EXERCISES

[1] Certain tubes manufactured by a company have a mean lifetime of 800 hours and a standard deviation of 40 hours. Find the probability that a random sample of 36 tubes taken from the group will have a mean lifetime.

a- Between 790 and 810 hours, **b-** More than 815 hours.

[2] A and B manufacture two types of cables, having mean breaking strengths of 4000 and 4500 pounds and standard deviations of 300 and 200 pounds respectively. If 100 cables of brand A and 50 cables of brand B are tested, what is the probability that the mean breaking strength of B will be

a- At least 600 pounds more than A, **b-** At least 450 pounds more than A.

[3] Find **a-** $P(-t_{0.005} < t < t_{0.01})$ **b-** Find $P(t > -t_{0.025})$.

[4] Given a random sample of size 24 from a normal distribution, find, K such that

a- $P(-2.069 < t < K) = 0.965$ **b-** $P(K < t < 2.807) = 0.095$.

c- $P(-K < t < K) = 0.90$.

[5] Consider the four independent random variables X, Y, U and V such that $X \sim N(0,16)$, $Y \sim N(5,4)$, $U \sim \chi^2(4)$ and $V \sim \chi^2(16)$.

State the distribution of each of the following variables

a- $\frac{X^2}{16} + \frac{(Y-5)^2}{4}$ **b-** $\frac{X}{\sqrt{V}}$ **c-** $\frac{4U}{V}$ **d-** $X+2Y$ **e-** $2X-Y$

[6] If $X_1, X_2, \ldots, X_n$ are i.i.d. $N(0,\sigma^2)$, state the distribution of each of the following variables:

a- $U = 3X_1 - 5X_2 + 8$ **b-** $V = \sum_{i=1}^{n} X_i$

c- $W = \left(\sum_{i=1}^{n} X_i\right)^2 \Big/ n\sigma^2$ **d-** $Y = \frac{2X_1^2}{X_2^2 + X_3^2}$ **e-** $Y = \frac{\sum X_i}{\sqrt{\sum X_i^2}}$

[7] If $X_1, X_2, \ldots, X_n$ are i.i.d. $N(0,\sigma^2)$, state the distribution of each of the following variables:

a- $Y = 5X_1 - 7X_2 + 2$ **b-** $Y = \frac{2X_1^2}{X_2^2 + X_3^2}$ **c-** $Y = \frac{\sum X_i}{\sqrt{\sum X_i^2}}$

[8] Suppose that X_1, X_2, X_3 and X_4 are i.i.d. $N(0,\sigma^2)$, then the distribution of the random

variable $\mathbf{Y = \frac{X_1 + X_2}{\sqrt{X_3^2 + X_4^2}}}$ **is**

a. χ^2 **(2)** **b.** t(2) **c.** F(2,2) **d.** None of the above.

[9] Consider the three independent random variables X, U and V such that $\mathbf{X \sim N(0,1)}$, $\mathbf{U \sim \chi^2(4)}$ and $\mathbf{V \sim \chi^2(16)}$. Find the distribution of $\mathbf{W = X^2 + U + V}$.

[10] Let $\overline{\mathbf{X}}$ and $\overline{\mathbf{Y}}$ be sample means of two independent random samples of sizes 10 and 20 from N(4,9) and N(5,16) respectively. Find mean, variance and distribution of $\mathbf{Z = \overline{X} - 2\overline{Y} + 3}$.

[11] Show that if **X** has a t distribution with v d.f., then $\mathbf{Y = X^2}$ has an **F** distribution with $\mathbf{\nu_1 = 1}$ and $\mathbf{\nu_2 = \nu}$ d.f.

[12] Circle the best answer from each of the following multiple-choice questions:
Let X ~ N(1,16), Y ~ N(0,4) and U ~ χ^2 (15) be three independent r.v's.

a- The distribution of 2X-3Y+5 is

i. N(7,28) ii. N(7,100) iii. N(2,105) iv. None of the above.

b- One of the following r.v.'s has F(16,1)

i. $\mathbf{\frac{U + Y^2/4}{(X-1)^2/16}}$ ii. $\mathbf{\frac{U + Y^2/4}{(X-1)^2}}$ iii. $\mathbf{\frac{(U + Y^2/4)/16}{(X-1)^2}}$ iv. None of the above.

c- The distribution of $\mathbf{\frac{X-1}{\sqrt{Z^2 + U}}}$ is

i. t(3) **ii.** t(15) **iii.** t(16) iv. None of the above.

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