> *#Вариант 1*
> *#Задание 1*

> $de\_system := \{diff(y1(x), x) = -2\, y1(x) + 2\, y2(x), diff(y2(x), x) = 7\, y1(x) + 3\, y2(x)\}$

$$de\_system := \left\{ \frac{d}{dx}\, y1(x) = -2\, y1(x) + 2\, y2(x), \frac{d}{dx}\, y2(x) = 7\, y1(x) + 3\, y2(x) \right\} \quad (1)$$

> $solve(\{-2\, y1 + 2\, y2 = 0, 7\, y1 + 3\, y2 = 0\})$

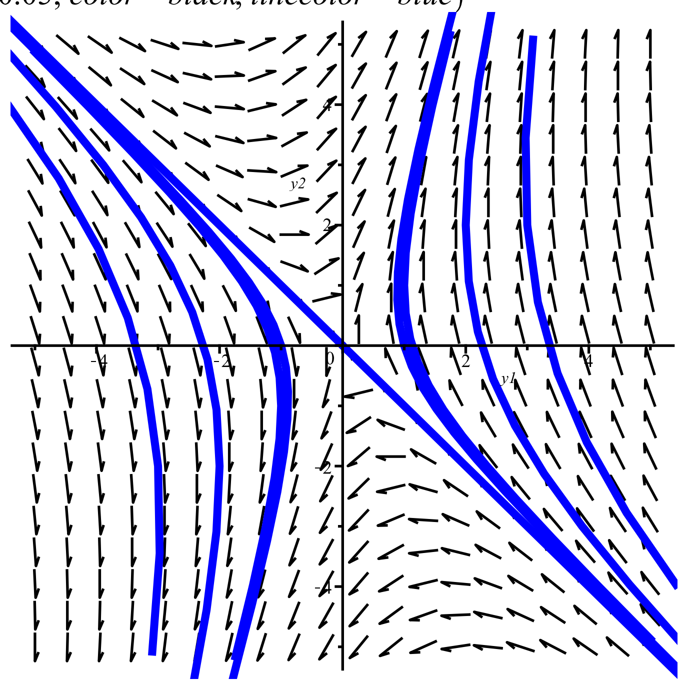
$$\{y1 = 0, y2 = 0\} \quad (2)$$

> *#Точка покоя - (0, 0)*

> *# Построим фазовый портрет:*

> $with(DETools)$:

> $phaseportrait(de\_system, [y1(x), y2(x)], x = -5..5, [[1, 1, 1], [-1, -1, -1], [1, 1, 0], [-1, -1, 0], [0, 2, 2], [0, -2, -2], [0, 3, 2], [0, -3, -2], [2, 1, -1], [-2, -1, 1]], y1 = -5..5, y2 = -5..5, stepsize = 0.05, color = black, linecolor = blue)$



> *# Найдем собственные значения матрицы системы:*

> $Matrix([[-2 - \lambda, 2], [7, 3 - \lambda]])$

$$\begin{bmatrix} -2 - \lambda & 2 \\ 7 & 3 - \lambda \end{bmatrix} \quad (3)$$

> $solve(LinearAlgebra[Determinant](\%) = 0)$

$$5, -4 \quad (4)$$

> *# Тип точки покоя — седло*

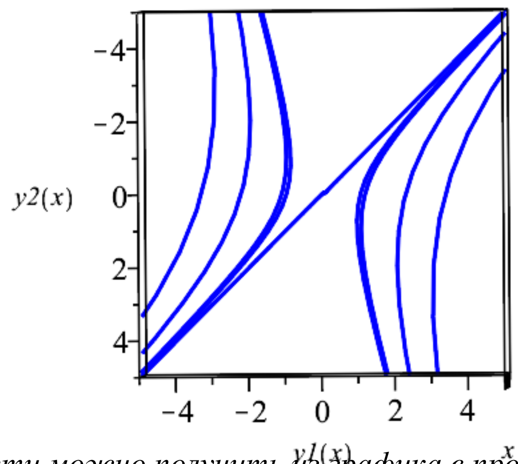
> $dsolve(de\_system, \{y1(x), y2(x)\})$

$$\left\{ y1(x) = \_C1\, e^{5x} + \_C2\, e^{-4x}, y2(x) = \frac{7\, \_C1\, e^{5x}}{2} - \_C2\, e^{-4x} \right\} \quad (5)$$

> *# Построим пространственные кривые, удовлетворяющие заданной системе:*

> $DEplot3d(de\_system, \{y1(x), y2(x)\}, x = -5..5, [[1, 1, 1], [-1, -1, -1], [1, 1, 0], [-1, -1,$

0], [0, 2, 2], [0, -2, -2], [0, 3, 2], [0, -3, -2], [2, 1, -1], [-2, -1, 1]], $y1 = -5..5$, $y2 = -5..5$, $stepsize = 0.05$, $linecolor = blue$, $scene = [x, y1(x), y2(x)]$)



\> # График на плоскости можно получить из графика в пространстве, взяв проекцию на плоскость $y_1 y_2$.

\> restart

\> # Перейдем к ДУ 1 - го порядка относительно функции $y_2(y_1)$:

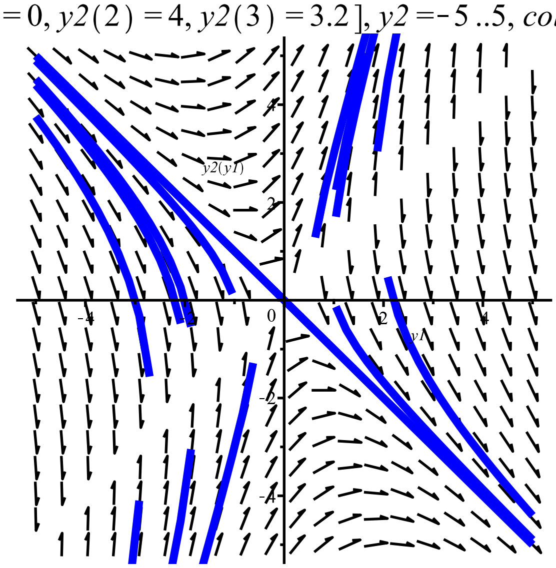
\> $de := diff(y2(y1), y1) = \dfrac{7\, y1 + 3\, y2(y1)}{-2\, y1 + 2\, y2(y1)}$

$$de := \frac{d}{dy1}\, y2(y1) = \frac{7\, y1 + 3\, y2(y1)}{-2\, y1 + 2\, y2(y1)} \tag{6}$$

\> # Построим график:

\> with(DETools) :

DEplot([de], y2(y1), y1 =-5..5, [y2(1) = 0, y2(1) = 2, y2(1) = 1.01, y2(1) = 3, y2(-1) = 1, y2(-1) =-3, y2(-1) = 0, y2(-2) = 0, y2(-2) =-4, y2(-2) =-1, y2(-3) =-5, y2(2) = 1, y2(3) = 2, y2(-3) = 0, y2(2) = 4, y2(3) = 3.2], y2 =-5..5, color = black, linecolor = blue)



\> # График схож с фазовым портретом исходной системы.

> #Задание 2

> restart

> $de\_system := \{diff(y1(x), x) = 5\,y1(x) + 3\,y2(x), diff(y2(x), x) = 4\,y1(x) + 9\,y2(x)\}$

$$de\_system := \left\{\frac{d}{dx}\,y1(x) = 5\,y1(x) + 3\,y2(x), \frac{d}{dx}\,y2(x) = 4\,y1(x) + 9\,y2(x)\right\} \quad (7)$$

> $dsolve(de\_system)$

$$\left\{y1(x) = \_C1\,e^{3x} + \_C2\,e^{11x}, y2(x) = -\frac{2\,\_C1\,e^{3x}}{3} + 2\,\_C2\,e^{11x}\right\} \quad (8)$$

> #Задание 3

> restart

> $de\_system := \{diff(x(t), t) = x(t) + 2\,y(t), diff(y(t), t) = 2\,x(t) + y(t) + 1\}$

$$de\_system := \left\{\frac{d}{dt}\,x(t) = x(t) + 2\,y(t), \frac{d}{dt}\,y(t) = 2\,x(t) + y(t) + 1\right\} \quad (9)$$

> $dsolve(de\_system)$

$$\left\{x(t) = e^{3t}\,\_C2 + e^{-t}\,\_C1 - \frac{2}{3}, y(t) = e^{3t}\,\_C2 - e^{-t}\,\_C1 + \frac{1}{3}\right\} \quad (10)$$

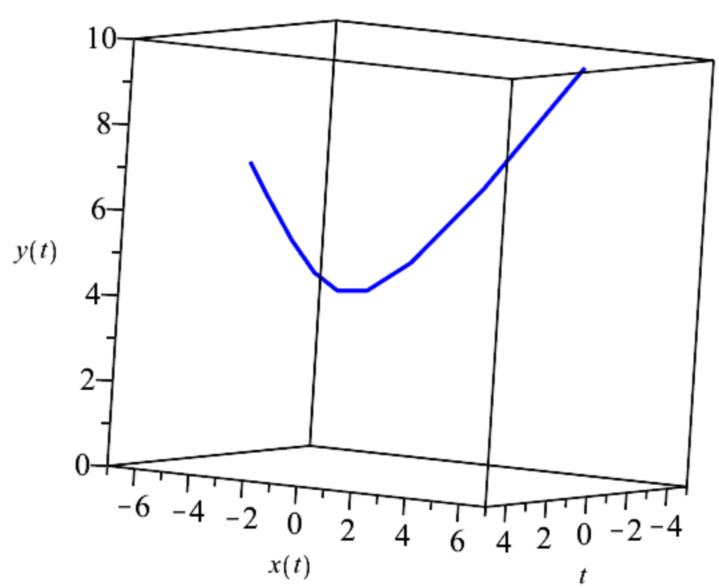
> $dsolve(\{de\_system[1], de\_system[2], x(0) = 0, y(0) = 5\})$

$$\left\{x(t) = \frac{8\,e^{3t}}{3} - 2\,e^{-t} - \frac{2}{3}, y(t) = \frac{8\,e^{3t}}{3} + 2\,e^{-t} + \frac{1}{3}\right\} \quad (11)$$

> # Построим график:

> $with(DETools):$

> $DEplot3d(de\_system, \{x(t), y(t)\}, t = -5..5, [[x(0) = 0, y(0) = 5]], x = -7..7, y = 0..10, linecolor = blue)$



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