> #Снежко Максим, 253505, вариант 1, "Обыкновенные ДУ 1-го порядка"

> #Задача 1

> #Для данного дифференциального уравнения методом изоклин постройте интегральную кривую, проходящую через точку M

> $diff(y(x), x) = y(x) - x^2$

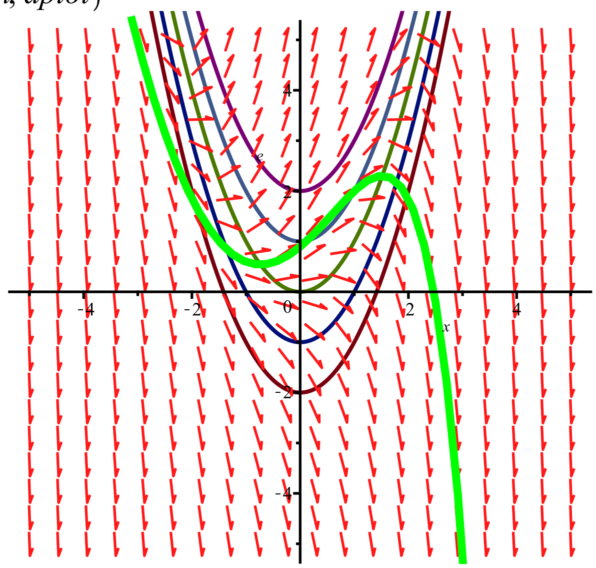
$$\frac{d}{dx} y(x) = y(x) - x^2 \qquad (1)$$

> $with(DETools)$:

> $isoclin := plot\left(\left[seq\left(K + x^2, K = -2 .. 2\right)\right], x = -5 .. 5, e = -5 .. 5\right)$:

> $dplot := DEplot\left(diff(y(x), x) = y(x) - x^2, y(x), x = -5 .. 5, y = -5 .. 5, [y(1) = 2], linecolor = green\right)$:

> $plots[display](isoclin, dplot)$



> $restart$

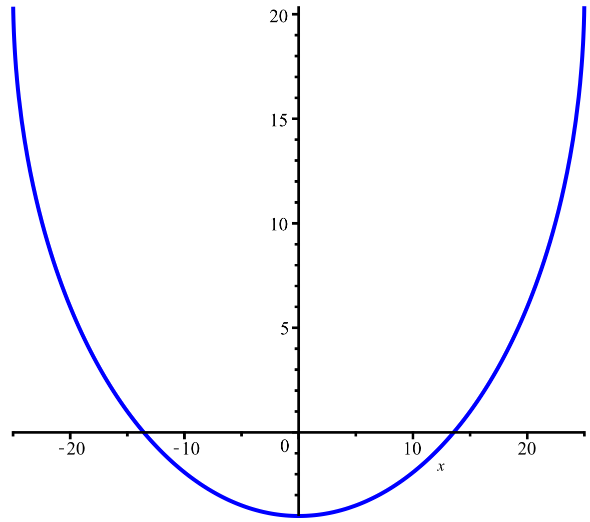
> #Задача 2.1

> $diffEquation := diff(y(x), x) = \frac{x}{\sqrt{625 - x^2}}$
> $solveEquation := dsolve(\{diffEquation, y(15) = 1\})$

$$diffEquation := \frac{d}{dx} y(x) = \frac{x}{\sqrt{-x^2 + 625}}$$

$$solveEquation := y(x) = \frac{(x - 25)\ (x + 25)}{\sqrt{-x^2 + 625}} + 21 \qquad (2)$$

> $plot(rhs(solveEquation), x = -25 .. 25, color = blue)$



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> #Задача 2.2

> restart

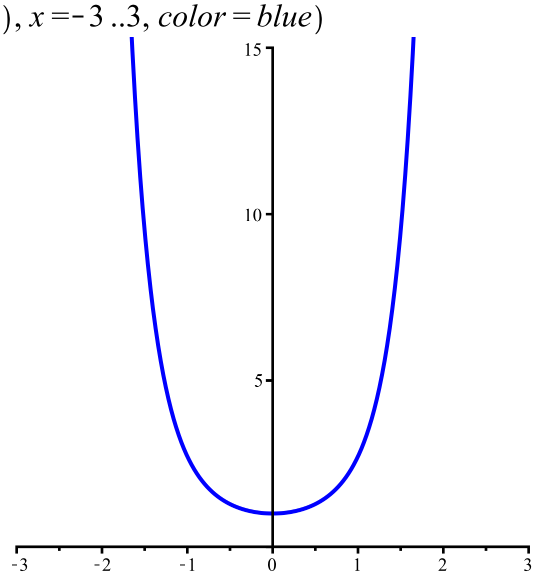
> $diffEquation := diff(y(x), x) = \frac{y(x) \cdot x}{\frac{1}{2}}$

$$diffEquation := \frac{d}{dx}\, y(x) = 2\, y(x)\, x \tag{3}$$

> $solveEquation := dsolve(\{diffEquation, y(1) = e^1\})$

$$solveEquation := y(x) = e^{x^2} \tag{4}$$

> $plot(rhs(solveEquation), x = -3 ..3, color = blue)$



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> #Задание 3

> *restart*

> $expr := diff(y(x), x) = \frac{4 \cdot x + 21 \cdot y(x) - 25}{24 \cdot x + y(x) - 25};$

$$expr := \frac{\mathrm{d}}{\mathrm{d}x}\, y(x) = \frac{4\,x + 21\,y(x) - 25}{24\,x + y(x) - 25} \qquad \textbf{(5)}$$

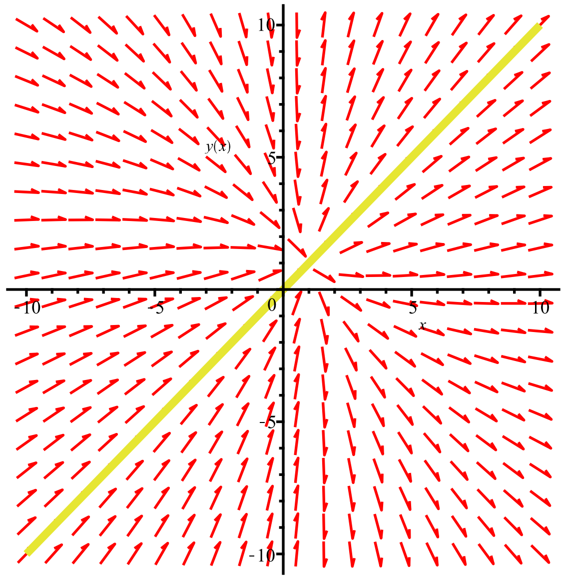
> $sol := dsolve(expr)$

$$sol := 4\ln\left(-\frac{-5 + y(x) + 4\,x}{x-1}\right) - 5\ln\left(\frac{-y(x) + x}{x-1}\right) - \ln(x-1) - \_C1 = 0 \qquad \textbf{(6)}$$

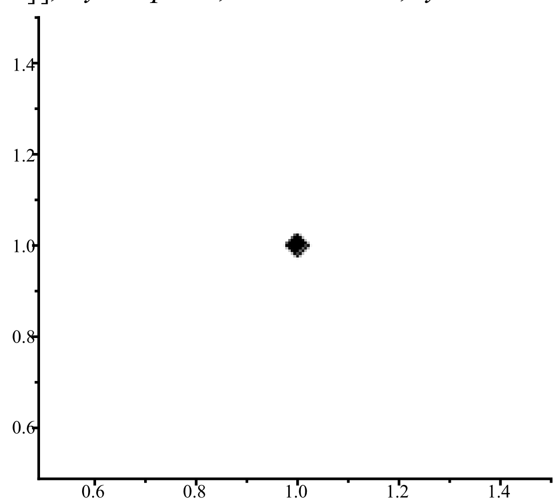
> $solve(\{4 \cdot x + 21 \cdot y - 25 = 0, 24 \cdot x + y - 25 = 0\})$

$$\{x = 1, y = 1\} \qquad \textbf{(7)}$$

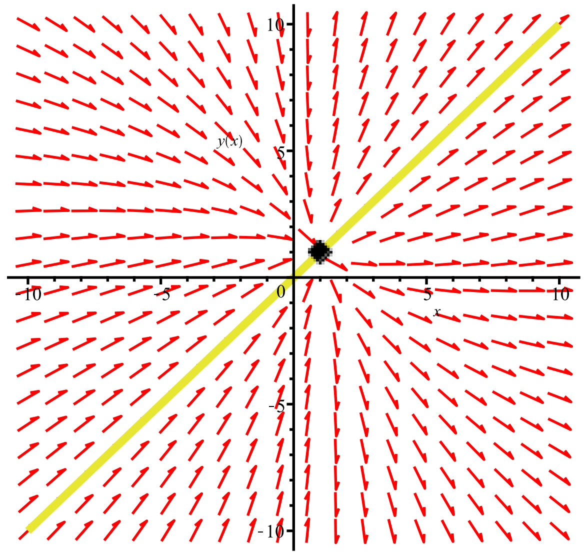
$dfield := DETools[DEplot](expr, y(x), x = -10..10, y = -10..10, [\,y(0) = 0], color = red):$



$suspectPoint := plot([[1, 1]], style = point, color = black, symbol = soliddiamond, symbolsize = 30):$



$plots[display](dfield, suspectPoint)$



> $matr := Matrix([[24-x, 1], [4, 21-x]])$

$$matr := \begin{bmatrix} 24-x & 1 \\ 4 & 21-x \end{bmatrix} \tag{8}$$

> $solve(\ LinearAlgebra[Determinant](matr) = 0\ )$

$$25, 20 \tag{9}$$

> #Так как оба корня положительны, то узел неустойчивый

> #Задание 4
#Найдите решение задачи Коши. Сделайте чертеж интегральной кривой.
restart

> $de := diff(y(x), x) + x \cdot y(x) = (1+x) e^{-x} \cdot y(x)^2$

$$de := \frac{d}{dx}\, y(x) + x\, y(x) = (1+x)\, e^{-x} y(x)^2 \tag{10}$$

> $dsolve(de)$

$$y(x) = \frac{1}{e^{\frac{x^2}{2}}\ \_C1 + e^{-x}} \tag{11}$$

> $dsolve(\{de, y(0) = 1\})$

$$y(x) = \frac{1}{e^{-x}} \tag{12}$$

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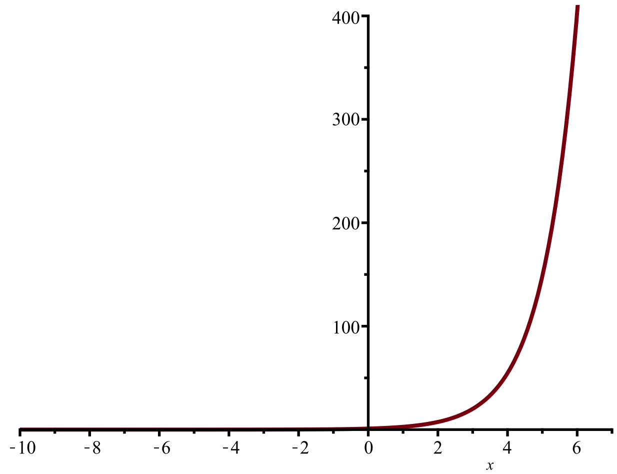
> $plot\left(\frac{1}{e^{-x}}\right)$



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\> *#Задание 5.1*
restart

#Решите дифференциальные уравнения. Постройте в одной системе координат интегральные кривые при целых значениях произвольной постоянной от –1 до 1.

\> $x = \mathrm{D}(y)(x)\arcsin(\mathrm{D}(y)(x)) + \sqrt{1 - (\mathrm{D}(y)(x))^2}$

$$x = \mathrm{D}(y)(x)\,\arcsin(\mathrm{D}(y)(x)) + \sqrt{1 - \mathrm{D}(y)(x)^2} \qquad \textbf{(13)}$$

\> *#Введем замену y'=t*

\> $x := t\cdot\arcsin(t) + \sqrt{1 - t^2}$

$$x := t\arcsin(t) + \sqrt{-t^2 + 1} \qquad \textbf{(14)}$$

\> $dy = t\cdot dx$

$$dy = t\,dx \qquad \textbf{(15)}$$

\> $dx = diff(x, t)\cdot dt$

$$dx = \arcsin(t)\,dt \qquad \textbf{(16)}$$

\> $subs(dx = \arcsin(t)\cdot dt, dy = t\cdot dx)$

$$dy = t\arcsin(t)\,dt \qquad \textbf{(17)}$$

\> $y := rhs(dsolve(\mathrm{D}(y)(t) = t\cdot\arcsin(t)))$

$$y := \frac{t^2\arcsin(t)}{2} + \frac{t\sqrt{-t^2+1}}{4} - \frac{\arcsin(t)}{4} + \_C1 \qquad \textbf{(18)}$$

\> $y1 := subs(\_C1 = -1, y) :$

\> $y2 := subs(\_C1 = 0, y) :$

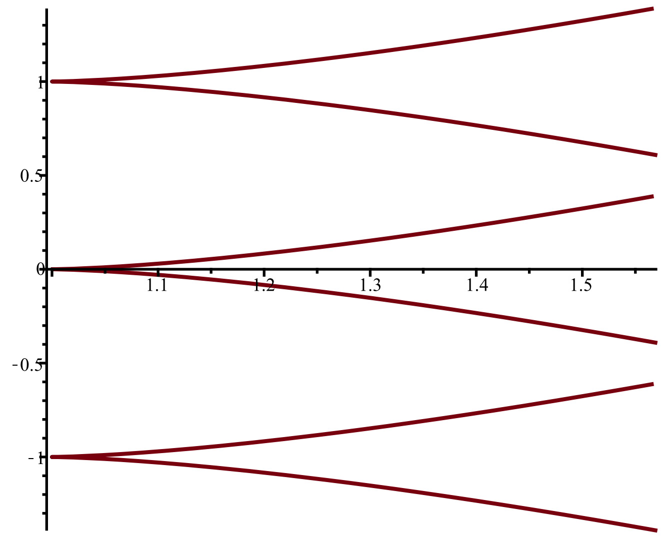
\> $y3 := subs(\_C1 = 1, y) :$

\> $pl1 := plot([x, y1, t = -20..20]) :$

\> $pl2 := plot([x, y2, t = -20..20]) :$

\> $pl3 := plot([x, y3, t = -20..20]) :$

\> $plots[display](pl1, pl2, pl3)$



> *#Задание 5.2*

> *restart*

> $y = \frac{1}{2} \cdot \ln\left(\left|\frac{(1 + D(y)(x))}{1 - D(y)(x)}\right|\right) - D(y)(x)$

$$y = \frac{\ln\left(\left|\frac{1 + D(y)(x)}{-1 + D(y)(x)}\right|\right)}{2} - D(y)(x) \qquad (19)$$

> *#Заменим y' на t*

> $y := \frac{1}{2} \cdot \left(\ln\left(\left|1 + t\right|\right) - \ln(|1 - t|)\right) - t$

$$y := \frac{\ln(|1 + t|)}{2} - \frac{\ln(|t - 1|)}{2} - t \qquad (20)$$

> $dx = \frac{dy}{t}$

$$dx = \frac{dy}{t} \qquad (21)$$

> $simplify(dy = diff(y, t) \cdot dt)$

$$dy = \left(\frac{\mathrm{abs}(1, 1 + t)}{2\,|1 + t|} - \frac{\mathrm{abs}(1, t - 1)}{2\,|t - 1|} - 1\right) dt \qquad (22)$$

> $subs\left(dy = \left(\frac{\mathrm{abs}(1, 1 + t)}{2 \cdot |1 + t|} - \frac{\mathrm{abs}(1, t - 1)}{2 \cdot |t - 1|} - 1\right) \cdot dt, dx = \frac{dy}{t}\right)$

$$dx = \frac{\left(\frac{\mathrm{abs}(1, 1 + t)}{2\,|1 + t|} - \frac{\mathrm{abs}(1, t - 1)}{2\,|t - 1|} - 1\right) dt}{t} \qquad (23)$$

> $x := rhs\left(dsolve\left(D(x)(t) = \frac{\left(\frac{\mathrm{abs}(1, 1 + t)}{2 \cdot |1 + t|} - \frac{\mathrm{abs}(1, t - 1)}{2 \cdot |t - 1|} - 1\right)}{t}\right)\right)$

$$x := -\frac{\left(\begin{cases} \ln(1-t)+\ln(1+t) & t<-1 \\ \mathit{undefined} & t=-1 \\ \ln(1-t)+\ln(1+t) & t<1 \\ \mathit{undefined} & t=1 \\ \ln(t-1)+\ln(1+t) & 1<t \end{cases}\right)}{2} + \_C1 \quad \mathbf{(24)}$$

> $x1 := subs(\_C1=-1, x):$

> $x2 := subs(\_C1=0, x):$

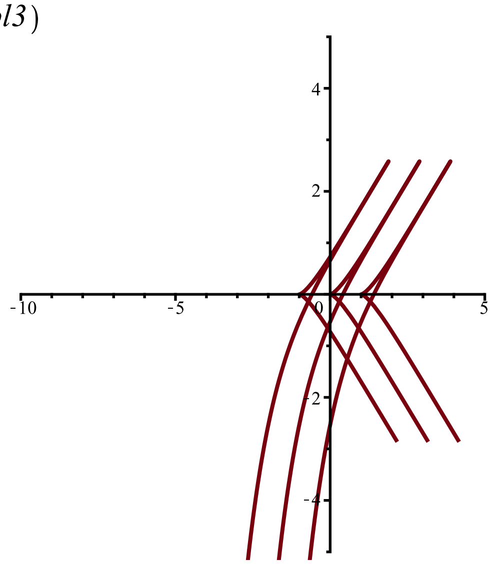
> $x3 := subs(\_C1=1, x):$

> $pl1 := plot([x1, y, t=-10..10], -10..5, -5..5):$

> $pl2 := plot([x2, y, t=-10..10], -10..5, -5..5):$

> $pl3 := plot([x3, y, t=-10..10], -10..5, -5..5):$

> $plots[display](pl1, pl2, pl3)$



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> *#Задание 6*

> *#Найдите все решения уравнения. Постройте в одной системе координат график особого решения и интегральных кривых при целых значениях произвольной постоянной от –3 до 3.*
restart

> $de := y(x) = x \cdot diff(y(x), x) + 2 \cdot (diff(y(x), x))^2 - 1$

$$de := y(x) = x\left(\frac{d}{dx}\, y(x)\right) + 2\left(\frac{d}{dx}\, y(x)\right)^2 - 1 \quad \mathbf{(25)}$$

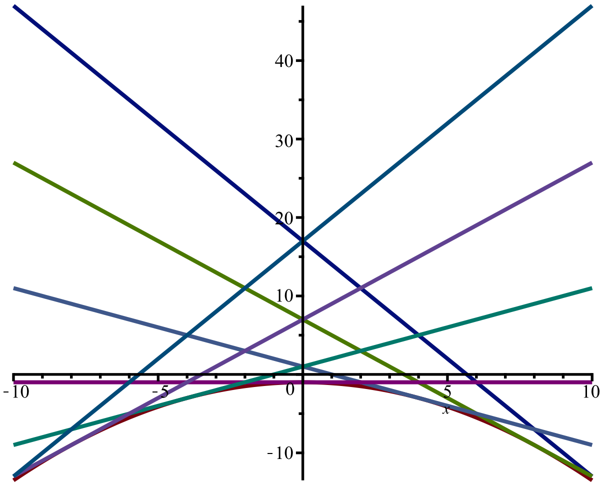
> $sol := dsolve(de)$

$$sol := y(x) = -\frac{x^2}{8} - 1,\ y(x) = 2\,\_C1^2 + x\,\_C1 - 1 \quad \mathbf{(26)}$$

> $sq := seq(2\,C^2 + x \cdot C - 1, C=-3..3)$

$$sq := -3\,x+17,\ -2\,x+7,\ -x+1,\ -1,\ x+1,\ 2\,x+7,\ 3\,x+17 \quad \mathbf{(27)}$$

> $plot\left(\left[-\frac{x^2}{8}-1, sq\right]\right)$



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