Project 2

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Part 1

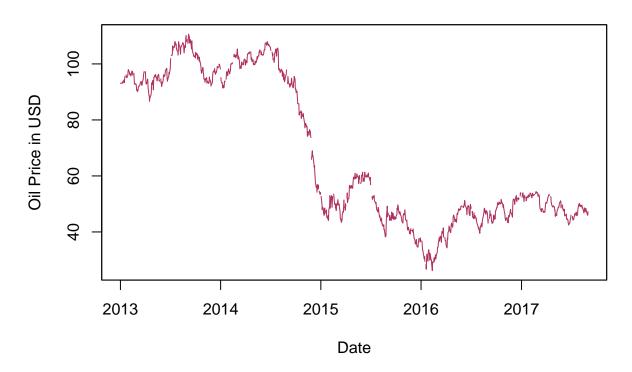
```
library(zoo)
oil_data <- read.csv("oil.csv")</pre>
```

Part 2

```
oil_data$date <- as.Date(oil_data$date, format = "%Y-%m-%d")
oil_ts <- zoo(oil_data$dcoilwtico, oil_data$date)

plot(oil_ts,
    main = "Raw Oil Price Time Series",
    xlab = "Date", ylab = "Oil Price in USD",
    col = "maroon", type = "l")</pre>
```

Raw Oil Price Time Series



Part 3

```
summary(oil_data)
##
         date
                            dcoilwtico
##
           :2013-01-01
                                : 26.19
    1st Qu.:2014-03-03
                          1st Qu.: 46.41
##
    Median :2015-05-02
                          Median : 53.19
##
##
           :2015-05-02
                          Mean
                                : 67.71
    Mean
##
    3rd Qu.:2016-06-30
                          3rd Qu.: 95.66
##
    Max.
           :2017-08-31
                          Max.
                                 :110.62
##
                          NA's
                                 :43
sum(is.na(oil_data$dcoilwtico))
## [1] 43
library(imputeTS)
imputed_oil_data <- na.interpolation(oil_data$dcoilwtico)</pre>
```

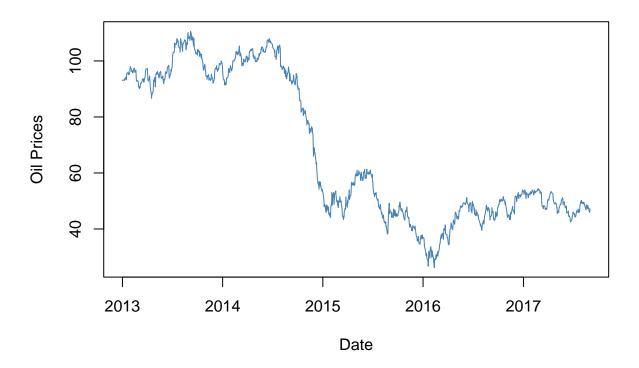
Part 4

```
imputed_oil_ts <- ts(imputed_oil_data, start = c(2013, 1), frequency = 365)

imputed_oil_data <- data.frame(
    date = oil_data$date,
    dcoilwtico = coredata(imputed_oil_ts)
)

plot(imputed_oil_data$date, imputed_oil_data$dcoilwtico, type = "l", col = "steelblue",
    main = "Daily Oil Prices After Imputation",
    xlab = "Date", ylab = "Oil Prices")</pre>
```

Daily Oil Prices After Imputation



After imputing missing values using linear interpolation, the time series is continuous without noticeable artifacts from imputation. The imputed series preserved the overall pattern and major shifts in oil prices, notably the sharp drop in mid-2014. We cannot see any clear seasonality but medium-term trends and cycles appear.

Part 5

The ETS models are of exponential smoothing methods that model time series data by decomposing it into three components namely error, trend, and seasonality. These models are useful for handling data with clear trend and seasonal patterns.

ETS models directly select the best combination of components based on information criteria like AIC. We will use the ets() function from the forecast package to fit these models and the forecast() function will be used to generate future values. The Holt-Winters method is a specific case of ETS models. It extends simple exponential smoothing by adding components for trend and seasonality. It has 2 forms namely additive and multiplicative. The additive version can be used when the seasonal variation is roughly constant over time, while the multiplicative version is appropriate when the variation changes proportionally with the level of the series. The method updates the level, trend, and seasonal components at each time step. We will implement it using HoltWinters() function. We can say that Holt-Winters is limited in flexibility and ETS models offer a broader range of configurations and support for damped trends. We can evaluate the model performance using tools like checkresiduals() and accuracy().

```
library(forecast)
ets_model <- ets(imputed_oil_ts)</pre>
summary(ets_model)
## ETS(A,N,N)
## Call:
## ets(y = imputed_oil_ts)
##
##
     Smoothing parameters:
##
       alpha = 0.9683
##
##
     Initial states:
##
       1 = 93.1395
##
##
     sigma: 1.1771
##
                           BIC
##
        AIC
                AICc
## 9055.133 9055.153 9070.448
##
## Training set error measures:
                                 RMSE
                                                         MPE
                                                                  MAPE
                          ME
                                             MAE
## Training set -0.03893581 1.176176 0.8950884 -0.08222931 1.548288 0.03256606
##
                          ACF1
## Training set -0.0007500942
hw_model <- HoltWinters(imputed_oil_ts, seasonal = "additive")</pre>
hw model
## Holt-Winters exponential smoothing with trend and additive seasonal component.
##
## Call:
## HoltWinters(x = imputed_oil_ts, seasonal = "additive")
## Smoothing parameters:
## alpha: 0.918104
## beta: 0
## gamma: 1
```

```
## Coefficients:
##
                [,1]
         51.08090472
## a
## b
         -0.09657653
## s1
         -2.40677664
## s2
         -2.51681642
## s3
         -3.31555151
## s4
         -3.41167794
## s5
         -2.27299329
## s6
         -1.29063747
         -1.76034147
## s7
         -3.07676790
## s8
## s9
         -3.30047111
## s10
         -6.85914575
## s11
        -10.64228675
## s12
         -8.07150106
## s13
         -9.38470294
## s14
         -8.90573563
## s15
         -9.41001142
## s16
        -10.14343447
## s17
        -12.68409010
       -12.17004565
## s18
## s19
        -14.39890182
## s20
       -15.28624093
## s21
        -17.25130505
## s22
        -19.07012773
## s23
        -18.87051882
## s24
        -18.26628566
        -20.24276615
## s25
## s26
        -17.82511323
## s27
        -18.85003149
## s28
        -17.46289210
## s29
        -18.25661528
## s30
        -18.66797912
## s31
        -19.10923033
## s32
        -19.95634394
## s33
        -19.29751894
## s34
        -19.63401669
        -19.94021592
## s35
## s36
        -20.11843642
## s37
        -22.48716819
## s38
        -24.44274823
        -23.80183770
## s39
## s40
        -23.50285202
        -23.79493809
## s41
## s42
        -25.66127133
## s43
        -25.83425906
        -23.33527602
## s44
## s45
        -25.06653692
## s46
        -23.18349675
        -23.56836234
## s47
## s48
        -24.25634339
## s49 -23.21542147
```

##

```
## s50
        -24.85461257
## s51
        -25.17112578
## s52
        -25.51952738
## s53
        -24.61274421
## s54
        -25.96791946
## s55
        -25.90479410
## s56
        -22.62012234
        -20.72343826
## s57
## s58
        -16.69535939
## s59
        -20.45162410
## s60
         -7.76761616
## s61
          5.37943426
## s62
          5.77261988
          4.42053585
## s63
## s64
          6.86361385
## s65
          5.62197785
## s66
          4.20478800
## s67
          2.88879021
## s68
          2.55283840
## s69
          1.53849206
## s70
          1.98507316
## s71
          1.76598191
## s72
          1.26158493
## s73
          0.88957655
## s74
          2.82907351
## s75
          1.95460344
## s76
          2.63432055
          1.86000856
## s77
## s78
          2.24580605
## s79
          0.68740959
## s80
          1.44443345
## s81
          0.86248316
## s82
          0.93412457
## s83
         -0.22574929
## s84
          0.79925092
## s85
         -0.48413413
## s86
         -0.79207434
## s87
         -1.98111123
## s88
         -3.68689841
## s89
         -4.51134534
## s90
         -4.76783209
## s91
         -4.07681199
         -2.86052163
## s92
## s93
         -3.22760938
## s94
         -4.44630646
## s95
         -4.92531727
## s96
         -6.95539002
         -7.25576779
## s97
## s98
         -8.15807115
## s99
         -7.12714637
## s100
         -7.47784829
## s101
         -7.13694346
## s102
        -6.73329958
## s103 -8.34818572
```

```
## s104 -8.46875603
## s105
        -8.33250502
## s106
        -8.04851626
## s107
        -8.82236014
## s108
        -8.61266962
## s109
        -8.44283074
## s110
        -6.80167525
        -7.67382286
## s111
## s112
        -8.07712281
## s113
        -8.79499582
## s114
        -9.94356767
## s115
        -9.81510288
## s116
        -9.75980245
## s117
        -8.75601953
## s118
        -6.57445594
## s119
        -5.19314328
## s120
        -4.93957927
## s121
        -4.93728124
## s122
        -4.97216398
## s123
        -4.01801413
## s124
        -4.53293978
## s125
        -5.00822681
## s126
        -5.73552778
## s127
        -5.03856733
## s128
       -5.06780229
## s129
        -4.60465771
## s130
        -3.91990777
        -3.28522487
## s131
## s132
        -3.54588798
## s133
        -3.44962194
## s134
        -3.15312090
## s135
        -2.93176270
## s136
        -2.49854974
## s137
        -3.29769870
## s138
        -3.99862173
## s139
        -5.24446912
## s140
        -6.95011599
## s141
        -8.15622631
## s142
        -9.06865749
## s143 -8.98862187
## s144 -10.35523079
## s145 -10.89596908
## s146 -10.06511817
## s147 -10.94061867
## s148 -10.12983963
## s149
        -8.78466375
## s150
        -8.84319871
## s151
        -8.50915088
```

s152

s153

s154

s155

-8.06903971

-7.75643519

-6.17036387

-5.18078589

s156 -5.64432203 ## s157 -6.23392568

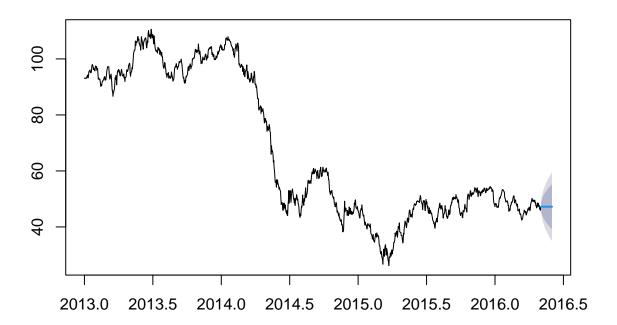
```
## s158
        -4.63877281
## s159
        -4.88657753
## s160
        -4.07155049
         -4.43542094
## s161
## s162
         -5.35799180
## s163
         -4.74706854
## s164
         -4.90798059
         -4.22722913
## s165
         -2.46008190
## s166
## s167
         -2.11488369
## s168
         -1.96057789
## s169
         -1.23874943
## s170
         -1.27637251
## s171
         -1.38347729
## s172
         -0.51651354
## s173
          0.64292231
## s174
          1.42802370
## s175
          1.62890090
## s176
          1.13002753
## s177
          1.61793305
## s178
          0.93693172
## s179
          1.17020381
## s180
          1.22956316
## s181
          1.61216422
## s182
          3.66993675
## s183
          2.73701704
## s184
          0.94812733
          0.80099444
## s185
## s186
          1.54026853
## s187
          0.50633439
## s188
         -0.37089844
## s189
         -2.16758320
## s190
         -2.27508388
## s191
         -1.67125365
## s192
         -2.09258063
## s193
         -0.91307199
## s194
          0.17626794
## s195
         -0.56512398
## s196
         -0.30218618
## s197
         -0.33046185
## s198
         -0.40104436
## s199
          0.49287035
## s200
          1.37327243
## s201
          1.83654565
## s202
          1.81695902
## s203
          0.56958731
## s204
          0.17404752
## s205
          1.30383672
## s206
          2.30069447
## s207
          1.99069321
## s208
          3.20968450
## s209
          4.72223145
## s210
          4.99745983
## s211
          5.20805098
```

```
## s212
          5.68543847
## s213
          5.64725422
          5.71615232
## s214
## s215
          6.50720430
## s216
          6.61091606
## s217
          6.68938996
## s218
          4.67065134
## s219
          4.51182922
## s220
          5.06839123
## s221
          4.02172312
## s222
          4.38804261
## s223
          4.87209418
## s224
          3.70780304
## s225
          3.52384612
## s226
          3.96268227
## s227
          3.76346203
## s228
          4.06970864
## s229
          5.17352019
## s230
          5.13143247
## s231
          5.05521615
## s232
          5.71698122
## s233
          7.01701154
## s234
          7.66254387
## s235
          7.24496847
          7.85484640
## s236
## s237
          8.35173379
## s238
          8.59126507
## s239
          9.87996945
## s240
         10.13995837
         11.23289752
## s241
## s242
         11.32284942
## s243
         10.49907421
## s244
         10.25193503
## s245
         11.07687484
## s246
         10.58559059
## s247
         10.32977265
## s248
         10.68834857
## s249
         11.10027072
## s250
         10.94744705
## s251
         11.24987881
## s252
         12.77782691
## s253
         13.39180276
## s254
         13.34163740
## s255
         15.28889558
## s256
         15.93735275
## s257
         16.17834582
         16.00997664
## s258
         15.66590902
## s259
## s260
         16.14716814
## s261
         17.09453427
## s262
         16.59248007
## s263
         16.27651998
## s264
         16.71532537
## s265
        16.46135581
```

```
## s266
        16.40979020
## s267
         16.27994364
## s268
         16.51554429
## s269
         15.69603284
## s270
         15.43949638
## s271
         15.24461019
## s272
         15.10197775
         15.20126860
## s273
## s274
         14.29145765
## s275
         14.94974009
## s276
         13.27067883
## s277
         13.14526286
         12.36621844
## s278
         13.56138030
## s279
## s280
         15.30919277
## s281
         15.88627340
## s282
         17.42176807
## s283
         17.10105626
## s284
         16.44785690
## s285
         15.55945001
## s286
         17.57916845
## s287
         18.48779092
## s288
         18.09497636
## s289
         17.58870807
## s290
         12.50127442
## s291
         11.39313083
## s292
         11.62636052
## s293
         11.13560982
## s294
         10.55155955
## s295
         10.84461358
## s296
         11.45506662
## s297
         11.93724003
## s298
         11.43480101
## s299
         11.41157184
## s300
         10.00820004
## s301
        11.33172231
## s302
         10.82869729
## s303
          9.20150656
## s304
         10.57263281
          9.10970444
## s305
## s306
          8.45834203
## s307
         10.05200407
         10.73558630
## s308
## s309
         10.69992598
## s310
         11.76935068
## s311
         13.23821949
         11.08803858
## s312
## s313
          8.77854866
## s314
         10.95311605
## s315
         10.51294967
## s316
          9.50249457
## s317
          8.67452825
## s318
          8.78715960
## s319
          8.30805416
```

```
## s320
          9.06323424
## s321
          8.76203868
## s322
          9.21806727
## s323
         11.07415644
## s324
         11.06416951
## s325
         10.05260235
## s326
          9.61609509
          8.69022701
## s327
## s328
          8.72643441
         10.65303610
## s329
## s330
         10.93574562
         12.58981585
## s331
         12.21691945
## s332
## s333
          9.36918692
## s334
          8.49395031
## s335
          8.78690832
## s336
          7.75791449
## s337
          8.33176605
## s338
          7.42918739
## s339
          5.83479959
## s340
          4.33587152
## s341
          4.39874233
## s342
          4.41915528
## s343
          0.76643750
## s344
          0.63161228
## s345
          1.00191417
## s346
          1.68874664
## s347
          1.63579001
## s348
          2.15899198
## s349
          0.34024683
## s350
          1.91672230
## s351
          0.93417145
## s352
          0.82148053
## s353
          1.19481854
## s354
          1.69317024
## s355
          1.20063541
## s356
          0.88437294
## s357
         -0.75906311
## s358
         -2.20870612
## s359
         -0.88514642
## s360
         -1.41038352
## s361
         -0.65270456
## s362
         -1.58817239
        -1.05197330
## s363
## s364
        -1.24186127
## s365
        -3.82090472
ets_forecast <- forecast(ets_model, h = 30)</pre>
plot(ets_forecast, main = "ETS Model Forecast")
```

ETS Model Forecast



```
hw_forecast <- forecast(hw_model, h = 30)
plot(hw_forecast, main = "Holt-Winters Forecast")</pre>
```

Holt-Winters Forecast



Part 6

Since we observed a long-term trend especially in 2014 when it first started to decline, we have recommended the following models:

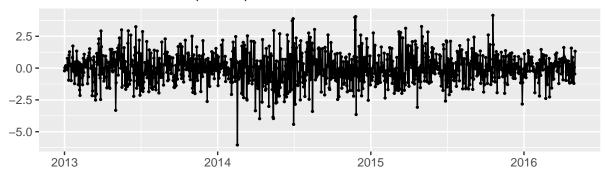
- 1) ETS(A,A,N) or Holt's Linear Trend Model: We recommended this model because it captures the level and an additive trend which corresponds with the long-term trend. It is also simple and deals with the trend directly therefore it can be useful for short-term forecasting. This model may not give indication of any medium-term cyclical patterns because it assumes a linear trend.
- 2) ETS(A,Ad,N) Holt's Damped Trend Model: It is a damped trend model which is a variation of Holt's linear trend that reduces the trend's influence over time. It can handle cyclical patterns better that aren't strictly linear. Thus this model can be suitable for oil prices, where the trend may stabilize or fluctuate. This model is more flexible than ETS(A,A,N) for capturing medium-term fluctuations and it also prevents over forecasting extreme trends but it can be slightly more complex to interpret due to the damping parameter.
- 3) ETS(A,N,N) Simple Exponential Smoothing: This model smooths the level without modeling trend or seasonality. We considered this model to test if the trend is weak enough to be ignored or if differencing the series makes it stationary. Our assumption is that this model may underperform as we are seeing a trend.
- 4) Holt-Winters Additive (ETS(A,A,A)): This is a test to confirm that seasonality is negligible, but we can expect it to not perform well due to the lack of fixed seasonal cycles.

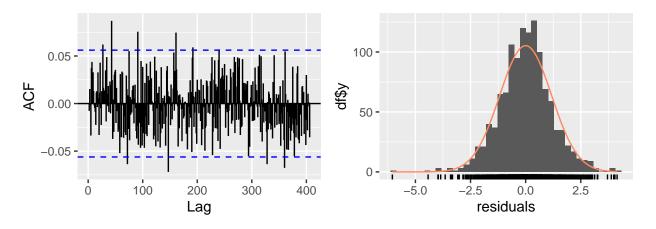
Part 7

checkresiduals(aan_ets)

```
aan_ets <- ets(imputed_oil_ts, model = "AAN")</pre>
summary(aan_ets)
## ETS(A,A,N)
##
## Call:
## ets(y = imputed_oil_ts, model = "AAN")
##
##
     Smoothing parameters:
##
       alpha = 0.9676
       beta = 1e-04
##
##
     Initial states:
##
##
       1 = 93.3935
##
       b = -0.0379
##
##
     sigma: 1.1775
##
##
        AIC
                AICc
                          BIC
## 9057.952 9058.002 9083.477
##
## Training set error measures:
                                                         MPE
                          ME
                                  RMSE
                                             MAE
                                                                 MAPE
## Training set 0.0002500013 1.175605 0.8948867 -0.01426733 1.546043 0.03255872
##
                         ACF1
## Training set -5.143552e-05
```

Residuals from ETS(A,A,N)





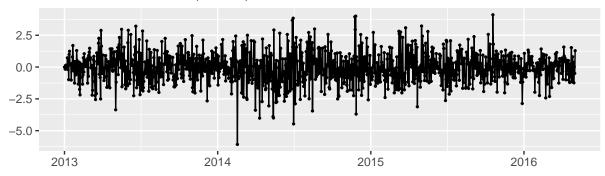
```
##
## Ljung-Box test
##
## data: Residuals from ETS(A,A,N)
## Q* = 259.56, df = 244, p-value = 0.2358
##
## Model df: 0. Total lags used: 244
ann_ets <- ets(imputed_oil_ts, model = "ANN")
summary(ann_ets)
## ETS(A,N,N)
##
## Call:</pre>
```

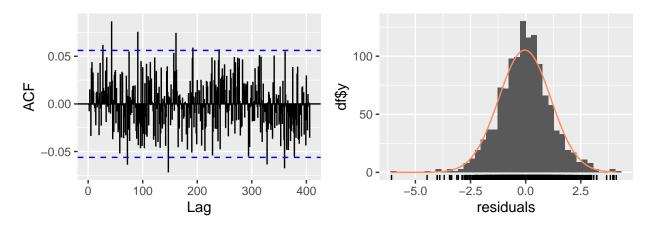
```
## Call:
  ets(y = imputed_oil_ts, model = "ANN")
##
##
     Smoothing parameters:
##
       alpha = 0.9683
##
##
     Initial states:
##
       1 = 93.1395
##
##
     sigma: 1.1771
##
```

```
## AIC AICc BIC
## 9055.133 9055.153 9070.448
##
## Training set error measures:
## Training set -0.03893581 1.176176 0.8950884 -0.08222931 1.548288 0.03256606
## Training set -0.0007500942
```

checkresiduals(ann_ets)

Residuals from ETS(A,N,N)





```
##
## Ljung-Box test
##
## data: Residuals from ETS(A,N,N)
## Q* = 259.69, df = 244, p-value = 0.234
##
## Model df: 0. Total lags used: 244

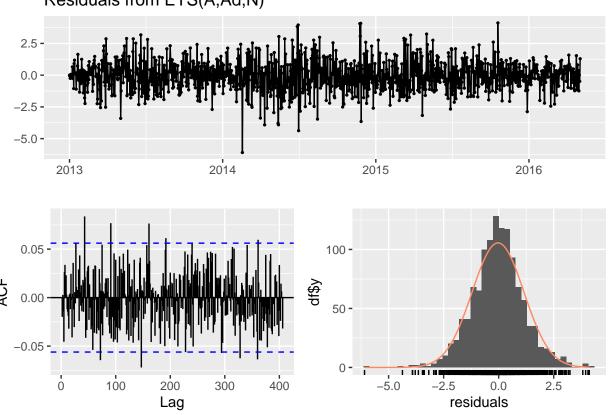
aadn_ets <- ets(imputed_oil_ts, model = "AAN", damped = TRUE)
summary(aadn_ets)</pre>
```

ETS(A,Ad,N)

```
##
## Call:
  ets(y = imputed_oil_ts, model = "AAN", damped = TRUE)
##
##
     Smoothing parameters:
##
       alpha = 0.9545
##
       beta = 0.0108
##
             = 0.9752
##
       phi
##
##
     Initial states:
##
       1 = 93.0527
       b = 0.0895
##
##
##
     sigma: 1.1772
##
##
        AIC
                AICc
                          BIC
## 9058.238 9058.308 9088.868
##
## Training set error measures:
                                                                           MASE
##
                                 RMSE
                                            MAE
                                                        MPE
                                                                MAPE
## Training set -0.02944197 1.174779 0.8936797 -0.05865279 1.54416 0.03251481
##
## Training set 0.0003706787
```

checkresiduals(aadn_ets)

Residuals from ETS(A,Ad,N)

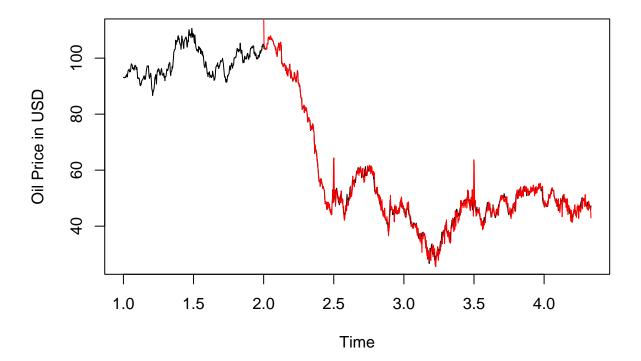


```
##
## Ljung-Box test
##
## data: Residuals from ETS(A,Ad,N)
## Q* = 259.43, df = 244, p-value = 0.2375
##
## Model df: 0. Total lags used: 244

oil_ts_daily <- ts(imputed_oil_ts, frequency = 365)
hw_additive <- HoltWinters(oil_ts_daily, seasonal = "additive")

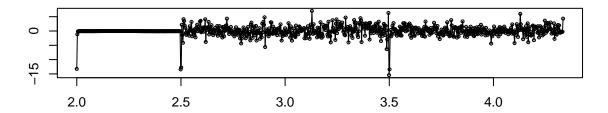
plot(oil_ts_daily, main = "Holt-Winters Additive Fit", ylab = "Oil Price in USD")
lines(fitted(hw_additive)[,1], col = "red")</pre>
```

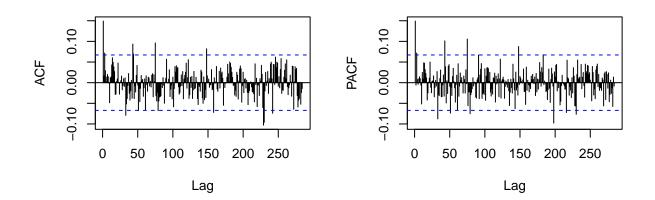
Holt-Winters Additive Fit



```
hw_residuals <- oil_ts_daily - fitted(hw_additive)[,1]
tsdisplay(hw_residuals, main = "Holt-Winters Residuals")</pre>
```

Holt-Winters Residuals





Step 7: Adequacy Analysis Based on Plots

1. ETS(A,N,N) - We can see residuals fluctuating around zero in the residual plot. For the most part, we did not notice any trend in the residuals over time. However, we did see large spikes, similar to around 2014, that we observe a drop in oil prices, which the model did not account for.

In the ACF Plot, most lags fall within the blue confidence bounds which suggests us that there is no significant autocorrelation. We also noticed that there was no significant spikes suggesting us the residuals are mostly white noise. This is very reassuring in terms of model adequacy since it can capture key patterns.

In the Histogram, the residuals are roughly normally distributed, around center 0, but with some very large negative residuals.

As we see no significant spikes in the ACF section, the Ljung-Box p-value > 0.05 tells us that the residuals are uncorrelated.

- 2. ETS(A,A,N) Model The residual plot is similar to that of ETS(A,N,N) residual plot, fluctuating around zero and having some significant outliers.
- The ACF plot shows no substantial autocorrelation, and the p-value is 0.2358 > 0.05 from the Ljung-Box test tells us that the residuals were white noise.

The ACF plot suggests a very slight sign of a normal distribution. The regressions errors are roughly near normal distribution and slightly skewed just like the other model at the 1% level. We can see that there is no autocorrelation in the residuals from the histogram plots.

3. ETS(A,Ad,N) - In the Residual Plot, the residuals vary about zero like in ETS(A,N,N), except that there are much larger spikes around 2014. These spikes may be a little smaller than with ETS(A,N,N) because the model is dampening the trend and is allowing for some of the

trend error to decrease. Also, there is no clear trend in the residuals which is a good thing. The ACF Plot is useful here again because most of the lags are within the blue confidence bounds with no meaningful autocorrelation in the residuals making it look like white noise In the histogram the residuals again are approximately normally distributed with slight skewness, similar to ETS(A,N,N) model.

We do not see any significant spikes in the ACF and also the Ljung-Box p-value is > 0.05, suggesting these residuals are uncorrelated.

4. Holt-Winters Additive (ETS(A,A,A))

In the Fitted Values Plot the red line (fitted values) follows the overall downward trend like drop in 2014 but smooths out much of the data's volatility and medium-term cyclical fluctuations. In the Residual Plot, residuals show clear patterns, large positive residuals early and negative residuals later, reflecting the model's systematic failure to fit the data. In the ACF Plot several lags exceed the blue confidence bounds like at lags 1, 2 indicating significant autocorrelation.

PACF Plot shows similar significant spikes, further confirming autocorrelation in residuals. Here Residuals are not white noise as there is significant autocorrelation, and the fitted values show systematic deviations. Thus Holt-Winters Additive is inadequate for this data due to the lack of seasonality and its inability to capture the data's volatility and cyclical patterns.

Part 8

```
n <- length(imputed_oil_ts)
train_size <- floor(0.9 * n)
train_data <- imputed_oil_ts[1:train_size]
test_data <- imputed_oil_ts[(train_size + 1):n]
h <- length(test_data)</pre>
```

```
ann_ets <- ets(train_data, model = "ANN")

fc_ann <- forecast(ann_ets, h = h)

errors_ann <- test_data - fc_ann$mean

rmse_ann <- sqrt(mean(errors_ann^2))

mae_ann <- mean(abs(errors_ann))

mape_ann <- mean(abs(errors_ann / test_data)) * 100

me_ann <- mean(errors_ann)

naive_errors <- diff(train_data)[1:(length(train_data)-1)]

scale <- mean(abs(naive_errors))

mase_ann <- mae_ann / scale

aicc_ann <- ann_ets$aicc</pre>
```

```
aan_ets <- ets(train_data, model = "AAN")

fc_aan <- forecast(aan_ets, h = h)

errors_aan <- test_data - fc_aan$mean
rmse_aan <- sqrt(mean(errors_aan^2))</pre>
```

```
mae_aan <- mean(abs(errors_aan))</pre>
mape_aan <- mean(abs(errors_aan / test_data)) * 100</pre>
me_aan <- mean(errors_aan)</pre>
naive_errors <- diff(train_data)[1:(length(train_data)-1)]</pre>
scale <- mean(abs(naive_errors))</pre>
mase_aan <- mae_aan / scale</pre>
aicc_aan <- aan_ets$aicc
aadn_ets <- ets(train_data, model = "AAN", damped = TRUE)</pre>
fc_aadn <- forecast(aadn_ets, h = h)</pre>
errors_aadn <- test_data - fc_aadn$mean
rmse_aadn <- sqrt(mean(errors_aadn^2))</pre>
mae aadn <- mean(abs(errors aadn))</pre>
mape_aadn <- mean(abs(errors_aadn / test_data)) * 100</pre>
me_aadn <- mean(errors_aadn)</pre>
naive_errors <- diff(train_data)[1:(length(train_data)-1)]</pre>
scale <- mean(abs(naive_errors))</pre>
mase_aadn <- mae_aadn / scale</pre>
aicc_aadn <- aadn_ets$aicc
hw_model <- HoltWinters(imputed_oil_ts, seasonal = "additive")</pre>
fc_hw <- forecast(hw_model, h = h)</pre>
errors hw <- test data - fc hw$mean
rmse hw <- sqrt(mean(errors hw^2))</pre>
mae_hw <- mean(abs(errors_hw))</pre>
mape_hw <- mean(abs(errors_hw / test_data)) * 100</pre>
me_hw <- mean(errors_hw)</pre>
naive_errors <- diff(train_data)[1:(length(train_data)-1)]</pre>
scale <- mean(abs(naive errors))</pre>
mase_hw <- mae_hw / scale</pre>
n <- length(imputed_oil_ts)</pre>
k <- 3 + frequency(imputed_oil_ts)</pre>
sse <- sum(residuals(hw_model)^2)</pre>
log_lik \leftarrow -n/2 * (log(2 * pi) + log(sse/n) + 1)
aic <- -2 * log_lik + 2 * k # AIC
aicc_hw \leftarrow aic + (2 * k * (k + 1)) / (n - k - 1)
metrics_table <- data.frame(</pre>
  Model = c("ETS(A,N,N)", "ETS(A,A,N)", "ETS(A,Ad,N)", "Holy-Winters"),
  RMSE = c(rmse_ann, rmse_aan, rmse_aadn, rmse_hw),
  MAE = c(mae_ann, mae_aan, mae_aadn, mae_hw),
  MAPE = c(mape_ann, mape_aan, mape_aadn, mape_hw),
  ME = c(me_ann, me_aan, me_aadn, me_hw),
  MASE = c(mase_ann, mase_aan, mase_aadn, mase_hw),
  AICc = c(aicc_ann, aicc_aan, aicc_aadn, aicc_hw)
print(metrics_table)
```

```
## 1 Model RMSE MAE MAPE ME MASE AICC
## 1 ETS(A,N,N) 2.467000 1.969859 4.073964 0.6237022 2.121357 8099.984
## 2 ETS(A,A,N) 3.927298 3.323798 6.795486 3.2083447 3.579425 8102.799
## 3 ETS(A,Ad,N) 3.150155 2.571321 5.226341 2.2554688 2.769076 8102.929
## 4 Holy-Winters 16.098853 13.350858 27.353083 12.3945754 14.377646 5518.061
```

ETS(A,N,N) is the best model by all performance metrics with the minimum RMSE (2.4670), along with MAE, MAPE, ME, MASE, and AICc. It has the best trade-off between accuracy and bias.