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MSc in Artificial Intelligence and Deep Learning

Master of Science Thesis

Development of estimation algorithms for navigation of unmanned aerial vehicles

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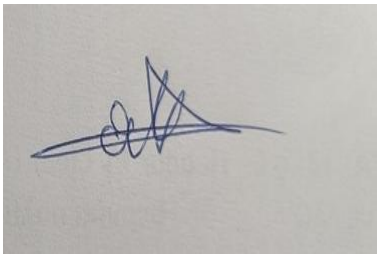

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Abstract

Accurate attitude estimation and prediction are critical for the autonomous navigation and advanced control of unmanned aerial vehicles (UAVs). While classical sensor fusion algorithms can estimate current vehicle orientation, they are fundamentally reactive and cannot anticipate future states. Modern control schemes such as Model Predictive Control (MPC), require multi-step ahead predictions of future attitude, motivating the development of predictive approaches based on deep learning.

This thesis develops a complete framework for UAV attitude prediction, centered on a Long Short-Term Memory (LSTM) Recurrent Neural Network (RNN) that predicts future roll, pitch and yaw angles over a 10-timesstep horizon using look—back windows of IMU measures and control inputs. To establish baselines and provide context for the deep learning approach, four classical tilt estimation filters were implemented in C++ using the Eigen library: the Complementary Filter, the Mahony Filter, the Explicit Complementary Filter and the Extended Kalama Filter (EKF). These filters fuse gyroscope and accelerometer data to estimate the attitude angles (ϕ, θ) and their hyperparameters were optimized using real flight data collect from a quadcopter platform. Experimental evaluation showed that LSTM approach not only achieves high estimation accuracy but also provides the essential multi-step prediction capability required for MCP-based flight controllers, bridging the gap between classical sensor fusion and modern deep learning for proactive flight control.

Keywords

UAVs, Complementary Filtering, EKF, Attitude estimation, Classic estimation methods, Neural Networks, Machine Learning, MPC, LSTM, RNN, Sensor Fusion

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Acronym Index

IMU: Inertial Measurement Unit
EKF: Extended Kalman Filter
UAV: Unmanned Aerial Vehicle
RNN: Recurrent Neural Network
LLM: Large Language Model
LSTM: Long Term Short-Memory
MPC: Model Predictive Protocol

INTRODUCTION

Unmanned Aerial Vehicles (UAVs), commonly known as drones, have emerged as transformative technologies across many sectors. From aerial photography, agriculture to package delivery, these autonomous platforms are reshaping how we approach tasks that were impossible in the past.

At the heart of every UAV lies a critical capability: the ability to know its own orientation in three-dimensional space. This ability, referred to as attitude, describes how the vehicle is tilted relative to the Earth's surface. Without accurate and continuous attitude information, a UAV cannot maintain stable flight, execute precise maneuvers, or respond appropriately to disturbances such as wind gusts. The attitude serves as the foundation upon which all flight control decisions are made.

Modern UAVs rely on Inertial Measurement Units (IMUs), compact electronic devices that combine multiple sensors to measure motion. A typical IMU contains gyroscopes, which measure rotational velocity, and accelerometers, which measure linear acceleration including gravity. These sensors are inexpensive, lightweight, and consume minimal power, making them ideal for small autonomous platforms. However, each sensor type has inherent limitations that prevent it from providing accurate attitude estimates on its own. Classical sensor fusion algorithms such as complementary filters and the Extended Kalman Filter (EKF) address this by combing gyroscope and accelerometer data to produce reliable estimates of the current tilt angles (ϕ, θ). While effective for real-time estimation these approaches are fundamentally reactive: they can estimate the current state and cannot anticipate future vehicle orientation.

However, modern advanced flight control strategies such as Model Predictive Control (MPC) require not just knowledge of the current attitude, but prediction of the future states over a finite horizon [12]. MPC computes optimal control actions by solving an optimization problem over predicted future trajectories, making the quality of the prediction critical to controller performance. This creates a need for algorithms that can provide accurate multi-step ahead attitude predictions.

Recent advances in deep learning have introduced new possibilities for time-series prediction in dynamical systems. Recurrent Neural Networks (RNNs) and in particular Long Short-Term Memory (LSTM) networks have demonstrated strong capabilities in learning temporal dependencies from sequential data [9]. In the field of UAV navigation, neural network-based approaches have been explored for attitude estimation as an alternative or complement to classical filters. Various techniques, algorithms and methods have been explored. Feedforward neural networks have been trained on IMU data and estimate orientation [10]. Also, RRN-based approaches have been used for inertial navigation [11]. However, the majority of these works focus on single-step estimation of the current state, rather than multi-step ahead prediction of future states. Furthermore, few works address the specific requirement of providing prediction horizons compatible with MPC-based flight controllers.

This thesis proposes an attention-based LSTM architecture for multi-step ahead attitude prediction that address this gap. Unlike existing neural networks approaches that estimate only the current state, the proposed method predicts the attitude angles (ϕ, θ, ψ) over a configurable future horizon of N timesteps $(k + 1\ to\ k + N)$, producing predictions directly suitable with MPC controllers. The attention mechanism enables the network to learn which timesteps within the input look-back window are most informative for the prediction task, improving prediction accuracy over standard LSTM architectures.

The subject of this thesis

This thesis addresses the challenge of predicting future attitude states of unmanned aerial vehicles, moving beyond the capabilities of classical estimation methods that can only determine the current orientation. The attitude of a UAV, described by the Euler angles (φ, θ, ψ), is essential for flight stability and control. While classical sensor fusion filters can estimate these angles in real-time from IMU measurements, they are inherently reactive and provide no information about the future evolution of the vehicle's state. Advanced control strategies such as Model Predictive Control (MPC) rely on predictions of future states over a finite horizon to compute optimal control actions, creating a fundamental need for accurate multi-step ahead attitude prediction.
To address this, an attention-based Long Short-Term Memory (LSTM) network is proposed that receives a look-back window of past sensor data and control inputs and predicts the attitude angles (φ, θ, ψ) for the next N timesteps into the future. The attention mechanism allows the network to automatically learn which past timesteps are most relevant for predicting the future, rather than relying only on the most recent observation. Training such a predictive model requires accurate attitude angle data. However, on a real UAV platform, the true attitude is not directly measurable — it must be estimated from raw inertial sensor data. For this reason, four classical tilt estimation filters are developed and optimized as part of this work: the Complementary Filter, the Mahony Passive Complementary Filter, the Explicit Complementary Filter, and the Extended Kalman Filter (EKF). These filters fuse gyroscope and accelerometer measurements to produce the attitude angle estimates that form the training dataset for the LSTM. The quality of these estimates is therefore directly linked to the quality of the LSTM predictions, making filter optimization an integral part of the overall methodology. The entire pipeline — from raw IMU processing through classical filtering to LSTM-based prediction — is implemented in C++ using the Eigen library and LibTorch (PyTorch C++ API), ensuring suitability for deployment on real-time embedded flight controllers

Aim and objectives

The primary aim of this thesis is to develop a complete estimation-to-prediction pipeline for UAV attitude, culminating in an attention-based LSTM network that provides multi-step ahead predictions suitable for integration with Model Predictive Control (MPC).
The specific objectives are:

1. Implement and optimize four classical tilt estimation filters to produce accurate attitude estimates from raw IMU data, establishing both baselines and the training dataset for the LSTM.

2. Design and train an attention-based LSTM architecture that predicts the attitude angles (φ, θ, ψ) over a future horizon of N timesteps from look-back windows of sensor and control data.

3. Evaluate prediction accuracy per step (k+1 through k+N) and per angle using RMSE and MAE metrics, quantifying how performance evolves across the prediction horizon.

4. Implement the complete pipeline in C++ using Eigen and LibTorch, demonstrating feasibility for real-time embedded deployment.

Methodology

The methodology follows two phases that form a complete estimation-to-prediction pipeline.

The first phase involves the implementation and optimization of four classical tilt estimation filters in C++ using the Eigen library. These filters process raw gyroscope and accelerometer data collected from a quadcopter platform at 50 Hz to estimate the tilt angles (φ, θ). Each filter's hyperparameters are optimized using grid search against ground truth measurements. The filters are compared using RMSE and MAE metrics, establishing that the EKF achieves the best estimation accuracy. The filter-estimated angles produced in this phase provide the attitude data required for training the LSTM model.

The second and primary phase is the development of the LSTM multi-step ahead prediction model. An attention-based LSTM network is designed to process look-back windows of K=10 timesteps containing attitude angles estimated by the filters, gyroscope measurements, and control torques from the PID controller (9 input features total), and predict the attitude angles (φ, θ, ψ) for the next N=10 timesteps. The model is trained on flight data collected from a quadcopter platform at a sampling rate of 33.3 Hz (Ts = 0.03 s). Training uses Mean Squared Error (MSE) loss with the Adam optimizer, and evaluation metrics are computed per prediction step to assess how accuracy degrades over the prediction horizon.

Innovation

The key innovation of this thesis lies in the application of an attention-based LSTM network for multi-step ahead attitude prediction of UAVs. While existing neural network approaches in the literature primarily focus on single-step attitude estimation as replacement for classical filters [10][11], this work targets a different objective: predicting future attitude states over a configurable horizon of N timesteps.

This distinction is critical because multi-step ahead predictions are perquisite for Model Predictive Control, which computes optimal control actions based on predicted future trajectories. By providing prediction at timesteps k+1 to k+N, the proposed LSTM architecture can server as the prediction model with an MPC framework enabling proactive rather than reactive flight control.

A key aspect of the proposed methodology is that the LSTM is trained entirely on filter-estimated attitude angles rather than externally provided ground truth. The classical tilt estimation filters developed in this work process raw IMU data to produce the attitude estimates that form the LSTM's training dataset. This makes the approach self-contained and directly applicable to real-world scenarios where ground truth orientation is unavailable — the same filters that generate training data offline can provide real-time input to the LSTM during deployment. The incorporation of an attention mechanism further distinguishes this work from standard LSTM-based approaches. Rather than relying solely on the final hidden state, the attention layer learns to weight the contributions of all timesteps in the look-back window, allowing the network to focus on the most informative inputs for each prediction.

Additionally, the complete pipeline is implemented entirely in C++ using LibTorch and Eigen, making it deployable on embedded systems. This is a practical requirement for real-world UAV applications that existing Python-based research implementations do not address.

Structure

Briefly outline the structure of the thesis, in chapters.

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1 CHAPTER 1: Theoretical background

This chapter establishes the theoretical foundation necessary to understand the attitude estimation problem for UAVs. An introduction is being made to the attitude estimation in UAV, what sensors are using to measure motion and explore different mathematical representations of orientations. Later fundamental concepts of sensor fusion are discussed which motivate the classic filter algorithms developed in section 2. Finally neural network architectures are introduced for sequential learning, providing the theoretical background for the deep learning approach presented in Chapter 3.

1.1 The role of attitude in UAV flight

The ability of a drones to operate autonomously has a critical dependency on their ability to determine and control their orientation in three-dimensional space.

The attitude of a UAV describes its rotation orientation relative to a fixed reference frame, typically defined with respect to the Earth's surface. In aviation, attitude is conventionally expressed using three angular coordinates known as Euler angles:

1) Roll (φ): Rotation around the longitudinal axis (forward-backward direction)
2) Pitch (θ): Rotation around the lateral axis (left-right direction)
3) Yaw (ψ): Rotation around the vertical axis (up-down direction)

These angles directly influence the vehicle's ability to maintain stable flight, execute precise maneuvers, and respond appropriately to environmental disturbances such as wind gusts. Without accurate attitude information a UAV can't, maintain level flight, execute controlled turns, provide stable platform for cameras and sensors, implement autonomous navigation algorithms.

Unlike position (which can be directly measured by GPS) or velocity (which can be derived from GPS updates), attitude can not be directly measured by a single sensor. Instead, it must be estimated by fusing measurements from multiple sensors, each with different characteristics and limitations.

1.2 IMU and Sensors

An Inertial Measurement Unit (IMU) is a compact electronic device that combines multiple motion sensors to measure a vehicle's kinematic state. A typical 6-axis IMU contains an 3-axis gyroscope and 3-axis accelerometer.

1.2.1 Gyroscope

A gyroscope is an angular rate sensor that measures how fast an object rotates around one or more axes, and in UAVs it is one of core sensors for attitude estimations. In practice gyroscopes output angular velocity (e.g. in degrees or rad per second). Angular velocity is measured as $\omega \in R^3$ (rad/s). Each axis output corresponds to the rate of rotation about the x, y, z axes of the sensor frame.

Gyroscopes have key advantages and limitations. They provide very smooth short-term attitude information and are insensitive to linear accelerations, which makes them reliable during aggressive maneuvers where accelerometers are heavily corrupted by non-gravitational forces. However, their measurements contain bias and noise, so integration causes drift that grows over time, meaning gyro-only attitude estimates must be regularly corrected using other sensors such as accelerometers, magnetometers, or GNSS within a sensor fusion algorithm. [1]

1.2.2 Accelerometer

Accelerometers are fundamentally different from what their name suggests. They do not measure kinematic acceleration. Instead they measure specific force, also called proper acceleration. It is measured as $\alpha \in R^3$ (m/s^2). Each axis output corresponds to the specific force experienced along the x, y, z axes of the sensor frame, including gravity during static conditions.

Accelerometers excel at providing long-term absolute reference through the gravity vector, allowing computation of roll and pitch via inverse trigonometry when the vehicle is mostly stationary or slow moving. Accelerometer measures $a = [ax, ay, az]$ and we can calculate roll and pitch via these equations

Equation 1 Roll from accelerometer $\boldsymbol{roll = atan2(ay, az)}$

Equation 2 Pitch from accelerometer $\boldsymbol{pitch = arsinc(ax/g)}$

1.3 Attitude Representation

Representing three-dimensional rotations mathematically is a non-trivial problem in attitude estimation. While the physical concept of orientation is intuitive, capturing it numerically requires careful consideration of mathematical properties, computational efficiency, and singularity avoidance. This section introduces three common representations used in attitude estimation algorithms, each with distinct advantages and limitations that influence their suitability for different filtering approaches.

1.3.1 Euler Angles

Euler angles are the most intuitive representation of orientation, describing attitude as a sequence of three rotations around specified axes. The aerospace convention, known as the ZYX (yaw-pitch-roll) sequence, applies rotations in the following order [2]:

1. Rotate by ψ (yaw) around the Z-axis (vertical)
2. Rotate by θ (pitch) around the new Y-axis
3. Rotate by φ (roll) around the new X-axis

It only has 3 parameters which it make’s it more human interpretable and lesser memory footprint for computer calculations.

Euler angles suffer from non-uniqueness (multiple angle combinations can represent the same physical orientation) and require complex trigonometric transformations when computing angular velocity from angle rates

The Complementary Filter (Chapter 2) operates directly on roll and pitch Euler angles, taking advantage of their simplicity for the basic weighted fusion approach. Furthermore, all filters in this thesis output their final estimates as Euler angles for interpretability and comparison with ground truth measurements. For UAV flight control applications where extreme pitch angles (near ±90°) are rare, Euler angles provide an acceptable representation.

1.3.2 Rotation Matrices (SO(3))

A rotation matrix R∈SO(3) is a 3×3 orthonormal matrix that transforms vectors from one coordinate frame to another. The notation SO(3) stands for "Special Orthogonal group in 3 dimensions," representing all possible rotations in three-dimensional space.

A rotation matrix can be constructed from Euler angles using the product of elementary rotation matrices [3]

Equation 3 Rotation matrix to euler angles: $\boldsymbol{R(\varphi,\theta,\psi) = Rz(\psi)\cdot\ Ry(\theta)\cdot Rx(\varphi)}$

Where Rx, Ry, Rz represent rotations around the X, Y, and Z axes respectively. The resulting 3×3 matrix contains nine elements, though only three are independent due to orthonormality constraints.

One drawback is redundancy: storing nine numbers to represent three degrees of freedom is inefficient. Also, they require periodic orthonormalization some common methods are Gram-Schmidt or singular value decomposition (SVD)[4].

The Mahony Passive Complementary Filter (Chapter 2) operates directly on rotation matrices, leveraging the geometric properties of SO(3) for error correction. The filter computes a cross-product-based correction term that exploits the manifold structure of rotation matrices, providing superior performance compared to Euler angle approaches. To maintain orthonormality, the Mahony filter implementation applies Gram-Schmidt orthonormalization after each prediction step, ensuring the rotation matrix remains valid throughout the flight.

1.3.3 Quaternions

A quaternion q is a four-parameter representation of rotation, consisting of one scalar and one vector component:

Equation 4 Quaternion $\boldsymbol{q = [q_0, q_1, q_2, q_3]^T = [q_0, q_v]^T}$

Where:
q_0 : Scalar part
$q_v = [q_1, q_2, q_3]^T$: Vector part

Like rotation matrices, quaternions require normalization after numerical integration to maintain the unit constraint. However, this is computationally cheaper than matrix orthonormalization (single division versus full matrix projection).

Extended Kalman Filter (EKF) uses quaternions for the state representation, witch a 7-dimensional state vector that includes the quaternion and gyroscope bias estimates.

1.3.4 Sensor Fusion Fundamentals

Sensor fusion combines measurements from multiple sensors to produce estimates superior to any single sensor. For attitude estimation, gyroscopes and accelerometers exhibit complementary error characteristics: gyroscopes provide smooth, high-frequency measurements but suffer from integration drift, while accelerometers offer a long-term gravity reference but are noisy and corrupted by linear acceleration.

The fundamental challenge is designing algorithms that:

1. Trust gyroscope measurements for short-term dynamics (responsiveness)
2. Use accelerometer measurements for long-term drift correction (stability)
3. Reject accelerometer corruption during vehicle acceleration

Fusion strategies range from simple weighted averaging (complementary filters) to nonlinear filters operating on manifolds (Mahony filter on SO(3)) to probabilistic state estimation (Extended Kalman Filter). Performance is evaluated using Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) against ground truth measurements.

1.4 Neural Networks for Time-Series Predictions

While classical filtering approaches rely on explicit mathematical models of sensor behavior and system dynamics, modern deep learning techniques offer an alternative paradigm: learning patterns directly from data. This section introduces the theoretical foundations of neural network architectures designed for sequential data, which form the basis for the LSTM-based attitude predictor developed in Chapter 3.

1.4.1 Time-Series Data and Sequential Learning

Attitude estimation is fundamentally a time-series problem. IMU measurements arrive sequentially at fixed sampling intervals and the vehicle's current state depends on its past state. Unlike static classification tasks (e.g., image recognition), attitude estimation requires the model to understand temporal dependencies, how the current orientation relates to previous orientations and control inputs.

Traditional feedforward neural networks (e.g., multilayer perceptrons) process each timestep independently, ignoring temporal structure. They lack memory of previous inputs, making them unsuitable for sequential prediction. Recurrent architectures address this limitation by maintaining internal state across timesteps.

1.4.2 Recurrent Neural Networks (RNN)

A Recurrent Neural Network (RNN) extends feedforward network with recurrent connections that allow information to persist across time. At each timestep t, the RNN process input x(t) while maintain a hidden state h(t) that summarizes information from all previous timesteps [5]. The main advantage of this approach is that the calculation is very efficient and the state information may be stored infinitely in theory.

Despite their theoretical elegance, RNNs suffer from the vanishing gradient problem during training. When learning long-term dependencies (e.g., patterns spanning dozens of timesteps), gradients propagated backward through time become exponentially small, preventing the network from learning correlations between distant events [6]. For attitude estimation requiring lookback windows of 10+ timesteps (0.3+ seconds), this limitation is critical.

1.4.3 Long Short-Term Memory (LSTM)

Long Short-Term Memory networks, introduced by Hochreiter and Schmidhuber (1997) [7], solve the vanishing gradient problem through a sophisticated gating mechanism that controls information flow across time. LSTMs replace the simple hidden state of RNNs with two components:

1) Cell state(c): Carris information across long time intervals with minimal modification
2) Hidden state(h): A filtered version of the cell state used for output prediction

For IMU-based attitude estimation, LSTMs offer several advantages over classical filters.

- Automatic feature learning: No manual tuning of filter gains or covariance matrices
- Long-term pattern recognition: Can remember sensor characteristics, bias patterns, and motion dynamics over extended periods
- Adaptive sensor fusion: Gates automatically learn when to trust accelerometer measurements (during low dynamics) versus gyroscope integration (during aggressive maneuvers)
- Multi-step prediction capability: Can forecast future attitudes based on learned temporal patterns in sensor and control data

This thesis implements an LSTM-based multi-step ahead predictor that forecasts UAV attitude angles (roll, pitch, yaw) over a 10-timestep prediction horizon (0.30 seconds at 33.3 Hz sampling rate). The network processes a lookback window of past sensor measurements, ground truth angles, and control inputs to predict future vehicle orientation. This predictive capability enables proactive flight control and disturbance rejection strategies that anticipate vehicle motion rather than merely reacting to current measurements.

2 CHAPTER 2: Tilt Estimation Filters

This chapter presents the implementation, optimization, and comparative analysis of four classical sensor fusion algorithms for UAV attitude estimation. Building upon the theoretical foundations established in Chapter 1, we now examine how different mathematical approaches translate into practical filtering solutions, each offering distinct trade-offs between computational complexity, estimation accuracy, and robustness to sensor noise.

The four algorithms implemented in this work represent a progression of increasing mathematical sophistication. The Complementary Filter provides a baseline approach through simple weighted fusion of gyroscope and accelerometer measurements in Euler angle space. The Mahony Passive Complementary Filter advances this concept by operating on the SO(3) manifold using rotation matrices, incorporating geometric error correction through proportional feedback. The Explicit Complementary Filter extends the Mahony approach by adding integral feedback for gyroscope bias estimation, providing adaptive correction for sensor drift. Finally, the Extended Kalman Filter (EKF) implements a full probabilistic state estimation framework using quaternion representation, offering optimal estimation under the assumptions of Gaussian noise and linear measurement models.

All implementations are developed in C++ using the Eigen library for efficient matrix operations, with careful attention to computational efficiency suitable for real-time embedded deployment. Each filter processes 3-axis gyroscope and accelerometer measurements from a 6-DOF IMU operating at 50 Hz sampling rate. The algorithms are validated against ground truth measurements from a dataset containing 1409 samples collected during quadcopter flight operations, covering a range of dynamic conditions from hover to aggressive maneuvers.

For each filter, this chapter presents the mathematical formulation, implementation details, hyperparameter tuning methodology, and performance evaluation using Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) metrics for both roll and pitch estimation. The analysis investigates how each algorithm handles the fundamental sensor fusion challenge: balancing short-term gyroscope accuracy against long-term accelerometer stability while rejecting accelerometer corruption during dynamic flight.

2.1 Complementary Filtering

The Complementary Filter is one of the most fundamental and widely used sensor fusion algorithms for attitude estimation in low-cost UAV applications due to its computational simplicity and effectiveness. It addresses the inherent limitations of individual inertial sensors by fusing their measurements in the frequency domain [8].

The algorithm relies on the observation that gyroscope and accelerometer errors exhibit complementary spectral characteristics. Gyroscopes provide precise, low-noise measurements of angular velocity that are reliable in the high-frequency domain (short term) but suffer from low-frequency drift due to the integration of bias and noise over time. Conversely, accelerometers provide a stable absolute reference to the gravity vector that is reliable in the low-frequency domain (long term) but are subject to high-frequency noise and corruption from external linear accelerations (non-gravitational forces) during dynamic maneuvers.

The discrete-time implementation for the roll angle p is given by:

$$\varphi(t) = \alpha \cdot \left(\phi(t-1) + \omega_{gyro} \cdot dt\right) + (1-\alpha) \cdot \phi_{accel}$$

2.1.1 Experimental results

The Complementary Filter was implemented with a gain coefficient $a = 0.79$ and tested against the ground truth dataset. The results for the Roll and Pitch estimation are presented below.

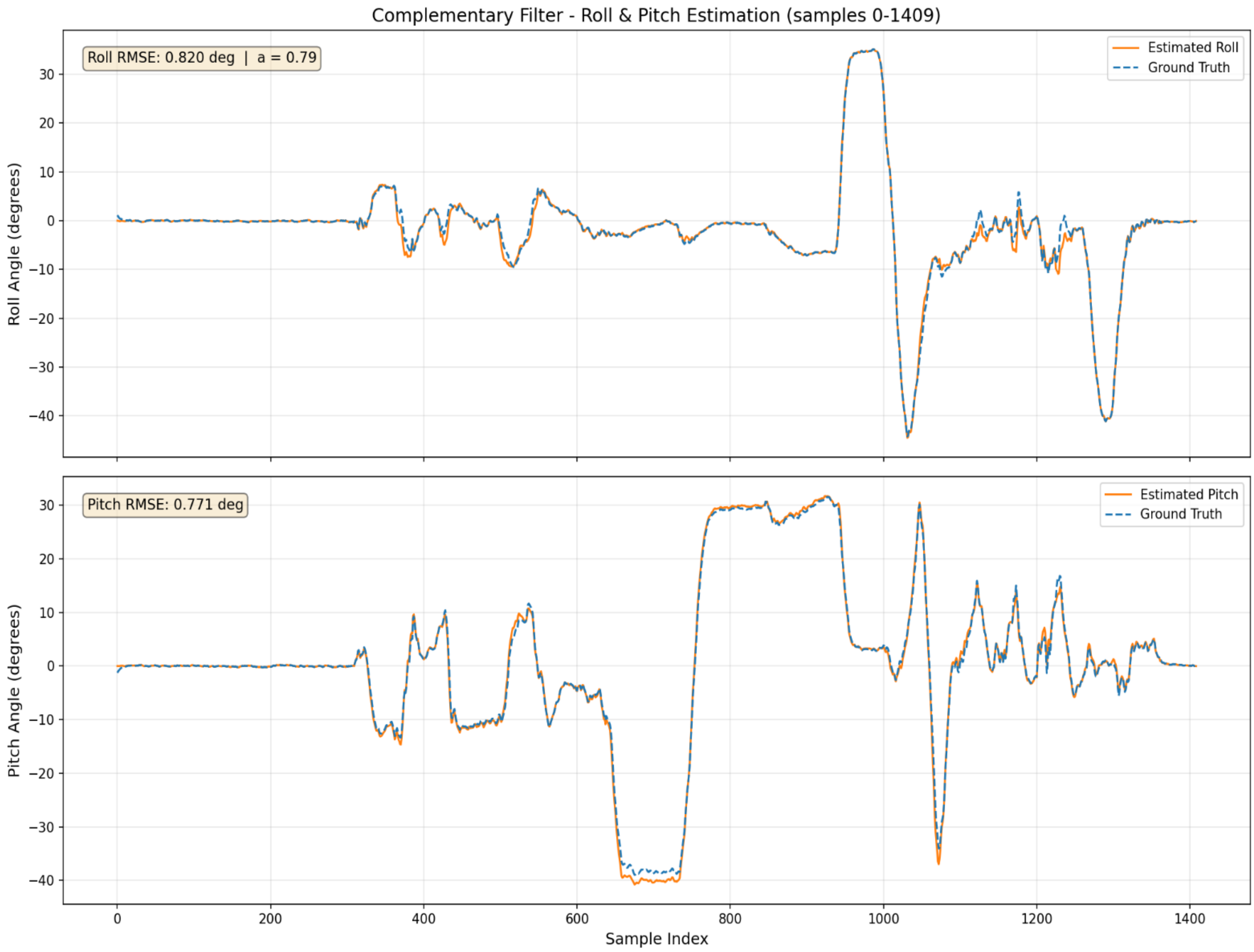

Figure 1 Complementary Filter Roll and Pitch Estimation

Figure 1 represent the global performance of the Complementary Filter over the entire flight duration (1409 samples). The estimated Roll and Pitch angles show a strong correlation with the ground ruth data. The low RMSE values 0.820° for Roll and 0.771° for Pitch confirm that the filter successfully understand high-frequence gyroscope data with the low-frequency accelerometer data.

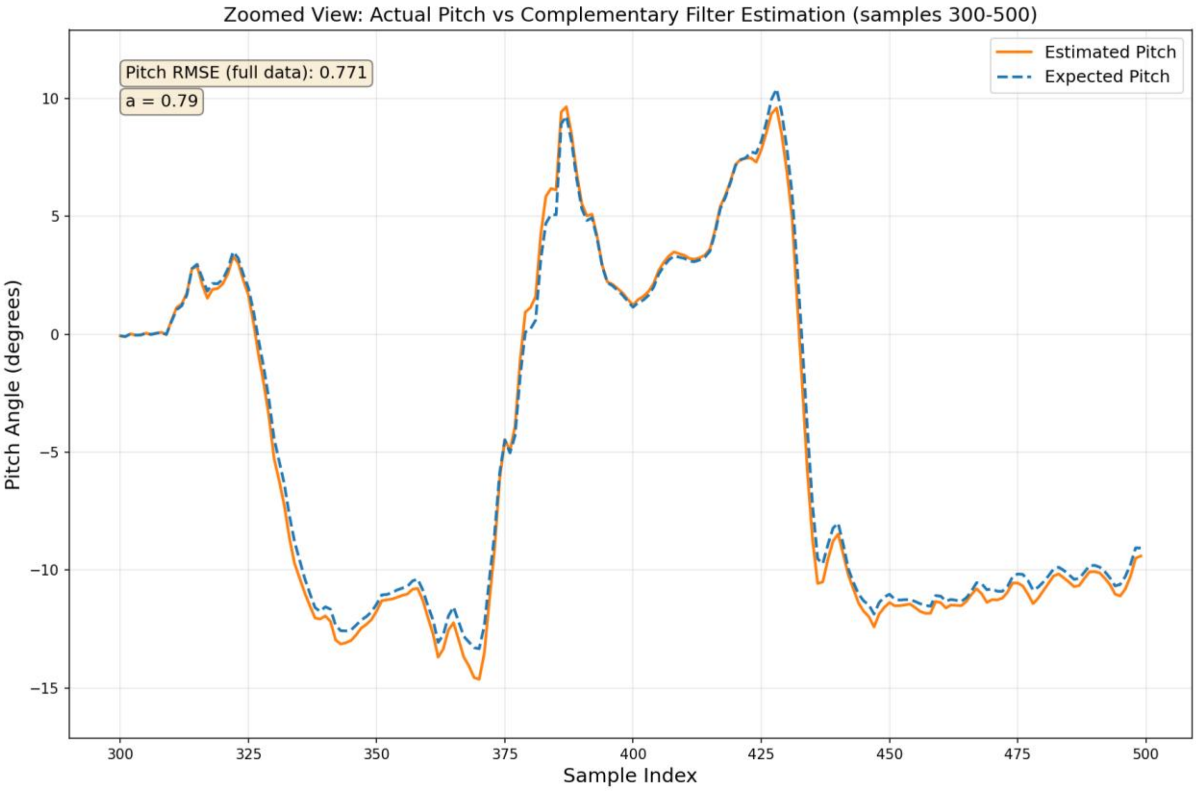

Figure 2 Zoomed view (samples 300-500) Pitch Estimation

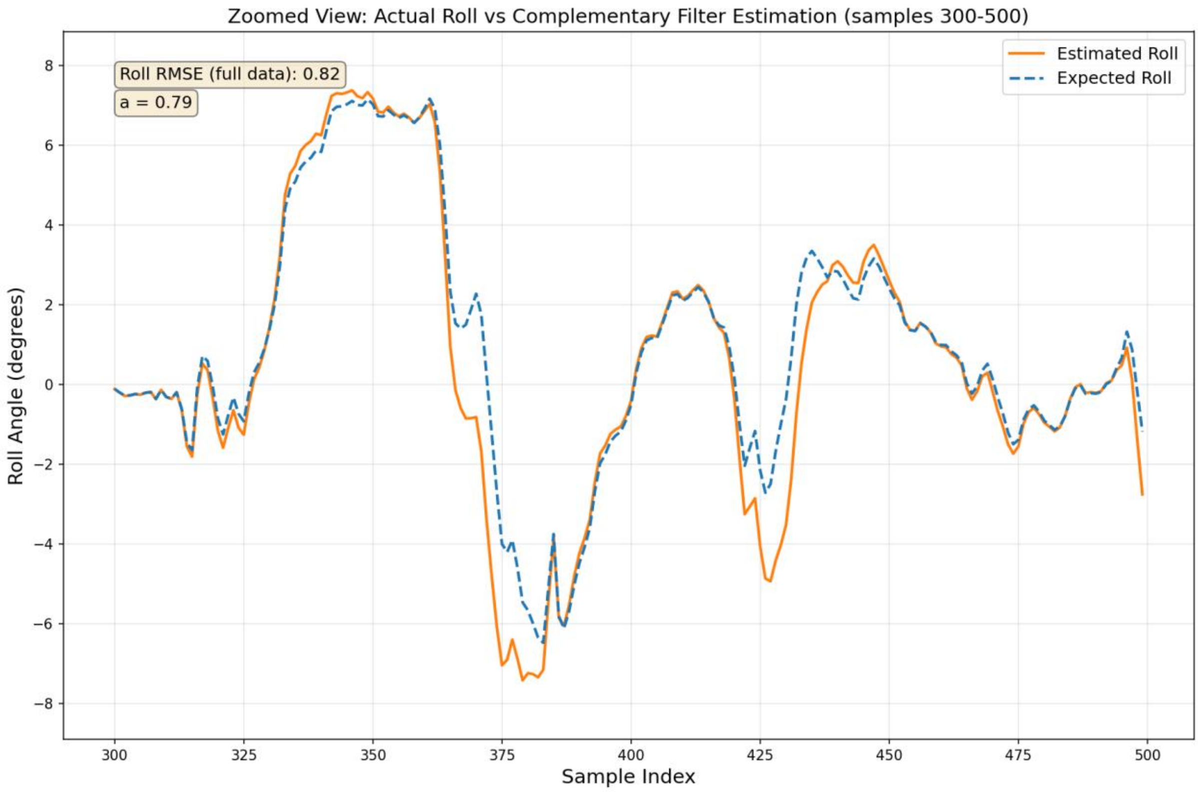

Figure 3 Zoomed view (samples 300-500) Roll Estimation

Figure 2 provides a more detailed view of the Pitch estimation during a dynamic maneuvering phase (samples 300-500). It can be observed that the Pitch still follows close enough the truth data. Figure 3 illustrates the Roll estimation for the same interval. Generally it performs almost as well as Pitch with a minor exception at around sample 375 which is a little bit off.

In conclusion, the Complementary Filter proves to be a robust and computationally efficient solution for general UAV attitude estimation, It successfully mitigates gyroscope drift and provides a smooth estimate suitable for flight control. The biggest drawback it’s the reliance on a fixed α gain means it cannot adapt to various flight conditions that are encountered often in the real world.

2.2 Mahony Filtering

The Mahony filter is a nonlinear complementary filter that operates directly on the Special Orthogonal group $SO(3)$, representing orientation using rotation matrices [3]. Unlike the standard Complementary Filter which operates on Euler angles, the Mahony filter avoids singularities (gimbal lock) and provides a more geometric approach to error correction.

The core principle of the Mahony filter is to estimate the rotation matrix $\hat{R}$ by comparing the measured direction of gravity (from the accelerometer) with the estimated direction of gravity.

The error between these two vectors is used to generate a correction term that "steers" the gyroscope integration towards the true vertical.

The implementation done here is a "Passive" Mahony filter as described in [3].

2.2.1 Sub-sub-title

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2.3 Explicit Complementary Filtering

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2.4 Extended Kalman Filtering (EKF)

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3 CHAPTER 3: Deep learning approach

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Conclusions

Future Work

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Appendix A

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Appendix B

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Appendix C

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