



Master of Science Thesis

Development of estimation algorithms for navigation of unmanned aerial vehicles

Space to insert image, picture or diagram related to the thesis subject
(optional)

Student: Kostakis Ilias-Odysseas
Registration Number: AIDL-0063

MSc Thesis Supervisor

Alexandridis Alexandros Professor

Insert supervisor name in latin characters, in the form Surname, name
Insert supervisor grade as: Professor, Assoc. Professor, Assist. Professor, Lecturer

ATHENS-EGALEO, February 2025

This MSc Thesis has been accepted, evaluated and graded by the following committee:

Supervisor	Member	Member
SIGNATURE	SIGNATURE	SIGNATURE
Surname, Name	Surname, Name	Surname, Name
Grade	Grade	Grade
Department or School	Department or School	Department or School
Institution	Institution	Institution

Copyright © All rights reserved.

University of West Attica and (Name and Surname of the student)

Month, Year

You may not copy, reproduce or distribute this work (or any part of it) for commercial purposes. Copying/reprinting, storage and distribution for any non-profit educational or research purposes are allowed under the conditions of referring to the original source and of reproducing the present copyright note. Any inquiries relevant to the use of this thesis for profit/commercial purposes must be addressed to the author.

The opinions and the conclusions included in this document express solely the author and do not express the opinion of the MSc thesis supervisor or the examination committee or the formal position of the Department(s) or the University of West Attica.

Declaration of the author of this MSc thesis

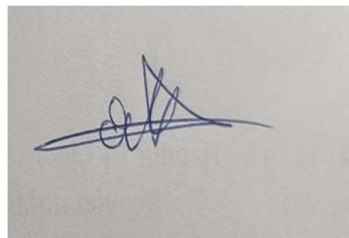
I, Ilias-Odyseas Kostakis, Pantelis with the following student registration number: mscaidl-0063, postgraduate student of the MSc program in “Artificial Intelligence and Deep Learning”, which is organized by the Department of Electrical and Electronic Engineering and the Department of Industrial Design and Production Engineering of the Faculty of Engineering of the University of West Attica, hereby declare that:

I am the author of this MSc thesis and any help I may have received is clearly mentioned in the thesis. Additionally, all the sources I have used (e.g., to extract data, ideas, words or phrases) are cited with full reference to the corresponding authors, the publishing house or the journal; this also applies to the Internet sources that I have used. I also confirm that I have personally written this thesis and the intellectual property rights belong to myself and to the University of West Attica. This work has not been submitted for any other degree or professional qualification except as specified in it.

Any violations of my academic responsibilities, as stated above, constitutes substantial reason for the cancellation of the conferred MSc degree.

The author

Ilias-Odyseas Kostakis





{Dedication page (optional)}

{Acknowledgements page (optional)}

Abstract

Accurate attitude estimation is essential for autonomous navigation and control of unmanned aerial vehicles (UAVs). The attitude, defined by the roll, pitch, and yaw angles, directly influences the vehicle's ability to maintain stable flight and respond effectively to control inputs. Reliable estimation enables precise navigation, robust disturbance rejection and accurate sensor and camera alignment during autonomous operations.

This thesis implements and presents a comparative analysis of four classical sensor fusion algorithms for estimating attitude from data collected by an inertial measurement unit (IMU). The four filters are: the Complementary Filter, the Extended Kalman Filter (EKF), the Mahony Passive Filter and the Explicit Complementary Filter. Each filter has its own unique approach to estimating the attitude.

The primary objective of this work is to implement, optimize, and evaluate these four filters using real flight data collected from a quadcopter platform. Each filter processes the 3-axis readings from the gyroscope and accelerometer at a 50Hz sampling rate to estimate vehicle orientation. The implementation is done in C++ using the mathematical library Eigen for efficient matrix operations, with emphasis on computational performance suitable for real-time embedded systems.

All four algorithms were implemented and validated against ground truth measurements. Comparative evaluation reveals distinct performance characteristics. The Complementary Filter offers computational simplicity, the Mahony Filter provides improved estimation via geometric error correction on rotation matrices, the explicit complementary filter further reduces the Mahony error by bias estimation and the EKF achieves the highest precision in estimation. The analysis demonstrates the fundamental trade-offs between algorithmic complexity and estimation accuracy.

Beyond classical filtering approach, this thesis investigates the application of deep learning techniques to the attitude estimation problem. We will explore an architecture that uses Recurrent Neural Networks (RNN) and specific Long Short-Term Memory networks that will use the current IMU data and try to predict the attitude in the future. Implementation is carried

out in C++ using the LibTorch framework ensuring compatibility with real-time embedded systems while maintaining computational efficient required for onboard UAV deployment

Keywords

UAVs, Complementary Filtering, EKF, Attitude estimation, Classic estimation methods, Neural Networks, Machine Learning

Table of Contents

List of Tables.....	9
List of figures	9
Acronym Index	9
INTRODUCTION	10
The subject of this thesis	10
Aim and objectives	11
Methodology	12
Innovation	12
Structure.....	12
1 CHAPTER 1: Theoretical background.....	13
1.1 The role of attitude in UAV flight	13
1.2 IMU and Sensors	13
1.2.1 Gyroscope	13
1.2.2 Accelerometer	14
1.3 Attitude Representation.....	14
1.3.1 Euler Angles	14
1.3.2 Rotation Matrices (SO(3))	15
1.3.3 Quaternions	15
1.3.4 Sensor Fusion Fundamentals.....	16
1.4 Neural Networks for Time-Series Predictions	16
1.4.1 Time-Series Data and Sequential Learning.....	16
1.4.2 Recurrent Neural Networks (RNN)	17
2 CHAPTER 2: Classic Filters.....	19
2.1 Complementary Filtering.....	19
2.1.1 Sub-sub-title.....	19
2.2 Mahony Filtering.....	19
2.2.1 Sub-sub-title.....	19
2.3 Explicit Complementary Filtering	19
2.3.1 Sub-sub-title.....	19
2.4 Extended Kalman Filtering (EKF)	19
2.4.1 Sub-sub-title.....	19
3 CHAPTER 3: Deep learning approach.....	20
3.1 Sub-title.....	20
Conclusions	22
Future Work	22
Bibliography – References – Online sources.....	23
Appendix A	24
Appendix B.....	24
Appendix C	24

List of Tables

Table 1.1 Table description (include reference to source, if applicable)[12]

Table 2.1 Table description (include reference to source, if applicable)[13]

List of figures

Acronym Index

IMU: Inertial Measurement Unit
EKF: Extended Kalman Filter
UAV: Unmanned Aerial Vehicle
RNN: Recurrent Neural Network
LLM: Large Language Model
LSTM: Long Term Short-Memory

INTRODUCTION

Unmanned Aerial Vehicles (UAVs), commonly known as drones, have emerged as transformative technologies across many sectors. From aerial photography, agriculture to package delivery, these autonomous platforms are reshaping how we approach tasks that were impossible in the past.

At the heart of every UAV lies a critical capability: the ability to know its own orientation in three-dimensional space. This ability, referred to as attitude, describes how the vehicle is tilted relative to the Earth's surface. Without accurate and continuous attitude information, a UAV cannot maintain stable flight, execute precise maneuvers, or respond appropriately to disturbances such as wind gusts. The attitude serves as the foundation upon which all flight control decisions are made.

Modern UAVs rely on Inertial Measurement Units (IMUs), compact electronic devices that combine multiple sensors to measure motion. A typical IMU contains gyroscopes, which measure rotational velocity, and accelerometers, which measure linear acceleration including gravity. These sensors are inexpensive, lightweight, and consume minimal power, making them ideal for small autonomous platforms. However, each sensor type has inherent limitations that prevent it from providing accurate attitude estimates on its own.

The challenge of combining imperfect sensor measurements to produce reliable attitude estimates has motivated decades of research in sensor fusion algorithms. These algorithms integrate data from multiple sources, using the strength of each different sensor while mitigating their individual weaknesses. The field encompasses approaches ranging from simple weighted averaging to sophisticated probabilistic frameworks, each offering different trade-offs between computational complexity, estimation accuracy, and robustness.

Most recently the deep learning and AI domain has started to have impact on this field too. Combining neural network techniques and sensor fusion algorithms is something that can be explored in order to further improve the attitude estimation. These new approaches could potentially explore new patterns that would be difficult to be done via classic filters.

The subject of this thesis

This thesis addresses the fundamental challenge of real-time attitude estimation for unmanned aerial vehicles using inertial sensors. The orientation of a vehicle in three-dimensional space is described by three angles: roll, pitch and yaw. This work focuses specifically on roll and pitch estimation, which are essential for maintaining level flight and controlling the vehicle's response to disturbances.

The core challenge lies in the inherent limitations of individual inertial sensors. Gyroscopes provide precise short-term measurements but suffer from drift over time due to integration errors. Accelerometers can estimate orientation from the gravity vector but become unreliable during dynamic maneuvers when additional accelerations corrupt the measurements. No single sensor can provide accurate attitude estimates independently, creating a fundamental sensor fusion problem.

Accurate attitude estimation is critical for UAV flight stability and autonomous operations. As commercial applications expand—from package delivery to infrastructure inspection—reliable estimation becomes increasingly important. Modern embedded flight controllers demand efficient algorithms that balance estimation accuracy against computational constraints. However, the increasing complexity of flight scenarios and the recent advances in deep learning techniques create new opportunities to explore new approaches use neural networks. Using past data we possibly could predict quite accurately the attitude in the future.

Aim and objectives

The primary aim of this thesis is to implement, optimize and compare four classical sensor fusion algorithms for UAV attitude estimation. This comprehensive analysis establishes a fundamental understanding of their performance characteristics, computational requirements, and practical trade-offs. The analysis serves both as a validation of the implementations and as a foundation for future work exploring hybrid approaches that integrate classical filtering with deep learning.

This work implements four distinct attitude estimation filters in C++, each representing a different point in the design space of sensor fusion algorithms with distinct mathematical foundations and computational characteristics. All implementations utilize the Eigen library for efficient matrix operations and are designed with real-time embedded deployment in mind, avoiding dynamic memory allocation and minimizing computational overhead.

The Complementary Filter serves as a baseline approach, implementing simple weighted fusion between gyroscope integration and accelerometer measurements. The Mahony Passive Complementary Filter improves upon this by operating on rotation matrices (SO(3) space) rather than Euler angles, using geometric error correction through a proportional gain (kp) applied to the cross product between measured and estimated gravity vectors. The Explicit Complementary Filter takes a direct vector approach that works with acceleration measurements without reconstructing attitude. It adds integral feedback (ki) for gyroscope bias estimation, providing three-axis bias correction alongside attitude estimation. Finally, the Extended Kalman Filter (EKF) implements a quaternion-based approach using a 7-dimensional state vector (quaternion + 3-axis gyro bias), featuring a full predict-update cycle with Jacobian computation, process noise covariance (Q) and measurement noise covariance (R) matrices.

The implementations are validated against ground truth measurements from a dataset of 1409 samples collected at 50Hz from a quadcopter platform equipped with a 6-axis IMU (3-axis gyroscope and 3-axis accelerometer). Ground truth roll and pitch angles were captured independently during the test, providing reference values for validation.

For each filter, Root Mean Square Error (RMSE) and Mean Error Absolute (MEA) metrics are computed for both roll and pitch to quantify estimation accuracy. The analysis investigates how simple filters like the Complementary Filter perform against optimal estimators like the

EKF, and identifies which filters excel during high-dynamics versus low-dynamics flight conditions. Visual comparisons through graphs support these quantitative assessments.

Beyond classical filtering approaches, this thesis investigates the potential of integrating Recurrent Neural Networks (RNNs), specifically Long Short-Term Memory (LSTM) networks to predict attitude using past data. The implementation will be done using LibTorch (PyTorch C++ API) to main consistency with the C++ classical filter codebase and ensure compatibility with real-time embedded systems.

Methodology

The first step is to implement the four classical filters. Main objective is implementing all the complicated mathematics with C++ and Eigen library to have a ready go-to solution for embedded systems. The next step was to verify the functionality of each algorithm and optimize the hyperparameters. This was done by running the filters on our collected data and compare the output, pitch and roll estimation, with the actual positioning values we had. The check was done by having low RMSE/MSE value between estimation and ground truth. Afterwards the hyperparameters of each filter were optimized, using regular grid search. The final step of the classical filters was to compare all them and provide graphs and analysis of the behavior. Lastly, Long Short-Term Memory (LSTM) architectures were explored. They are combining raw IMU data and try to predict the next state of the drone.

Innovation

While classic sensor fusion algorithms for attitude estimation are well-established the proposed LSTM architecture that using raw IMU data and tries to predict the future, is something that has not been fully explored in the sector. The architecture predicts the state at time $k+1$ to $k+10$, enabling proactive attitude estimation.

Structure

Briefly outline the structure of the thesis, in chapters.

TODO After we finish writing just outline the structure ..

1 CHAPTER 1: Theoretical background

This chapter establishes the theoretical foundation necessary to understand the attitude estimation problem for UAVs. An introduction is being made to the attitude estimation in UAV, what sensors are using to measure motion and explore different mathematical representations of orientations. Later fundamental concepts of sensor fusion are discussed which motivate the classic filter algorithms developed in section 2. Finally neural network architectures are introduced for sequential learning, providing the theoretical background for the deep learning approach presented in Chapter 3.

1.1 The role of attitude in UAV flight

The ability of a drones to operate autonomously has a critical dependency on their ability to determine and control their orientation in three-dimensional space.

The attitude of a UAV describes its rotation orientation relative to a fixed reference frame, typically defined with respect to the Earth's surface. In aviation, attitude is conventionally expressed using three angular coordinates known as Euler angles:

- 1) Roll (ϕ): Rotation around the longitudinal axis (forward-backward direction)
- 2) Pitch (θ): Rotation around the lateral axis (left-right direction)
- 3) Yaw (ψ): Rotation around the vertical axis (up-down direction)

These angles directly influence the vehicle's ability to maintain stable flight, execute precise maneuvers, and respond appropriately to environmental disturbances such as wind gusts. Without accurate attitude information a UAV can't, maintain level flight, execute controlled turns, provide stable platform for cameras and sensors, implement autonomous navigation algorithms.

Unlike position (which can be directly measured by GPS) or velocity (which can be derived from GPS updates), attitude can not be directly measured by a single sensor. Instead, it must be estimated by fusing measurements from multiple sensors, each with different characteristics and limitations.

1.2 IMU and Sensors

An Inertial Measurement Unit (IMU) is a compact electronic device that combines multiple motion sensors to measure a vehicle's kinematic state. A typical 6-axis IMU contains an 3-axis gyroscope and 3-axis accelerometer.

1.2.1 Gyroscope

A gyroscope is an angular rate sensor that measures how fast an object rotates around one or more axes, and in UAVs it is one of core sensors for attitude estimations. In practice gyroscopes output angular velocity (e.g. in degrees or rad per second). Angular velocity is measured as $\omega \in R^3$ (rad/s). Each axis output corresponds to the rate of rotation about the x, y, z axes of the sensor frame.

Gyroscopes have key advantages and limitations. They provide very smooth short-term attitude information and are insensitive to linear accelerations, which makes them reliable during aggressive maneuvers where accelerometers are heavily corrupted by non-gravitational forces. However, their measurements contain bias and noise, so integration causes drift that grows over time, meaning gyro-only attitude estimates must be regularly corrected using other sensors such as accelerometers, magnetometers, or GNSS within a sensor fusion algorithm. [1]

1.2.2 Accelerometer

Accelerometers are fundamentally different from what their name suggests. They do not measure kinematic acceleration. Instead they measure specific force, also called proper acceleration. It is measured as $\alpha \in R^3 (m/s^2)$. Each axis output corresponds to the specific force experienced along the x, y, z axes of the sensor frame, including gravity during static conditions.

Accelerometers excel at providing long-term absolute reference through the gravity vector, allowing computation of roll and pitch via inverse trigonometry when the vehicle is mostly stationary or slow moving. Accelerometer measures $a = [ax, ay, az]$ and we can calculate roll and pitch via these equations

Equation 1 Roll from accelerometer $roll = atan2(ay, az)$

Equation 2 Pitch from accelerometer $pitch = arcsin(ax/g)$

1.3 Attitude Representation

Representing three-dimensional rotations mathematically is a non-trivial problem in attitude estimation. While the physical concept of orientation is intuitive, capturing it numerically requires careful consideration of mathematical properties, computational efficiency, and singularity avoidance. This section introduces three common representations used in attitude estimation algorithms, each with distinct advantages and limitations that influence their suitability for different filtering approaches.

1.3.1 Euler Angles

Euler angles are the most intuitive representation of orientation, describing attitude as a sequence of three rotations around specified axes. The aerospace convention, known as the ZYX (yaw-pitch-roll) sequence, applies rotations in the following order [2]:

1. Rotate by ψ (yaw) around the Z-axis (vertical)
2. Rotate by θ (pitch) around the new Y-axis
3. Rotate by ϕ (roll) around the new X-axis

It only has 3 parameters which make it more human interoperable and lesser memory footprint for computer calculations.

Euler angles suffer from non-uniqueness (multiple angle combinations can represent the same physical orientation) and require complex trigonometric transformations when computing angular velocity from angle rates

The Complementary Filter (Chapter 2) operates directly on roll and pitch Euler angles, taking advantage of their simplicity for the basic weighted fusion approach. Furthermore, all filters in this thesis output their final estimates as Euler angles for interpretability and comparison with ground truth measurements. For UAV flight control applications where extreme pitch angles (near $\pm 90^\circ$) are rare, Euler angles provide an acceptable representation.

1.3.2 Rotation Matrices (SO(3))

A rotation matrix $R \in \text{SO}(3)$ is a 3×3 orthonormal matrix that transforms vectors from one coordinate frame to another. The notation $\text{SO}(3)$ stands for "Special Orthogonal group in 3 dimensions," representing all possible rotations in three-dimensional space.

A rotation matrix can be constructed from Euler angles using the product of elementary rotation matrices [3]

Equation 3 Rotation matrix to euler angles: $R(\varphi, \theta, \psi) = R_z(\psi) \cdot R_y(\theta) \cdot R_x(\varphi)$

Where R_x , R_y , R_z represent rotations around the X, Y, and Z axes respectively. The resulting 3×3 matrix contains nine elements, though only three are independent due to orthonormality constraints.

One drawback is redundancy: storing nine numbers to represent three degrees of freedom is inefficient. Also, they require periodic orthonormalization some common methods are Gram-Schmidt or singular value decomposition (SVD)[4].

The Mahony Passive Complementary Filter (Chapter 2) operates directly on rotation matrices, leveraging the geometric properties of $\text{SO}(3)$ for error correction. The filter computes a cross-product-based correction term that exploits the manifold structure of rotation matrices, providing superior performance compared to Euler angle approaches. To maintain orthonormality, the Mahony filter implementation applies Gram-Schmidt orthonormalization after each prediction step, ensuring the rotation matrix remains valid throughout the flight.

1.3.3 Quaternions

A quaternion q is a four-parameter representation of rotation, consisting of one scalar and one vector component:

Equation 4 Quaternion $q = [q_0, q_1, q_2, q_3]^T = [q_0, q_v]^T$

Where:

q_0 : Scalar part

$q_v = [q_1, q_2, q_3]^T$: Vector part

Like rotation matrices, quaternions require normalization after numerical integration to maintain the unit constraint. However, this is computationally cheaper than matrix orthonormalization (single division versus full matrix projection).

Extended Kalman Filter (EKF) uses quaternions for the state representation, with a 7-dimensional state vector that includes the quaternion and gyroscope bias estimates.

1.3.4 Sensor Fusion Fundamentals

Sensor fusion combines measurements from multiple sensors to produce estimates superior to any single sensor. For attitude estimation, gyroscopes and accelerometers exhibit complementary error characteristics: gyroscopes provide smooth, high-frequency measurements but suffer from integration drift, while accelerometers offer a long-term gravity reference but are noisy and corrupted by linear acceleration.

The fundamental challenge is designing algorithms that:

1. Trust gyroscope measurements for short-term dynamics (responsiveness)
2. Use accelerometer measurements for long-term drift correction (stability)
3. Reject accelerometer corruption during vehicle acceleration

Fusion strategies range from simple weighted averaging (complementary filters) to nonlinear filters operating on manifolds (Mahony filter on $SO(3)$) to probabilistic state estimation (Extended Kalman Filter). Performance is evaluated using Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) against ground truth measurements.

1.4 Neural Networks for Time-Series Predictions

While classical filtering approaches rely on explicit mathematical models of sensor behavior and system dynamics, modern deep learning techniques offer an alternative paradigm: learning patterns directly from data. This section introduces the theoretical foundations of neural network architectures designed for sequential data, which form the basis for the LSTM-based attitude predictor developed in Chapter 3.

1.4.1 Time-Series Data and Sequential Learning

Attitude estimation is fundamentally a time-series problem. IMU measurements arrive sequentially at fixed sampling intervals ($T_s = 0.02s$ for 50 Hz), and the vehicle's current state depends on its past state. Unlike static classification tasks (e.g., image recognition), attitude estimation requires the model to understand temporal dependencies, how the current orientation relates to previous orientations and control inputs.

Traditional feedforward neural networks (e.g., multilayer perceptrons) process each timestep independently, ignoring temporal structure. They lack memory of previous inputs, making them unsuitable for sequential prediction. Recurrent architectures address this limitation by maintaining internal state across timesteps.

1.4.2 Recurrent Neural Networks (RNN)

A Recurrent Neural Network (RNN) extends feedforward network with recurrent connections that allow information to persist across time. At each timestep t , the RNN process input $x(t)$ while maintain a hidden state $h(t)$ that summarizes information from all previous timesteps [5]. The main advantage of this approach is that the calculation is very efficient and the state information may be stored infinitely in theory.

Despite their theoretical elegance, RNNs suffer from the vanishing gradient problem during training. When learning long-term dependencies (e.g., patterns spanning dozens of timesteps), gradients propagated backward through time become exponentially small, preventing the network from learning correlations between distant events [6]. For attitude estimation requiring lookback windows of 10+ timesteps (0.3+ seconds), this limitation is critical.

1.4.3 Long Sort-Term Memory (LSTM)

Long Short-Term Memory networks, introduced by Hochreiter and Schmidhuber (1997) [7], solve the vanishing gradient problem through a sophisticated gating mechanism that controls information flow across time. LSTMs replace the simple hidden state of RNNs with two components:

- 1) Cell state(c): Carries information across long time intervals with minimal modification
- 2) Hidden state(h): A filtered version of the cell state used for output prediction

For IMU-based attitude estimation, LSTMs offer several advantages over classical filters.

- Automatic feature learning: No manual tuning of filter gains or covariance matrices
- Long-term pattern recognition: Can remember sensor characteristics, bias patterns, and motion dynamics over extended periods
- Adaptive sensor fusion: Gates automatically learn when to trust accelerometer measurements (during low dynamics) versus gyroscope integration (during aggressive maneuvers)
- Multi-step prediction capability: Can forecast future attitudes based on learned temporal patterns in sensor and control data

This thesis implements an LSTM-based multi-step ahead predictor that forecasts UAV attitude angles (roll, pitch, yaw) over a 10-timestep prediction horizon (0.30 seconds at 33.3 Hz sampling rate). The network processes a lookback window of past sensor measurements, ground truth angles, and control inputs to predict future vehicle orientation. This predictive capability enables proactive flight control and disturbance rejection strategies that anticipate vehicle motion rather than merely reacting to current measurements.

Implementation is performed in C++ using the LibTorch framework (PyTorch C++ API), maintaining consistency with the classical filter codebase and ensuring compatibility with real-time embedded UAV systems. The architecture, training methodology, and comparative performance analysis against classical filters are detailed in Chapter 3.

2 CHAPTER 2: Classic Filters

Text

2.1 Complementary Filtering

Text

2.1.1 Sub-sub-title

2.2 Mahony Filtering

Text

2.2.1 Sub-sub-title

Text

2.3 Explicit Complementary Filtering

Text

2.3.1 Sub-sub-title

Text

2.4 Extended Kalman Filtering (EKF)

Text

2.4.1 Sub-sub-title

Text

3 CHAPTER 3: Deep learning approach

Text

3.1 Sub-title

Text



Conclusions

Future Work

Bibliography – References – Online sources

1. Zhou, X., Chen, L., Sun, C., Jia, W., Yi, N., & Sun, W. (2025). Highly Accurate Attitude Estimation of Unmanned Aerial Vehicle Payloads Using Low-Cost MEMS. *Micromachines*, 16(6), 632. <https://doi.org/10.3390/mi16060632>
2. Weisstein, Eric W. "Euler Angles." From *MathWorld*--A Wolfram Resource. <https://mathworld.wolfram.com/EulerAngles.html>
3. Rotation Matrix to Euler Angles. <https://learnopencv.com/rotation-matrix-to-euler-angles/>
4. R. Mahony, Tarek Hamel, Jean-Michel Pflimlin. Nonlinear Complementary Filters on the Special Orthogonal Group. *IEEE Transactions on Automatic Control*, 2008, 53 (5), pp.1203-1217. [10.1109/TAC.2008.923738](https://doi.org/10.1109/TAC.2008.923738). [hal-00488376](https://hal.archives-ouvertes.fr/hal-00488376)
5. Goodfellow, I., Bengio, Y., & Courville, A. (2016). Deep learning. MIT Press.
6. Bengio, Y., Simard, P., & Frasconi, P. (1994). Learning long-term dependencies with gradient descent is difficult. *IEEE Transactions on Neural Networks*, 5(2), 157-166.
7. Hochreiter, S., & Schmidhuber, J. (1997). Long short-term memory. *Neural Computation*, 9(8), 1735-1780
8. Test
9. Test
10. As
- 11.

Appendix A

.....

Appendix B

.....

Appendix C

.....