Dec 10

Ch.4: Tests of Hypothesis

Recall

[1] Tests Concerning Population mean

σ^2 is known

Use Z-test (Z_c)

σ^2 is unknown, $n \geq 30$

Z-test (Z_c), replace $\sigma \to S$

σ^2 is unknown, $n < 30$

Use T-test (T_c)

reject H_0 when Z_c/T_c falls in the re region (specified by critical points depend on α)

or when p-value $\leq \alpha$

Do not reject H_0 when

Z_c/T_c outside the rejection region

or

P-value $> \alpha$

2 Tests Concerning two population mean μ_1, μ_2

σ_1^2, σ_2^2 are known

use: Z-test (Z_c)

σ_1^2, σ_2^2 are unknown

$n_1 \geq 30, n_2 \geq 30$

Z-test

$\sigma_1 \to s_1, \sigma_2 \to s_2$

σ_1^2, σ_2^2 are unknown

$n_1, n_2 < 30$

T-test

Assume

$\sigma_1^2 = \sigma_2^2 = \sigma^2$

(we need here test of variances) (Dec10)

, estimate σ^2 by S_p^2

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3 Paired Comparisons

σ_1, σ_2 are unknown , $n < 30$

Assume that you have one random sample, for each object of sample you take two observations one before treatment and one after treatment

by examining the differences of all pairs of measurements we hope to draw a conclusion about the effectiveness of the treatment

Observations are taken in pairs :

$(X_1, Y_1), (X_2, Y_2), (X_3, Y_3), \dots (X_n, Y_n)$ is

on one object of sample

One random sample with two observations (before, after) treatment

each pair being observed under the same experimental conditions

let $d_1 = X_1 - Y_1$, $d_2 = X_2 - Y_2$, -- $d_n = X_n - Y_n$

Since $X_1, X_2, \dots X_n \sim N(\mu_1, \sigma_1^2)$

$Y_1, Y_2, \dots Y_n \sim N(\mu_2, \sigma_2^2)$

$d_1, d_2, \dots d_n \sim N(\mu_d, \sigma_d^2)$

$\mu_d = \mu_1 - \mu_2$

$H_0 : \mu_1 - \mu_2 = 0$ vs H_a $\mu_d < 0$ one sided

$\mu_d = 0$ No effect

$\mu_d > 0$

$\mu_d \neq 0$ two-sided

Usually , $n < 30$ and σ_1, σ_2 are unknow

Use - T-test

$$T_{c,n-1} = \frac{\bar{d} - d_0}{\frac{S_d}{\sqrt{n}}}$$

hyp difference (usually $= 0$)

$\boxed{\text{if one sample } T_c = \frac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}}}$

If $T_{c,n-1}$ falls in rejection region

$\Rightarrow$ reject H_0

otherwise Do not reject H_0

Ex (4.5) Nine adults agreed to test the efficiency of a new diet program to decrease the weight. Their weights (per pounds) were measured before and after the program and found to be as

Adult	1	2	3	4	5	6	7	8	9	
Before	132	139	126	114	122	132	142	119	126	x_i
After	124	141	118	116	114	132	145	123	121	y_i

Test the efficiency of this program at level of significance $\alpha = 0.01$

$H_0: \mu_1 = \mu_2$ vs. $H_a: \mu_1 > \mu_2$

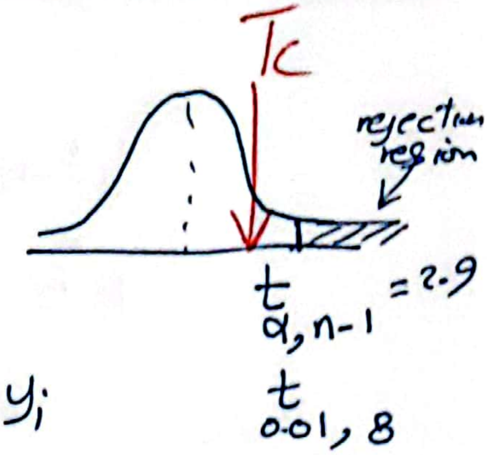
$\mu_1 - \mu_2 = 0$ $\qquad$ $\mu_1 - \mu_2 > 0$

diet is not effective $\qquad$ diet is effective

[one sided test]

$\alpha = 0.01$

$d.o.f = n - 1 = 8$



1- get critical point $t_{0.01, 8} = 2.9$

2- measure differences $d_i = x_i - y_i$

i	1	2	3	4	5	6	7	8	9
di	8	-2	8	-2	8	0	-3	-4	5

3- from di

Compute $\bar{d} = \dfrac{\sum_{i=1}^{9} d_i}{9} = \dfrac{18}{9} = 2$

$$S_d^2 = \frac{\sum_{i=1}^{9} (d_i - \bar{d})^2}{8} = 26.75$$

$S_d = 5.17$ [by Calculator: statistics mode]

hyp difference $\mu_1 - \mu_2 = 0$

Calculated value $T_{c,8} = \dfrac{\bar{d} - d_0}{\frac{S_d}{\sqrt{n}}} = \dfrac{2 - 0}{\frac{5.17}{\sqrt{9}}} = 1.16$

Since $T_c < t_{\alpha, n-1}$ (outside the rejection region)

Do not reject H_0

$\Rightarrow$ this diet is not effective with 99% level of Confidence by Minitab

Stat / Basic statistics / Paired t /

$$P\text{-value} = 0.14 > \overset{\alpha}{0.01}$$

Do not reject H_0 ✓✓

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[4] Tests for the equality of Two Variances

very useful before solving ex 4.4

[When two populations σ_1, σ_2 are unknown $, n_1, n_2 < 30$]

[To assume $\sigma_1^2 = \sigma_2^2 = \sigma^2 \Rightarrow$ you must test this hyp first]

$$H_0: \sigma_1^2 = \sigma_2^2 \quad \text{vs} \quad H_a: \sigma_1^2 \neq \sigma_2^2$$

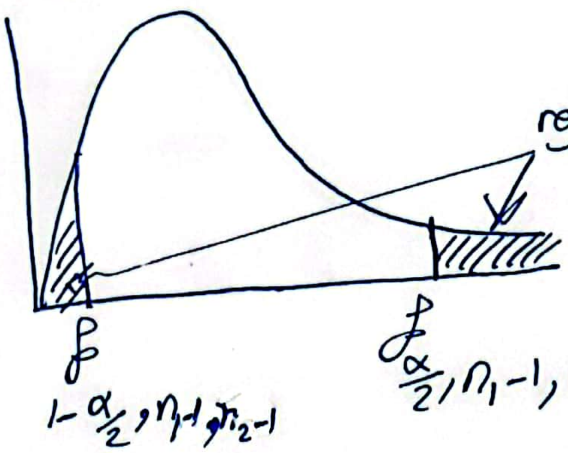
[two sided test]

From α level of sign.

two critical points

$f_{\frac{\alpha}{2}, n_1-1, n_2-1}$

$f_{1-\frac{\alpha}{2}, n_1-1, n_2-1}$



$\boxed{\phi(f_{\frac{\alpha}{2}}) = 1 - \frac{\alpha}{2}}$

Remmember $f_{1-\frac{\alpha}{2}, n_1-1, n_2-1} = \dfrac{1}{f_{\frac{\alpha}{2}, n_2-1, n_1-1}}$

F - Test

$$F_c = \frac{S_1^2}{S_2^2}$$

Calculated F value

If F_c falls in rejection region reject H_o

If F_c falls outside rejection region do not reject H_o

or if P-value $< \alpha$ reject H_o
P-value $> \alpha$ do not reject H_o

Ex (4.7) Test the hypothesis that the variances of two populations in Ex (4.4) are equal, use $\alpha = 0.02$

$n_1 = 9$, $S_1 = 3.27 \Rightarrow$
$n_2 = 11$, $S_2 = 2.68 \Rightarrow$

$H_o: \sigma_1^2 = \sigma_2^2$ vs $H_a: \sigma_1^2 \neq \sigma_2^2$

two critical points

$f_{0.01,8,10} = 5.05$

$F_{0.99,8,10} = \frac{1}{F_{0.01,10,8}} = 0.172$

$$F_c = \frac{S_1^2}{S_2^2} = \frac{(3.27)^2}{(2.68)^2} = 1.49$$

Since $0.172 < F_c < 5.05$

do not reject Ho

$\sigma_1^2 = \sigma_2^2$ with 98% level of Conf.

<u>For $\alpha = 0.05$ (as Ex 4.4)</u>

Two critical Points

$$F_{0.025, 8, 10} = 3.854$$

$$F_{0.975, 8, 10} = \frac{1}{F_{0.025, 10, 8}} = 0.232$$

Since $0.232 < F_c < 3.85$

do not reject Ho $\therefore \sigma_1^2 = \sigma_2^2$

with 95% level of Conf.

$\therefore$ there is insufficient evidence that the variances differ

Regards

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