477.1001/677.1001 Design and Analysis of Algorithms

University of Nevada, Las Vegas

Spring 2020

Assignment 7

Due: Saturday, April 11, 2020, by email

1. Show the steps of the $O(n^{1.59})$ divide-and-conquer multiplication algorithm (Section 7.1 in the textbook) for the binary numbers $x = 1100$ and $y = 1010$.

2. Assume that `Algorithm 1` uses $n^{1.59}$ many steps and `Algorithm 2` uses n^2 many steps to solve the same problem. Your computer uses one nanosecond per step. What are the run times of `Algorithm 1` and `Algorithm 2` for $n = 1000000$?

3. Carry out the steps to find the median in linear time using `select(12,S)`, where `S` is the sequence of keys below. Assume that the recursion stops when the set has 5 elements or fewer, at which point it returns the correct value.

   `S= 6 2 27 24 12 22 41 19 38 31 46 35 15 10 30`

4. Run the dynamic program for a knapsack problem with 5 items where the values v_i are $(1, 2, 4, 5, 7)$, the weights w_i are $(1, 2, 3, 5, 6)$ and the capacity is 8.

5. The English coinage before decimalization included half-crowns (30 pennies), florins (24 pennies), shillings (12 pennies), sixpences (6 pennies), threepences (3 pennies) and pennies (1 penny). Show that with these coins the greedy algorithm does not necessarily produce an optimal solution even when an unlimited supply of coins of each denomination is available.

6. To celebrate the number 12, a new 12 cent coin is introduced and the quarter is abolished. Therefore, we have now the following coins: 1,5,10,12 cents. Set up the dynamic programming solution that finds the minimum number of coins necessary to make change. Specifically, what is the best way to make change for 16 cents?

How to submit. Create one PDF file with your solutions. Email this file as an attachment to the TA, Mahdi Hajiali, Hajiali@unlv.nevada.edu. Subject of your email must be

`CS477 Bein Assignment 7, <your name>, <your student ID number>`.

Lunes

$W = 11$

$X = 00$

$Y = 10$

$Z = 10$

$P = WY = 11 \cdot 10 = 110$

$q = XZ = 00 \cdot 10 = 00$

$r = (X+X)(Y+Z) = 11 \cdot 100 = 1100$

$1100 \cdot 1010 = (1000 \cdot p) + (100 \cdot (r-p-q)) + q$

$= 1100000 + 11000 + 0$

$= 1111000$

Solutions Asgn 7

1] see previous page

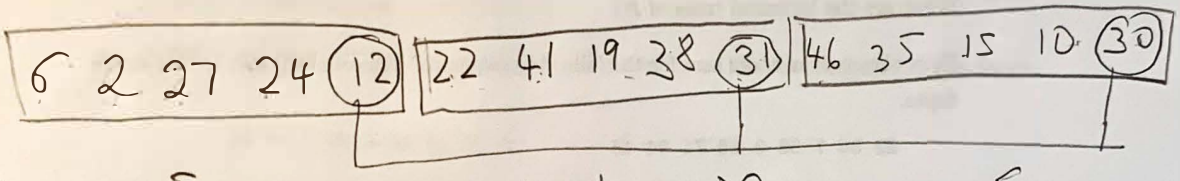
2) $n = 1{,}000{,}000$

$n^{1.59} = 3.47 \cdot 10^9$ steps $= 3.47$ sec

$n^2 = 10^{12}$ steps $= 1000$ sec

factor is 288.18

3]



$S =$ S_1 pivot 30 S_2

6 2 27 24 12 22 19 15 10 | 30 | 41 38 31 46 35

pivot

select (12, S)

= select (2, S_2) = 35

42nd smallest in S_2

5] Greedy: $51 = 30 + 12 + 6 + 3$ Greedy

opt: $24 + 24 + 3$

6] not given

Knapsack solution

0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	1	**2**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**
0	1	2	**4**	**5**	**6**	**7**	**7**	**7**	**7**	**7**	**7**	**7**	**7**	**7**
0	1	2	4	5	6	7	7	**9**	**10**	**11**	**12**	**12**	**12**	**12**
0	1	2	4	5	6	7	**8**	9	**11**	**12**	**13**	**14**	**14**	**16**