Sheet 5 Solution

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Question 1

$$J = \cos(u)$$
$$u = u_1 + u_2 = \sin(x^2) + 3x^2$$
$$u_1 = \sin(t), u_2 = 3t$$
$$t = x^2$$

$$\begin{aligned}
\frac{\delta t}{\delta x} &= 2x \\
\frac{\delta u_2}{\delta t} &= 3 \\
\frac{\delta u_1}{\delta t} &= \sin(t) \\
\frac{\delta u}{\delta t} &= \frac{\delta u_1}{\delta t} + \frac{\delta u_2}{\delta t} \\
\frac{\delta t}{\delta x} &= 2x \\
\frac{\delta J}{\delta x} &= -\sin(u)\frac{\delta u}{\delta t} \cdot \frac{\delta t}{\delta x} \\
&= -\sin(u)(\frac{\delta u_1}{\delta t} \cdot \frac{\delta t}{\delta x} + \frac{\delta u_2}{\delta t} \cdot \frac{\delta t}{\delta x}) \\
&= -\sin(\sin(x^2) + 3x^2)(\sin(x^2) \cdot 2x + 3 \cdot 2x) \\
&= -\sin(\sin(x^2) + 3x^2)(2x\cos(x^2) + 6x)
\end{aligned}$$

Question 2

$$J = x^2 + y^2 + z^2$$
$$x = u + v$$
$$y = u - v$$
$$z = u^2 + v^2$$

Forward Path

$$\begin{aligned} J &= (u+v)^2 + (u-v)^2 + (u^2+v^2)^2 \\ &= u^4 + v^4 + 2u^2 + 2v^2 + 2u^2v^2 \end{aligned}$$

Back-propagation

$$\begin{aligned} \frac{\delta x}{\delta u} &= 1 \\ \frac{\delta y}{\delta u} &= 1 \\ \frac{\delta z}{\delta u} &= 2u \\ \frac{\delta J}{\delta u} &= 2x\frac{\delta x}{\delta u} + 2y\frac{\delta y}{\delta u} + 2z\frac{\delta z}{\delta u} \\ &= 2(u+v)(1) + 2(u-v)(1) + 2(u^2+v^2)(2u) \\ &= 4u^3 + 4uv^2 + 4u^3 \end{aligned}$$

$$\begin{aligned} \frac{\delta x}{\delta v} &= 1 \\ \frac{\delta y}{\delta v} &= -1 \\ \frac{\delta z}{\delta v} &= 2v \\ \frac{\delta J}{\delta v} &= 2x\frac{\delta x}{\delta v} + 2y\frac{\delta y}{\delta v} + 2z\frac{\delta z}{\delta v} \\ &= 2(u+v)(1) + 2(u-v)(-1) + 2(u^2+v^2)(2v) \\ &= 4v^3 + 4vu^2 + 4v \end{aligned}$$

Question 3

$$\begin{aligned} f(x,y) &= x^3 - y^2 + 3xy^2 \\ \frac{\delta x}{\delta u} &= (2x)^{-1} \\ \frac{\delta y}{\delta u} &= -(2y)^{-1} \\ \frac{\delta f}{\delta u} &= 3x^2 \cdot \frac{\delta x}{\delta u} - 2y\frac{\delta y}{\delta u} + 3(\frac{\delta x}{\delta u})y^2 + 3(2y\frac{\delta y}{\delta u})x \\ &= \frac{3}{2}x + \frac{3}{2}\frac{y^2}{x} - 3 - \frac{3}{2}y \end{aligned}$$

$$\begin{aligned}
f(x,y) &= x^3 - y^2 + 3xy^2 \\
\frac{\delta x}{\delta u} &= (2x)^{-1} \\
\frac{\delta y}{\delta u} &= -(2y)^{-1} \\
\frac{\delta f}{\delta u} &= 3x^2 \cdot \frac{\delta x}{\delta u} - 2y\frac{\delta y}{\delta u} + 3(\frac{\delta x}{\delta u})y^2 + 3(2y\frac{\delta y}{\delta u})x \\
&= \frac{3}{2}x + \frac{3}{2}\frac{y^2}{x} - 3 - \frac{3}{2}y
\end{aligned}$$

$$\begin{aligned}
f(x,y) &= x^3 - y^2 + 3xy^2 \\
\frac{\delta x}{\delta v} &= (2y)^{-1} \\
\frac{\delta y}{\delta u} &= (2x)^{-1} \\
\frac{\delta f}{\delta v} &= 3x^2 \cdot \frac{\delta x}{\delta v} - 2y\frac{\delta y}{\delta v} + 3(\frac{\delta x}{\delta v})y^2 + 3(2y\frac{\delta y}{\delta v})x \\
&= \frac{3}{2}\frac{x^2}{y} - \frac{3}{2}\frac{y^2}{x} + \frac{9}{2}y
\end{aligned}$$

Question 4

$$J = \hat{y}\ln(y + (1-\hat{y})) - \ln(1-y)$$
$$y = \frac{1}{1+e^{-\alpha}}$$
$$\alpha = \sum_{i=0}^{n} \theta_i x_i$$

forward path:

$$\begin{aligned}
J &= \hat{y}\ln(y + (1-\hat{y})) - \ln(1-y) \\
&= \hat{y}\ln\left(\frac{1}{1+e^{-\sum_{i=0}^{n}\theta_i x_i}} + (1-\hat{y})\right) - \ln\left(1 - \frac{1}{1+e^{-\sum_{i=0}^{n}\theta_i x_i}}\right)
\end{aligned}$$

back-propagation

$$
\begin{aligned}
\frac{\delta\alpha}{\delta\theta} &= \sum_{i=0}^{n} x_i \\
\frac{\delta y}{\delta\alpha} &= y(1-y) \\
\frac{\delta J}{\delta\theta} &= \hat{y}\left(\frac{\frac{\delta y}{\delta\theta}}{y+(1-\hat{y})}\right) + \frac{\frac{\delta y}{\delta\theta}}{1-y} \\
&= \hat{y}\left(\frac{y(1-y)\sum_{i=0}^{n} x_i}{y+(1-\hat{y})}\right) + \frac{y(1-y)\sum_{i=0}^{n} x_i}{1-y} \\
&= \hat{y}\left(\frac{\frac{1}{1+e^{-\sum_{i=0}^{n}\theta_i x_i}}(1-\frac{1}{1+e^{-\sum_{i=0}^{n}\theta_i x_i}})\sum_{i=0}^{n} x_i}{\frac{1}{1+e^{-\sum_{i=0}^{n}\theta_i x_i}}+(1-\hat{y})}\right) + \frac{1}{1+e^{-\sum_{i=0}^{n}\theta_i x_i}}\sum_{i=0}^{n} x_i
\end{aligned}
$$