Formula Sheet for
Probability and Statistical (II)

Chapter 1

If $X_1, X_2, \dots, X_n$ constitute a R.S. from a $N(\mu, \sigma^2)$, then

(1) $Z = \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1).$

(2) $T = \dfrac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$

(3) $U = \dfrac{(n-1)S^2}{\sigma^2} = \cancel{\sum_{i=1}\left(\frac{X_i-\mu}{\sigma}\right)^2} \sim \chi^2_{n-1}$

(4) $F = \dfrac{U_1/(n_1-1)}{U_2/(n_2-1)} = \dfrac{S_1^2\,\sigma_2^2}{S_2^2\,\sigma_1^2} \sim F_{n_1-1,n_2-1}$

Chapter 2

Point Estimation

(1) $\mu_k = E(X^k)$, $m_k = \dfrac{1}{n}\sum_{i=1}^{n} X_i^k$, $k = 1, 2, \dots, r$

(2) $MSE(\hat{\theta}) = E(\hat{\theta}-\theta)^2$

Special Probability Distributions

A- Discrete Distributions

Name of Disn	*PMF*	*parameters*	*Mean*	*Variance*
Bernoulli	$f(x,p) = p^x q^{1-x}$, $x = 0, 1$	p	p	pq
Binomial	$f(x,n,p) = \binom{n}{x} p^x q^{n-x}$, $x = 0, 1, 2, \dots, n$	n, p	np	npq
Poisson	$f(x;\lambda) = \dfrac{e^{-\lambda}\lambda^x}{x!}$ for $x = 0, 1, 2, \dots$	λ	λ	λ
Geometric	$f(x;p) = p\,q^{x-1}$ for $x = 1, 2, 3, \dots$	p	$1/p$	q/p^2
-ve Binomial	$f(x;k,p) = \binom{x-1}{k-1} p^k q^{x-k}$, $x = k, k+1, k+2, \dots$	k, p	$\dfrac{k}{p}$	$\dfrac{k\,q}{p^2}$

B- Discrete Distributions

Name of Disn	*PDF*	*parameters*	*Mean*	*Variance*
Uniform	$f(x) = \dfrac{1}{\beta-\alpha}$ for $\alpha < x < \beta$	α, β	$(\alpha+\beta)/2$	$(\beta-\alpha)/12$
Exponential	$f(x;\theta) = \dfrac{1}{\theta} e^{-x/\theta}$ for $x > 0$	θ	θ	θ^2
Gamma	$f(x;\theta) = \dfrac{1}{\theta^n \Gamma(n)} x^{n-1} e^{-x/\theta}$ for $x > 0$ if n is +ve integer, $\Gamma(n) = (n-1)!$	θ	$n\theta$	$n\theta^2$
Normal	$f(x,\mu,\sigma) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$	μ, σ^2	μ	σ^2

Chapter 3

Summary of Confidence Interval

Parameter	Point Estimate	100(1- α) % CI
μ σ^2 known	$\bar{x}$	$\bar{x} - z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}}$ $E = \dfrac{z_{\alpha/2}\,\sigma}{\sqrt{n}}$ $\quad n = \left[\dfrac{z_{\alpha/2}\,\sigma}{E}\right]^2$, $\sigma \sim R/4$
μ σ^2 unknown	$\bar{x}$	$\bar{x} - t_{\frac{\alpha}{2}}\dfrac{S}{\sqrt{n}} \le \mu \le \bar{x} + t_{\frac{\alpha}{2}}\dfrac{S}{\sqrt{n}}$
$\mu_1 - \mu_2$ σ_1 & σ_2 known	$\bar{X}_1 - \bar{X}_2$	$\left((\bar{X}_1 - \bar{X}_2) - z_{\alpha/2}\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}},\ (\bar{X}_1 - \bar{X}_2) + z_{\alpha/2}\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}\right)$
$\mu_1 - \mu_2$ $\sigma_1 = \sigma_2 = \sigma$ but unknown	$\bar{X}_1 - \bar{X}_2$	$\left((\bar{x}_1 - \bar{x}_2) - t_{\alpha/2}S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}},\ (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2}S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}\right)$
p	$\hat{p} = \dfrac{x}{n}$	$\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$, $E = z_{\alpha/2}\sqrt{\dfrac{pq}{n}}$, $n = \dfrac{Z_{\alpha/2}^2\,pq}{E^2}$

Chapter 4

Summary of Hypothesis Testing on Means and Variances

H_0	Test Statistic	H_a	Critical Region
$\mu = \mu_0$ σ^2 known	$Z_c = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$	$\mu < \mu_o$ $\mu > \mu_o$ $\mu \ne \mu_o$	$Z_c < -z_\alpha$ $Z_c > z_\alpha$ $\lvert Z_c \rvert > z_{\alpha/2}$
$\mu = \mu_0$ σ^2 unknown	$T_c = \dfrac{\bar{x} - \mu_0}{S/\sqrt{n}}$, $\nu = n-1$	$\mu < \mu_o$ $\mu > \mu_o$ $\mu \ne \mu_o$	$T_c < -t_\alpha$ $T_c > t_\alpha$ $\lvert T_c \rvert > t_{\alpha/2}$
$\mu_1 - \mu_2 = d_o$ σ_1 & σ_2 known	$Z = \dfrac{(\bar{X}_1 - \bar{X}_2) - d_o}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	$\mu_1 - \mu_2 < d_o$ $\mu_1 - \mu_2 > d_o$ $\mu_1 - \mu_2 \ne d_o$	$Z_c < -z_\alpha$ $Z_c > z_\alpha$ $\lvert Z_c \rvert > z_{\alpha/2}$
$\mu_1 - \mu_2 = d_o$ $\sigma_1 = \sigma_2 = \sigma$ but unknown	$T_c = \dfrac{(\bar{X}_1 - \bar{X}_2) - d_o}{S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$, $\nu = n_1 + n_2 - 2$ $S_p^2 = \dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$	$\mu_1 - \mu_2 < d_o$ $\mu_1 - \mu_2 > d_o$ $\mu_1 - \mu_2 \ne d_o$	$T_c < -t_\alpha$ $T_c > t_\alpha$ $\lvert T_c \rvert > t_{\alpha/2}$
$\mu_1 - \mu_2 = d_o$ Paired Comparisons	$T_c = \dfrac{\bar{d} - d_0}{S_d/\sqrt{n}}$, $\nu = n-1$	$\mu_d < d_o$ $\mu_d > d_o$ $\mu_d \ne d_o$	$T_c < -t_\alpha$ $T_c > t_\alpha$ $\lvert T_c \rvert > t_{\alpha/2}$
$\sigma_1^2 = \sigma_2^2$	$F_c = S_1^2/S_2^2$ $\nu_1 = n_1 - 1$, $\nu_2 = n_2 - 1$	$\sigma_1^2 \ne \sigma_2^2$	$F_c > F_{\alpha/2, n_1-1, n_2-1}$ or $F_c < F_{1-\alpha/2, n_1-1, n_2-1}$

Chapter 5

ANOVA

$$\hat{\sigma}^2 = MSE,\ \hat{\alpha}_i = \bar{x}_{i.} - \bar{x}_{..}\ ,\ \sum_{i=1}^{k}\hat{\alpha}_i = 0 \quad \text{and} \quad \hat{\beta}_j = \bar{x}_{.j} - \bar{x}_{..}\ ,\ \sum_{j=1}^{n}\hat{\beta}_j = 0$$