



Master of Science Thesis

Development of estimation algorithms for navigation of unmanned aerial vehicles

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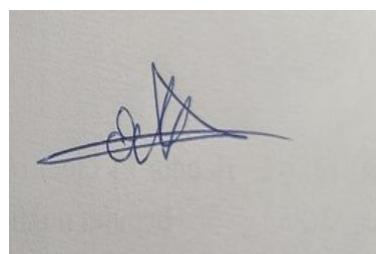
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Abstract

Accurate attitude estimation and prediction are critical for the autonomous navigation and advanced control of unmanned aerial vehicles (UAVs). While classical sensor fusion algorithms effectively estimate current vehicle orientation, they are fundamentally reactive and cannot anticipate future states. Modern control strategies, such as Model Predictive Control (MPC), require multi-step ahead predictions of future attitude, creating a need for predictive models based on deep learning. This thesis develops a comprehensive framework for UAV attitude prediction centered on an attention-based Long Short-Term Memory (LSTM) network. The proposed architecture predicts future roll ϕ , pitch θ , and yaw ψ angles over a 10-timestep horizon using look-back windows of IMU measurements and control inputs. To establish a robust training dataset and baseline performance, four classical tilt estimation filters—the Complementary Filter, Mahony Filter, Explicit Complementary Filter, and Extended Kalman Filter (EKF)—were implemented in C++ and optimized using real flight data. By utilizing the EKF-estimated angles as training labels, the framework becomes self-contained and applicable to real-world scenarios where external ground-truth systems are unavailable. Experimental evaluation demonstrates that the attention-based LSTM not only achieves high estimation accuracy but also provides the essential multi-step prediction capability required for proactive MPC-based flight controllers, effectively bridging the gap between classical sensor fusion and modern deep learning for UAV navigation.

Keywords

UAVs, Complementary Filtering, EKF, Attitude estimation, Classic estimation methods, Neural Networks, Machine Learning, MPC, LSTM, RNN, Sensor Fusion

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Acronym Index

IMU: Inertial Measurement Unit
EKF: Extended Kalman Filter
UAV: Unmanned Aerial Vehicle
RNN: Recurrent Neural Network
LLM: Large Language Model
LSTM: Long Term Short-Memory
MPC: Model Predictive Control

INTRODUCTION

Unmanned Aerial Vehicles (UAVs), commonly known as drones, have emerged as transformative technologies across many sectors. From aerial photography, agriculture to package delivery, these autonomous platforms are reshaping how we approach tasks that were impossible in the past.

At the heart of every UAV lies a critical capability: the ability to know its own orientation in three-dimensional space. This ability, referred to as attitude, describes how the vehicle is tilted relative to the Earth's surface. Without accurate and continuous attitude information, a UAV cannot maintain stable flight, execute precise maneuvers, or respond appropriately to disturbances such as wind gusts. The attitude serves as the foundation upon which all flight control decisions are made.

Modern UAVs rely on Inertial Measurement Units (IMUs), compact electronic devices that combine multiple sensors to measure motion. A typical IMU contains gyroscopes, which measure rotational velocity, and accelerometers, which measure linear acceleration including gravity. These sensors are inexpensive, lightweight, and consume minimal power, making them ideal for small autonomous platforms. However, each sensor type has inherent limitations that prevent it from providing accurate attitude estimates on its own. Classical sensor fusion algorithms such as complementary filters and the Extended Kalman Filter (EKF) address this by combining gyroscope and accelerometer data to produce reliable estimates of the current tilt angles (ϕ, θ). While effective for real-time estimation these approaches are fundamentally reactive: they can estimate the current state and cannot anticipate future vehicle orientation.

However, modern advanced flight control strategies such as Model Predictive Control (MPC) require not just knowledge of the current attitude, but prediction of the future states over a finite horizon [12]. MPC computes optimal control actions by solving an optimization problem over predicted future trajectories, making the quality of the prediction critical to controller performance. This creates a need for algorithms that can provide accurate multi-step ahead attitude predictions.

Recent advances in deep learning have introduced new possibilities for time-series prediction in dynamical systems. Recurrent Neural Networks (RNNs) and in particular Long Short-Term Memory (LSTM) networks have demonstrated strong capabilities in learning temporal dependencies from sequential data [9]. In the field of UAV navigation, neural network-based approaches have been explored for attitude estimation as an alternative or complement to classical filters. Various techniques, algorithms and methods have been explored. Feedforward neural networks have been trained on IMU data and estimate orientation [10]. Also, RNN-based approaches have been used for inertial navigation [11]. However, the majority of these works focus on single-step estimation of the current state, rather than multi-step ahead prediction of future states. Furthermore, few works address the specific requirement of providing prediction horizons compatible with MPC-based flight controllers.

This thesis proposes an attention-based LSTM architecture for multi-step ahead attitude prediction that address this gap. Unlike existing neural networks approaches that estimate only the current state, the proposed method predicts the attitude angles (ϕ, θ, ψ) over a configurable future horizon of N timesteps ($k + 1$ to $k + N$), producing predictions directly suitable with MPC controllers. The attention mechanism enables the network to learn which timesteps within the input look-back window are most informative for the prediction task, improving prediction accuracy over standard LSTM architectures.

The subject of this thesis

This thesis addresses the challenge of predicting future attitude states of unmanned aerial vehicles, moving beyond the capabilities of classical estimation methods that can only determine the current orientation. The attitude of a UAV, described by the Euler angles (ϕ, θ, ψ), is essential for flight stability and control. While classical sensor fusion filters can estimate these angles in real-time from IMU measurements, they are inherently reactive and provide no information about the future evolution of the vehicle's state. Advanced control strategies such as Model Predictive Control (MPC) rely on predictions of future states over a finite horizon to compute optimal control actions, creating a fundamental need for accurate multi-step ahead attitude prediction.

To address this, an attention-based Long Short-Term Memory (LSTM) network is proposed that receives a look-back window of past sensor data and control inputs and predicts the attitude angles (ϕ, θ, ψ) for the next N timesteps into the future. The attention mechanism allows the network to automatically learn which past timesteps are most relevant for predicting the future, rather than relying only on the most recent observation. Training such a predictive model requires accurate attitude angle data. However, on a real UAV platform, the true attitude is not directly measurable — it must be estimated from raw inertial sensor data. For this reason, four classical tilt estimation filters are developed and optimized as part of this work: the Complementary Filter, the Mahony Passive Complementary Filter, the Explicit Complementary Filter, and the Extended Kalman Filter (EKF). These filters fuse gyroscope and accelerometer measurements to produce the attitude angle estimates that form the training dataset for the LSTM. The quality of these estimates is therefore directly linked to the quality of the LSTM predictions, making filter optimization an integral part of the overall methodology. The entire pipeline — from raw IMU processing through classical filtering to LSTM-based prediction — is implemented in C++ using the Eigen library and LibTorch (PyTorch C++ API), ensuring suitability for deployment on real-time embedded flight controllers

Aim and objectives

The primary aim of this thesis is to develop a complete estimation-to-prediction pipeline for UAV attitude, culminating in an attention-based LSTM network that provides multi-step ahead predictions suitable for integration with Model Predictive Control (MPC).

The specific objectives are:

1. Implement and optimize four classical tilt estimation filters to produce accurate attitude estimates from raw IMU data, establishing both baselines and the training dataset for the LSTM.
2. Design and train an attention-based LSTM architecture that predicts the attitude angles (ϕ , θ , ψ) over a future horizon of N timesteps from look-back windows of sensor and control data.
3. Evaluate prediction accuracy per step ($k+1$ through $k+N$) and per angle using RMSE and MAE metrics, quantifying how performance evolves across the prediction horizon.
4. Implement the complete pipeline in C++ using Eigen and LibTorch, demonstrating feasibility for real-time embedded deployment.

Methodology

The methodology follows two phases that form a complete estimation-to-prediction pipeline.

The first phase involves the implementation and optimization of four classical tilt estimation filters in C++ using the Eigen library. These filters process raw gyroscope and accelerometer data collected from a quadcopter platform at 50 Hz to estimate the tilt angles (ϕ , θ). Each filter's hyperparameters are optimized using grid search against ground truth measurements. The filters are compared using RMSE and MAE metrics, establishing that the EKF achieves the best estimation accuracy. The filter-estimated angles produced in this phase provide the attitude data required for training the LSTM model.

The second and primary phase is the development of the LSTM multi-step ahead prediction model. An attention-based LSTM network is designed to process look-back windows of $K=10$ timesteps containing attitude angles estimated by the filters, gyroscope measurements, and control torques from the PID controller (9 input features total), and predict the attitude angles (ϕ , θ , ψ) for the next $N=10$ timesteps. The model is trained on flight data collected from a quadcopter platform at a sampling rate of 33.3 Hz ($T_s = 0.03$ s). Training uses Mean Squared Error (MSE) loss with the Adam optimizer, and evaluation metrics are computed per prediction step to assess how accuracy degrades over the prediction horizon.

Innovation

The key innovation of this thesis lies in the application of an attention-based LSTM network for multi-step ahead attitude prediction of UAVs. While existing neural network approaches in the literature primarily focus on single-step attitude estimation as replacement for classical filters [10][11], this work targets a different objective: predicting future attitude states over a configurable horizon of N timesteps.

This distinction is critical because multi-step ahead predictions are prerequisite for Model Predictive Control, which computes optimal control actions based on predicted future trajectories. By providing prediction at timesteps $k + 1$ to $k + N$, the proposed LSTM architecture can serve as the prediction model with an MPC framework enabling proactive rather than reactive flight control.



A key aspect of the proposed methodology is that the LSTM is trained entirely on filter-estimated attitude angles rather than externally provided ground truth. The classical tilt estimation filters developed in this work process raw IMU data to produce the attitude estimates that form the LSTM's training dataset. This makes the approach self-contained and directly applicable to real-world scenarios where ground truth orientation is unavailable — the same filters that generate training data offline can provide real-time input to the LSTM during deployment. The incorporation of an attention mechanism further distinguishes this work from standard LSTM-based approaches. Rather than relying solely on the final hidden state, the attention layer learns to weight the contributions of all timesteps in the look-back window, allowing the network to focus on the most informative inputs for each prediction.

Additionally, the complete pipeline is implemented entirely in C++ using LibTorch and Eigen, making it deployable on embedded systems. This is a practical requirement for real-world UAV applications that existing Python-based research implementations do not address.

Structure

Briefly outline the structure of the thesis, in chapters.

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1 CHAPTER 1: Theoretical background

This chapter establishes the theoretical foundation necessary to understand the attitude estimation problem for UAVs. An introduction is being made to the attitude estimation in UAV, what sensors are using to measure motion and explore different mathematical representations of orientations. Later fundamental concepts of sensor fusion are discussed which motivate the classic filter algorithms developed in section 2. Finally neural network architectures are introduced for sequential learning, providing the theoretical background for the deep learning approach presented in Chapter 3.

1.1 The role of attitude in UAV flight

The ability of a drones to operate autonomously has a critical dependency on their ability to determine and control their orientation in three-dimensional space.

The attitude of a UAV describes its rotation orientation relative to a fixed reference frame, typically defined with respect to the Earth's surface. In aviation, attitude is conventionally expressed using three angular coordinates known as Euler angles:

- 1) Roll (ϕ): Rotation around the longitudinal axis (forward-backward direction)
- 2) Pitch (θ): Rotation around the lateral axis (left-right direction)
- 3) Yaw (ψ): Rotation around the vertical axis (up-down direction)

These angles directly influence the vehicle's ability to maintain stable flight, execute precise maneuvers, and respond appropriately to environmental disturbances such as wind gusts. Without accurate attitude information a UAV can't, maintain level flight, execute controlled turns, provide stable platform for cameras and sensors, implement autonomous navigation algorithms.

Unlike position (which can be directly measured by GPS) or velocity (which can be derived from GPS updates), attitude can not be directly measured by a single sensor. Instead, it must be estimated by fusing measurements from multiple sensors, each with different characteristics and limitations.

1.2 IMU and Sensors

An Inertial Measurement Unit (IMU) is a compact electronic device that combines multiple motion sensors to measure a vehicle's kinematic state. A typical 6-axis IMU contains an 3-axis gyroscope and 3-axis accelerometer.

1.2.1 Gyroscope

A gyroscope is an angular rate sensor that measures how fast an object rotates around one or more axes, and in UAVs it is one of core sensors for attitude estimations. In practice gyroscopes output angular velocity (e.g. in degrees or rad per second). Angular velocity is measured as $\omega \in R^3$ (rad/s). Each axis output corresponds to the rate of rotation about the x, y, z axes of the sensor frame.

Gyroscopes have key advantages and limitations. They provide very smooth short-term attitude information and are insensitive to linear accelerations, which makes them reliable during aggressive maneuvers where accelerometers are heavily corrupted by non-gravitational forces. However, their measurements contain bias and noise, so integration causes drift that grows over time, meaning gyro-only attitude estimates must be regularly corrected using other sensors such as accelerometers, magnetometers, or GNSS within a sensor fusion algorithm. [1]

1.2.2 Accelerometer

Accelerometers are fundamentally different from what their name suggests. They do not measure kinematic acceleration. Instead they measure specific force, also called proper acceleration. It is measured as $\alpha \in R^3 (m/s^2)$. Each axis output corresponds to the specific force experienced along the x, y, z axes of the sensor frame, including gravity during static conditions.

Accelerometers excel at providing long-term absolute reference through the gravity vector, allowing computation of roll and pitch via inverse trigonometry when the vehicle is mostly stationary or slow moving. Accelerometer measures $a = [ax, ay, az]$ and we can calculate roll and pitch via these equations

Equation 1 Roll from accelerometer $roll = atan2(ay, az)$

Equation 2 Pitch from accelerometer $pitch = arsin(a_x/g)$

1.3 Attitude Representation

Representing three-dimensional rotations mathematically is a non-trivial problem in attitude estimation. While the physical concept of orientation is intuitive, capturing it numerically requires careful consideration of mathematical properties, computational efficiency, and singularity avoidance. This section introduces three common representations used in attitude estimation algorithms, each with distinct advantages and limitations that influence their suitability for different filtering approaches.

1.3.1 Euler Angles

Euler angles are the most intuitive representation of orientation, describing attitude as a sequence of three rotations around specified axes. The aerospace convention, known as the ZYX (yaw-pitch-roll) sequence, applies rotations in the following order [2]:

1. Rotate by ψ (yaw) around the Z-axis (vertical)
2. Rotate by θ (pitch) around the new Y-axis
3. Rotate by ϕ (roll) around the new X-axis

It only has 3 parameters which make's it more human interpretable and lesser memory footprint for computer calculations.

Euler angles suffer from non-uniqueness (multiple angle combinations can represent the same physical orientation) and require complex trigonometric transformations when computing angular velocity from angle rates

The Complementary Filter (Chapter 2) operates directly on roll and pitch Euler angles, taking advantage of their simplicity for the basic weighted fusion approach. Furthermore, all filters in this thesis output their final estimates as Euler angles for interpretability and comparison with ground truth measurements. For UAV flight control applications where extreme pitch angles (near $\pm 90^\circ$) are rare, Euler angles provide an acceptable representation.

1.3.2 Rotation Matrices (SO(3))

A rotation matrix $R \in SO(3)$ is a 3×3 orthonormal matrix that transforms vectors from one coordinate frame to another. The notation SO(3) stands for "Special Orthogonal group in 3 dimensions," representing all possible rotations in three-dimensional space.

A rotation matrix can be constructed from Euler angles using the product of elementary rotation matrices [3]

$$\text{Equation 3 Rotation matrix to euler angles: } R(\varphi, \theta, \psi) = R_z(\psi) \cdot R_y(\theta) \cdot R_x(\varphi)$$

Where R_x , R_y , R_z represent rotations around the X, Y, and Z axes respectively. The resulting 3×3 matrix contains nine elements, though only three are independent due to orthonormality constraints.

One drawback is redundancy: storing nine numbers to represent three degrees of freedom is inefficient. Also, they require periodic orthonormalization some common methods are Gram-Schmidt or singular value decomposition (SVD)[4].

The Mahony Passive Complementary Filter (Chapter 2) operates directly on rotation matrices, leveraging the geometric properties of SO(3) for error correction. The filter computes a cross-product-based correction term that exploits the manifold structure of rotation matrices, providing superior performance compared to Euler angle approaches. To maintain orthonormality, the Mahony filter implementation applies Gram-Schmidt orthonormalization after each prediction step, ensuring the rotation matrix remains valid throughout the flight.

1.3.3 Quaternions

A quaternion q is a four-parameter representation of rotation, consisting of one scalar and one vector component:

$$\text{Equation 4 Quaternion } q = [q_0, q_1, q_2, q_3]^T = [q_0, q_v]^T$$

Where:

q_0 : Scalar part

$q_v = [q_1, q_2, q_3]^T$: Vector part

Like rotation matrices, quaternions require normalization after numerical integration to maintain the unit constraint. However, this is computationally cheaper than matrix orthonormalization (single division versus full matrix projection).

Extended Kalman Filter (EKF) uses quaternions for the state representation, with a 7-dimensional state vector that includes the quaternion and gyroscope bias estimates.

1.3.4 Sensor Fusion Fundamentals

Sensor fusion combines measurements from multiple sensors to produce estimates superior to any single sensor. For attitude estimation, gyroscopes and accelerometers exhibit complementary error characteristics: gyroscopes provide smooth, high-frequency measurements but suffer from integration drift, while accelerometers offer a long-term gravity reference but are noisy and corrupted by linear acceleration.

The fundamental challenge is designing algorithms that:

1. Trust gyroscope measurements for short-term dynamics (responsiveness)
2. Use accelerometer measurements for long-term drift correction (stability)
3. Reject accelerometer corruption during vehicle acceleration

Fusion strategies range from simple weighted averaging (complementary filters) to nonlinear filters operating on manifolds (Mahony filter on SO(3)) to probabilistic state estimation (Extended Kalman Filter). Performance is evaluated using Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) against ground truth measurements.

1.4 Neural Networks for Time-Series Predictions

While classical filtering approaches (detailed in Chapter 2) rely on explicit mathematical models of sensor behavior, modern deep learning offers a data-driven paradigm. This section establishes the theoretical foundation for the LSTM-based predictor developed in Chapter 3, moving from basic sequential learning to the attention mechanisms that enable robust multi-step ahead prediction.

1.4.1 Time-Series Data and Sequential Learning

Attitude estimation is fundamentally a time-series problem. IMU measurements arrive sequentially at fixed intervals T_s , and the vehicle's state at time t is highly dependent on its state at $t-1$. Traditional feedforward networks process each sample independently, ignoring this temporal structure. For a UAV, where dynamics are continuous, the model must understand these dependencies to distinguish between sensor noise and actual motion.

1.4.2 Recurrent Neural Networks (RNN)

RNNs address the limitations of feedforward networks by maintaining a hidden state h_t that acts as a "memory" of previous inputs. At each timestep t , the RNN processes the current input x_t and the previous hidden state h_{t-1} . While theoretically capable of learning long sequences, standard RNNs suffer from the vanishing gradient problem, where information

from earlier timesteps is lost during training [6]. For attitude prediction requiring look-back windows of $K \geq 10$ samples, this makes standard RNNs unreliable.

1.4.3 Long Short-Term Memory (LSTM)

The LSTM architecture, introduced by Hochreiter and Schmidhuber [7], solves the vanishing gradient problem through a complex gating mechanism. It maintains a cell state C_t that carries information across long intervals. The flow of information is controlled by three gates:

Forget Gate (f_t): Determines which historical data to discard

Input Gate (i_t): Updates the cell state with new information

Output Gate (o_t): Filter the cell state to produce the hidden state h_t

For IMU data, LSTMs can learn to ignore high-frequency vibration noise (via the forget gate) while focusing on the persistent gravity vector (via the input gate).

1.4.4 Attention Mechanisms

Standard LSTMs use only the final hidden state h_k to generate predictions, which can create an information bottleneck. To improve prediction accuracy over a 10-step horizon ($n=10$) this incorporates an **Attention Mechanism**. This layer calculate a score for every hidden state in the look-back window, allowing the model to “attend” to specific past event (e.g., a sudden change in ω_{gyro}) that are critical for predicting the future state. The final context vector c is a weighted sum:

$$c = \sum_{i=1}^T a_i h_i$$

where a_i represents the learned import of the t -th timestep. The weights a_i are computed by passing each hidden state through a scoring function followed by a softmax normalization, ensuring they sum to one. This allows the network to dynamically focus on different parts of the input sequence depending on the prediction task.

1.4.5 Related Work

Recent literature has explored deep learning as an alternative or complement to classic filters for IMU-based attitude estimation. Liu et al. [10] demonstrated LSTM-based orientation estimation for UAVs, showing that recurrent networks can learn temporal patterns in IMU data. Weber et al. [13] proposed RIANN, a GRU-based network that takes raw 6-DOF IMU data and outputs quaternion orientation, consistently outperforming Mahony, Madgwick, and complementary filters across diverse motion types and sensor hardware without per-dataset tuning. Golroudbari and Sabour [14] developed hybrid CNN-LSTM architectures evaluated across seven public datasets totaling over 120 hours of IMU measurements, achieving state-of-the-art accuracy for real-time attitude estimation. Brossard et al. [15] took a different approach, training a convolutional network to denoise raw gyroscope signals which are then integrated through dead reckoning, achieving accuracy comparable to visual-inertial odometry systems. Cohen and Klein [11] similarly investigated hybrid models for sensor denoising.

However, a common limitation of all these approaches is that they focus exclusively on **single-step estimation** of the current attitude state. None of them address the problem of **multi-step ahead prediction**, where the model must forecast future attitude angles over a prediction horizon of N timesteps. This distinction is critical: while accurate current-state estimation is sufficient for reactive controllers, advanced control strategies such as Model Predictive Control (MPC) require predictions of future states ($k+1$ through $k+N$) to compute optimal control actions [12].

2 CHAPTER 2: Tilt Estimation Filters

This chapter presents the implementation, optimization, and comparative analysis of four classical sensor fusion algorithms for UAV attitude estimation. Building upon the theoretical foundations established in Chapter 1, we now examine how different mathematical approaches translate into practical filtering solutions, each offering distinct trade-offs between computational complexity, estimation accuracy, and robustness to sensor noise.

The four algorithms implemented in this work represent a progression of increasing mathematical sophistication. The Complementary Filter provides a baseline approach through simple weighted fusion of gyroscope and accelerometer measurements in Euler angle space. The Mahony Passive Complementary Filter advances this concept by operating on the $\text{SO}(3)$ manifold using rotation matrices, incorporating geometric error correction through proportional feedback. The Explicit Complementary Filter extends the Mahony approach by adding integral feedback for gyroscope bias estimation, providing adaptive correction for sensor drift. Finally, the Extended Kalman Filter (EKF) implements a full probabilistic state estimation framework using quaternion representation, offering optimal estimation under the assumptions of Gaussian noise and linear measurement models.

All implementations are developed in C++ using the Eigen library for efficient matrix operations, with careful attention to computational efficiency suitable for real-time embedded deployment. Each filter processes 3-axis gyroscope and accelerometer measurements from a 6-DOF IMU operating at 50 Hz sampling rate. The algorithms are validated against ground truth measurements from a dataset containing 1409 samples collected during quadcopter flight operations, covering a range of dynamic conditions from hover to aggressive maneuvers.

For each filter, this chapter presents the mathematical formulation, implementation details, hyperparameter tuning methodology, and performance evaluation using Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) metrics for both roll and pitch estimation. The analysis investigates how each algorithm handles the fundamental sensor fusion challenge: balancing short-term gyroscope accuracy against long-term accelerometer stability while rejecting accelerometer corruption during dynamic flight.

2.1 Complementary Filtering

The Complementary Filter is one of the most fundamental and widely used sensor fusion algorithms for attitude estimation in low-cost UAV applications due to its computational simplicity and effectiveness. It addresses the inherent limitations of individual inertial sensors by fusing their measurements in the frequency domain [8].

The algorithm relies on the observation that gyroscope and accelerometer errors expose complementary spectral characteristics. Gyroscopes provide precise, low-noise measurements of angular velocity that are reliable in the high-frequency domain (short term) but suffer from low-frequency drift due to the integration of bias and noise over time. Conversely, accelerometers provide a stable absolute reference to the gravity vector that is reliable in the low-frequency domain (long term) but are subject to high-frequency noise and corruption from external linear accelerations (non-gravitational forces) during dynamic maneuvers.

The discrete-time implementation for the roll angle p is given by:

$$\varphi(t) = \alpha \cdot (\phi(t-1) + \omega_{gyro} \cdot dt) + (1 - \alpha) \cdot \phi_{accel}$$

2.1.1 Parameters

The parameter α control the trust that is placed on each sensor. It takes values in the range of $(0,1)$ and acts as a weighting factor that determines how much the filter relies on gyroscope integration versus accelerometer measurements. When α is close to 1, the filter places greater trust on the gyroscope term. On the other side when close to 0 the filter relies on the accelerometer.

To determine the optimal value, a grid search was performed over $\alpha \in [0.01, 0.99]$ with a step size of 0.01. For each candidate value, the Complementary Filter was executed on the full dataset and the RMSE of the φ estimation against ground truth was computed. The search identified $\alpha = 0.79$ as the value minimizing the roll RMSE.

2.1.2 Experimental results

The Complementary Filter was implemented with a gain coefficient $a = 0.79$ and tested against the ground truth dataset. The results for the Roll and Pitch estimation are presented below.



Figure 1 Complementary Filter Roll and Pitch Estimation

Figure 1 represents the global performance of the Complementary Filter over the entire flight duration (1409 samples). The estimated Roll and Pitch angles show a strong correlation with the ground truth data. The low RMSE values 0.820° for Roll and 0.771° for Pitch confirm that the filter successfully understands high-frequency gyroscope data with the low-frequency accelerometer data.

Zoomed View: Pitch - Complementary Filter (samples 300-500)

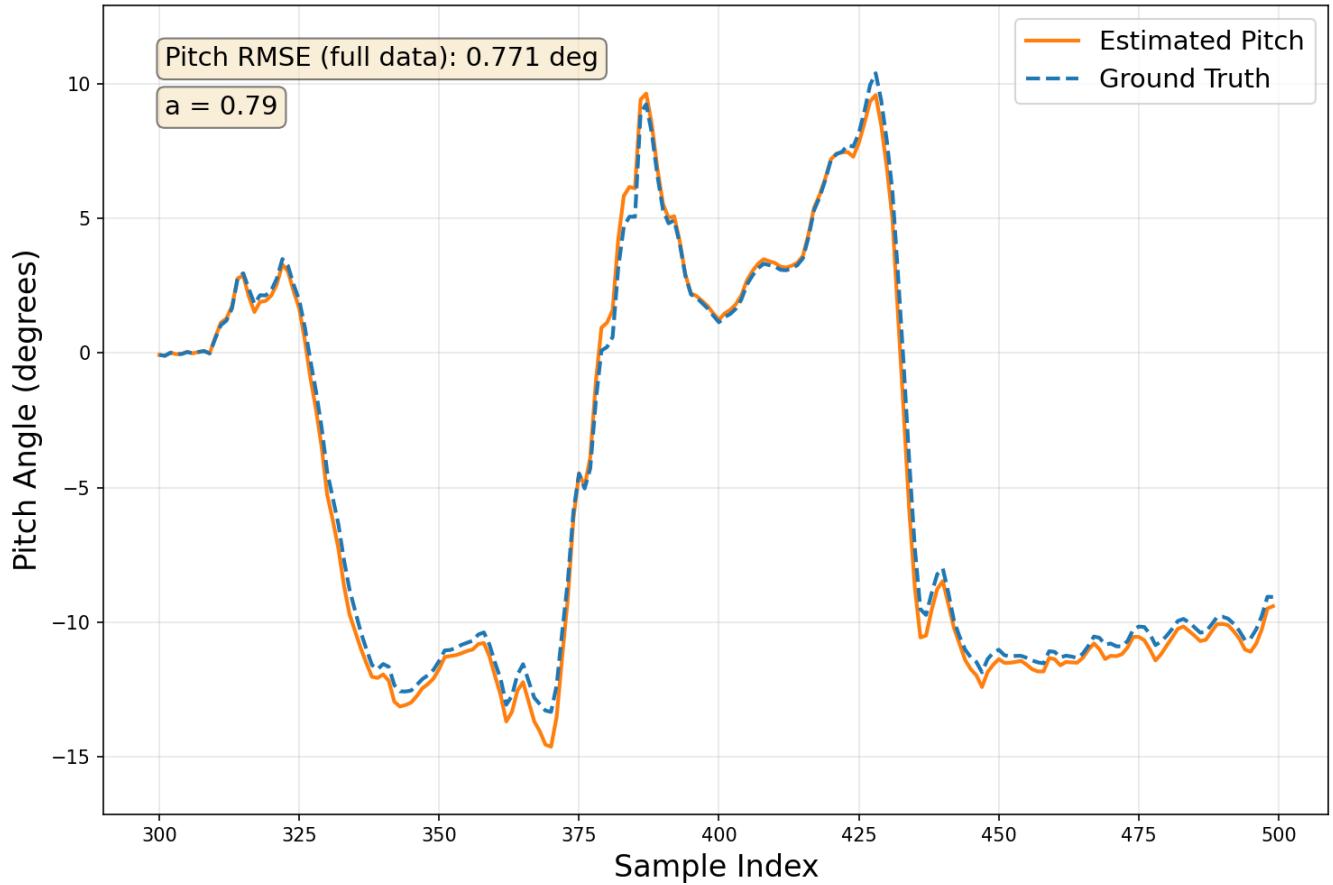


Figure 2 Complementary Filter Zoomed view (samples 300-500) Pitch Estimation

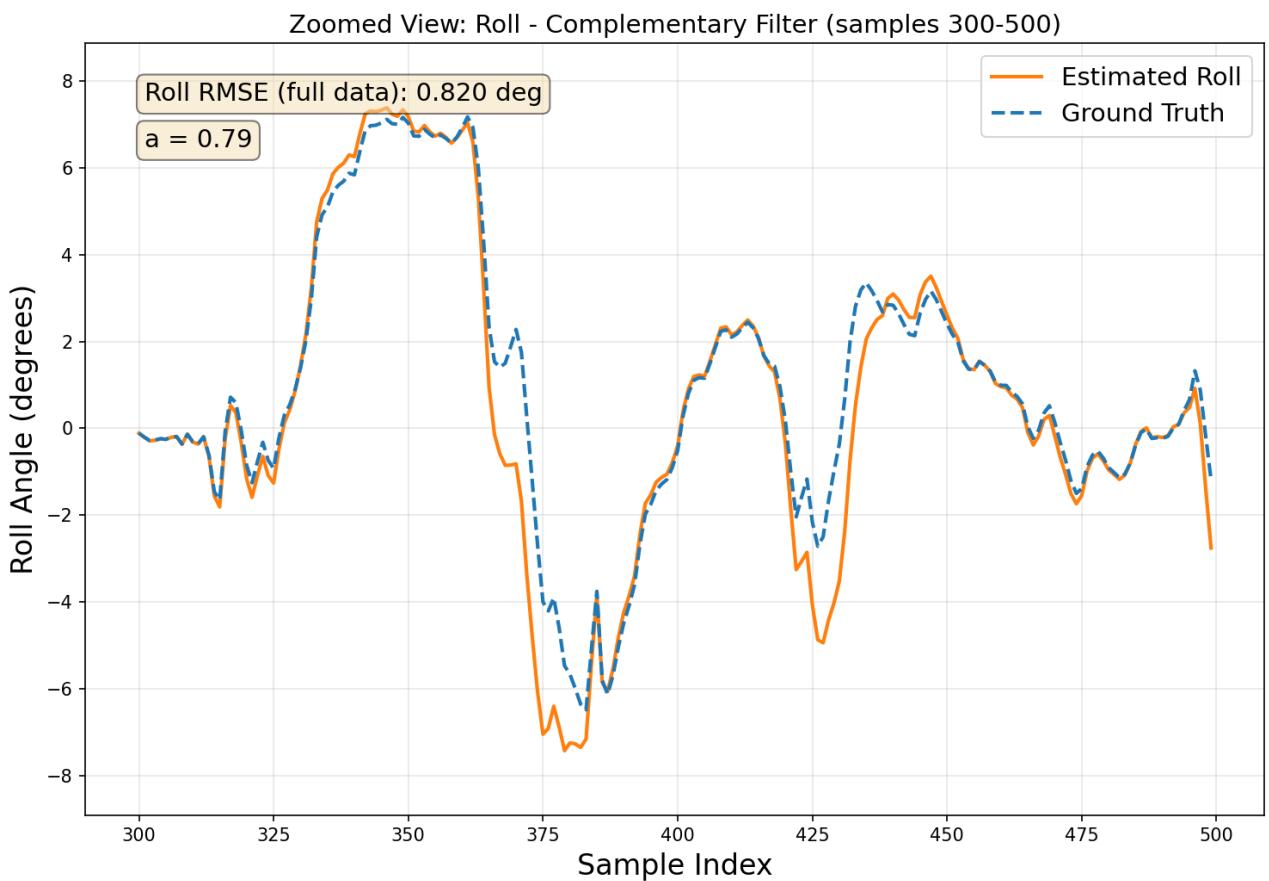


Figure 3 Complementary Filter Zoomed view (samples 300-500) Roll Estimation

Figure 2 provides a more detailed view of the Pitch estimation during a dynamic maneuvering phase (samples 300-500). It can be observed that the Pitch still follows close enough the truth data. Figure 3 illustrates the Roll estimation for the same interval. Generally, it performs almost as well as Pitch with a minor exception at around sample 375 which is a little bit off.

In conclusion, the Complementary Filter proves to be a robust and computationally efficient solution for general UAV attitude estimation. It successfully mitigates gyroscope drift and provides a smooth estimate suitable for flight control. The biggest drawback it's the reliance on a fixed α gain means it cannot adapt to various flight conditions that are encountered often in the real world.

2.2 Mahony Filter

The Mahony filter is a nonlinear complementary filter that operates directly on the Special Orthogonal group $SO(3)$, representing orientation using rotation matrices [3]. Unlike the standard Complementary Filter which operates on Euler angles, the Mahony filter avoids singularities (gimbal lock) and provides a more geometric approach to error correction.

The core principle of the Mahony filter is to estimate the rotation matrix \hat{R} by comparing the

measured direction of gravity (from the accelerometer) with the estimated direction of gravity. The error between these two vectors is used to generate a correction term that “steers” the gyroscope integration towards the true vertical.

The implementation done here is a “Passive” Mahony filter as described in [3].

2.2.1 Algorithm Overview

The core principle of the Mahony filter is to maintain an estimated rotation matrix \hat{R} that represents the current attitude. At each timestep, the filter compares the measured direction of gravity (derived from the accelerometer) with the direction prediction by the current estimate. The discrepancy between these two is used to generated a correction term that steers they gyroscope integration towards the true orientation.

2.2.2 Parameters

The Mahony filter has a single tunable hyperparameter: the proportional gain k_p . This parameter controls the of the accelerometer-based correction applied to the gyroscope integration at each timestep.

When k_p is small (close to zero), the filter relies almost entirely on the gyroscope for attitude estimation. The gyroscope provides accurate short-term measurements but accumulates drift over time. When k_p is large, the filter applies aggressive corrections based on the accelerometer measurement. While this eliminates gyroscope drift, it make the estimate highly sensitive to accelerometer noise and non-gravitational acceleration that occur during maneuvers[4].

The optimal value of k_p must balance these two effects: it should be large enough to prevent gyroscope drift within the operating timeframe, yet small enough to maintain smooth estimates during dynamic flight. Unlike the Complementary Filter's a parameter which directly weights two angle estimates, k_p operates in the angular velocity domain, scaling the correction signal that is added to the gyroscope before integration. This provides a more natural correction mechanism that respects the geometric structure of rotations.

To determine the optimal value, a grid search was performed over $k_p \in [1,100]$ with integer steps. For each candidate value, the Mahony filter was executed on the full dataset and the RMSE of both ϕ and θ was computed against ground truth. The optimization criterion was the average of the roll and pitch RMSE, ensuring that both angles are estimated well simultaneously. The search identified $k_p = 11$ as the optimal value.

2.2.3 Experimental Results

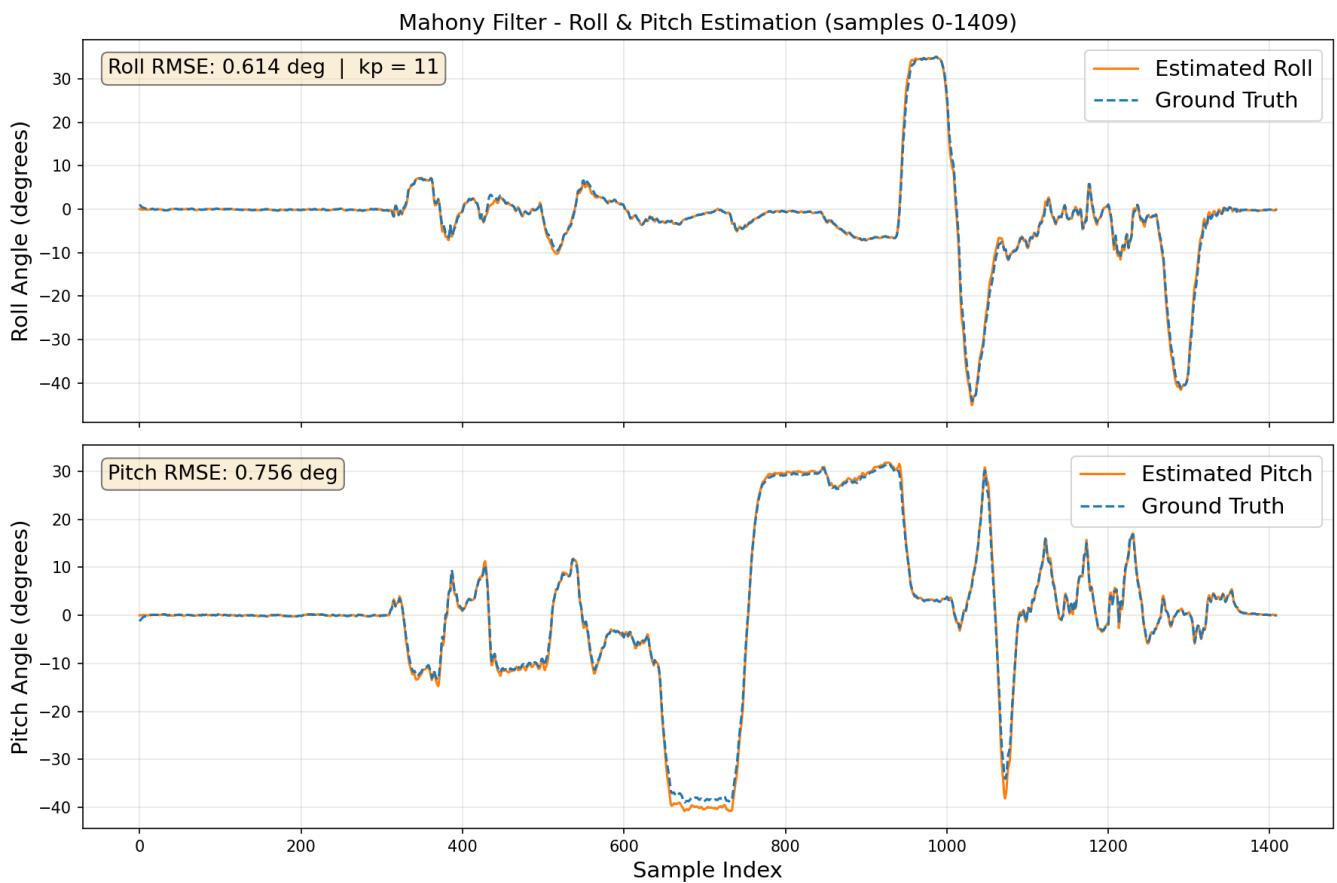


Figure 4 Mahony Filter Roll and Pitch Estimation

Figure 4 represents the global performance of the Mahony filter over the entire flight duration (1409) samples. The estimated ϕ and θ angles show a strong correlation with the ground truth data. The filter achieves an RMSE of 0.614 for ϕ and 0.756 θ , representation a significant improvement over the Complementary Filter. The improvement is particularly notable for the roll angle ϕ , where the filter reduces the RMSE 25% approximately.

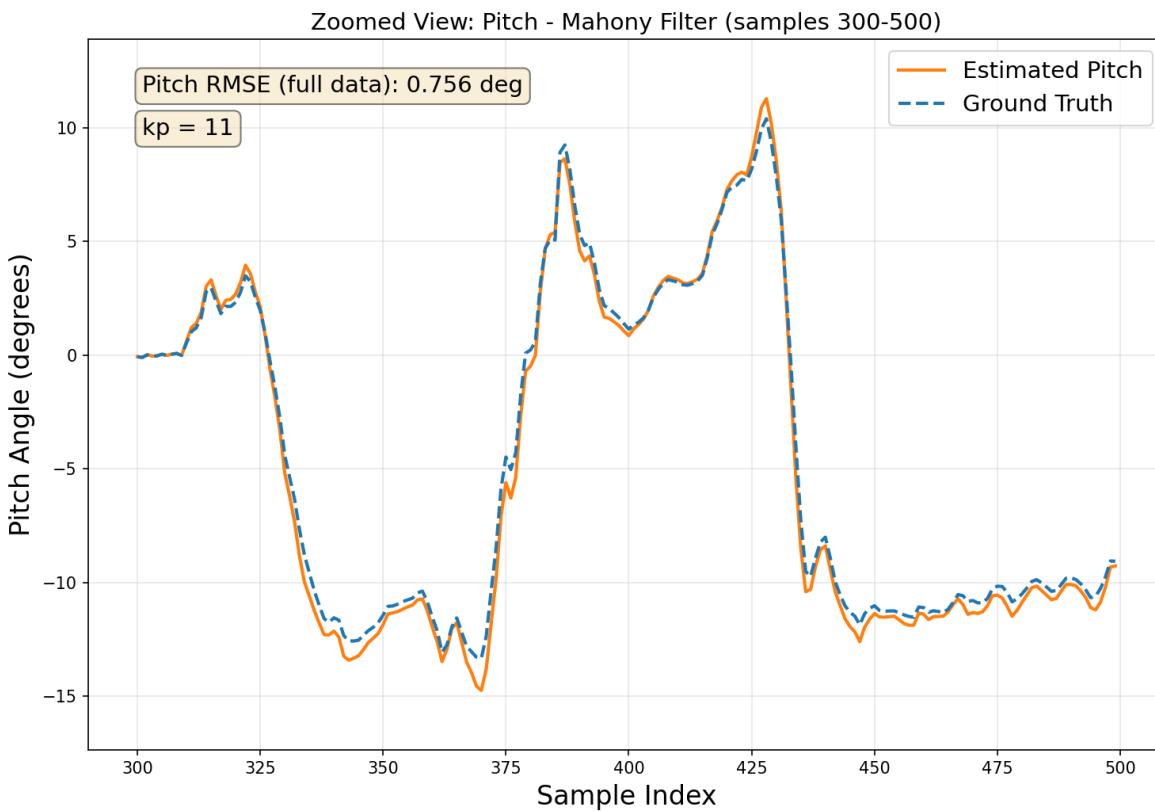


Figure 5 Mahony Filter Zoomed view (samples 300-500) Pitch Estimation

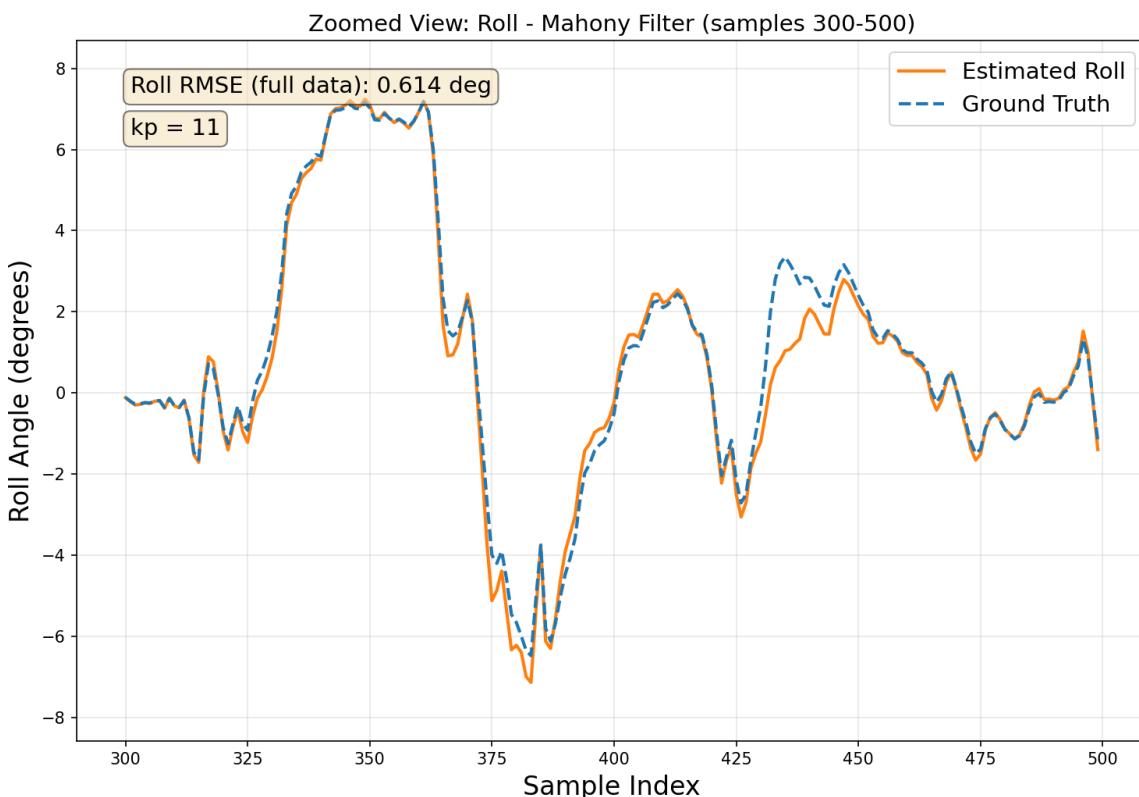


Figure 6 Mahony Filter Zoomed view (samples 300-500) Roll Estimation

Figure 5 and Figure 6 provide zoomed view of the attitude angles during a maneuvering phase (samples 300-500). Compared to the Complementary Filter, the Mahony filters the rapid angles changes more closely. The geometric error correction on $SO(3)$ allows the filter to handle large rotation more gracefully than the Euler angle-based approach of the Complementary Filter.

In conclusion, the Mahony Passive Complementary Filter demonstrates improved estimation accuracy over the basic Complementary Filter while maintaining computational efficiency suitable for real-time applications. Operating on $SO(3)$ with rotation matrices provides inherent robustness against gimbal lock and allows geometric error correction. The single hyperparameter k_p makes tuning straightforward, and the SVD-based orthonormalization ensures numerical stability during prolonged operation. However, the proportional-only correction means that persistent biases in the gyroscope may not be fully eliminated over very long time periods — a limitation addressed by the Explicit Complementary Filter in the following section.

2.3 Explicit Complementary Filtering

Text

2.3.1 Sub-sub-title

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2.4 Extended Kalman Filtering (EKF)

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2.4.1 Sub-sub-title

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3 CHAPTER 3: Deep learning approach

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3.1 Sub-title

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Conclusions

Future Work

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Appendix A

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Appendix B

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Appendix C

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