Vaksin sebagai input

u = Pemberian Vaksin

$\dot{S} = aS - \propto SI - \gamma S - \mu S$

$\dot{I} = \propto SI + \gamma S - \beta I - \mu I - \mu_c I - u$

$\dot{R} = \beta I - \mu R$

Pelinieran disekitar $(S^*, I^*, R^*) = (0,0,0)$

$$\begin{bmatrix} \dot{z_1} \\ \dot{z_2} \\ \dot{z_3} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{S}}{\partial S} & \frac{\partial \dot{S}}{\partial I} & \frac{\partial \dot{S}}{\partial R} \\ \frac{\partial \dot{I}}{\partial S} & \frac{\partial \dot{I}}{\partial I} & \frac{\partial \dot{I}}{\partial R} \\ \frac{\partial \dot{R}}{\partial S} & \frac{\partial \dot{R}}{\partial I} & \frac{\partial \dot{R}}{\partial R} \end{bmatrix}_{(S^*,I^*,R^*)} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} u$$

$$= \begin{bmatrix} a - \alpha I^* - \gamma - \mu & -\alpha S^* & 0 \\ \alpha I^* + \gamma & \alpha S^* - \beta - \mu - \mu_c & 0 \\ 0 & \beta & -\mu \end{bmatrix}_{(S^*,I^*,R^*)} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} u$$

$$= \begin{bmatrix} a - \gamma - \mu & -\alpha S^* & 0 \\ \gamma & -\beta - \mu - \mu_c & 0 \\ 0 & \beta & -\mu \end{bmatrix}_{(S^*,I^*,R^*)} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} u$$

Misalkan:

$X = a - \gamma - \mu$

$Y = -\beta - \mu - \mu_c$ (negatif)

$Z = -\mu$ (negatif)

KETERAMATAN

$$M_0 = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$$

$C = [0 \quad 0 \quad 1]$

$$CA = [0 \quad 0 \quad 1] \begin{bmatrix} X & 0 & 0 \\ \gamma & -Y & 0 \\ 0 & \beta & -Z \end{bmatrix}$$

$$= [0 \quad \beta \quad -Z]$$

$$CA^2 = [0 \quad \beta \quad -Z]\begin{bmatrix} X & 0 & 0 \\ \gamma & -Y & 0 \\ 0 & \beta & -Z \end{bmatrix}$$

$$= [\beta\gamma \quad -Y\beta - Z\beta \quad Z^2]$$

$$M_0 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & \beta & -Z \\ \beta\gamma & -Y\beta - Z\beta & Z^2 \end{bmatrix}$$

$$\det(M_0) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & \beta & -Z \\ \beta\gamma & -Y\beta - Z\beta & Z^2 \end{bmatrix} = -(\beta^2\,\gamma)$$

Karena det $(M_0) \neq 0$, maka Rank $M_0 = 3 = n$

Jadi sistem teramati