Projectile Mechanics

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| **Velocity Components** |
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|  |
| **Governing Equation** |
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|  |
| **Max Height** |
|  |
|  |
| **Time of Flight** |
|  |
|  |
| **Final Position** |
|  |

# Projectile Target

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| --- |
| **Known Variables** |
| |  |  | | --- | --- | |  | Launch height | |  | Target height | |  | Distance from target | |  | Angle of target | |
|  |
| **Unknowns** |
| |  |  | | --- | --- | |  | Launch Angle | |  | Launch Velocity | |
|  |
| **System of Equations** |
|  |
|  |
| **Time of Flight** |
|  |
|  |
| **Final Position** |
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|  |

If at x final, we want 45deg target hit

# Projectile Target with fixed initial velocity

the launch should be at the angle halfway between the target and Zenith (vector opposite to Gravity)

# Projectile Target with Ceiling

Priority to ceiling determines launch angle and target angle. Distance from target and height of launch and target zones are known. Maximize target angle of attack to reduce roll. To maximize target roll, find minimum launch angle to satisfy target – remember, projectile dimensions must be considered when calculating heights.

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| **Known Variables** |
| |  |  | | --- | --- | |  | Launch height | |  | Target height | |  | Distance from target | |  | Max projectile height | |
|  |
| **Unknowns** |
| |  |  | | --- | --- | |  | Target Angle | |  | Launch Velocity | |

With respect to the target angle

Or with respect to peak

Relative height with respect to launchpad set launch height to 0. Otherwise, follow the given delta measurement to find the height of the ceiling from the target to compute max angle of attack. The parabola is symmetric about the point where ymax exists.

This just leads back to the equation for the parabola!

Since we don’t know xmax

# Projectile with Drag

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| --- | --- | --- | --- |
| Stokes Drag | At low speeds | Linear |  |
| Newton Drag | At higher speeds | Quadratic |  |

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| --- | --- |
|  | **FD** is the drag force  **c** is the drag coefficient  **V** is the relative velocity  **ρ** is the air density  **A** is the cross sectional area of the projectile  **μ** = k/m = cρA/(2m) |

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| **Component** | **Horizontal** | **Vertical** |
| **Displacement** |  |  |
| **Velocity** |  |  |
| **Acceleration** |  |  |

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| --- | --- | --- |
| **Component** | **Horizontal** | **Vertical** |
| **Displacement** |  |  |
| **Velocity** |  |  |
| **Acceleration** |  |  |

**Numerical Solution**

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With linear air resistance you will get projectiles which will slow down exponentially, so you will have to solve the differential equation with resistance

**Launch Angle:**

**Target Angle:**

**Ceiling Height:**

Flow rate:

Diagram

Description automatically generated

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| --- | --- |
|  |  |
| Nonchoked regime | Choked regime |
| Flow rate nonchoked | Flow rate limited by geometry of the valve |
|  |  |

|  |  |
| --- | --- |
|  | Specific gravity of air |
|  | Temperature in the reservoir assumed constant |
|  | The compressibility factor |
|  | Engineering constant to convert the quantity into units of molecules per time |

Ideal gas law where and are the number of molecules in the tank and barrel, respectively.

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| --- | --- | --- | --- |
| Reservoir |  |  |  |
|  |  |  |  |
| Barrel |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| Reservoir |  |  |  |
|  |  |  |  |
| Barrel |  |  |  |

involving the solution of 4 nonlinear differential equations

pressure loss across the valve, potentially turbulent air flow, and gas velocities approaching the speed of sound reduce the velocity estimates substantially (e.g., 50% loss is not uncommon) in real life.

<https://aircannonplans.com/pdf/air-cannon-velocity.pdf>

<https://aircannonplans.com/pdf/acp-exit-volocity.pdf>

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Solve for

<http://persweb.wabash.edu/facstaff/madsenm/publications/AJP_80_24_rohrbach_air_cannon.pdf>

<https://arxiv.org/pdf/1106.2803.pdf>

<https://physics.stackexchange.com/questions/518845/how-would-one-solve-this-system-of-differential-equations-for-muzzle-velocity-as>

Flow is modeled as a function of the pressure differential between the tank pressure and the barrel pressure

Model the gas expansion in the barrel and the tank using the Ideal Gas Law

The number of molecules in the tank and barrel are governed by the flow of molecules between them through the valve.