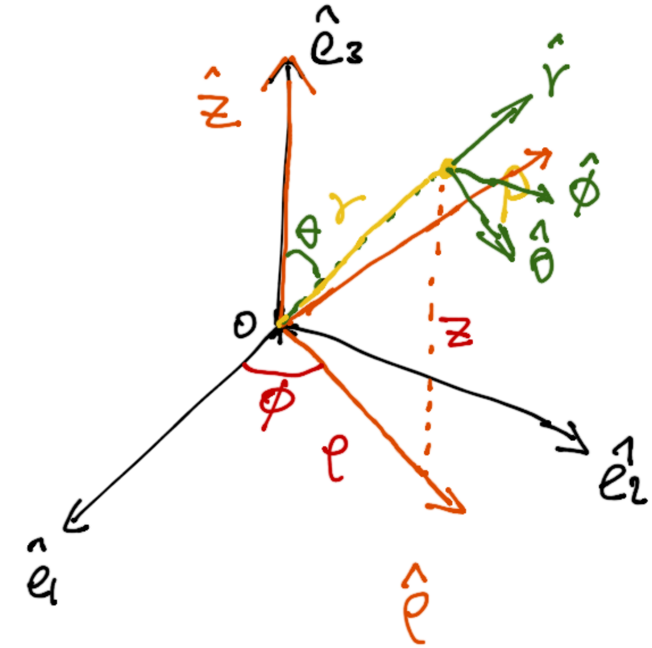
Cartesian system

Cylindrical polar system

Spherical polar system



$$\vec{OP} = \vec{r} = x_1\hat{e}_1 + x_2\hat{e}_2 + x_3\hat{e}_3$$

$$= \rho\hat{\rho} + z\hat{z}$$

$$= r\hat{r}$$

$$\vec{V} = \dot{\vec{r}} = \dot{x}_1\hat{e}_1 + \dot{x}_2\hat{e}_2 + \dot{x}_3\hat{e}_3 \qquad (\dot{\hat{e}}_1, \dot{\hat{e}}_2, \dot{\hat{e}}_3 = 0).$$

$$= \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{z}$$

$$= \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\sin\theta\dot{\phi}\hat{\phi}$$

$$\left[\dot{\rho}\hat{\rho} + \rho\dot{\hat{\rho}} = \dot{\rho}\hat{\rho} + \rho(\cos\phi\dot{\hat{e}}_1 + \sin\phi\dot{\hat{e}}_2)\right]$$

$$= \dot{\rho}\hat{\rho} + \rho\dot{\phi}(-\sin\phi\hat{e}_1 + \cos\phi\hat{e}_2)$$

$$= \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi}\ \Big]$$

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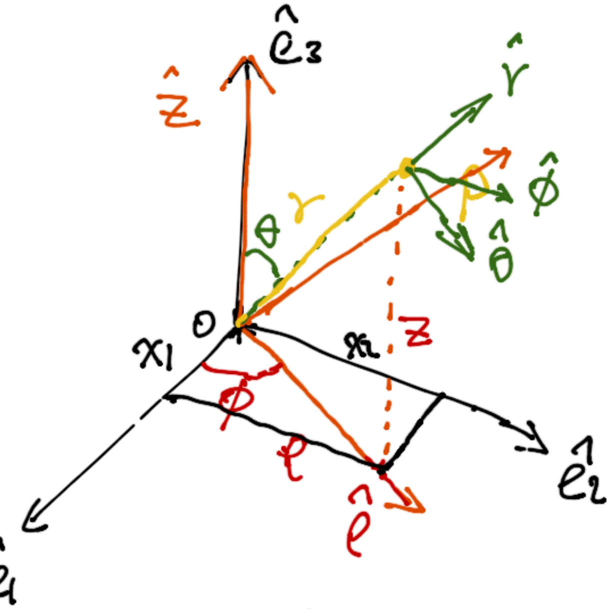
Coordinate transformation

$$^{\mathcal{E}}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad ^{C}\begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$

in C system
in terms of
$\mathcal{E}$

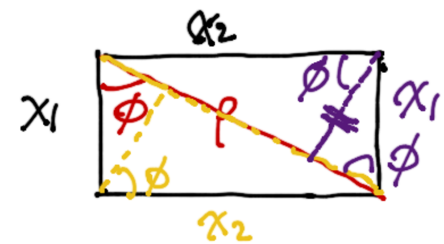
Right



Transformation Matrix.

$$M_C^{\mathcal{E}} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$^{C}\begin{bmatrix} ? \\ ? \\ ? \end{bmatrix} = M_C^{\mathcal{E}} \cdot {}^{\mathcal{E}}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



$$= {}^{C}\begin{bmatrix} \cos\phi\, x_1 + \sin\phi\, x_2 \\ -\sin\phi\, x_1 + \cos\phi\, x_2 \\ x_3 \end{bmatrix} \overset{?}{=} {}^{C}\begin{bmatrix} \rho \\ 0 \\ z \end{bmatrix}$$

Orbit: state, properties, characteristics. description,

transformation,

Orbit Determination: vector tricks under orbit knowledge.

Maneuvers: analyze, design multiple kinds of maneuver.

Intro to real astrodynamics: perturbation, R3BP.

$r(t)$ $\qquad\qquad$ $r(\theta)$

Energy, Momentum, $\longrightarrow$ solution of Keplerian problem.

$$E = \frac{1}{2}\dot{r}^2 + \frac{1}{2}\frac{H^2}{r} - \frac{\mu}{r}$$

Energy.

$$\frac{dE}{dt} = \frac{d^2r}{dt^2} - \frac{H^2}{r^3} + \frac{\mu}{r^2} = 0$$

$$H = r^2\dot{\theta}, \quad \dot{\theta} = \frac{H}{r^2}, \quad \dot{\theta}^2 = \frac{H^2}{r^4},$$

Momentum

$$\ddot{\theta} = -\frac{2H}{r^3}\dot{r} = -\frac{2H^2}{r^5}\frac{dr}{d\theta}$$

$$\frac{dr}{dt} = \frac{dr}{d\theta}\cdot\frac{d\theta}{dt} = \frac{dr}{d\theta}\cdot\dot{\theta}$$

$$\frac{d^2r}{dt^2} = \dot{\theta}^2\frac{d^2r}{d\theta^2} + \ddot{\theta}\frac{dr}{d\theta}$$

$$\left[\frac{H^2}{r^4}\frac{d^2r}{d\theta^2} - \frac{2H^2}{r^5}\frac{dr}{d\theta} - \frac{H^2}{r^3} + \frac{\mu}{r^2} = 0.\right]\cdot\frac{r^2}{H^2}$$

$$\frac{1}{r^2}\frac{d^2r}{d\theta^2} - \frac{2}{r^3}\frac{dr}{d\theta} - \frac{1}{r} + \frac{\mu}{H^2} = 0.$$

$$r = \frac{1}{s}.$$

$$\frac{dr}{d\theta} = \frac{d(\frac{1}{s})}{d\theta} = -\frac{1}{s^2}\frac{ds}{d\theta}$$

$$\frac{d^2r}{d\theta^2} = \frac{2}{s^3}\left(\frac{ds}{d\theta}\right)^2 - \frac{1}{s^2}\frac{d^2s}{d\theta^2}$$

$$s^2 \cdot \left[\frac{2}{s^3}\frac{ds}{d\theta} - \frac{1}{s^2}\frac{d^2s}{d\theta^2}\right] - 2s^3\left[-\frac{1}{s^2}\frac{ds}{d\theta}\right]^2 - s + \frac{\mu}{H^2} = 0.$$

$$\frac{2}{s}\left(\frac{ds}{d\theta}\right)^2 - \frac{d^2s}{d\theta^2} - \frac{2}{s}\left(\frac{ds}{d\theta}\right)^2 - s + \frac{\mu}{H^2} = 0.$$

$$\frac{d^2s}{d\theta^2} + s = \frac{\mu}{H^2}.$$

Non-homo 2nd order Diff. eqn.

$$s = A\cos(\theta-\theta_0) + \frac{\mu}{H^2}.$$

$$\frac{d^2s}{d\theta^2} = -A\cancel{\cos}(\theta-\theta_0)$$

$$p = \frac{H^2}{\mu}, \quad e = pA$$

$$r = \frac{1}{A\cos(\theta-\theta_0) + \frac{\mu}{H^2}}. = \frac{1}{\frac{e}{p}\cos(\theta-\theta_0) + \frac{1}{p}}$$

$$r = \frac{p}{1 + e\cos(\theta - \theta_0)}$$

$e = 0$ $r = p$ circle

$0 < e < 1, -1 < -e < e\cos(\theta - \theta_0) < e < 1.$ ellipse.

$1 + e\cos(\theta - \theta_0) > 0.$

$e \geq 1$ $1 + e\cos(\theta - \theta_0) \to 0.$ $r \to \infty$ parabola, hyperbola.

$e = 1$