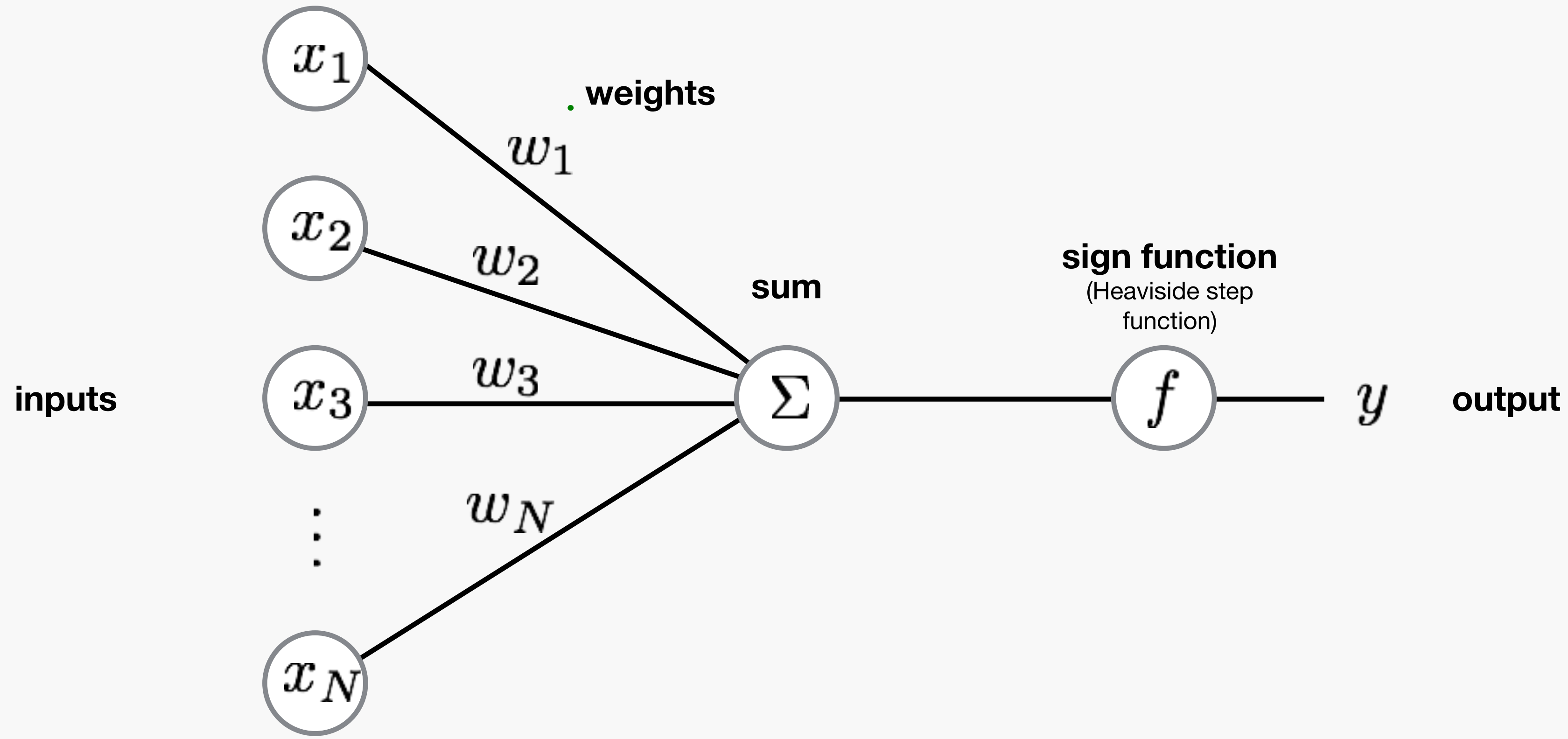
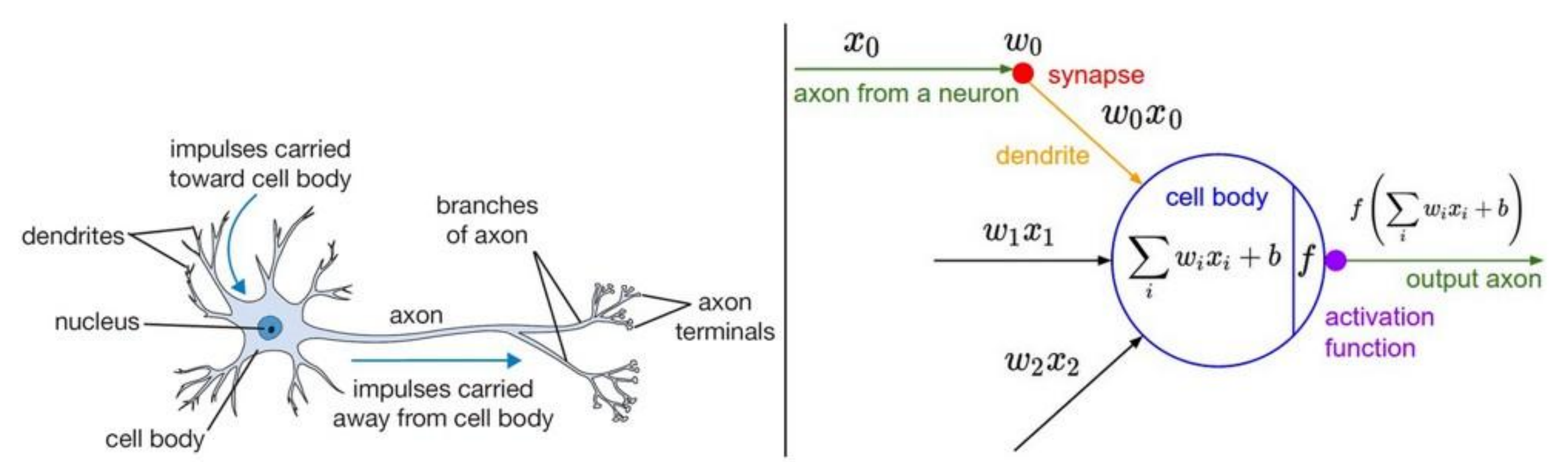
Perceptron

The Perceptron



Aside: Inspiration from Biology



A cartoon drawing of a biological neuron (left) and its mathematical model (right).

Neural nets/perceptrons are **loosely** inspired by biology.

But they certainly are **not** a model of how the brain works, or even how neurons work.

function PERCEPTRON ALGORITHM

$\boldsymbol{w}^{(0)} \leftarrow \mathbf{0}$

for $t = 1, \ldots, T$ **do**

RECEIVE$(\boldsymbol{x}^{(t)})$ $\boldsymbol{x} \in \{0,1\}^N$ N-d binary vector

$\hat{y}_A^{(t)} = \text{sign}\left(\langle \boldsymbol{w}^{(t-1)}, \boldsymbol{x}^{(t)} \rangle\right)$ perceptron is just one line of code!

sign of zero is +1

RECEIVE(y^t) $y \in \{1, -1\}$

$w_n^{(t)} = w_n^{(t-1)} + y_t \cdot x_n^{(t)} \cdot \mathbf{1}[y^{(t)} \neq \hat{y}^{(t)}]$

$\textsc{Receive}(\boldsymbol{x}^{(t)})$

$$\hat{y}_A^{(t)} = \mathrm{sign}\Big(\langle \boldsymbol{w}^{(t-1)}, \boldsymbol{x}^{(t)}\rangle\Big)$$

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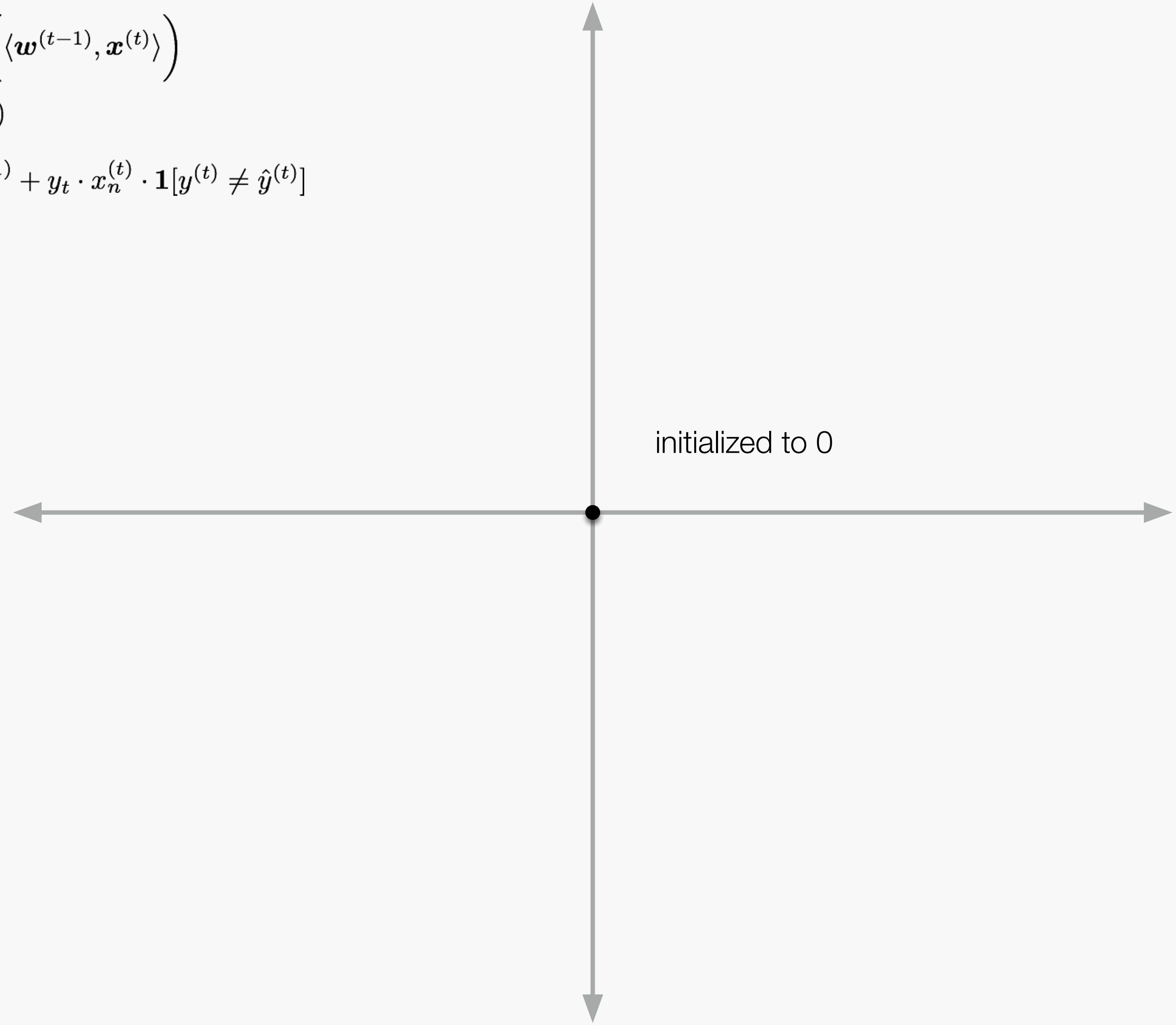

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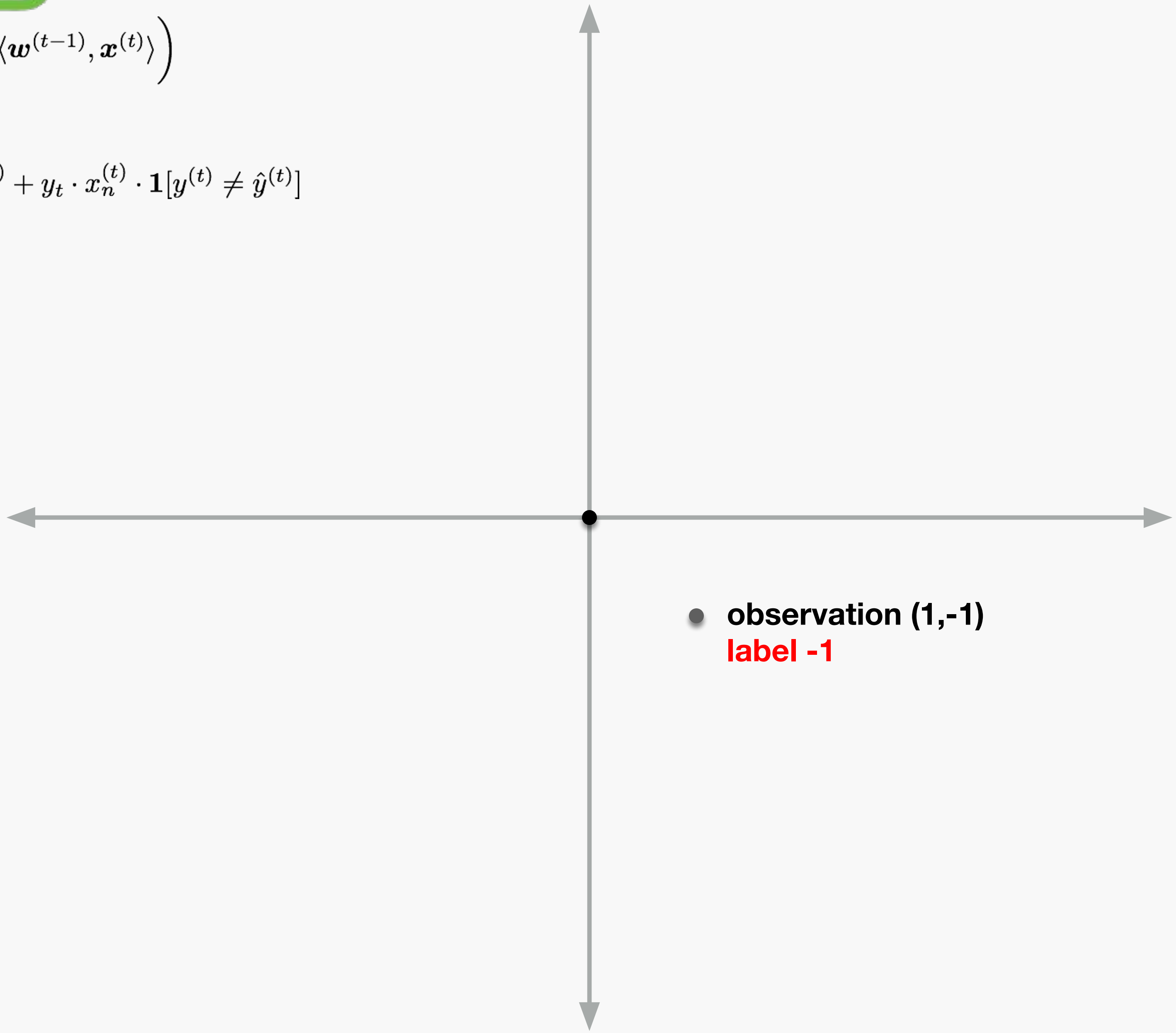

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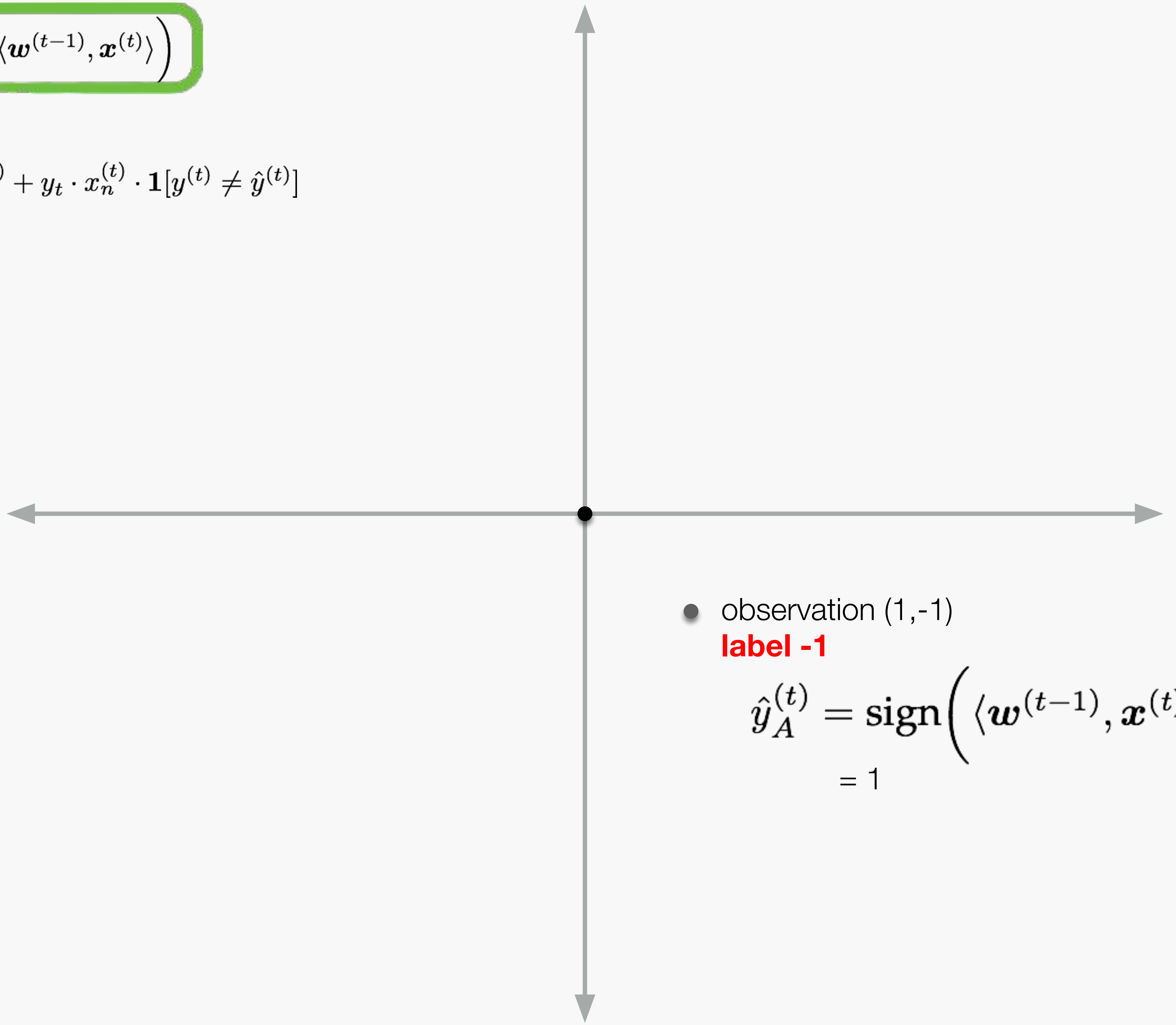

$$\hat{y}_A^{(t)} = \mathrm{sign}\Big(\langle \boldsymbol{w}^{(t-1)}, \boldsymbol{x}^{(t)} \rangle\Big)$$

$= 1$

$\textsc{Receive}(\boldsymbol{x}^{(t)})$

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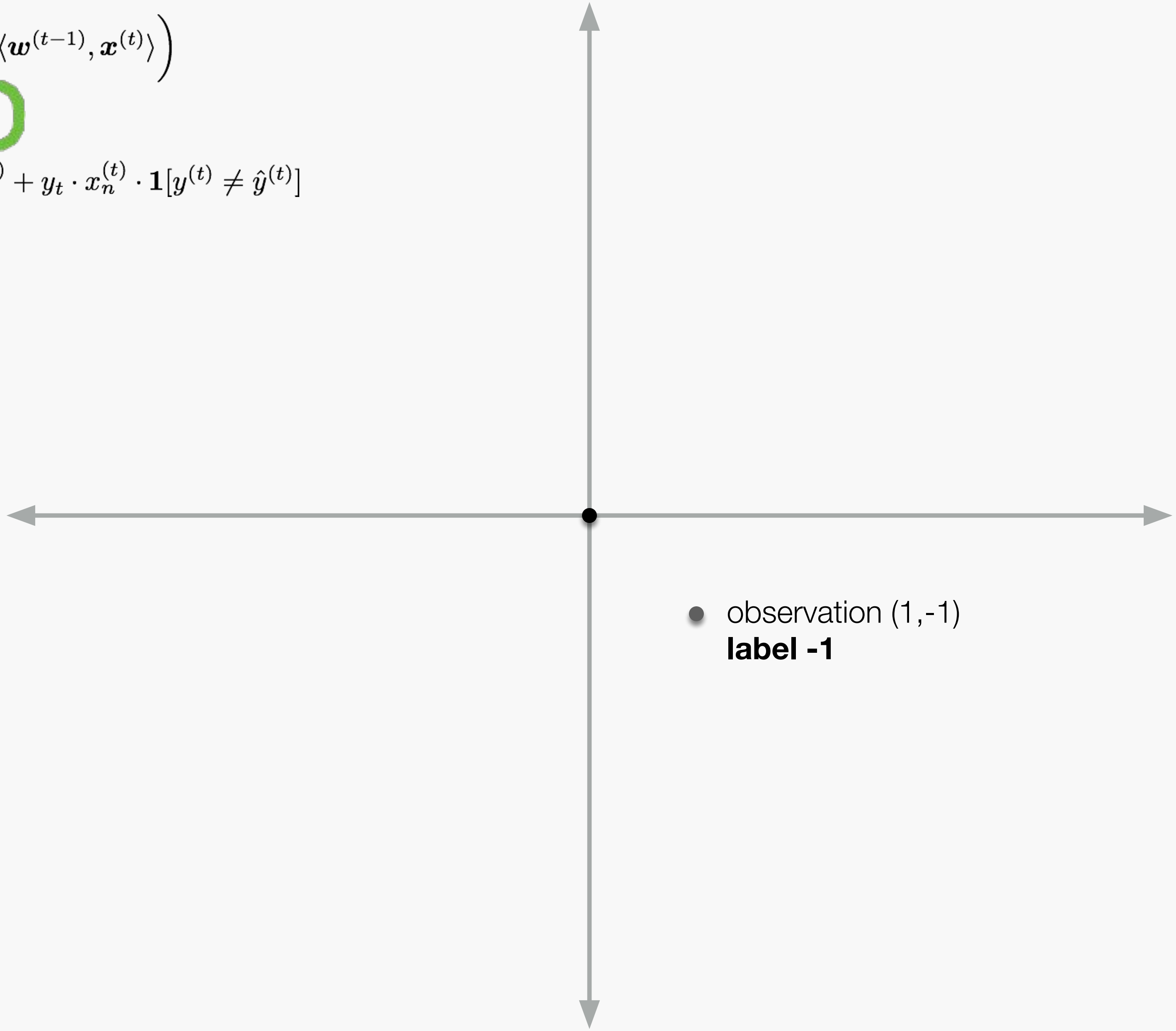

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update w

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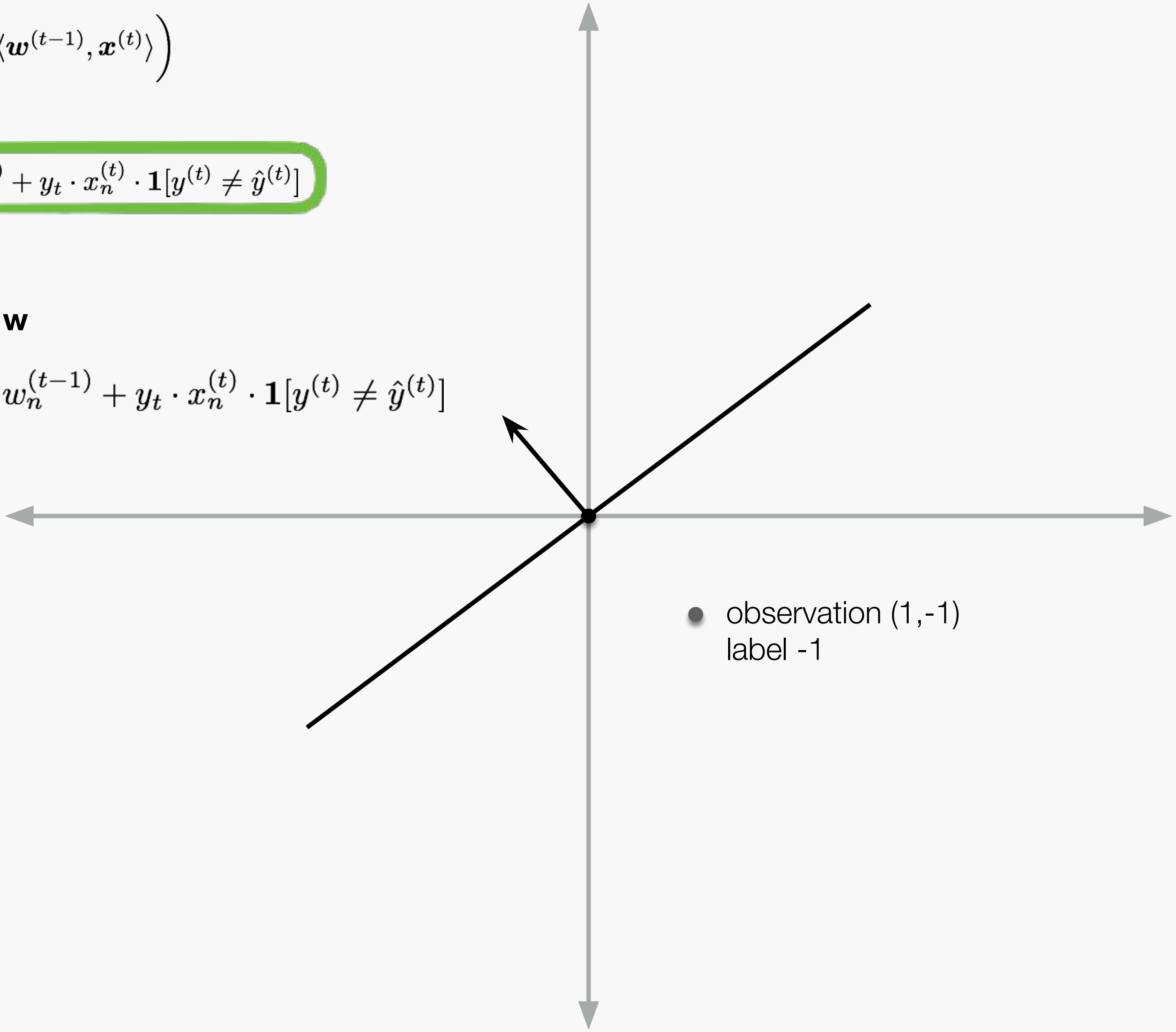

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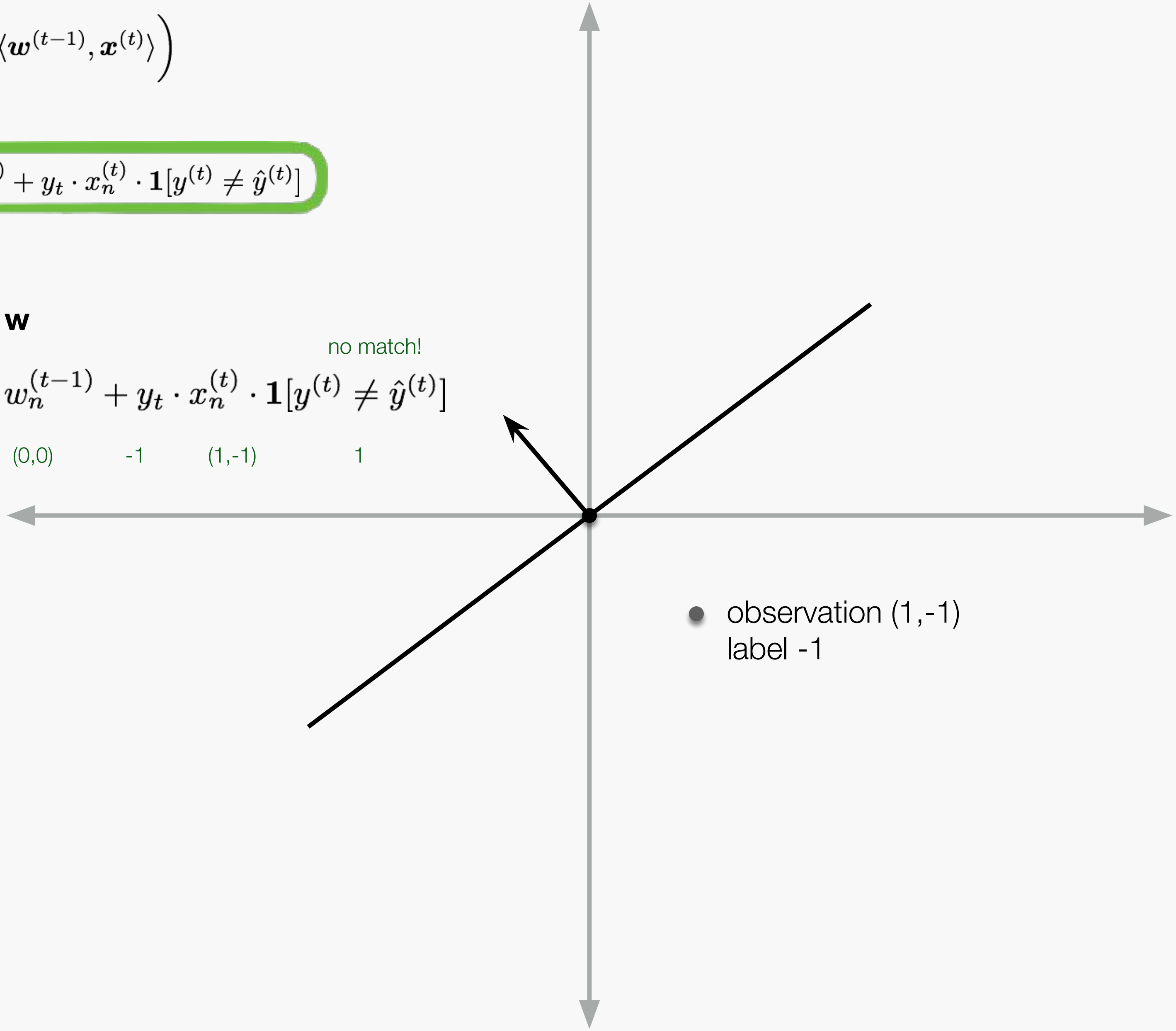
update w

no match!

$$w_n^{(t)} = w_n^{(t-1)} + y_t \cdot x_n^{(t)} \cdot \mathbf{1}[y^{(t)} \neq \hat{y}^{(t)}]$$

(-1,1) (0,0) -1 (1,-1) 1



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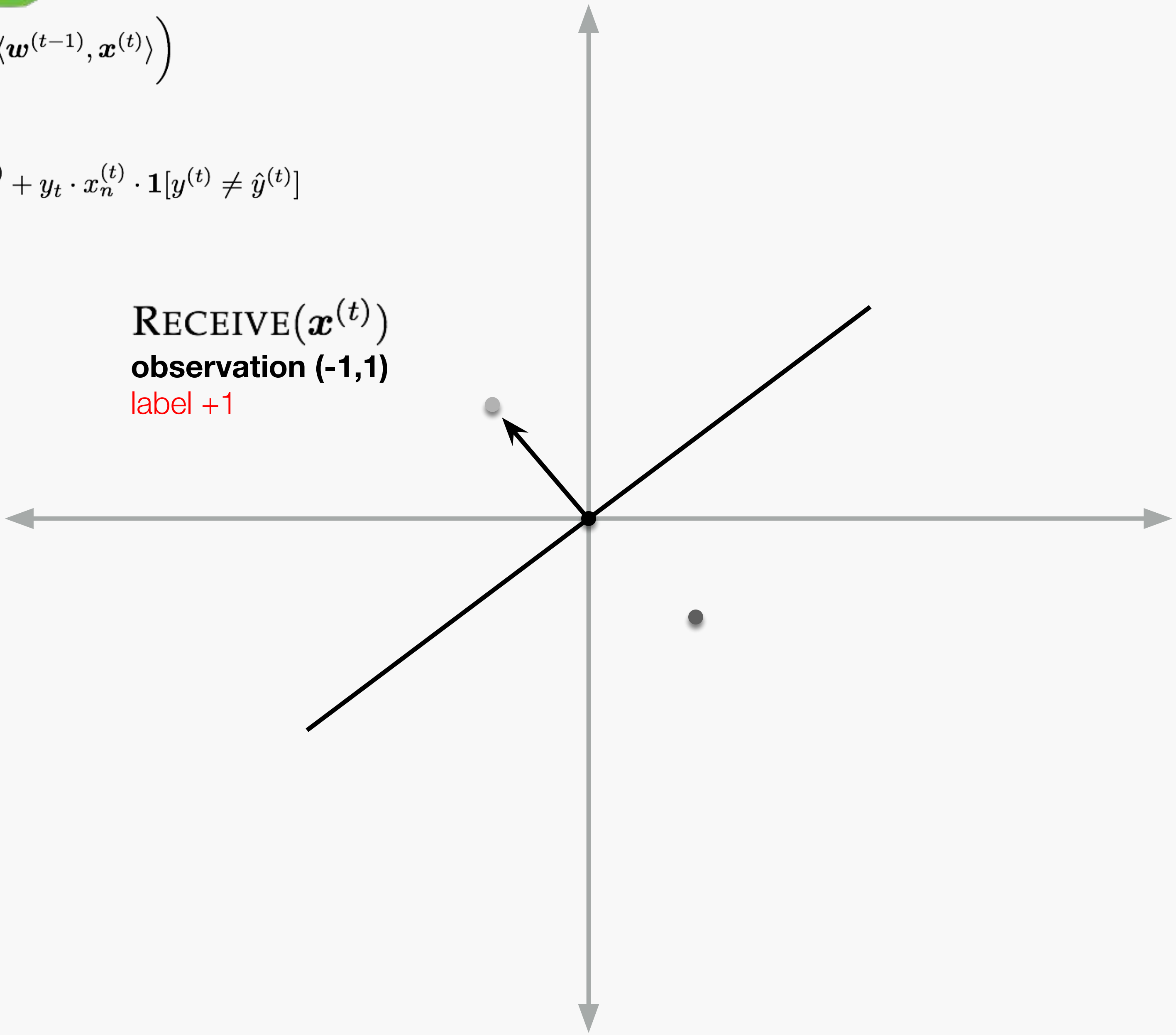
(-1,1)



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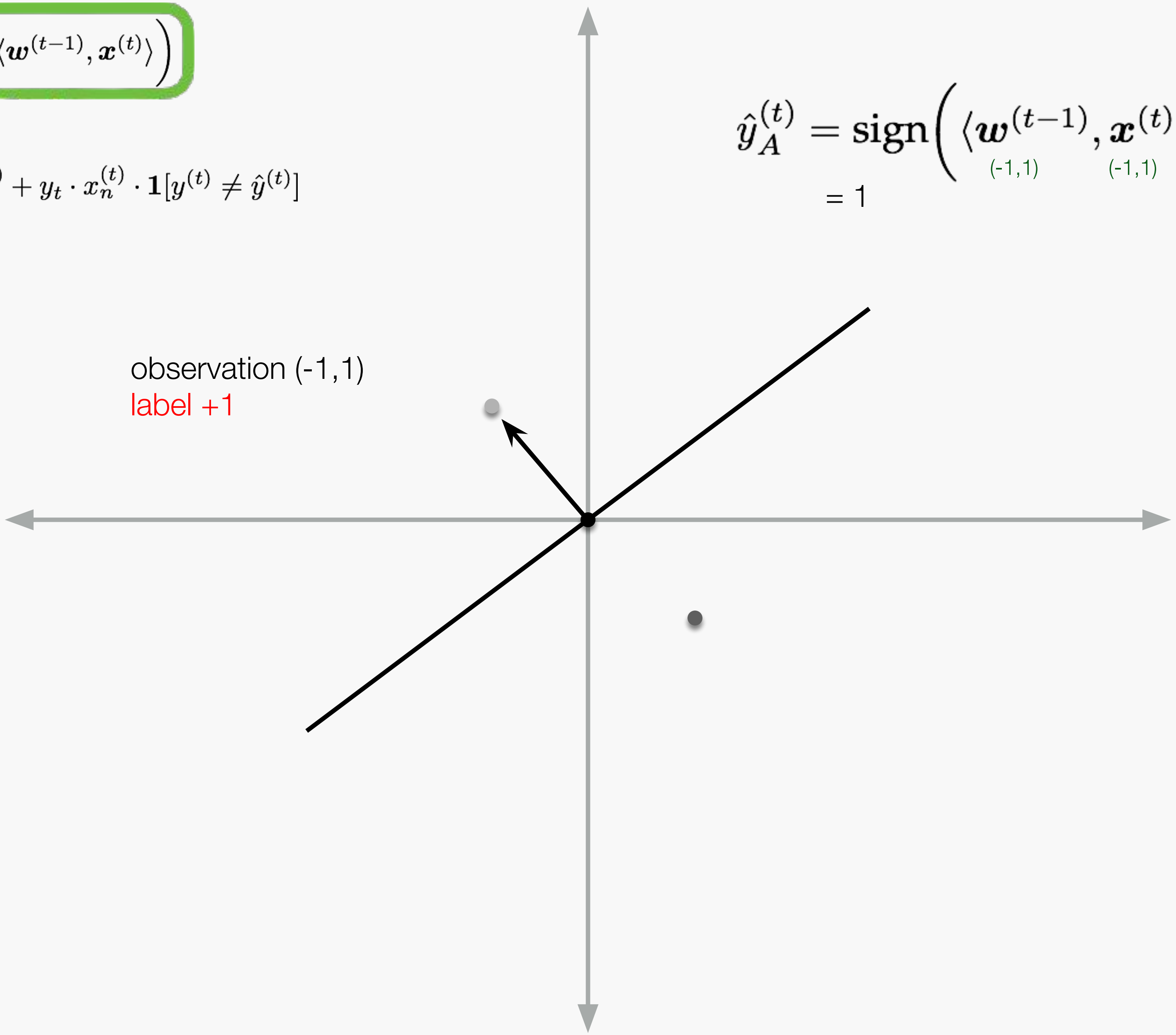

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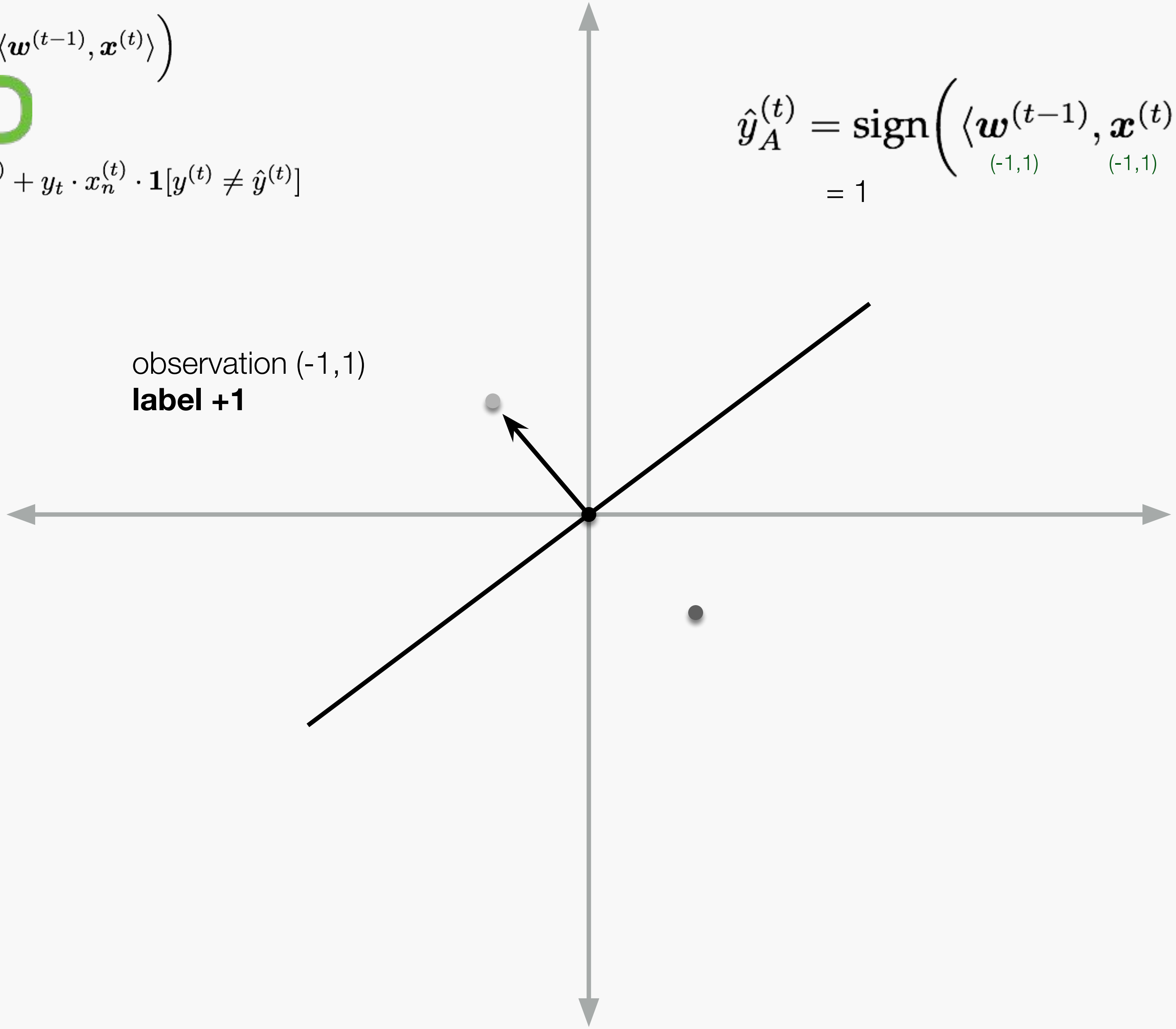

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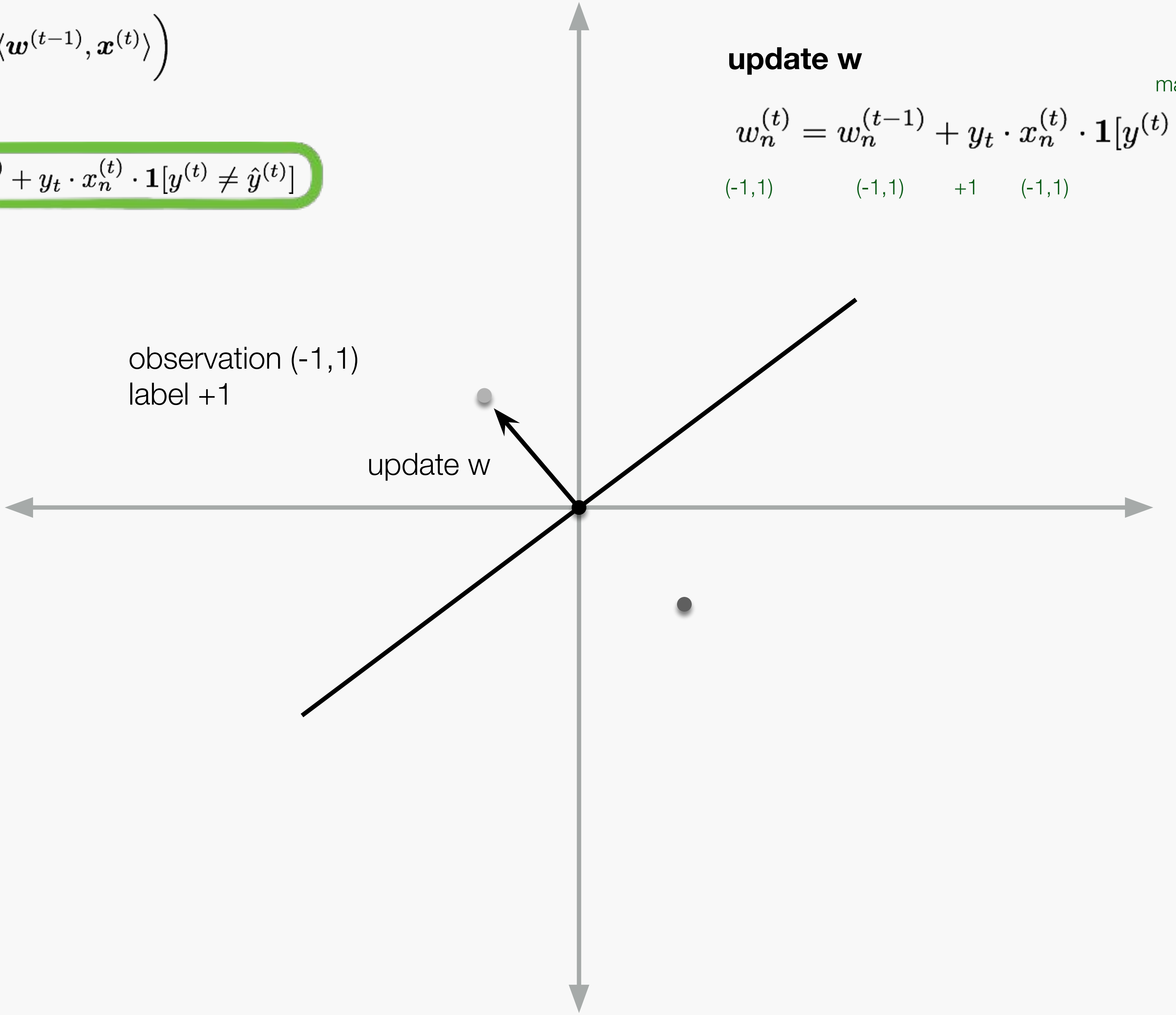
(-1,1) (-1,1) +1 (-1,1) 0



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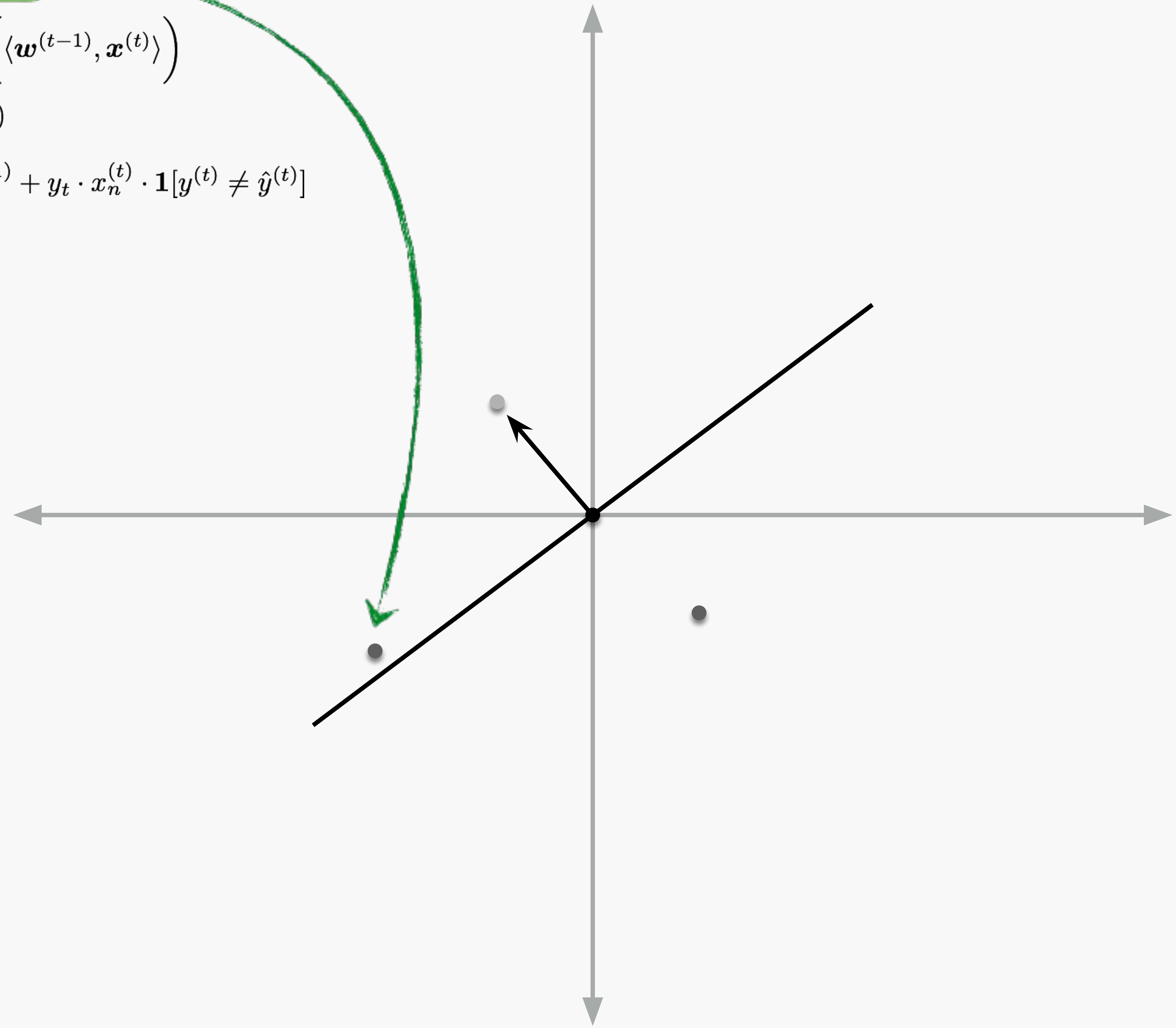

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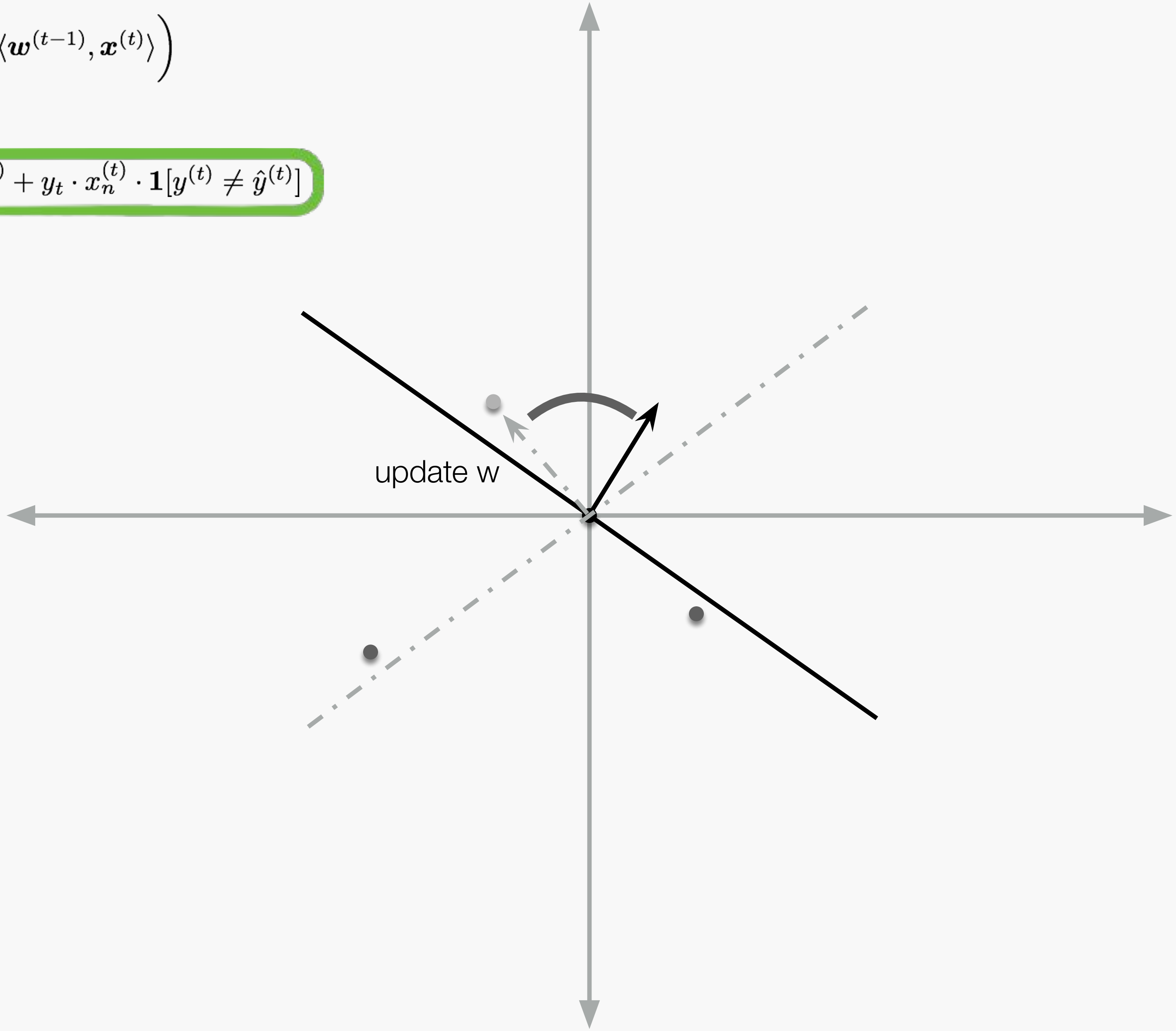

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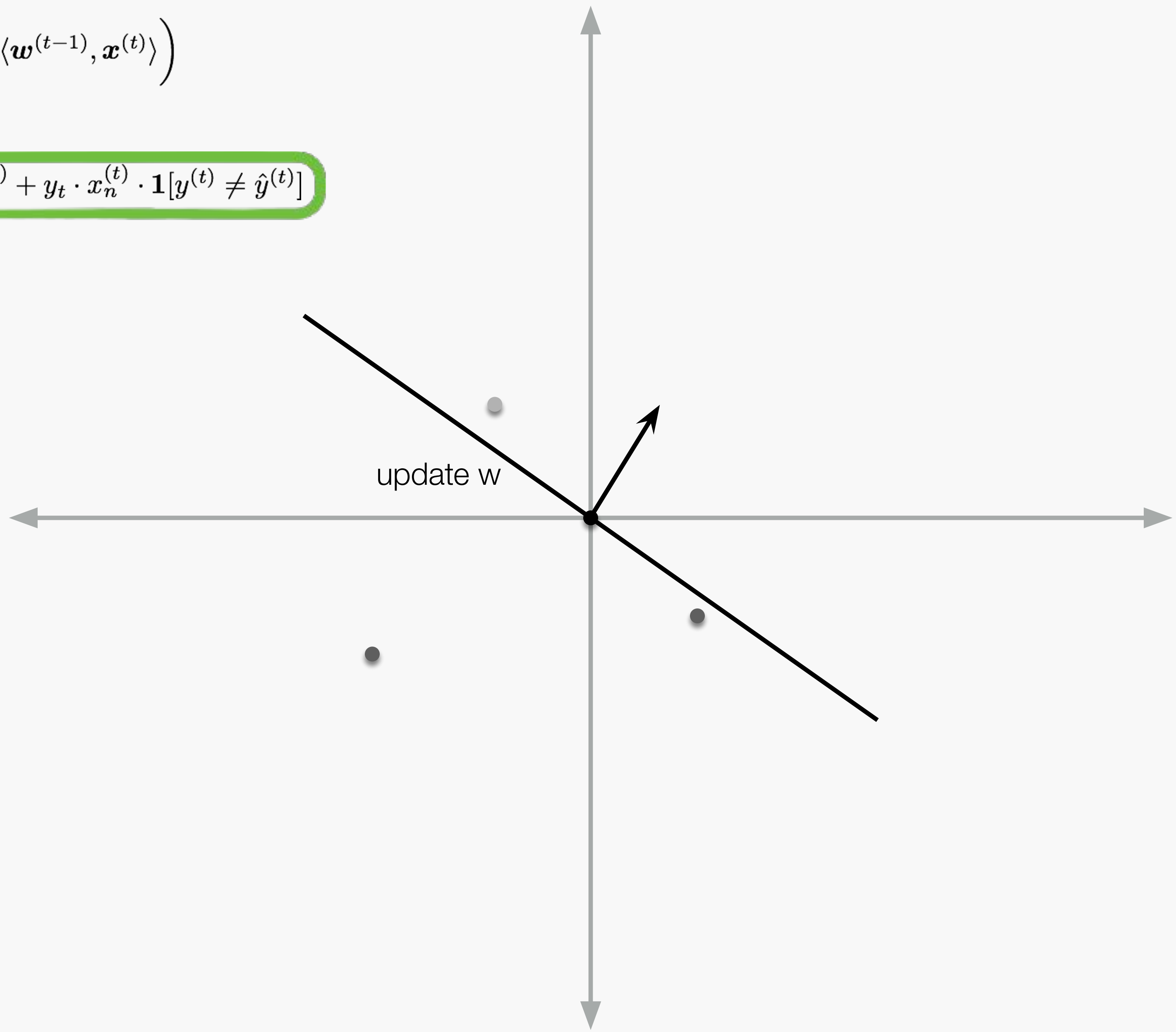

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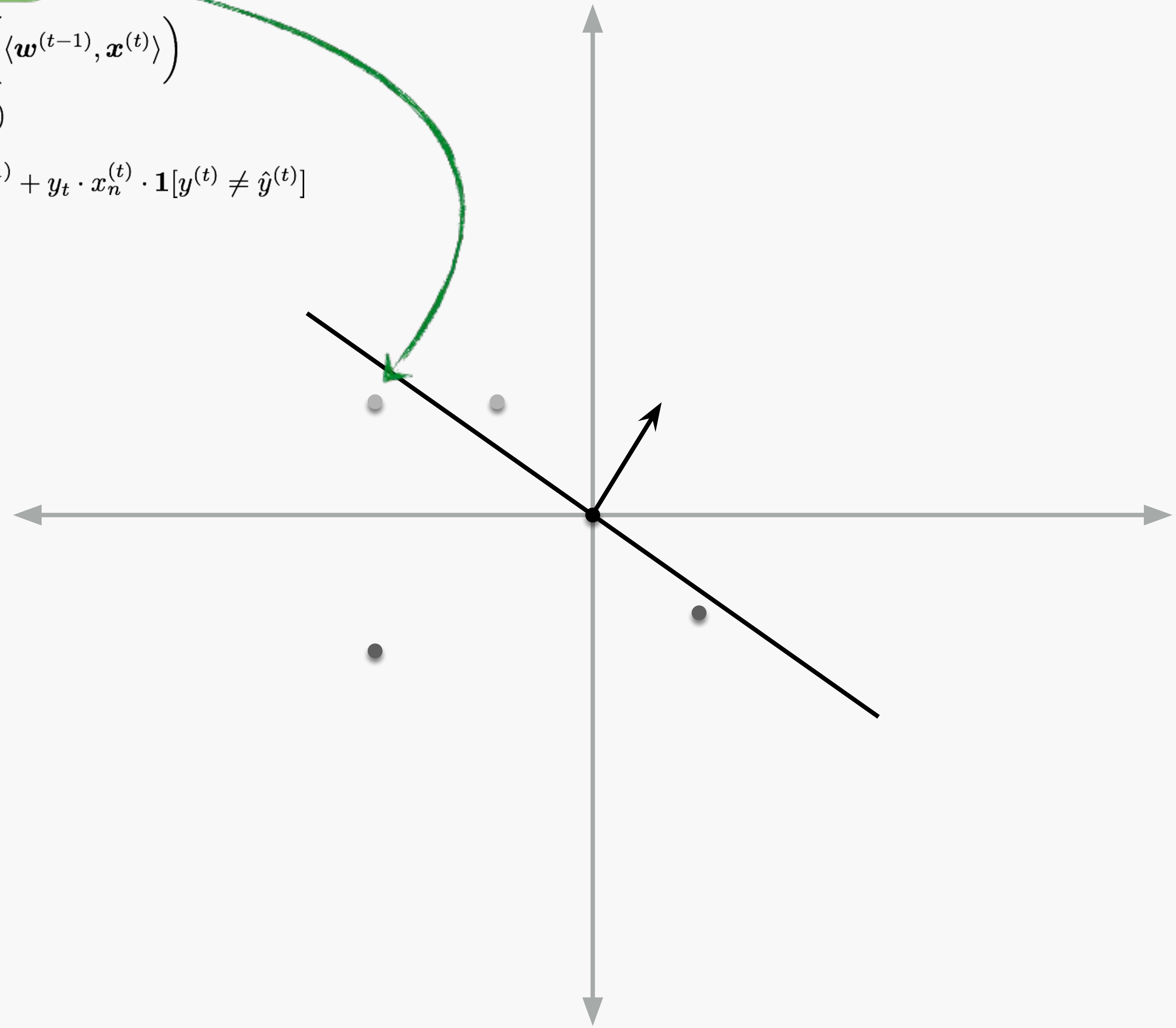

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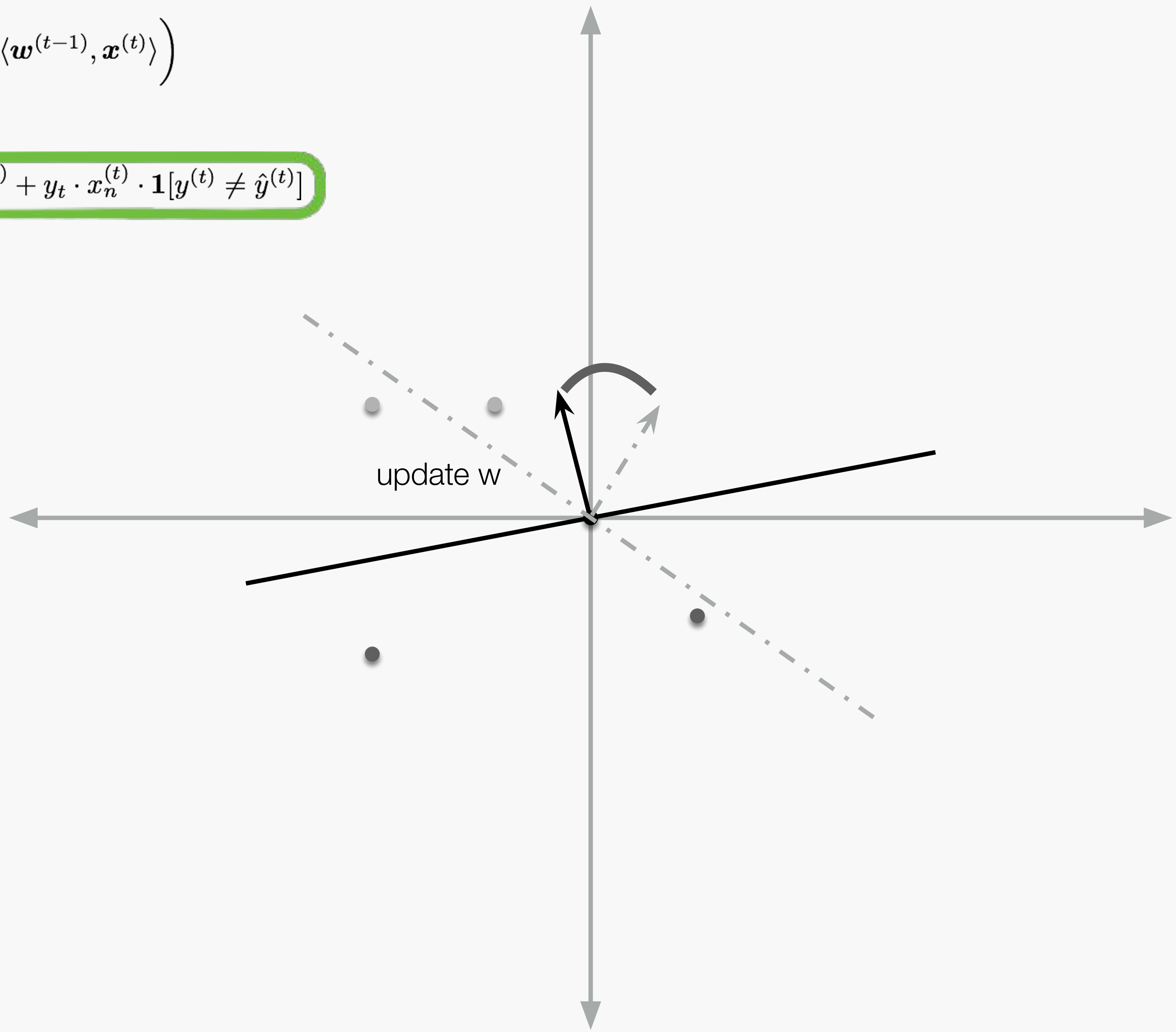

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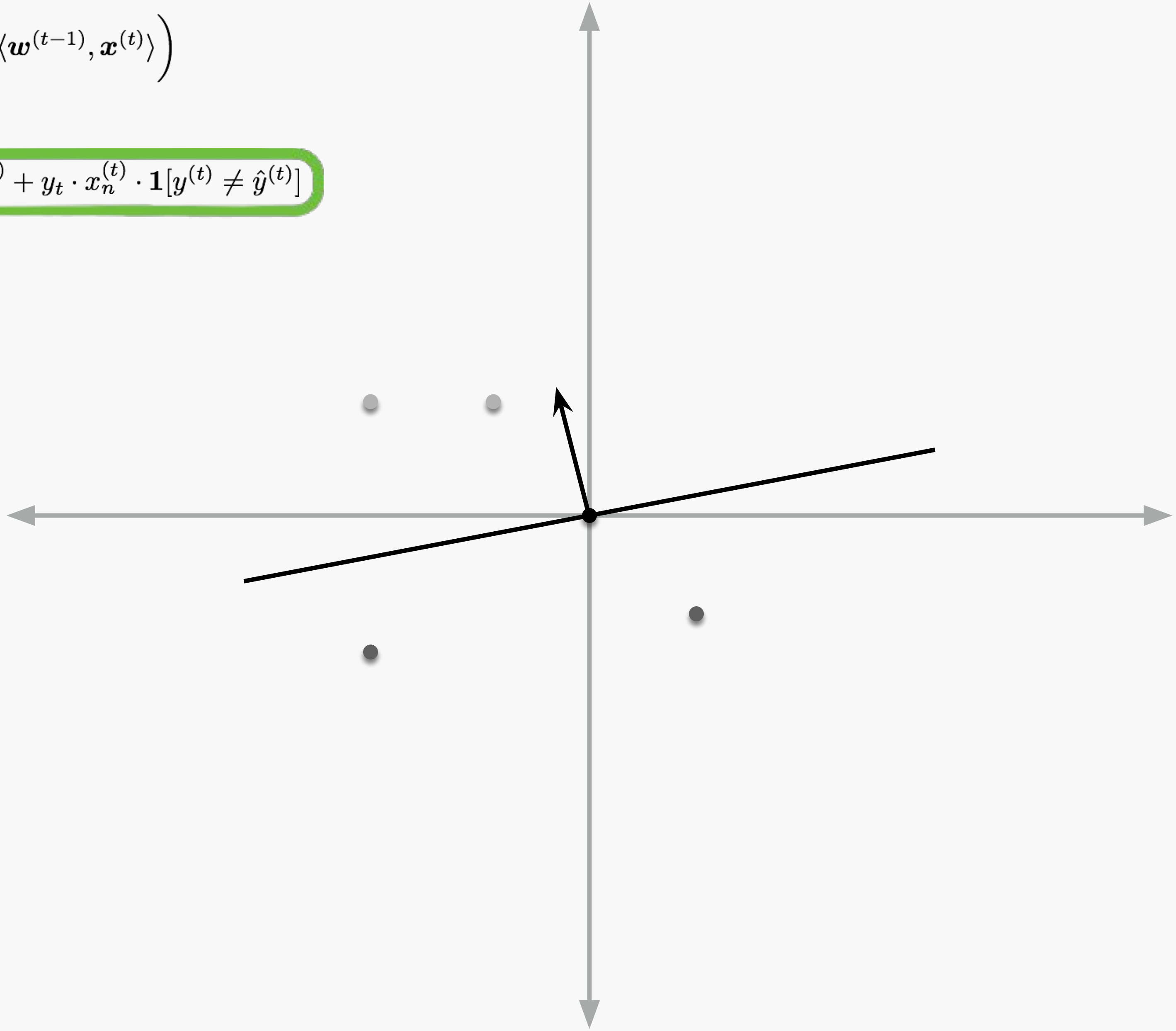
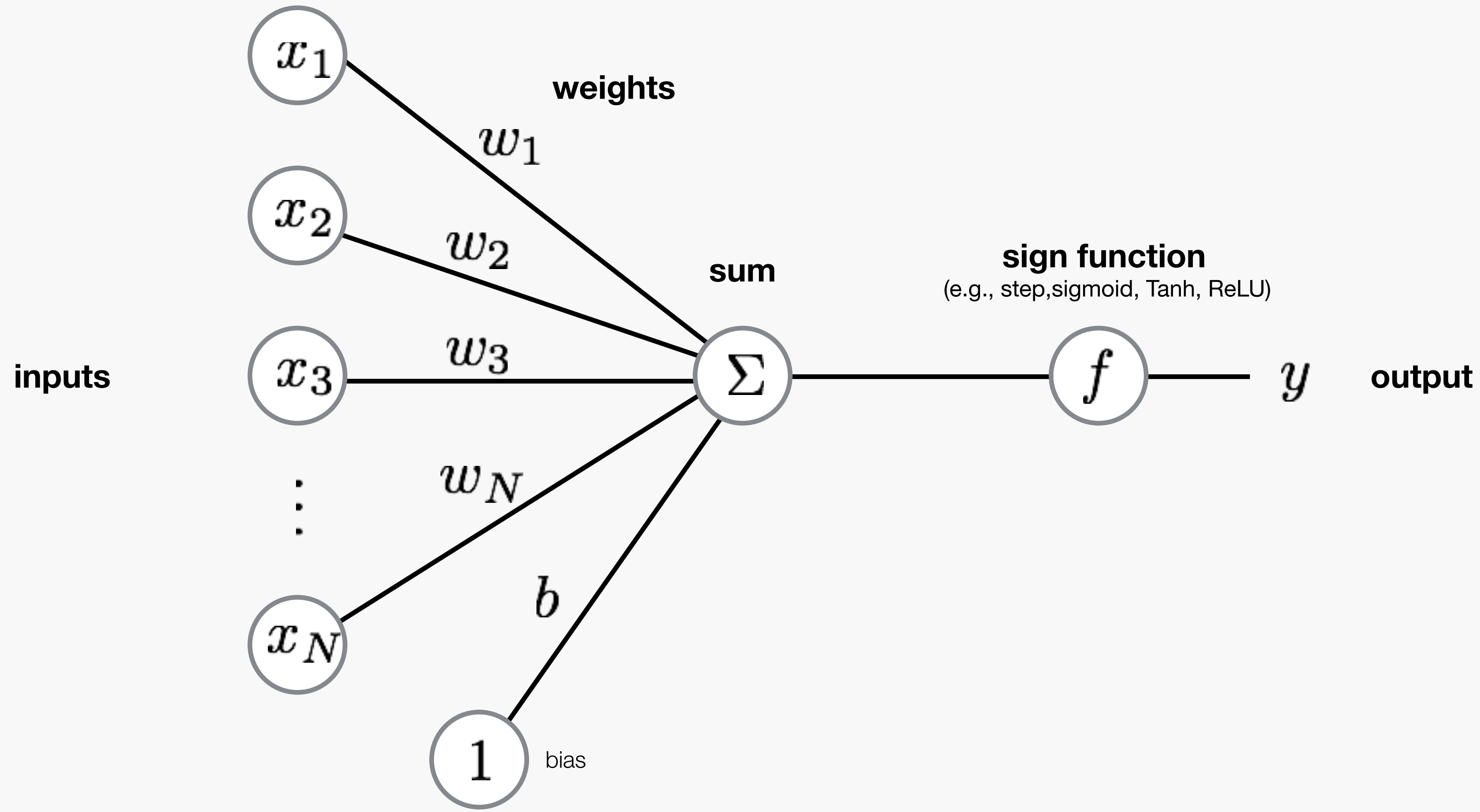
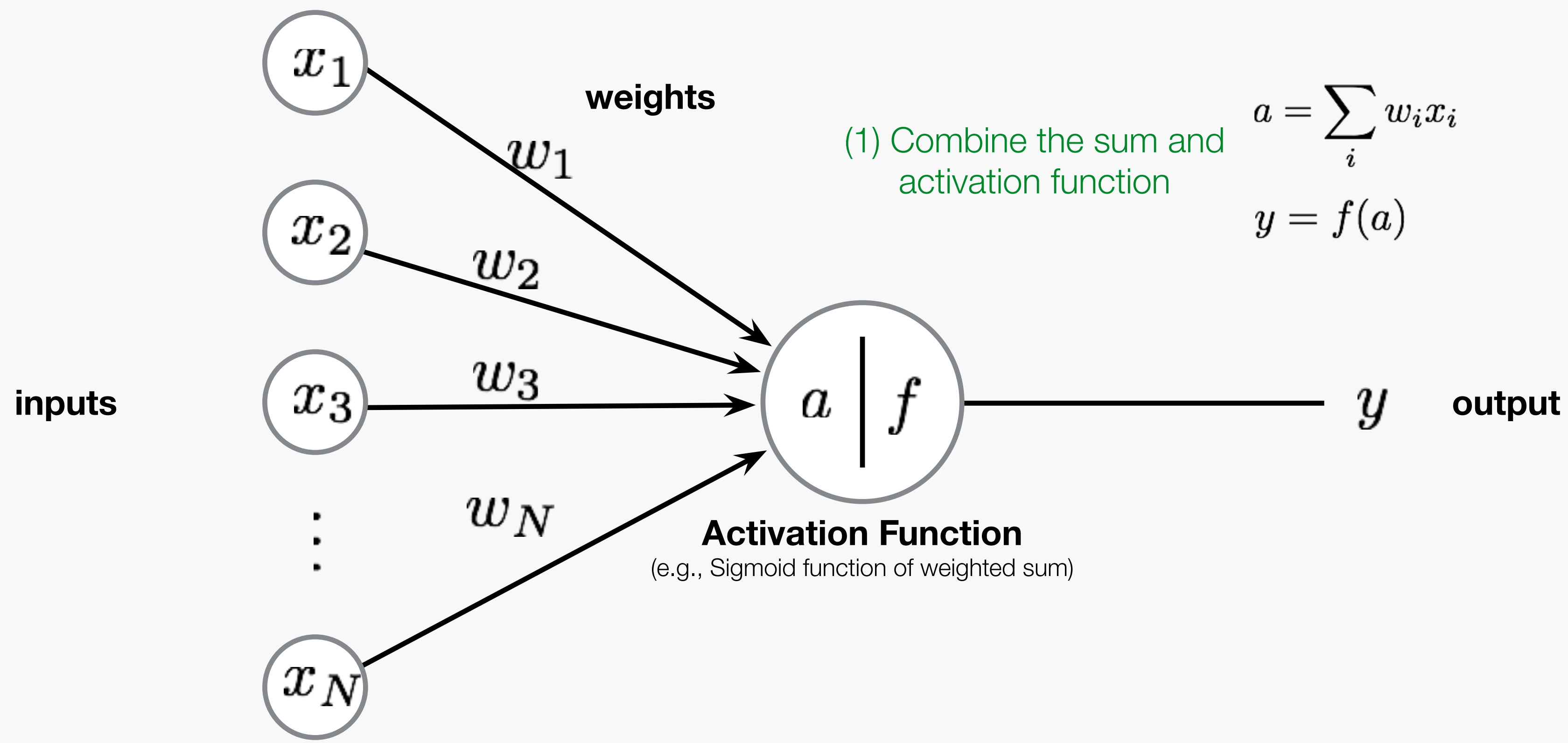


repeat …

The Perceptron



Another way to draw it…



(1) Combine the sum and activation function

$$a = \sum_i w_i x_i$$

$$y = f(a)$$

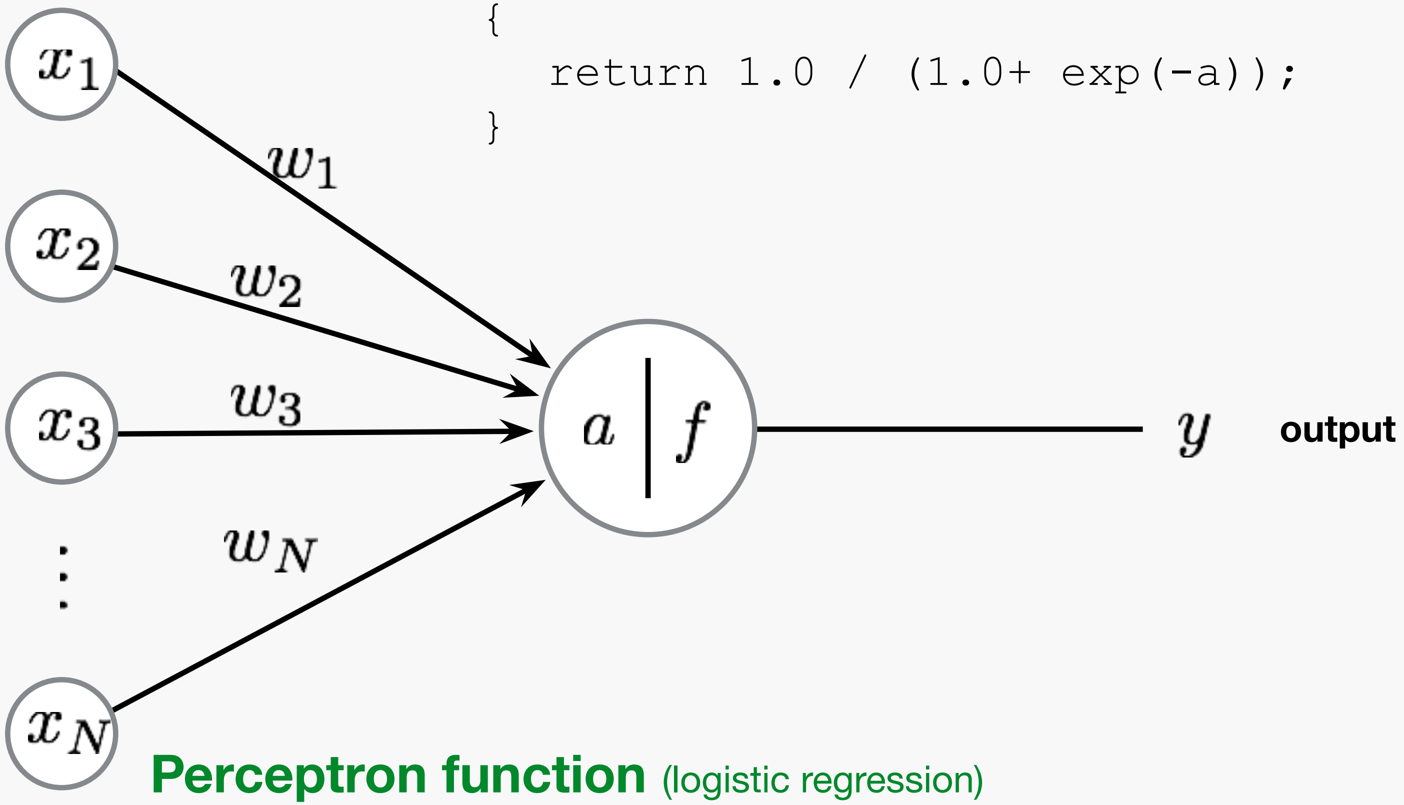
(2) suppress the bias term (less clutter)

$$x_N = 1$$
$$w_N = b$$

Programming the 'forward pass'

Activation function (sigmoid, logistic function)

```
float f(float a)
{
   return 1.0 / (1.0+ exp(-a));
}
```



Perceptron function (logistic regression)

```
float perceptron(vector<float> x, vector<float> w)
{
   float a  = dot(x,w);
   return f(a);
}
```

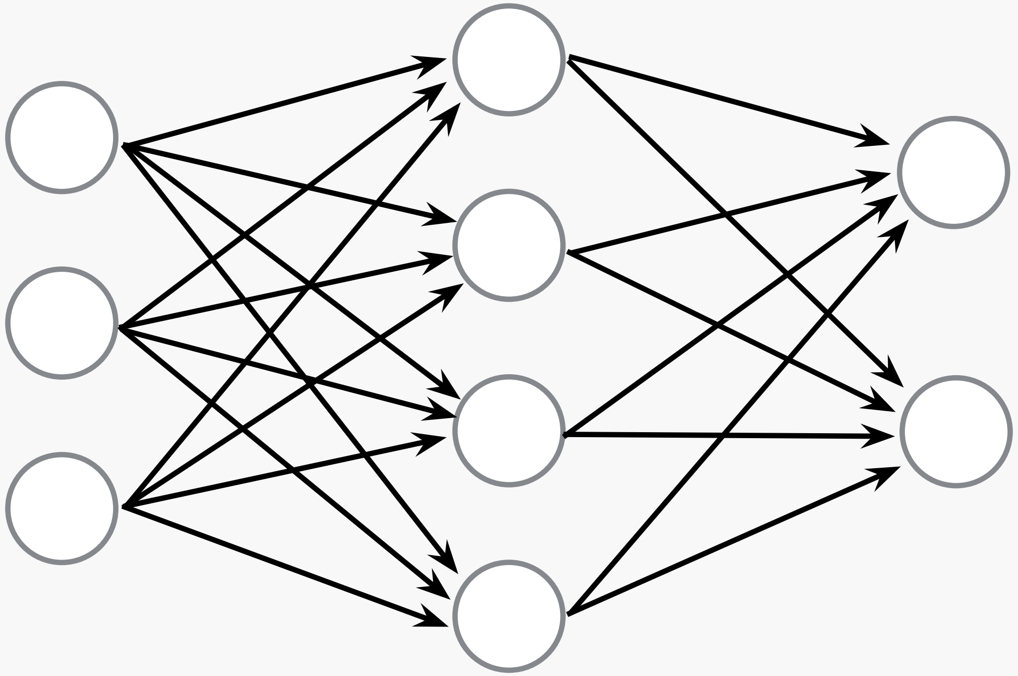
Neural networks

Connect a bunch of perceptrons together …

Connect a bunch of perceptrons together …

Neural Network

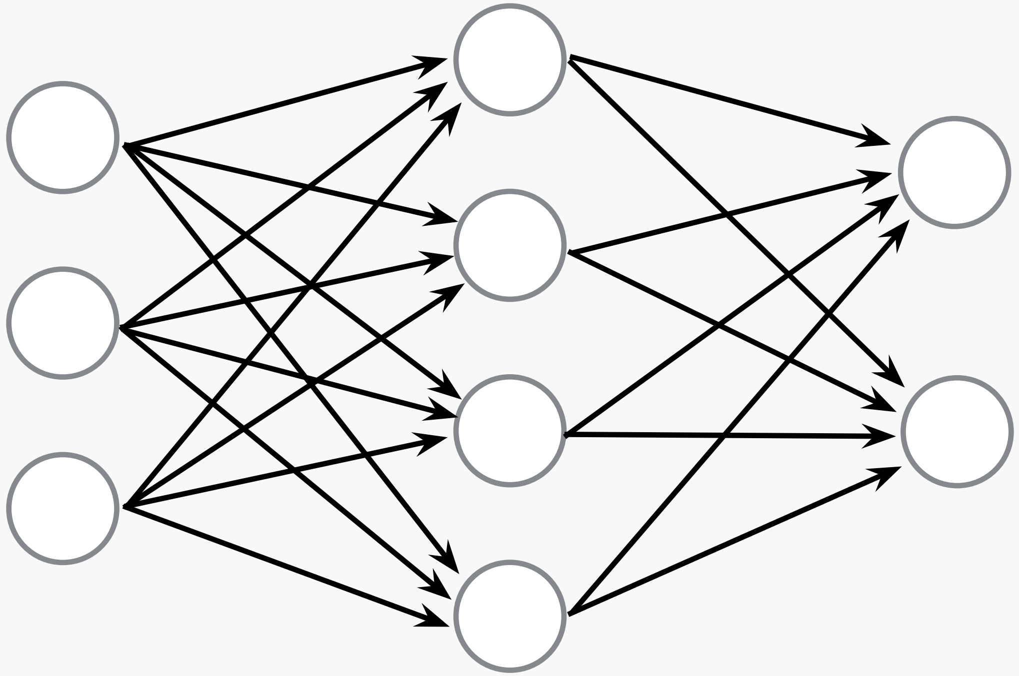
a collection of connected perceptrons



Connect a bunch of perceptrons together …

Neural Network

a collection of connected perceptrons

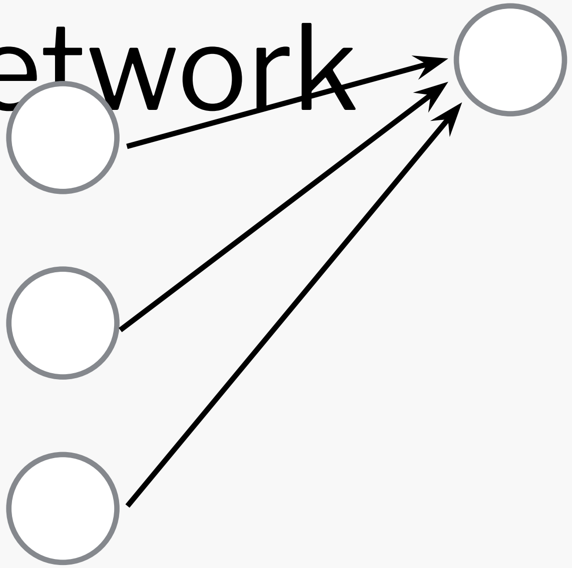


How many perceptrons in this neural network?

Connect a bunch of perceptrons together …

a collection of connected perceptrons

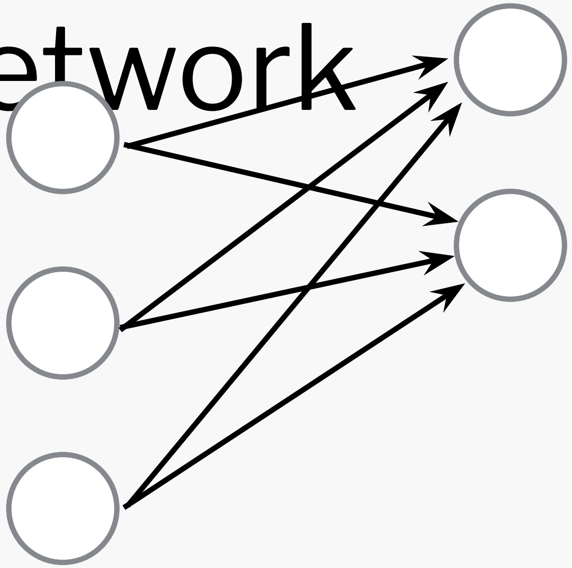
Neural Network



Connect a bunch of perceptrons together …

a collection of connected perceptrons

Neural Network

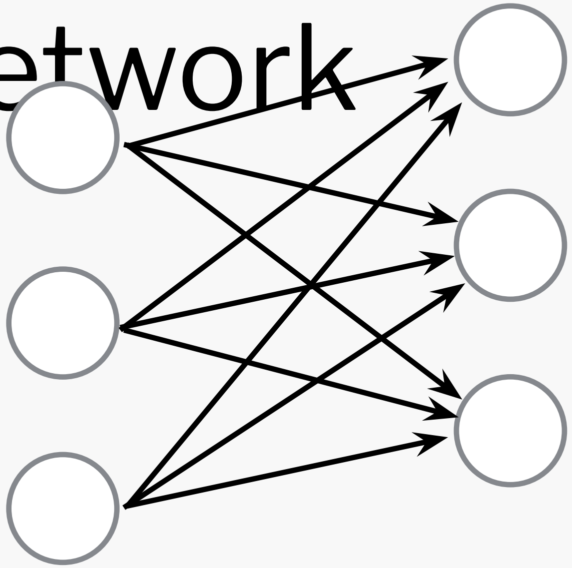


‘two
perceptrons’

Connect a bunch of perceptrons together …

a collection of connected perceptrons

Neural Network

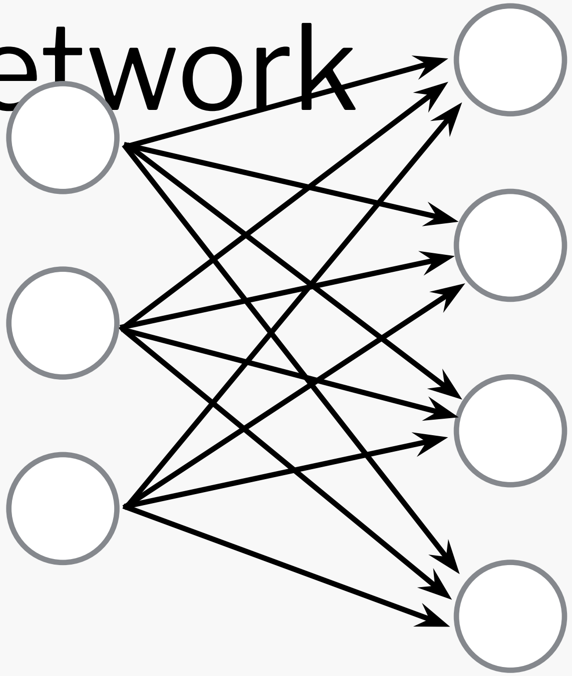


‘three perceptrons’

Connect a bunch of perceptrons together …

a collection of connected perceptrons

Neural Network

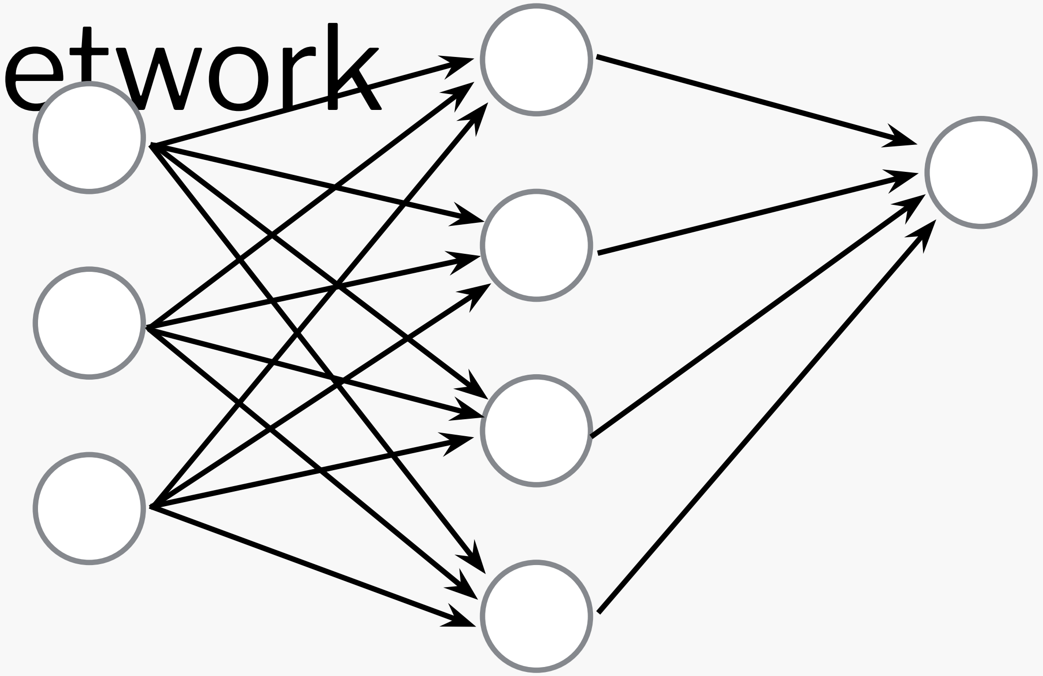


‘four
perceptrons’

Connect a bunch of perceptrons together …

a collection of connected perceptrons

Neural Network

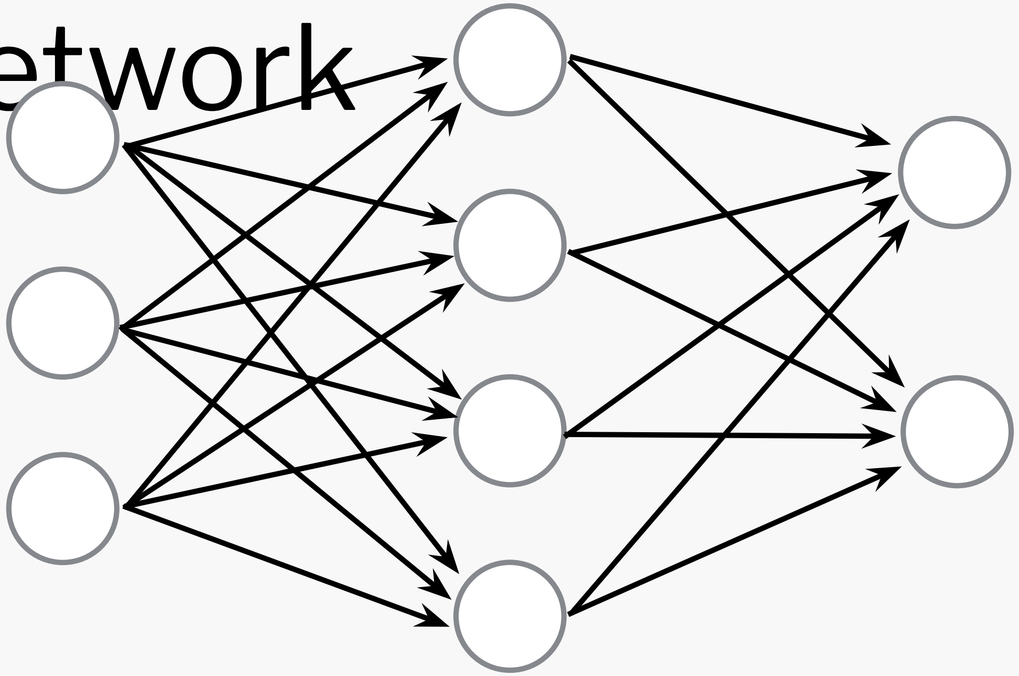


'five perceptrons'

Connect a bunch of perceptrons together …

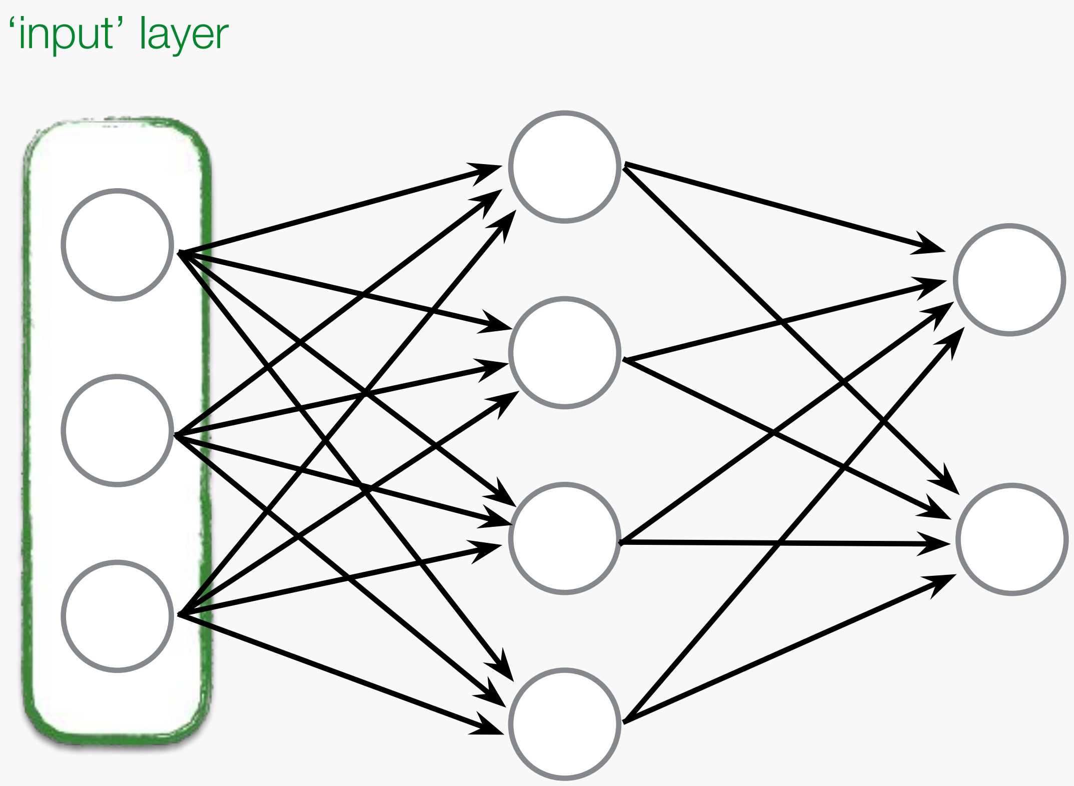
a collection of connected perceptrons

Neural Network



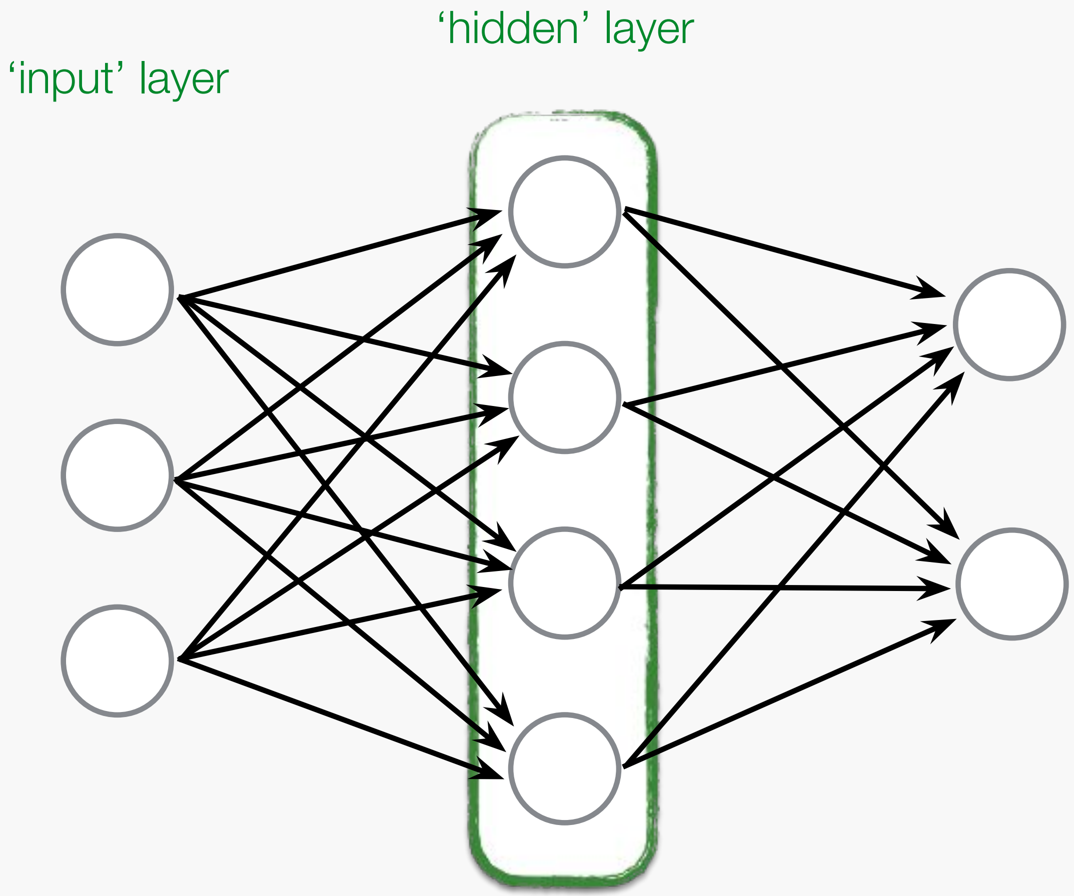
'six perceptrons'

Some terminology…



…also called a **Multi-layer Perceptron** (MLP)

Some terminology…



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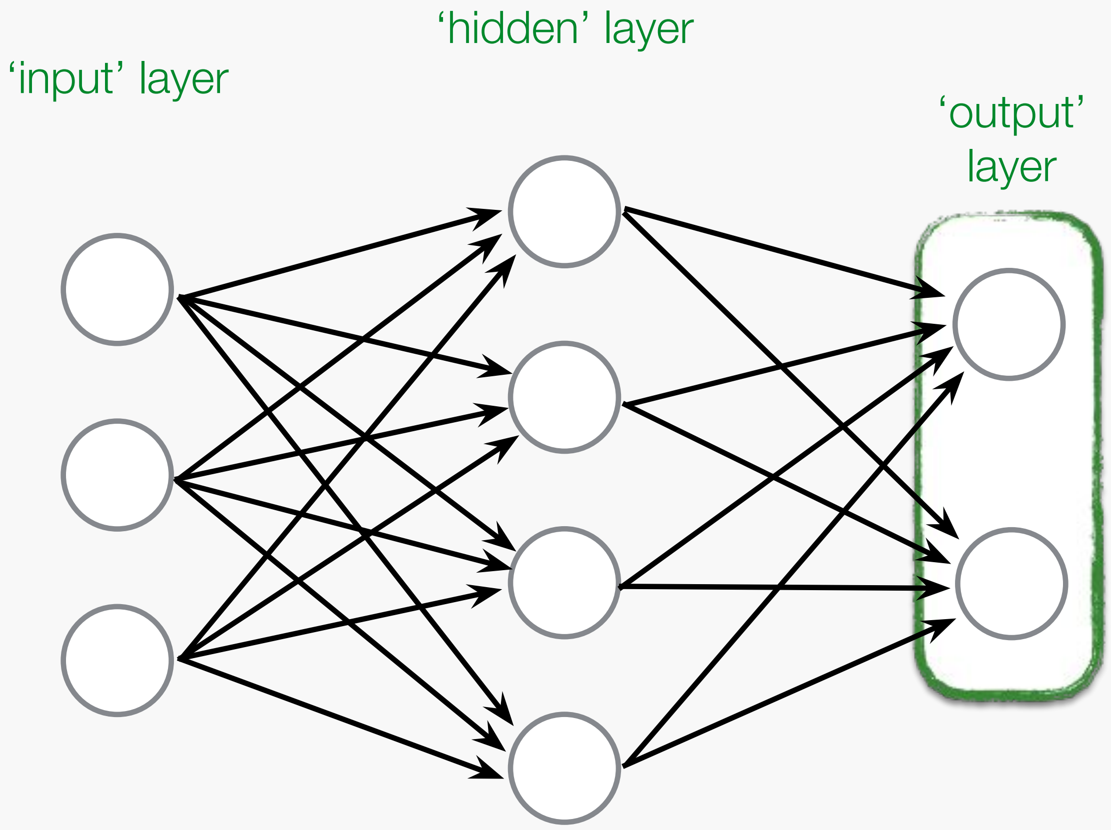
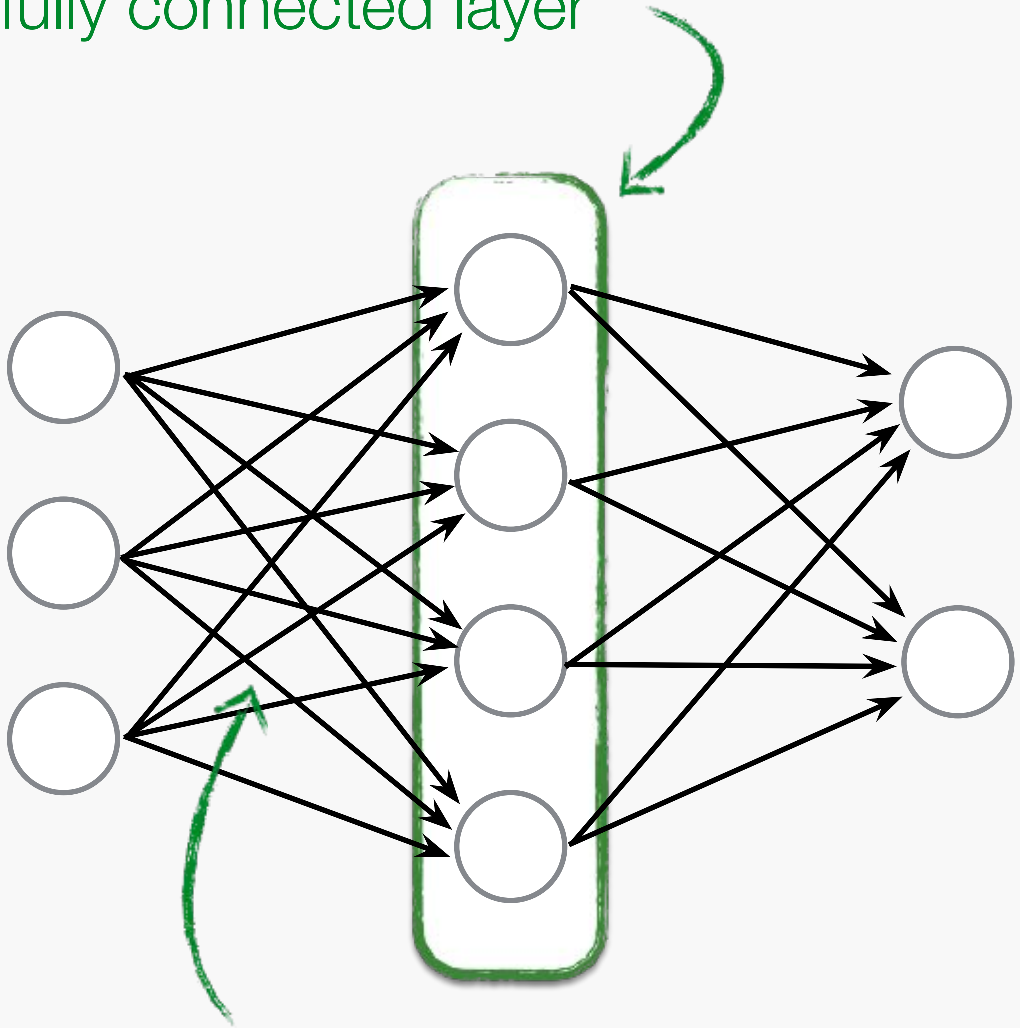
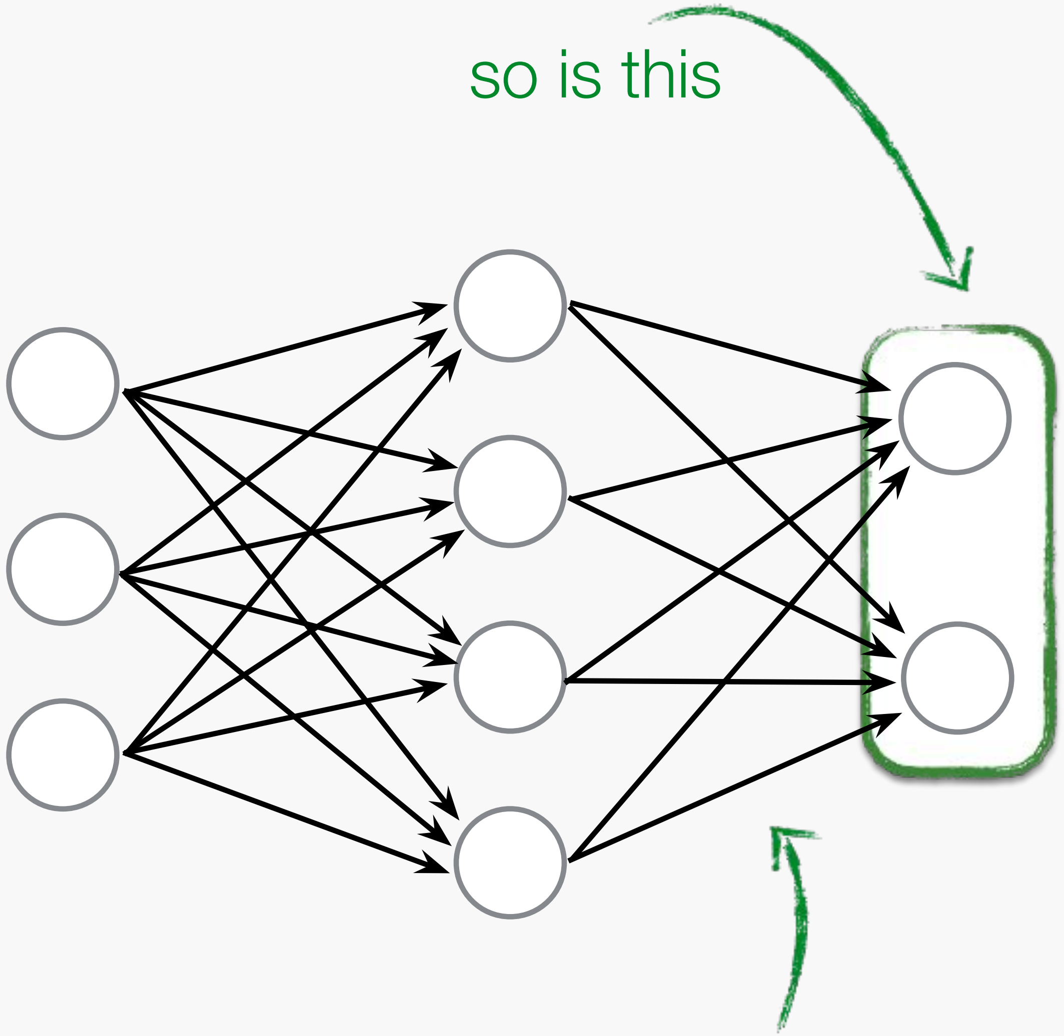


…also called a **Multi-layer Perceptron** (MLP)

this layer is a
‘fully connected layer’



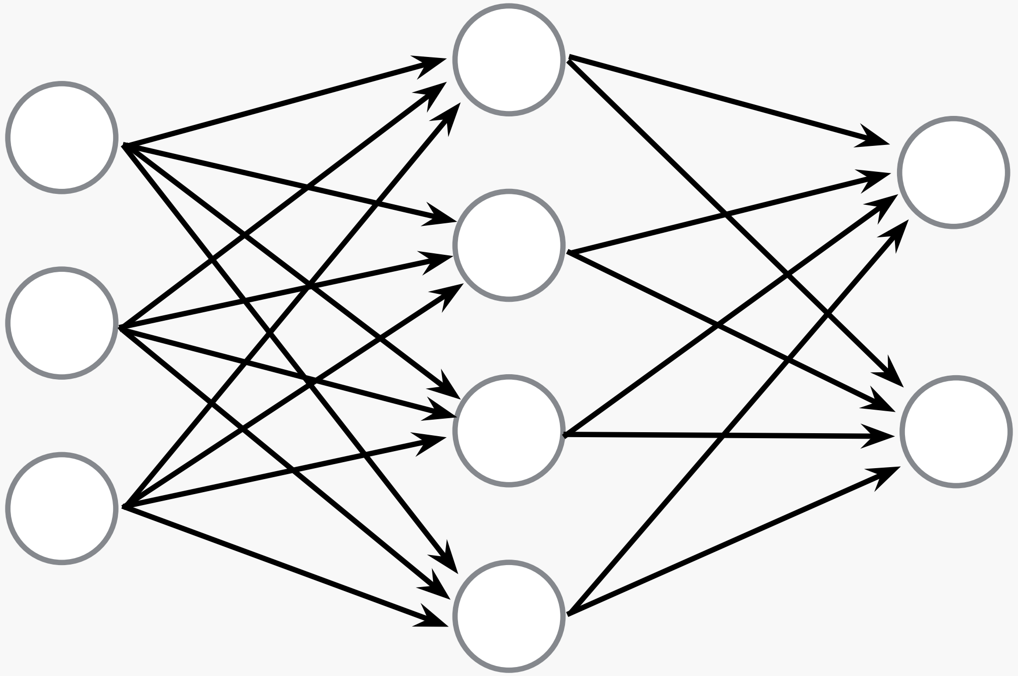
all pairwise neurons <u>between</u> layers are connected so is this



all pairwise neurons between layers are connected

How many neurons (perceptrons)?

How many weights (edges)?

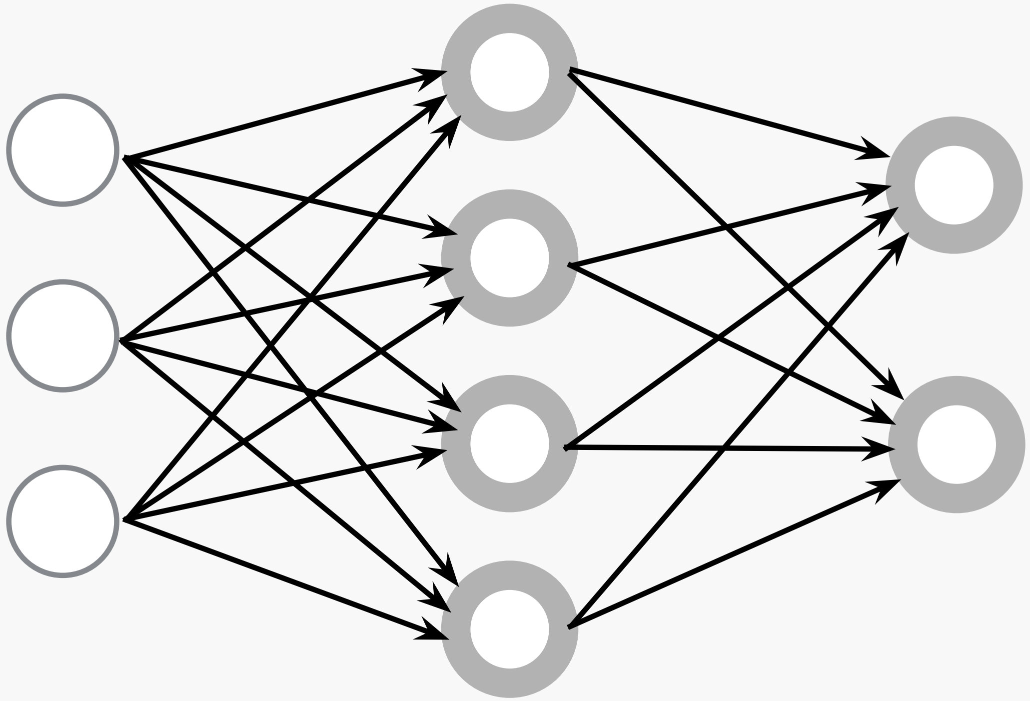


How many learnable parameters total?

How many neurons (perceptrons)?

4 + 2 =
6

How many weights (edges)?



How many learnable parameters total?

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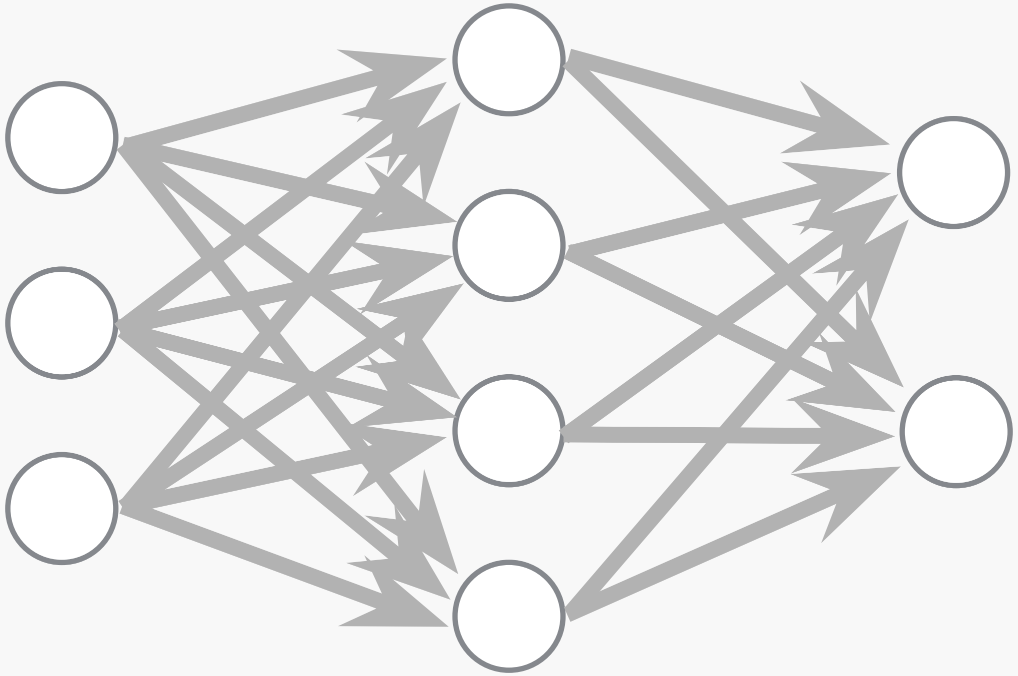
How many weights (edges)?

(3 x 4) + (4 x 2) = 20



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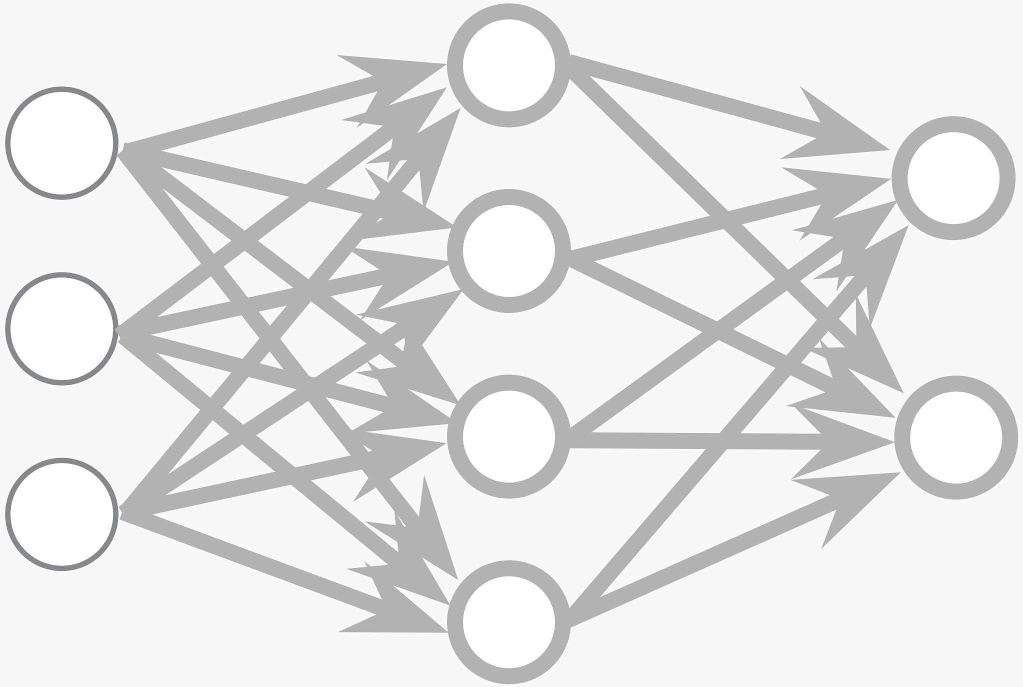
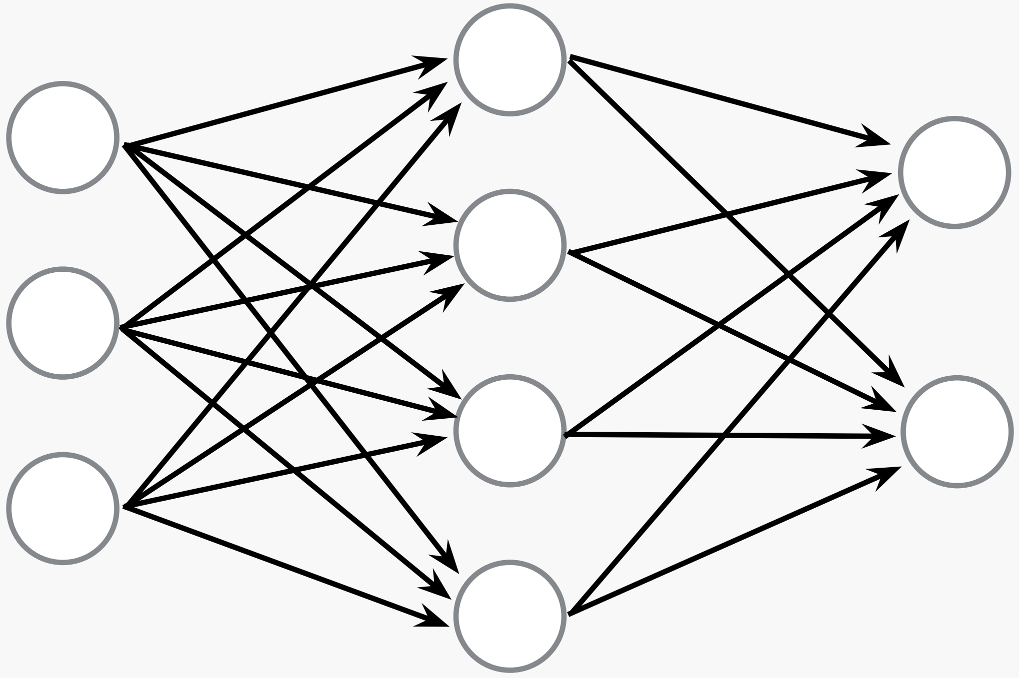


How many learnable parameters total?

20 + 4 + 2 =
26

bias terms

performance usually tops out at 2-3 layers,
deeper networks don't really improve performance...



...with the exception of **convolutional** networks for images