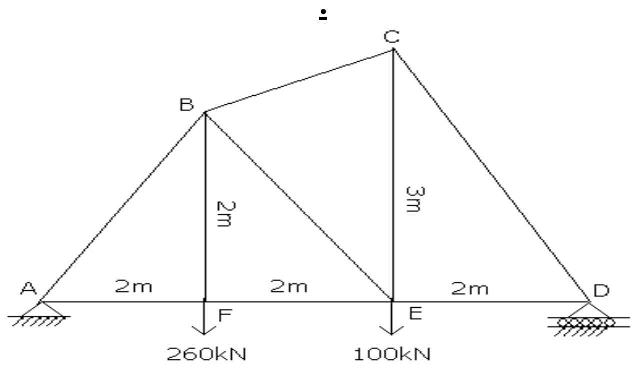
Flange : 125 mm X 25 mm

Web: 175 mm X25 mm

Calculate the maximum shear stress for the section of the beam

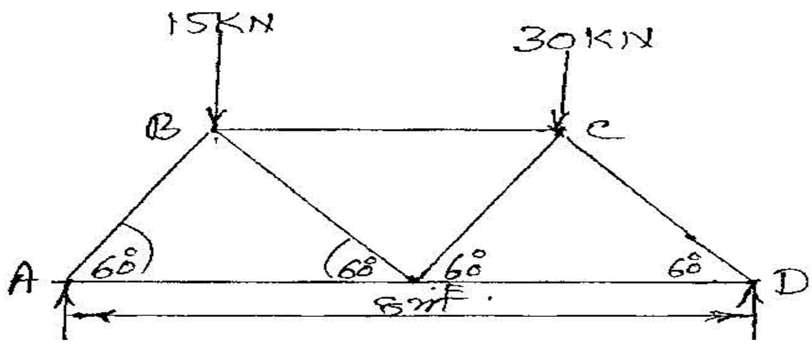
SECTION-IV

7 Analyse the frame shown in Fig **[14M]**



OR

8 Find the magnitude and nature of forces in all the members of the truss shown in Fig **[14M]**



SECTION-V

9 a) A solid steel shaft has to transmit 100 kW at 160 rpm. Taking allowable shear stress as 70 M Pa, find the suitable diameter of the shaft. The maximum torque transmitted in each revolution exceeds the mean by 20 %.(7M) **[14M]**

b) A cylindrical thin drum 800mm in diameter and 4m long is made of 10mm thick plates. If the drum is subjected to an internal pressure of 2.5MPa, determine its changes in diameter and length. Take E as 200GPa and poisons ratio as 0.25.(7M)

OR

10 a) Find the angle of twist per metre length of a hollow circular shaft of 100 mm external and 60 mm internal diameter, if the shear stress is not to exceed 35 M Pa. Take C = 85 G Pa. (7M) **[14M]**

b) A cylindrical vessel 2m long and 500mm in diameter with 10mm thick plates is subjected to an internal pressure of 3MPa.Calculate the change in volume of the vessel .Take E=200GPa and poissons ratio=0.3 for the vessel material.(7M)

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