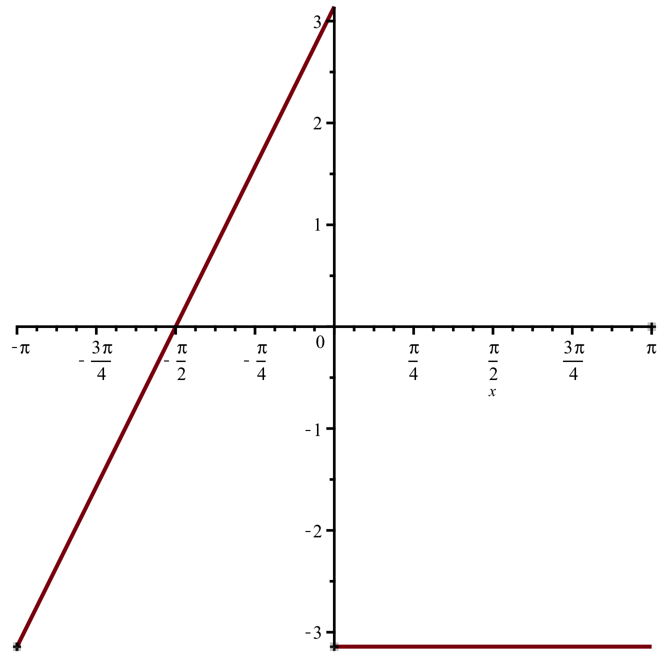
> # Lab2 Снежко Максим, группа 253505
> # Вариант 1

> # Задание 1

> $f := x \rightarrow piecewise(-\pi \le x < 0, \pi + 2 \cdot x, 0 \le x < \pi, -\pi)$

$$f := x \mapsto \begin{cases} \pi + 2 \cdot x & -\pi \le x < 0 \\ -\pi & 0 \le x < \pi \end{cases} \tag{1}$$

> $plot(f(x), x = -\pi..\pi, discont = true)$ # discont подсвечивает точки



> $a0 := simplify\left(\frac{1}{\pi} \cdot int(f(x), x = -\pi..\pi)\right)$ assuming $n :: posint$;

#В Maple, предложение assuming n::posint используется для задания допущения или предположения о переменной n. Конкретно, n::posint указывает, #что переменная n считается положительным целым числом (positive integer). Это означает, что Maple будет рассматривать n как целое число, #которое больше нуля.

$$a0 := -\pi \tag{2}$$

> $an := simplify\left(\frac{1}{\pi} \cdot int(f(x) \cdot \cos(n \cdot x), x = -\pi..\pi)\right)$ assuming $n :: posint$;

$$an := \frac{-2(-1)^n + 2}{\pi n^2} \tag{3}$$

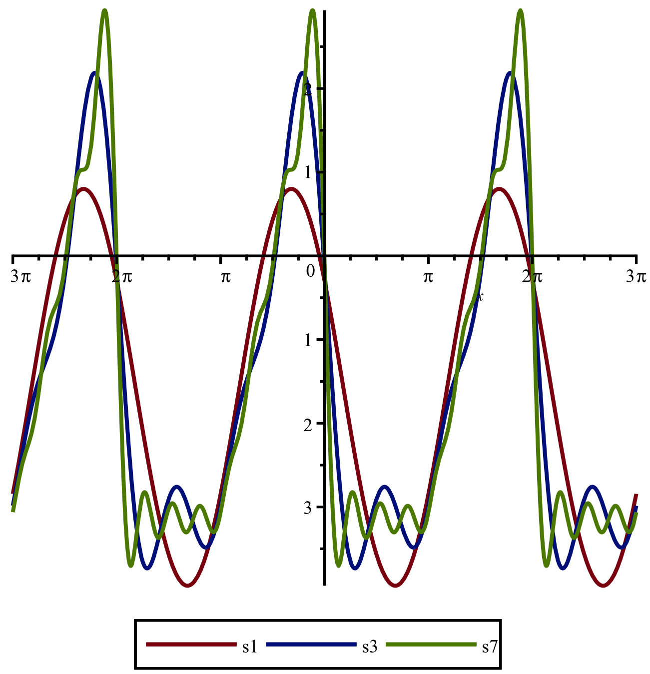
> $bn := simplify\left(\frac{1}{\pi} \cdot int(f(x) \cdot \sin(n \cdot x), x = -\pi..\pi)\right)$ assuming $n :: posint$;

$$bn := -\frac{2}{n} \tag{4}$$

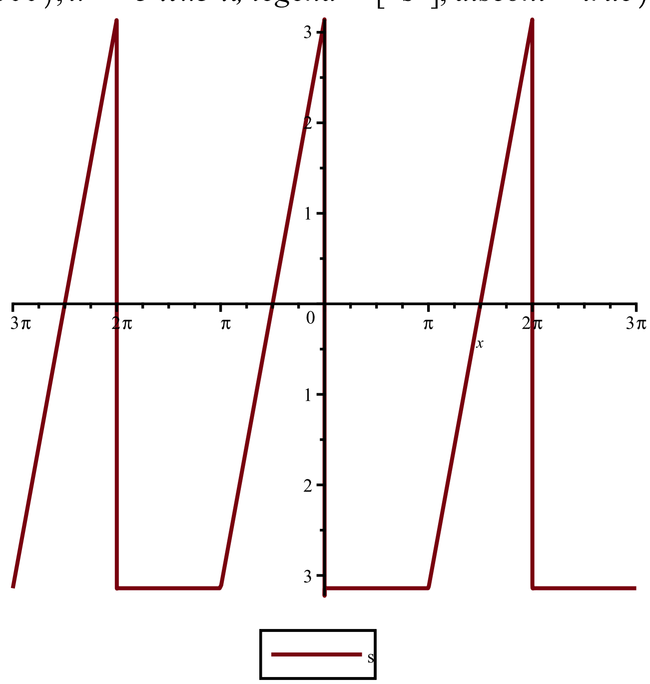
> $FurieSum :=$**proc**(f, k)

```
    local a0, an, bn, n;
  a0 := simplify(int(f(x), x = −π..π)/π);
  assume(n::posint);
  an := simplify(int(f(x)*cos(n*x), x = −π..π)/π);
  bn := simplify(int(f(x)*sin(n*x), x = −π..π)/π);
  return 1/2*a0 + sum(an*cos(n*x) + bn*sin(n*x), n = 1..k)
end proc
```

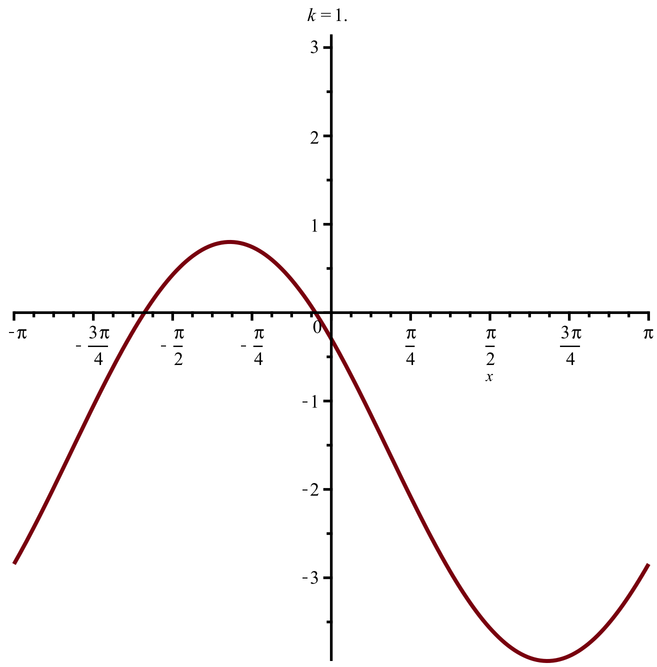
```
> plot([FurieSum(f, 1), FurieSum(f, 3), FurieSum(f, 7)], x =-3π..3 π, legend = ["s1", "s3",
  "s7"], discont = true)
```




```
> plot(FurieSum(f, 100000), x =   3 π..3 π, legend = ["s"], discont = true)
```




```
> plots[animate](plot, [FurieSum(f, k), x =   π..π], k = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10])
```



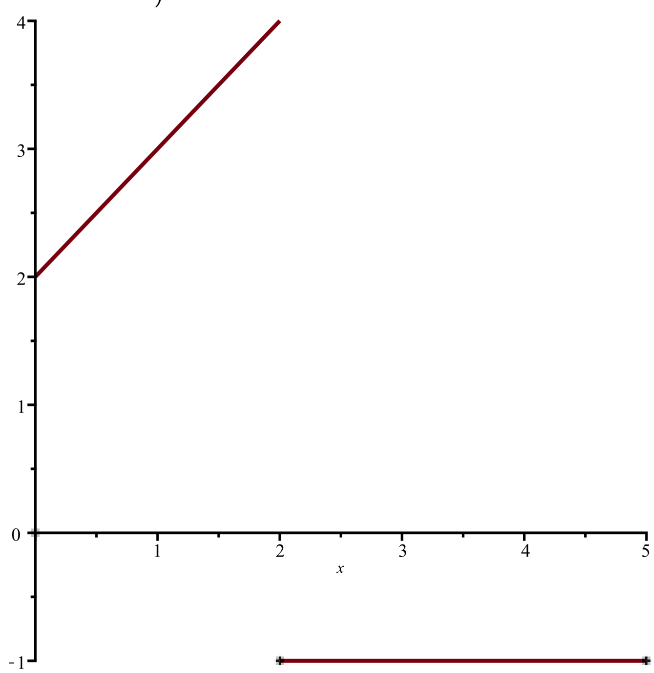
> *restart*;

> *#Задание 2*

> $f := x \rightarrow piecewise(0 < x < 2, x+2, 2 \le x \le 5, -1)$;

$$f := x \mapsto \begin{cases} x+2 & 0 < x < 2 \\ -1 & 2 \le x \le 5 \end{cases} \tag{5}$$

> $plot(f(x), x = 0..5, discont = true)$



> $l := \frac{5}{2}$: *# половина периода*

> $a0 := simplify\left(\frac{1}{l} \cdot int(f(x), x = 0..2 \cdot l)\right)$;

$$a0 := \frac{6}{5} \tag{6}$$

> $an := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right), x = 0..2 \cdot l\right)\right)$ assuming $n :: posint$

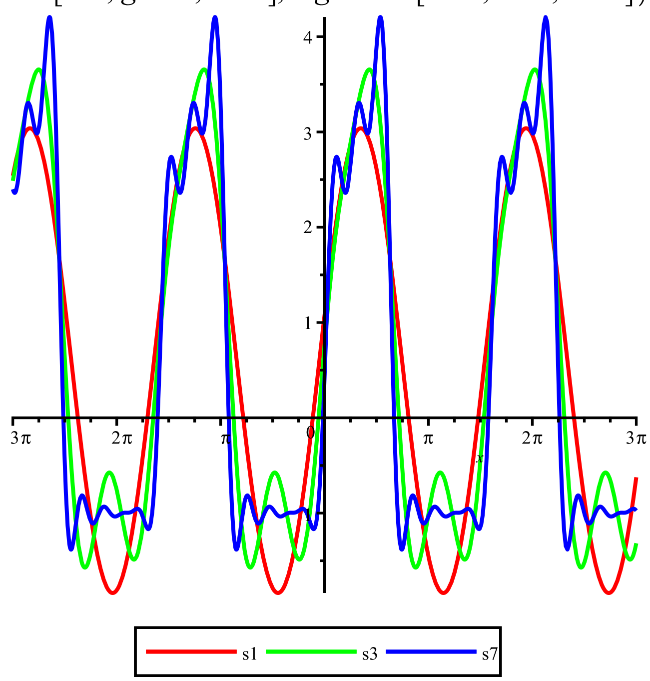
$$an := \frac{5 \left(2 \pi n \sin\left(\frac{4 \pi n}{5}\right) + \cos\left(\frac{4 \pi n}{5}\right) - 1\right)}{2 n^2 \pi^2} \tag{7}$$

> $bn := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right), x = 0..2 \cdot l\right)\right)$ assuming $n :: posint$

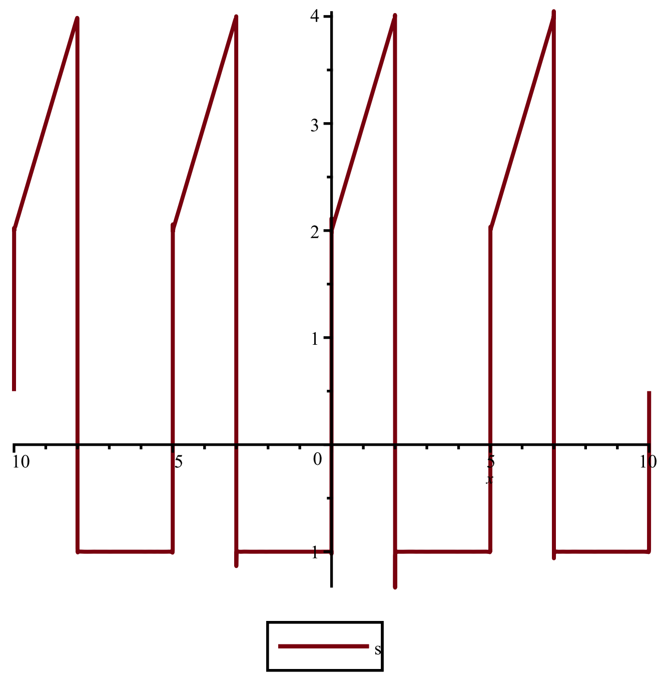
$$bn := \frac{-10 \pi n \cos\left(\frac{4 \pi n}{5}\right) + 6 \pi n + 5 \sin\left(\frac{4 \pi n}{5}\right)}{2 \pi^2 n^2} \tag{8}$$

```
> FurieSum :=proc(f, k, x1, x2)
    local a0, an, bn, n, l;
    l := 1/2*x2 − 1/2*x1;
    a0 := int(f(x), x = 0..2*l)/l;
    assume(n::posint);
    an := int(f(x)*cos(π*n*x/l), x = 0..2*l)/l;
    bn := int(f(x)*sin(π*n*x/l), x = 0..2*l)/l;
    return 1/2*a0 + sum(an*cos(π*n*x/l) + bn*sin(π*n*x/l), n = 1..k)
  end proc
```

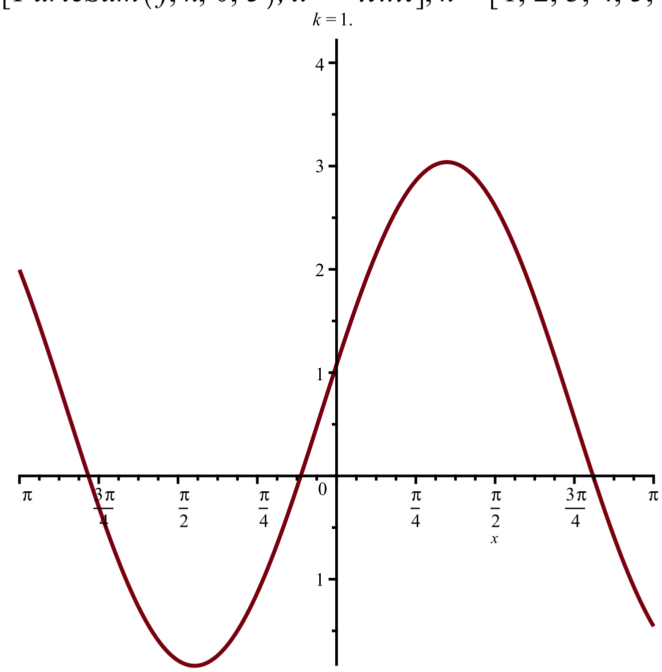
```
> plot([FurieSum(f, 1, 0, 5), FurieSum(f, 3, 0, 5), FurieSum(f, 7, 0, 5)], x = -3·Pi..3·Pi,
     discont = true, color = [red, green, blue], legend = ["s1", "s3", "s7"])
```




```
> plot(FurieSum(f, 10000, 0, 5), x =  10..10, legend = ["s"])
```



> $plots[animate](plot, [FurieSum(f, k, 0, 5), x = \ \pi..\pi], k = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10])$



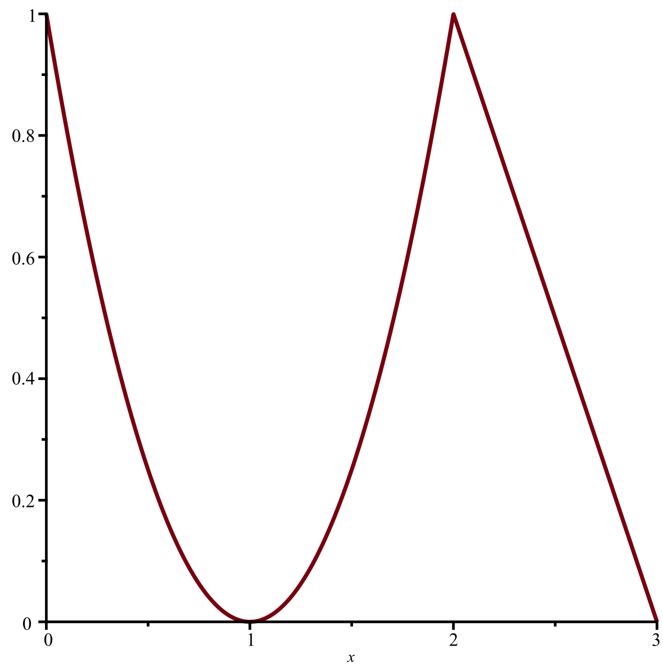
> $restart;$

> # *Задание 3*

> $f := x \rightarrow piecewise(0 \le x \le 2,\ (x \quad 1)^2,\ 2 < x < 3,\ (3 \quad x));$

$$f := x \mapsto \begin{cases} (x \quad 1)^2 & 0 \le x \le 2 \\ 3 \quad x & 2 < x < 3 \end{cases} \tag{9}$$

> $plot(f(x), x = 0..3)$



> $l := \frac{3}{2}$:

> $a0 := simplify\left(\frac{1}{l} \cdot int(f(x), x = 0 ..2 \cdot l)\right);$

$$a0 := \frac{7}{9} \tag{10}$$

> $an := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right), x = 0 ..2 \cdot l\right)\right)$ assuming $n :: posint$

$$an := \frac{9 \pi n \cos\left(\frac{4 \pi n}{3}\right) + 3 \pi n - 9 \sin\left(\frac{4 \pi n}{3}\right)}{2 \pi^3 n^3} \tag{11}$$

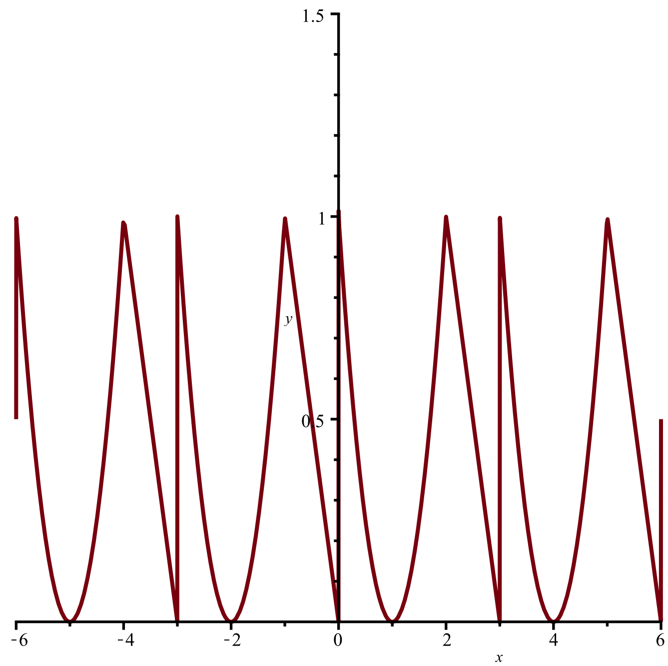
> $bn := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right), x = 0 ..2 \cdot l\right)\right)$ assuming $n :: posint$

$$bn := \frac{2 \pi^2 n^2 + 9 \pi n \sin\left(\frac{4 \pi n}{3}\right) + 9 \cos\left(\frac{4 \pi n}{3}\right) - 9}{2 \pi^3 n^3} \tag{12}$$

> $S := k \rightarrow \frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right) + bn \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), n = 1 ..k\right);$

$$S := k \mapsto \frac{a0}{2} + \sum_{n=1}^{k} \left(an \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right) + bn \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right)\right) \tag{13}$$

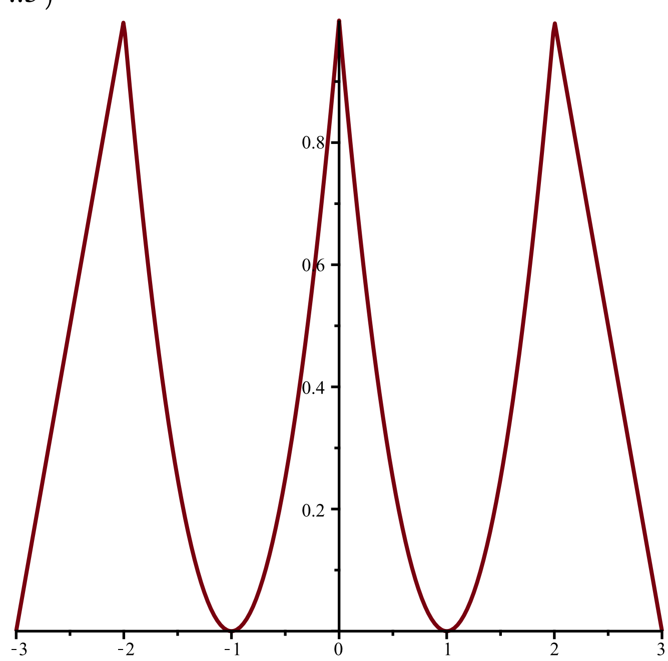
> $plot(S(100000), x = -6 ..6, y = 0 ..1.5, discont = true)$



> $fchetn := x \rightarrow piecewise\left(-3 < x < -2,\ (3 + x), -2 \leq x \leq 0,\ (-x - 1)^2, 0 \leq x \leq 2,\ (x - 1)^2,\right.$
$\left. 2 < x < 3,\ (3 - x)\right);$

$$fchetn := x \mapsto \begin{cases} 3 + x & -3 < x < -2 \\ (-x - 1)^2 & -2 \leq x \leq 0 \\ (x - 1)^2 & 0 \leq x \leq 2 \\ 3 - x & 2 < x < 3 \end{cases} \tag{14}$$

> $plot(fchetn(x), x = -3 ..3)$



> $l := 3$

$$l := 3 \tag{15}$$

> $a0 := simplify\left(\frac{1}{l} \cdot int(fchetn(x), x=-l..l)\right);$

$$a0 := \frac{7}{9} \tag{16}$$

> $an := simplify\left(\frac{1}{l} \cdot int\left(fchetn(x) \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right), x=-l..l\right)\right)$ assuming $n :: posint$

$$an := \frac{18\,\pi\, n \cos\left(\frac{2\,\pi\, n}{3}\right) - 6\,\pi\,(-1)^n\, n + 12\,\pi\, n - 36 \sin\left(\frac{2\,\pi\, n}{3}\right)}{n^3\,\pi^3} \tag{17}$$

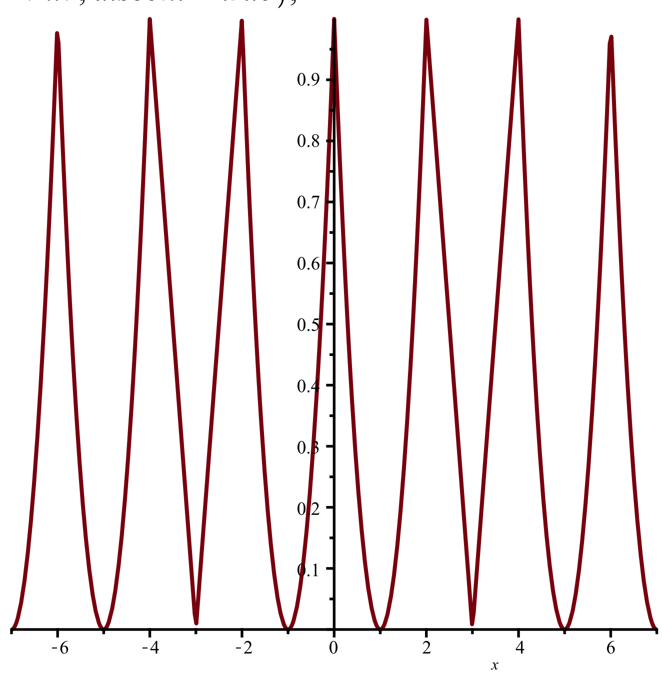
> $bn := simplify\left(\frac{1}{l} \cdot int\left(fchetn(x) \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right), x=-l..l\right)\right)$ assuming $n :: posint$

$$bn := 0 \tag{18}$$

> $S := k \rightarrow \frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right) + bn \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), n = 1..k\right)$

$$S := k \mapsto \frac{a0}{2} + \sum_{n=1}^{k} \left(an \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right) + bn \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right)\right) \tag{19}$$

> $plot(S(1000000), x = -7..7, discont = true);$



> $fnech := x \rightarrow piecewise\left(-3 < x < -2, -(3 + x), -2 \le x \le 0, -(-x - 1)^2, 0 \le x \le 2, (x - 1)^2, 2 < x < 3, (3 - x)\right);$

$$fnech := x \mapsto \begin{cases} -3 - x & -3 < x < -2 \\ -(-x - 1)^2 & -2 \le x \le 0 \\ (x - 1)^2 & 0 \le x \le 2 \\ 3 - x & 2 < x < 3 \end{cases} \tag{20}$$

> $plot(fnech(x), x=-3..3)$

> $a0 := simplify\left(\frac{1}{l}\cdot int(fnech(x), x=-l..l)\right);$

$$a0 := 0 \tag{21}$$

> $an := simplify\left(\frac{1}{l}\cdot int\left(fnech(x)\cdot\cos\left(\frac{\pi\cdot n\cdot x}{l}\right), x=-l..l\right)\right)$ assuming $n :: posint$

$$an := 0 \tag{22}$$

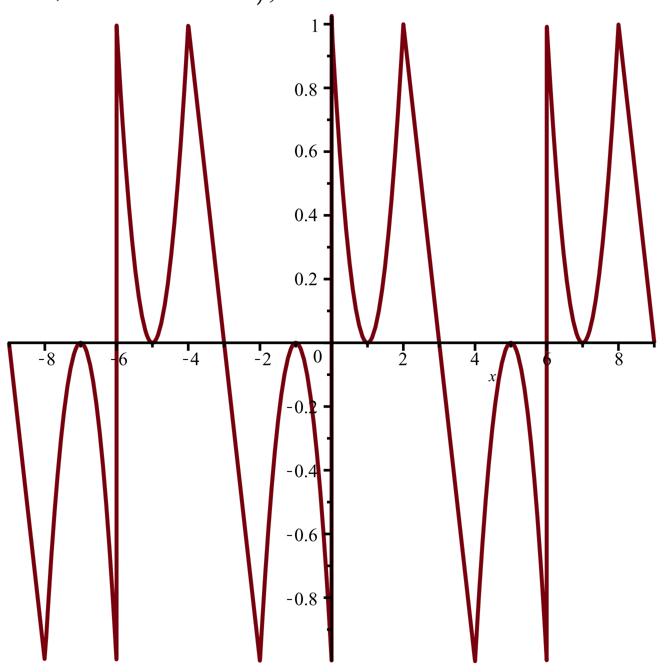
> $bn := simplify\left(\frac{1}{l}\cdot int\left(fnech(x)\cdot\sin\left(\frac{\pi\cdot n\cdot x}{l}\right), x=-l..l\right)\right)$ assuming $n :: posint$

$$bn := \frac{2\pi^2 n^2 + 18\pi n \sin\left(\frac{2\pi n}{3}\right) + 36\cos\left(\frac{2\pi n}{3}\right) - 36}{\pi^3 n^3} \tag{23}$$

> $S := k \rightarrow \frac{a0}{2} + sum\left(an\cdot\cos\left(\frac{\mathrm{Pi}\cdot n\cdot x}{l}\right) + bn\cdot\sin\left(\frac{\mathrm{Pi}\cdot n\cdot x}{l}\right), n=1..k\right)$

$$S := k \mapsto \frac{a0}{2} + \sum_{n=1}^{k}\left(an\cdot\cos\left(\frac{\pi\cdot n\cdot x}{l}\right) + bn\cdot\sin\left(\frac{\pi\cdot n\cdot x}{l}\right)\right) \tag{24}$$

> $plot(S(100000), x=-9..9, discont=true);$



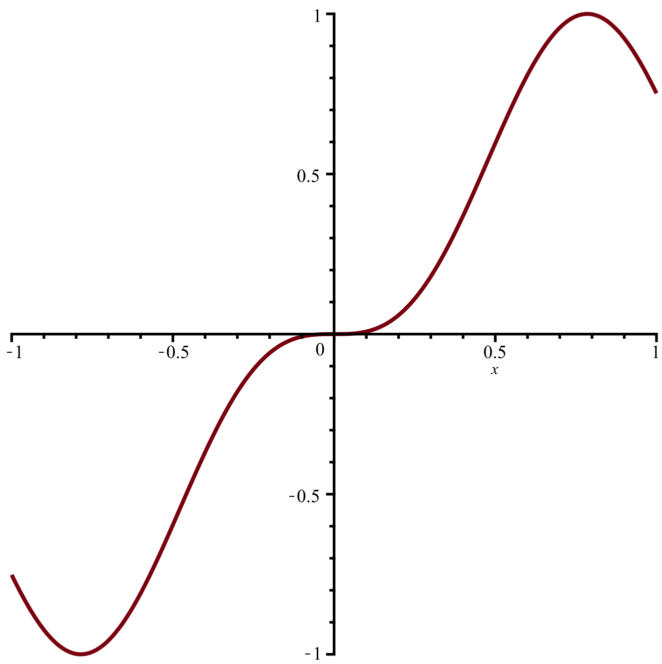
> # Задание 4

> restart

> $f := \sin(2x)^3$

$$f := \sin(2x)^3 \tag{25}$$

> $fplot := plot(f, x=-1..1)$



> $with(orthopoly)$

$$[G, H, L, P, T, U] \tag{26}$$

> **for** n **from** 0 **to** 7 **do** $c[n] := \frac{\left(\int_{-1}^{1} f \cdot P(n, x)\, dx\right)}{\int_{-1}^{1} P(n, x)^2\, dx}$; **end do**

$$c_0 := 0$$

$$c_1 := -\frac{\sin(2)^2 \cos(2)}{2} - \cos(2) + \frac{\sin(2)^3}{12} + \frac{\sin(2)}{2}$$

$$c_2 := 0$$

$$c_3 := -\frac{49 \sin(2)^2 \cos(2)}{72} + \frac{133 \cos(2)}{18} + \frac{77 \sin(2)}{36} + \frac{469 \sin(2)^3}{432}$$

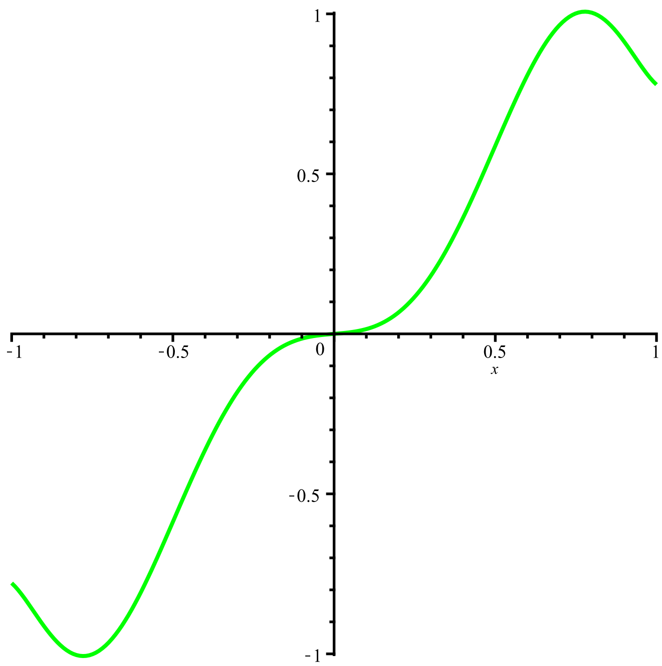
$$c_4 := 0$$

$$c_5 := \frac{715 \sin(2)^3}{576} + \frac{209 \sin(2)^2 \cos(2)}{96} - \frac{6215 \sin(2)}{96} - \frac{6721 \cos(2)}{48}$$

$$c_6 := 0$$

$$c_7 := -\frac{123305 \sin(2)^3}{20736} - \frac{8395 \sin(2)^2 \cos(2)}{3456} + \frac{681785 \cos(2)}{108} + \frac{2499805 \sin(2)}{864} \tag{27}$$

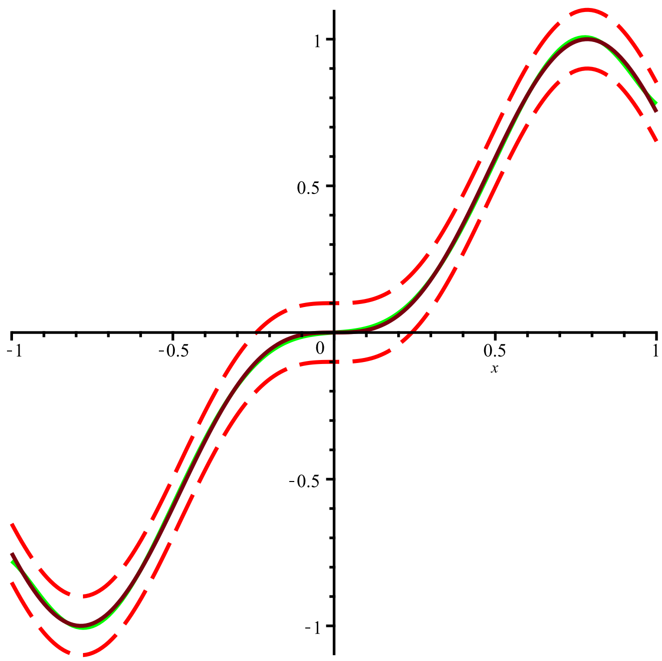
> $lej := plot(add(c[n] \cdot P(n, x), n = 0..7), x = -1..1, color = green)$



> $fdop1 := plot(f - 0.1, x = -1..1, linestyle = dash, color = red) :$

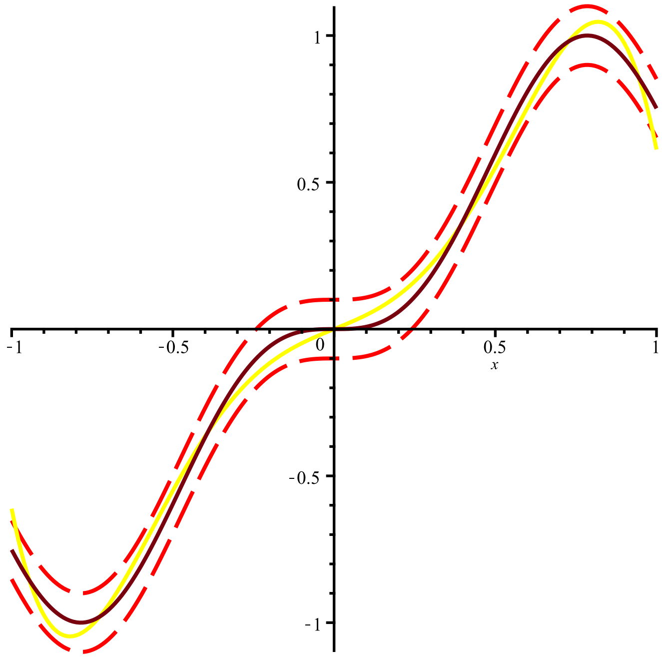
> $fdop2 := plot(f + 0.1, x = -1..1, linestyle = dash, color = red) :$

> $plots[display]([fdop1, fdop2, lej, fplot])$



> $mmin := plot(add(c[n] \cdot P(n, x), n = 0..6), x = -1..1, color = yellow) :$

> $plots[display]([fdop1, fdop2, mmin, fplot])$



> **for** n **from** 0 **to** 7 **do** $c[n] := \dfrac{\left(\displaystyle\int_{-1}^{1} \frac{f\cdot(\cos(n\cdot\arccos(x)))}{\sqrt{1-x^2}}\,\mathrm{d}x\right)}{\displaystyle\int_{-1}^{1} \frac{\cos(n\cdot\arccos(x))^2}{\sqrt{1-x^2}}\,\mathrm{d}x}$; **end do**

$$c_0 := 0$$

$$c_1 := \frac{2\left(\displaystyle\int_{-1}^{1} \frac{\sin(2\,x)^3\,x}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi}$$

$$c_2 := 0$$

$$c_3 := \frac{2\left(\displaystyle\int_{-1}^{1} \frac{\sin(2\,x)^3\cos(3\arccos(x))}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi}$$

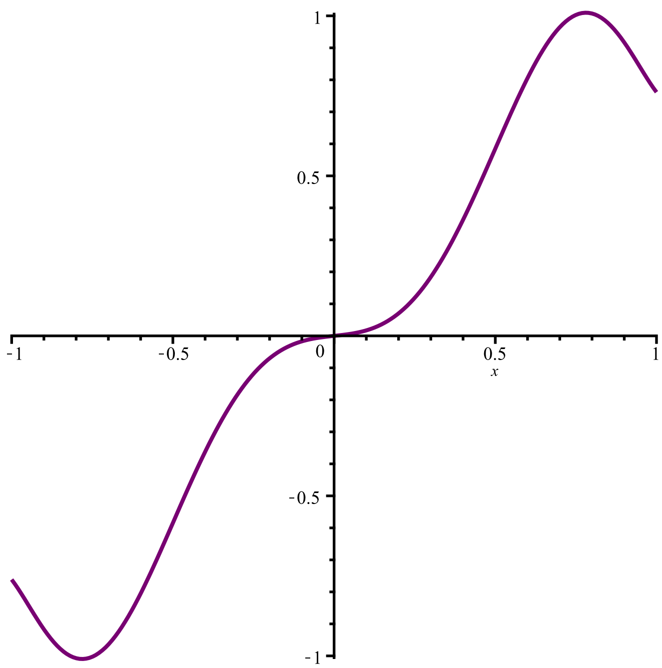
$$c_4 := 0$$

$$c_5 := \frac{2\left(\displaystyle\int_{-1}^{1} \frac{\sin(2\,x)^3\cos(5\arccos(x))}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi}$$

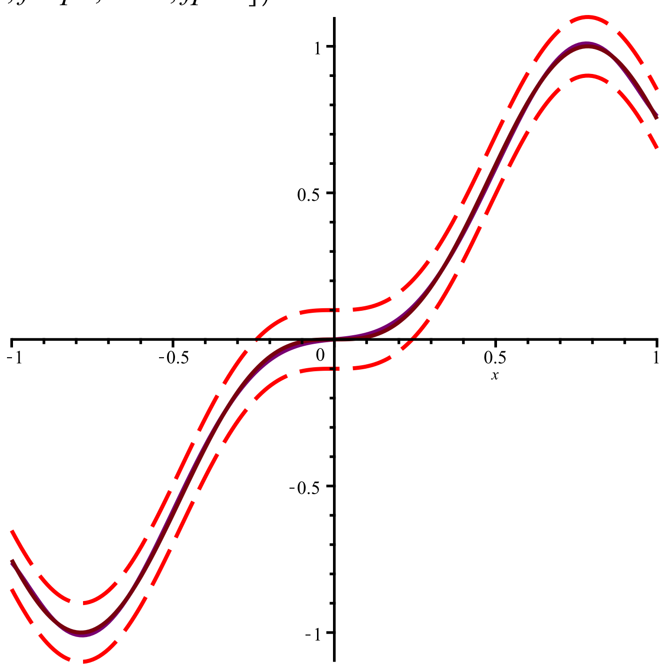
$$c_6 := 0$$

$$c_7 := \frac{2\left(\int_{-1}^{1} \frac{\sin(2x)^3 \cos(7\arccos(x))}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi} \tag{28}$$

> $cheb := plot\left(\frac{c[0]}{2} + add(c[n]\cdot T(n,x), n = 1..7), x = -1..1, color = purple\right)$

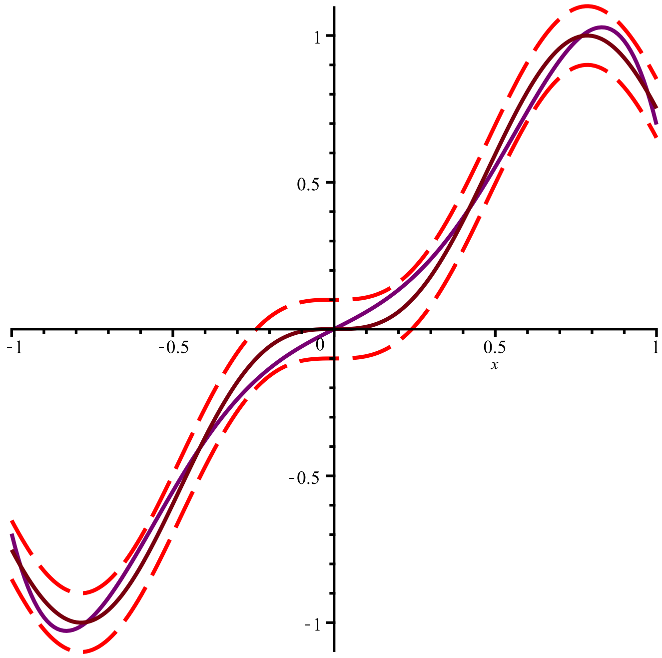


> $plots[display]([fdop1, fdop2, cheb, fplot])$



> $nmin := plot\left(\frac{c[0]}{2} + add(c[n]\cdot T(n,x), n = 1..5), x = -1..1, color = purple\right):$

> $plots[display]([fdop1, fdop2, nmin, fplot])$



> #Найдем коэффициенты Фурье. Так как функция нечетная, искать нужно только bn.

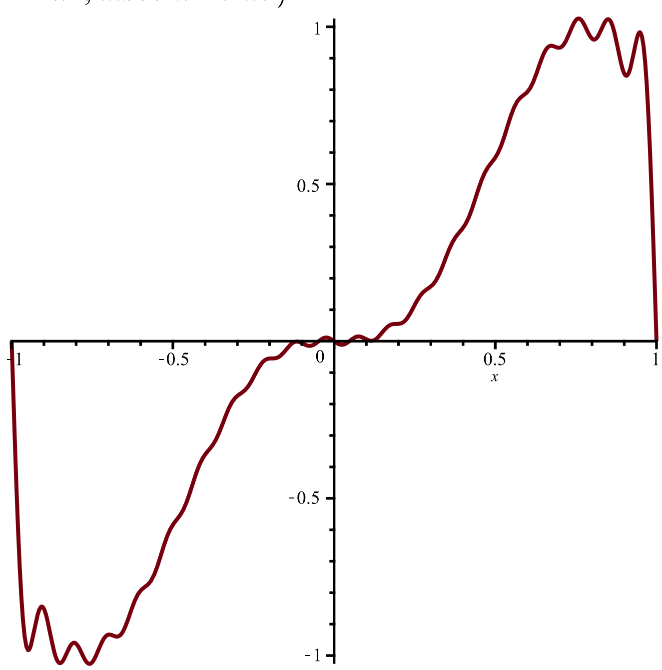
> $bn := simplify(int(f \cdot \sin(\mathrm{Pi} \cdot m \cdot x), x = -1..1))$ assuming $m :: posint$

$$bn := -\frac{3\,(-1)^m\, m\,\pi\left(\sin(2)\,\pi^2\, m^2 - \frac{\sin(6)\,\pi^2\, m^2}{3} - 36\sin(2) + \frac{4\sin(6)}{3}\right)}{2\,\pi^4\, m^4 - 80\,\pi^2\, m^2 + 288} \tag{29}$$

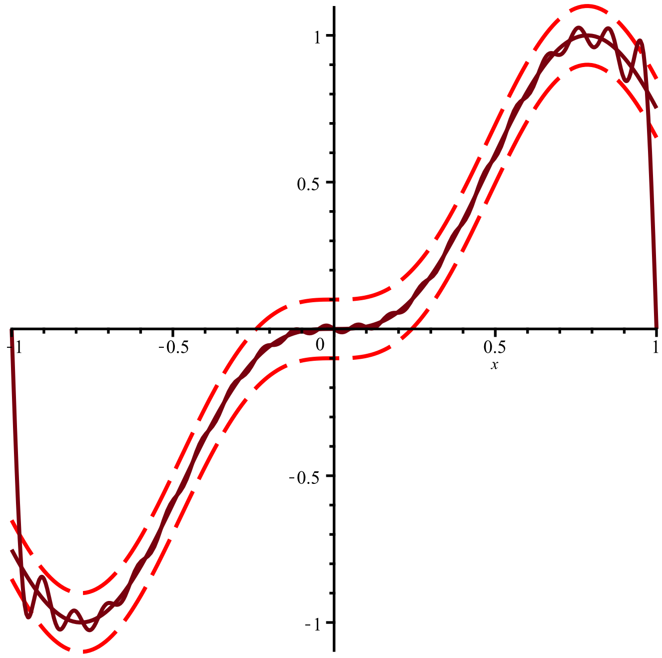
> $S := k \rightarrow sum(bn \cdot \sin(\mathrm{Pi} \cdot m \cdot x), m = 1..k)$

$$S := k \mapsto \sum_{m=1}^{k} bn \cdot \sin(\pi \cdot m \cdot x) \tag{30}$$

> $fur := plot(S(20), x = -1..1, discont = true)$



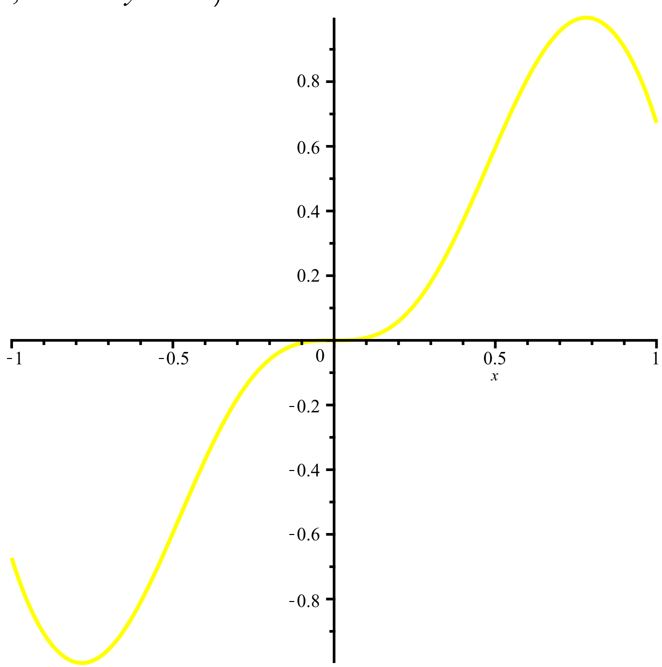
> $plots[display]([fdop1, fdop2, fur, fplot])$



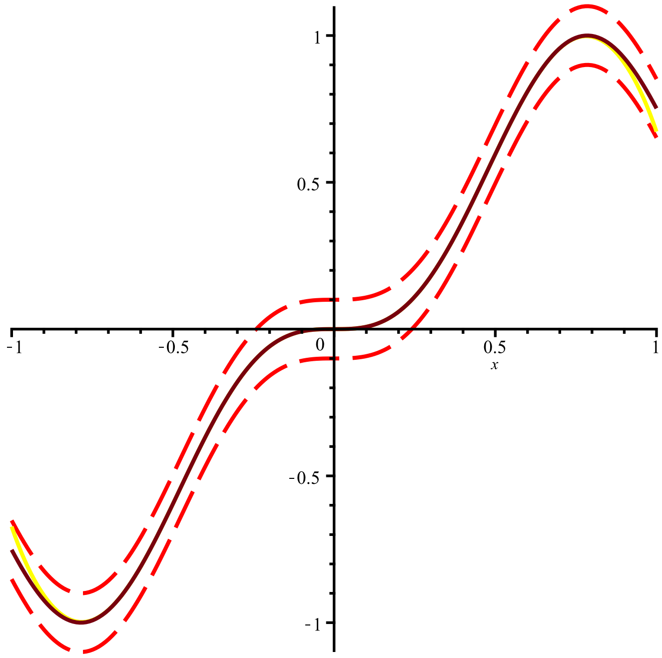
> $St := convert(taylor(f, x=0, 14), polynom)$

$$St := 8\,x^3 - 16\,x^5 + \frac{208}{15}\,x^7 - \frac{1312}{189}\,x^9 + \frac{10736}{4725}\,x^{11} - \frac{2336}{4455}\,x^{13} \qquad \textbf{(31)}$$

> $Stf := plot(St, x=-1..1, color=yellow)$



> $plots[display]([fdop1, fdop2, Stf, fplot])$



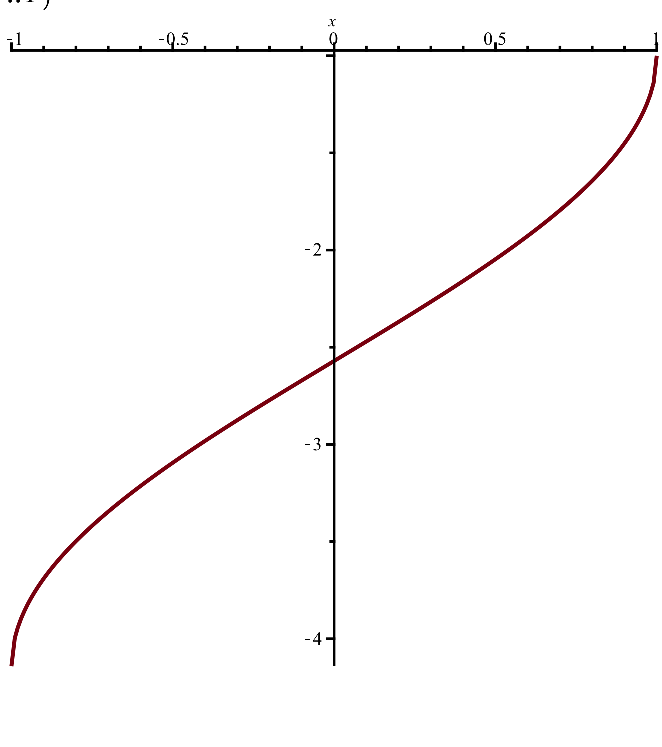
>

#Вторая функция
restart

> $f := -\arccos(x) - 1$

$$f := -\arccos(x) - 1 \qquad (32)$$

> $fplot := plot(f, x = -1 ..1)$



> $with(orthopoly)$

$$[G, H, L, P, T, U] \tag{33}$$

> **for** n **from** 0 **to** 7 **do** $c[n] := \dfrac{\left(\int_{-1}^{1} f \cdot P(n, x)\, \mathrm{d}x\right)}{\int_{-1}^{1} P(n, x)^2\, \mathrm{d}x}$; **end do**

$$c_0 := -1 - \frac{\pi}{2}$$

$$c_1 := \frac{3\,\pi}{8}$$

$$c_2 := 0$$

$$c_3 := \frac{7\,\pi}{128}$$

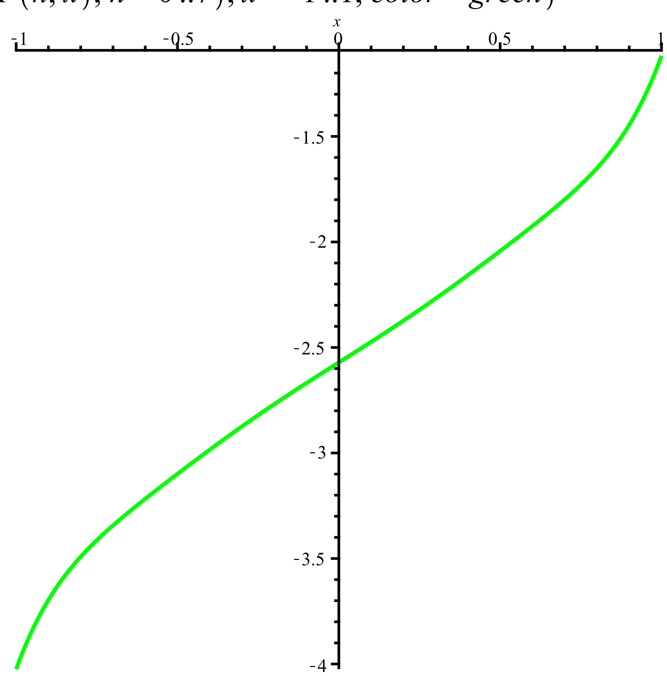
$$c_4 := 0$$

$$c_5 := \frac{11\,\pi}{512}$$

$$c_6 := 0$$

$$c_7 := \frac{375\,\pi}{32768} \tag{34}$$

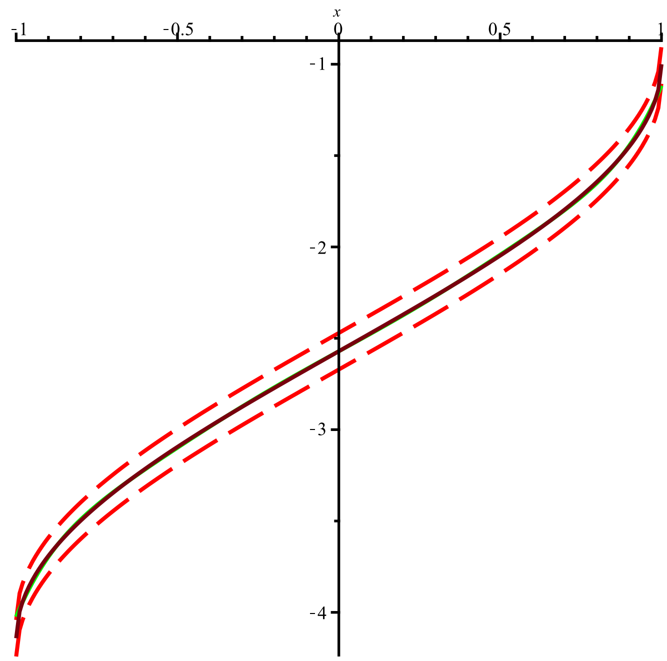
> $lej := plot(add(c[n] \cdot P(n, x), n = 0..7), x = -1..1, color = green)$



> $fdop1 := plot(f - 0.1, x = -1..1, linestyle = dash, color = red)$:

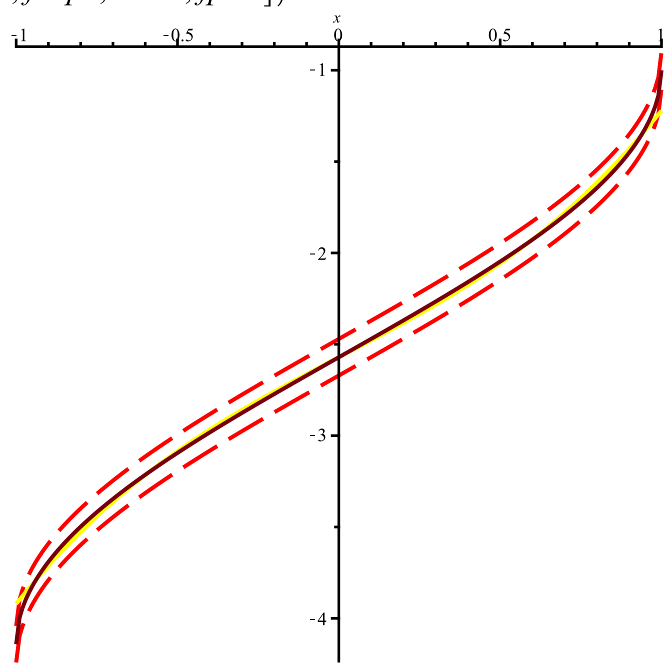
> $fdop2 := plot(f + 0.1, x = -1..1, linestyle = dash, color = red)$:

> $plots[display]([fdop1, fdop2, lej, fplot])$



> $mmin := plot(add(c[n] \cdot P(n, x), n = 0..4), x = -1..1, color = yellow)$:

> $plots[display]([fdop1, fdop2, mmin, fplot])$



> **for** n **from** 0 **to** 7 **do** $c[n] := \frac{2}{\mathrm{Pi}} \cdot \int_{-1}^{1} \frac{f \cdot T(n, x)}{\sqrt{1 - x^2}} \, \mathrm{d}x$; **end do**

$$c_0 := \frac{2\left(-\frac{1}{2}\pi^2 - \pi\right)}{\pi}$$

$$c_1 := \frac{4}{\pi}$$

$$c_2 := 0$$

$$c_3 := \frac{4}{9\,\pi}$$

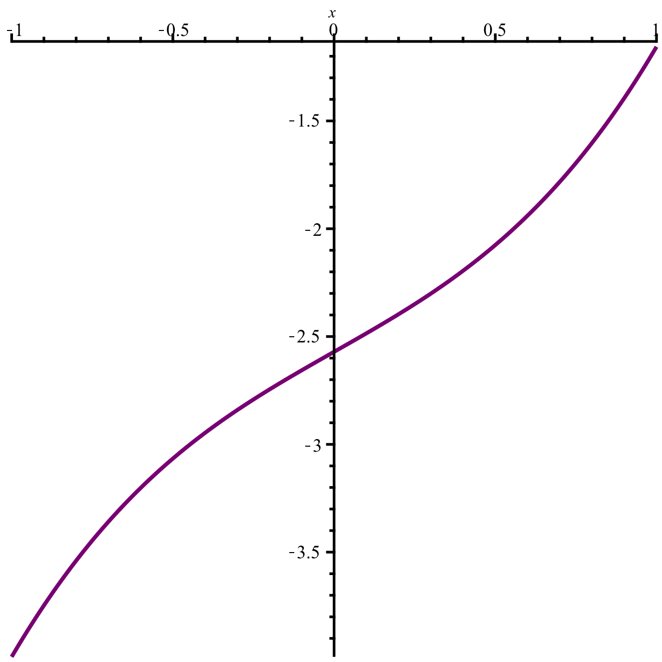
$$c_4 := 0$$

$$c_5 := \frac{4}{25\,\pi}$$

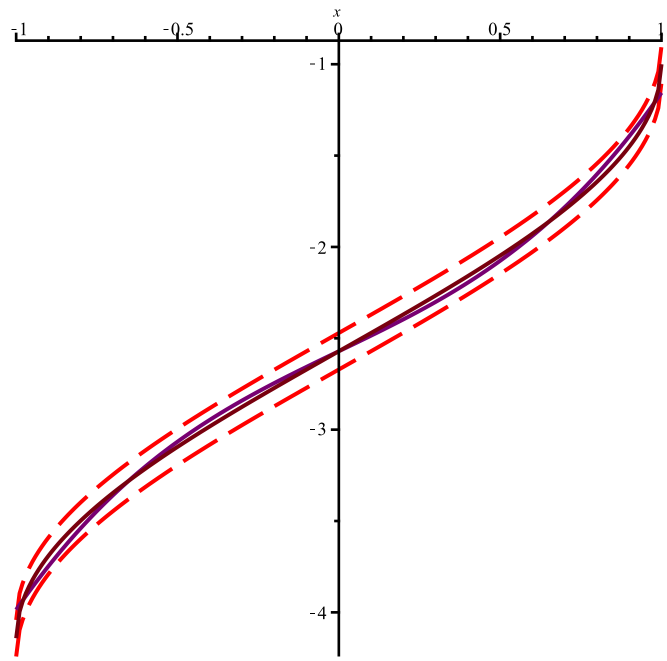
$$c_6 := 0$$

$$c_7 := \frac{4}{49\,\pi} \qquad \textbf{(35)}$$

> $cheb := plot\left(\frac{c[0]}{2} + add(c[n]\cdot T(n, x), n = 1 ..3), x = -1 ..1, color = purple\right)$

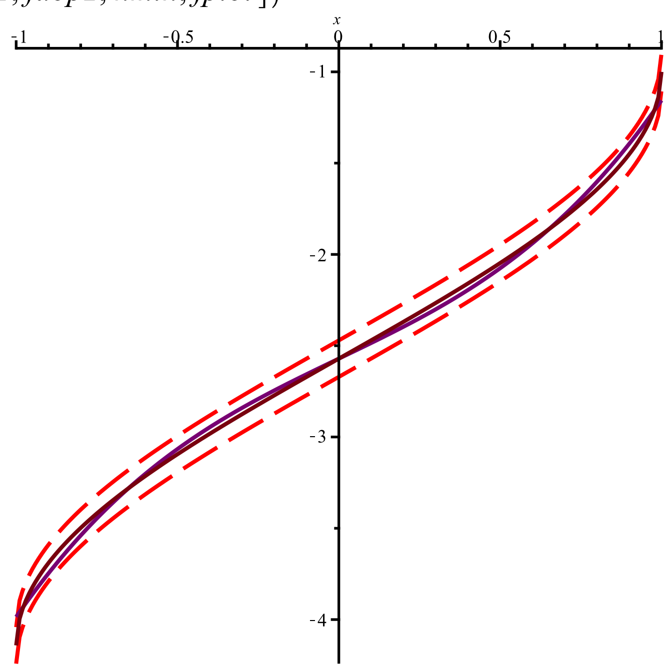


> $plots[display]([fdop1, fdop2, cheb, fplot])$



> $nmin := plot\left(\frac{c[0]}{2} + add(c[n]\cdot T(n,x), n=1..3), x=-1..1, color=purple\right):$

> $plots[display]([fdop1, fdop2, nmin, fplot])$



> $a0 := simplify(int(f, x=-1..1));$
$an := simplify(int(f\cdot\cos(\mathrm{Pi}\cdot m\cdot x), x=-1..1))$ assuming $m :: posint$

$$a0 := -2 - \pi$$

$$an := 0 \tag{36}$$

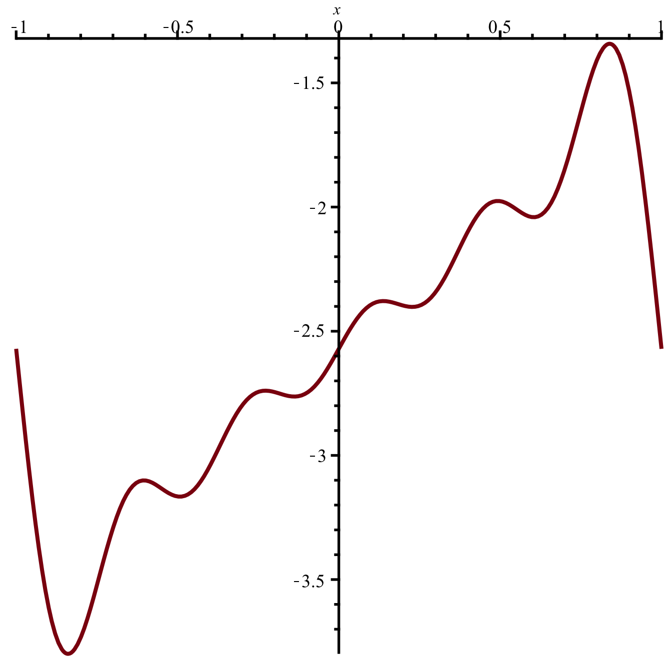
> $bn := simplify(int(f\cdot\sin(\mathrm{Pi}\cdot m\cdot x), x=-1..1))$ assuming $m :: posint$

$$bn := -\left(\int_{-1}^{1} (\arccos(x) + 1)\sin(\pi\, m\, x)\, dx\right) \tag{37}$$

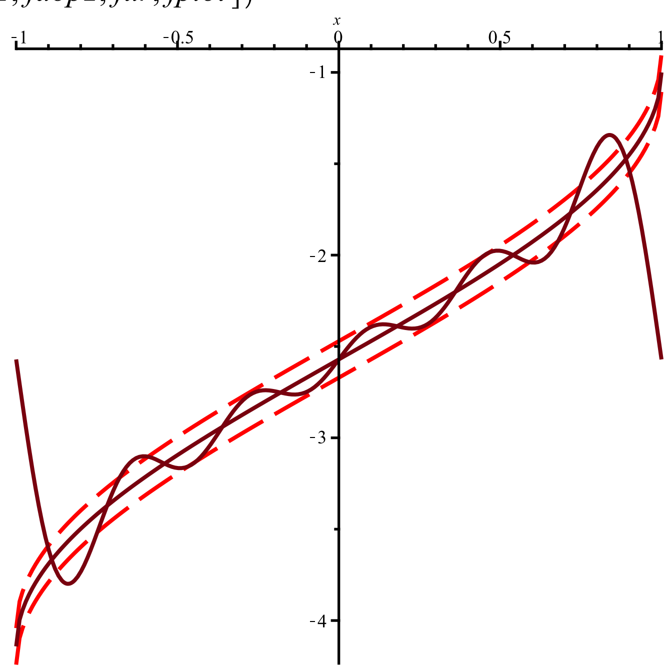
> $S := k \rightarrow \frac{a0}{2} + sum(bn \cdot \sin(\mathrm{Pi} \cdot m \cdot x), m = 1 .. k)$

$$S := k \mapsto \frac{a0}{2} + \left(\sum_{m=1}^{k} bn \cdot \sin(\pi \cdot m \cdot x) \right) \quad \textbf{(38)}$$

> $fur := plot(S(5), x = -1 .. 1, discont = true)$



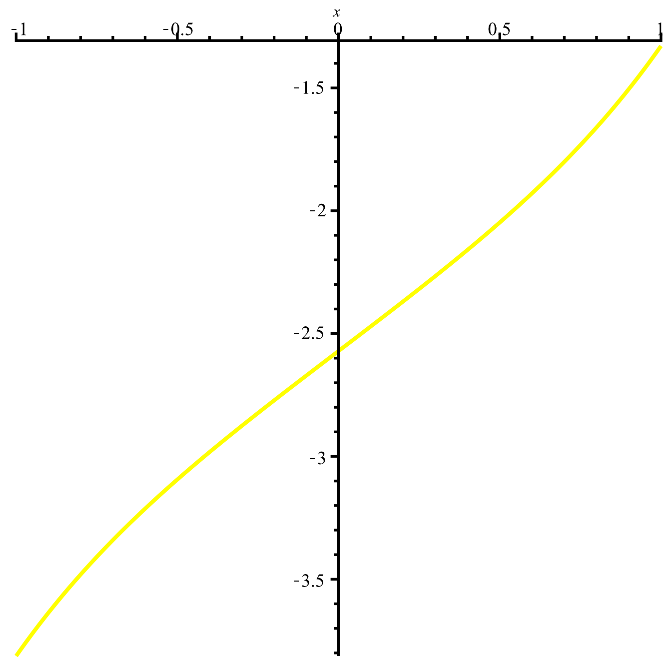
> $plots[display]([fdop1, fdop2, fur, fplot])$



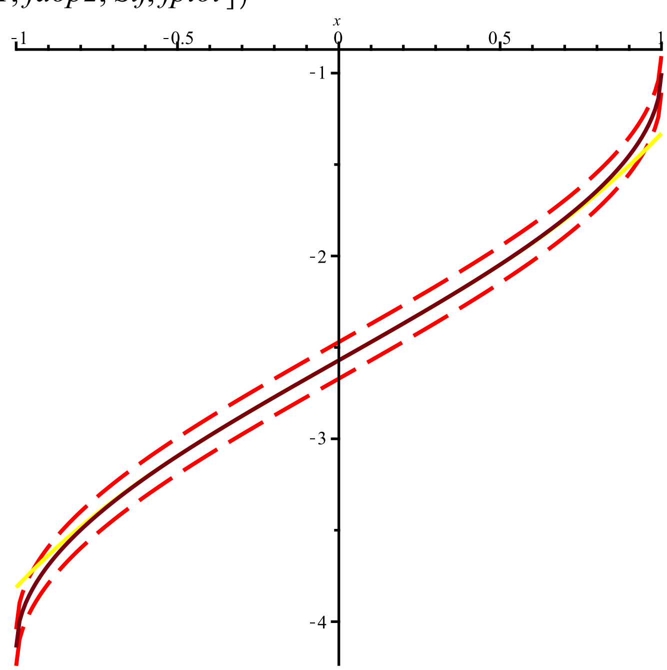
> $St := convert(taylor(f, x = 0, 6), polynom)$

$$St := -1 - \frac{1}{2}\pi + x + \frac{1}{6}x^3 + \frac{3}{40}x^5 \quad \textbf{(39)}$$

> $Stf := plot(St, x = -1 .. 1, color = yellow)$



> *plots*[*display*]([*fdop1*, *fdop2*, *Stf*, *fplot*])



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