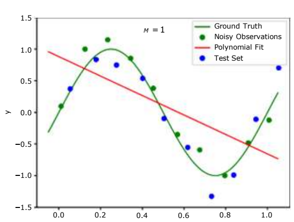
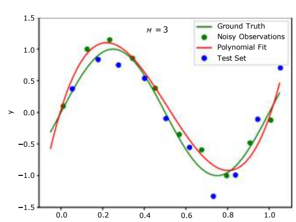
Regularization

1

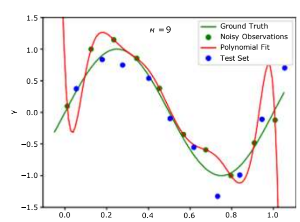
Recap: Capacity, Overfitting and Underfitting



Capacity too low



Capacity about right



Capacity too high

- ► **Underfitting:** Model too simple, does not achieve low error on training set
- ► **Overfitting:** Training error small, but test error (= generalization error) large
- ► **Regularization:** Take model from third regime (right) to second regime (middle)

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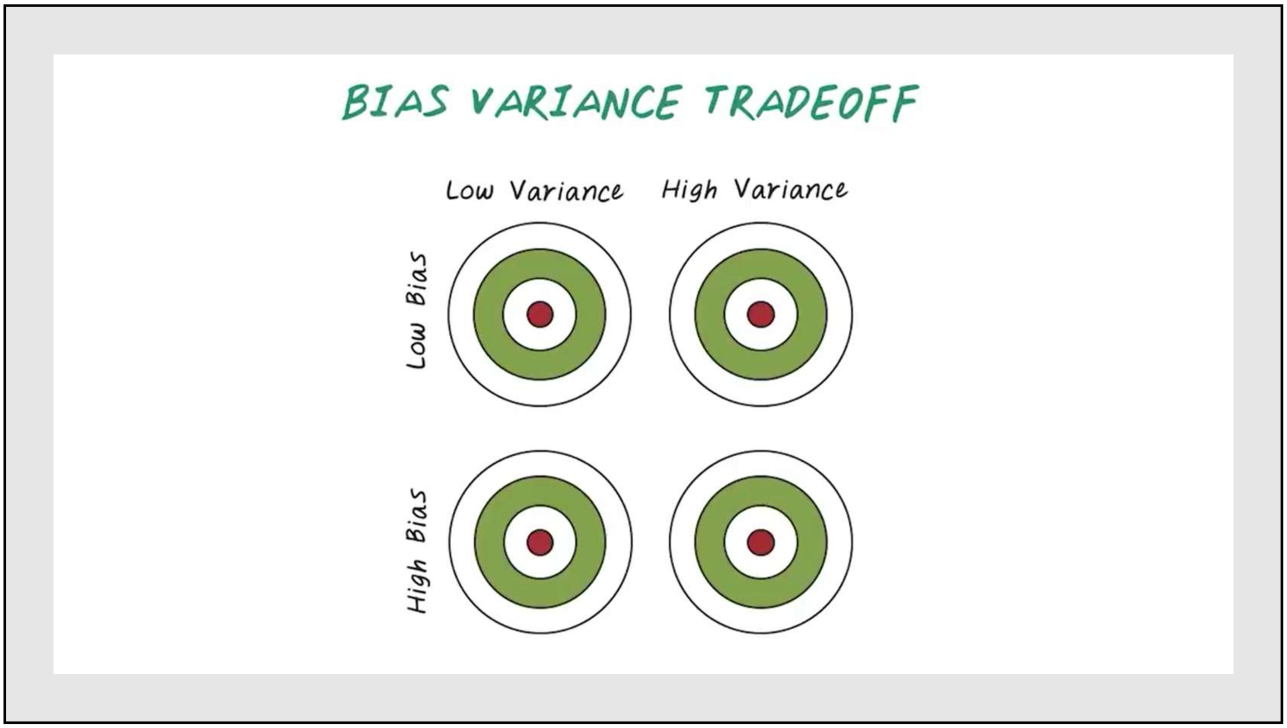
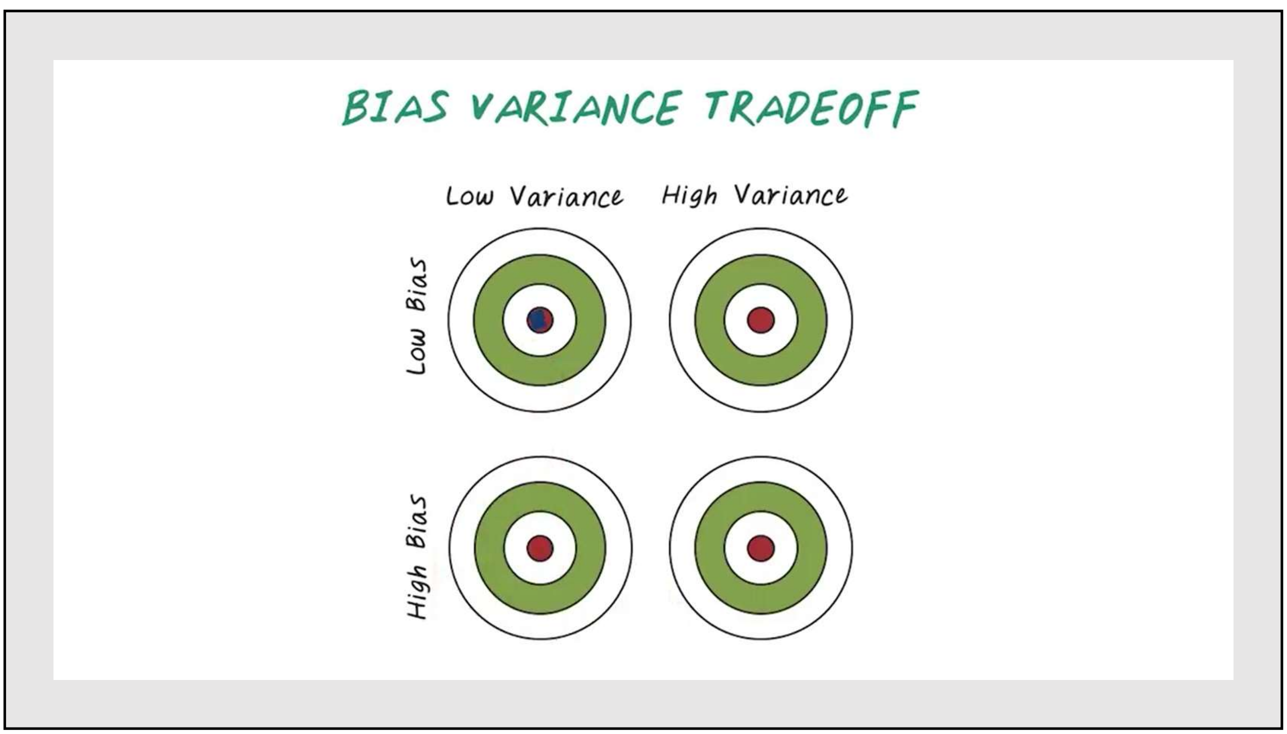

BIAS VARIANCE TRADEOFF
Low Variance
High Variance
Low Bias
High Bias

3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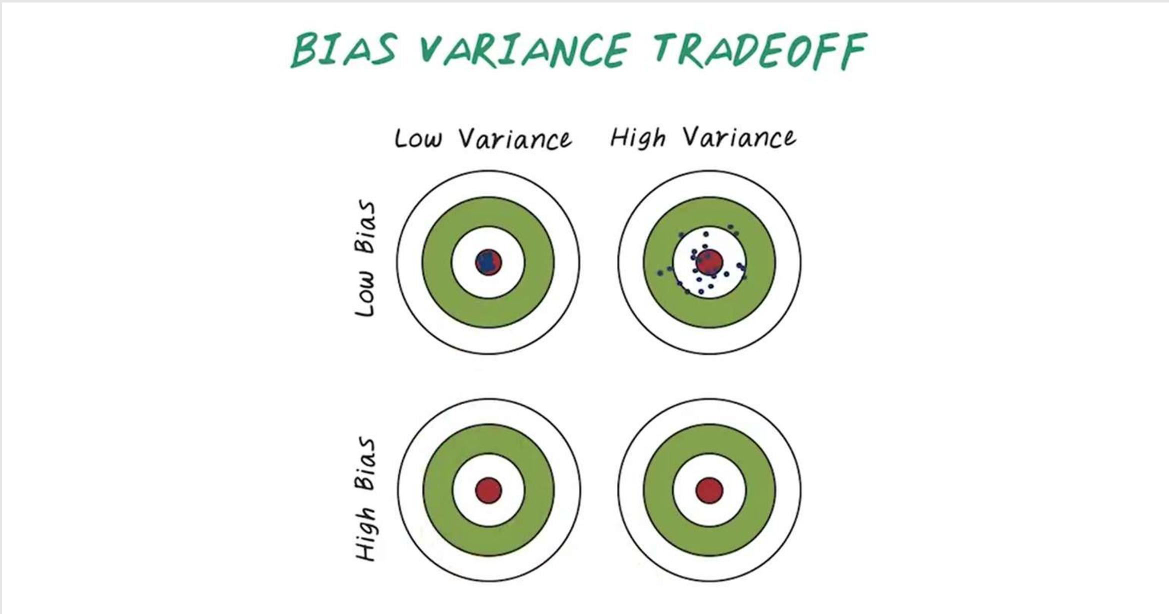
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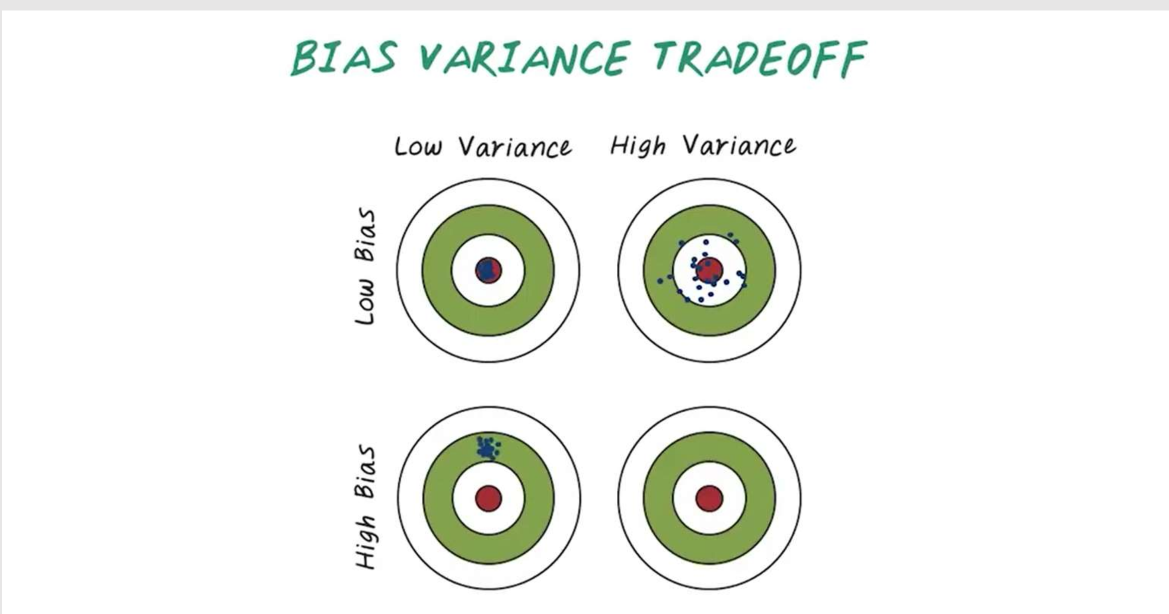
BIAS VARIANCE TRADEOFF
Low Variance
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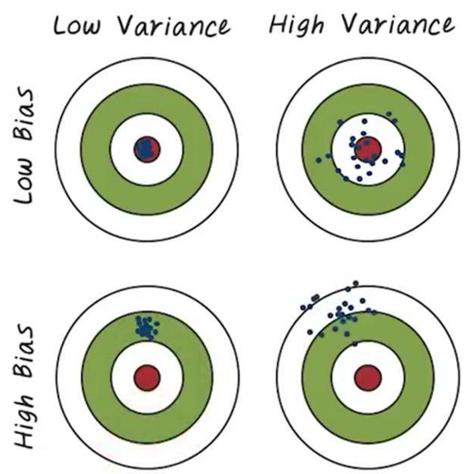
BIAS VARIANCE TRADEOFF



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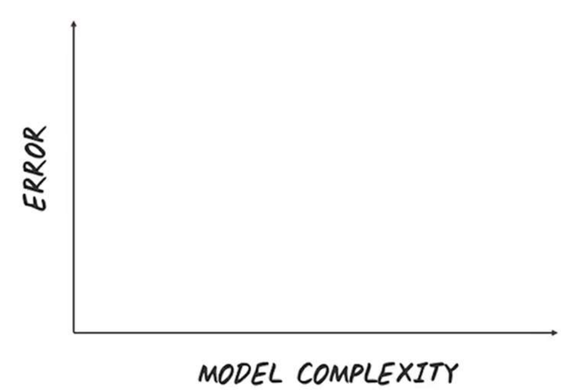
BIAS VARIANCE TRADEOFF



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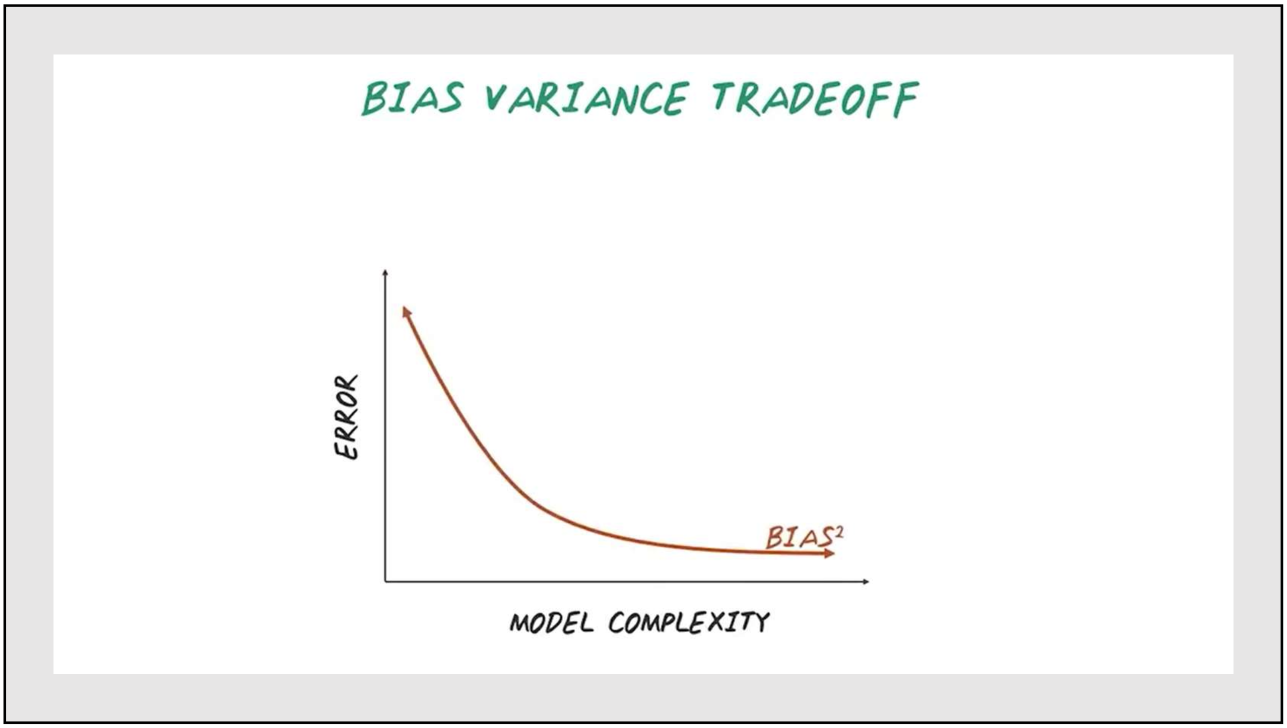
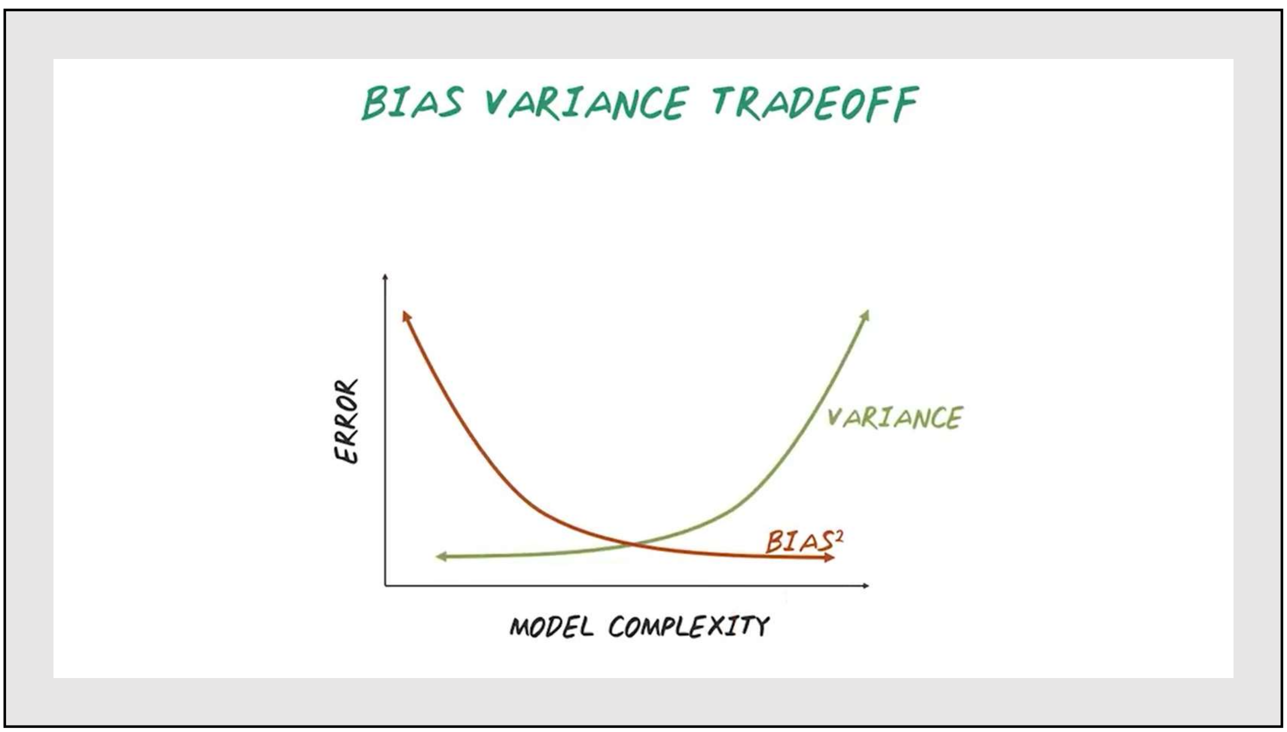

BIAS VARIANCE TRADEOFF
ERROR
BIAS²
MODEL COMPLEXITY

9


BIAS VARIANCE TRADEOFF
ERROR
VARIANCE
BIAS²
MODEL COMPLEXITY

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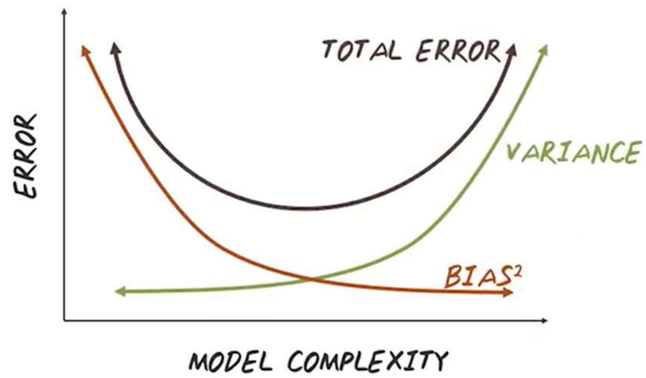
BIAS VARIANCE TRADEOFF



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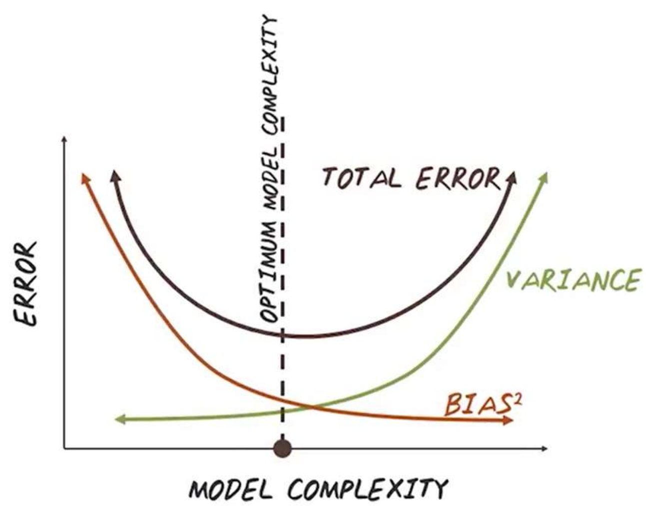
BIAS VARIANCE TRADEOFF



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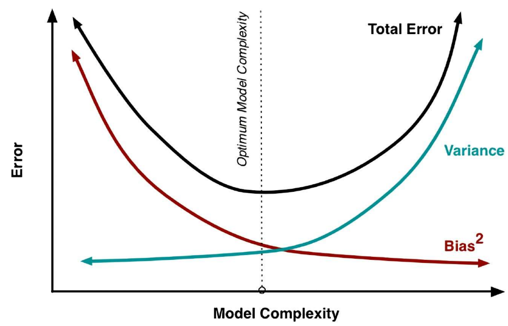
Regularization

- ▶ Trades **increased bias** for **reduced variance**
- ▶ Goal is to **minimize generalization error** despite using **large model family**



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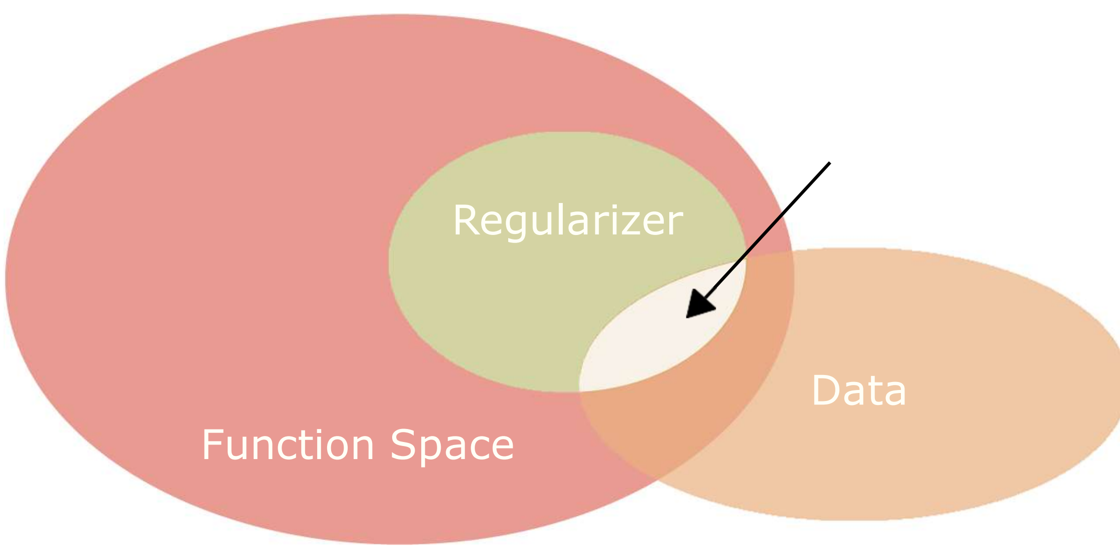
Function Space View



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Parameter Penalties

Parameter Penalties

Let $\mathcal{X}$ = (**X**, **y**) denote the dataset and **w** the model parameters. We can **limit the model capacity** by adding a parameter norm penalty $\mathcal{R}$ to the loss $\mathcal{L}$

$$\underbrace{\tilde{\mathcal{L}}(\mathcal{X}, \mathbf{w})}_{\text{Total Loss}} = \underbrace{\mathcal{L}(\mathcal{X}, \mathbf{w})}_{\text{Original Loss}} + \alpha \underbrace{\mathcal{R}(\mathbf{w})}_{\text{Regularizer}}$$

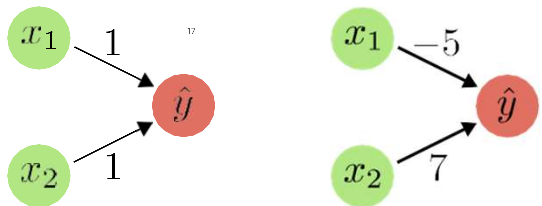
where $\alpha \in [0, \infty)$ controls the **strength of the regularizer.**

- ► $\mathcal{R}$ quantifies the size of the parameters / model capacity
- ► Minimizing $\tilde{\mathcal{L}}$ will decrease both $\mathcal{L}$ and $\mathcal{R}$
- ► Typically, $\mathcal{R}$ is applied only to the weights (not the bias) of the affine layers
- ► Often, $\mathcal{R}$ drives weights closer to the origin (in absence of prior knowledge)

Parameter Penalties

Why do we want the weights/inputs to be small?

- Suppose x_1 and x_2 are nearly identical.
  The following two networks make nearly the **same predictions:**



- But the second network might predict wrongly if the test distribution is slightly different (x_1 and x_2 match less closely) ⇒ **Worse generalization**

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L2 Regularization

- Weight Decay

$$Cost\ function = Loss + \frac{\lambda}{2m} * \sum \|w\|^2$$

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L1 Regularization

$$Cost\,function = Loss + \frac{\lambda}{2m} * \sum \|w\|$$

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L2 vs. L1 Regularization

Furry

Has two eyes

Has a tail

Has paws

Has two ears

L2 regularization will take all information into account to make decisions

Slide credits: Leal-Taixe and Niessner, I2DL.

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L2 vs. L1 Regularization

Furry

Has two eyes

Has a tail

Has paws

Has two ears

L1 regularization will focus all the attention to a few key features

Slide credits: Leal-Taixe and Niessner, I2DL.

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Early Stopping

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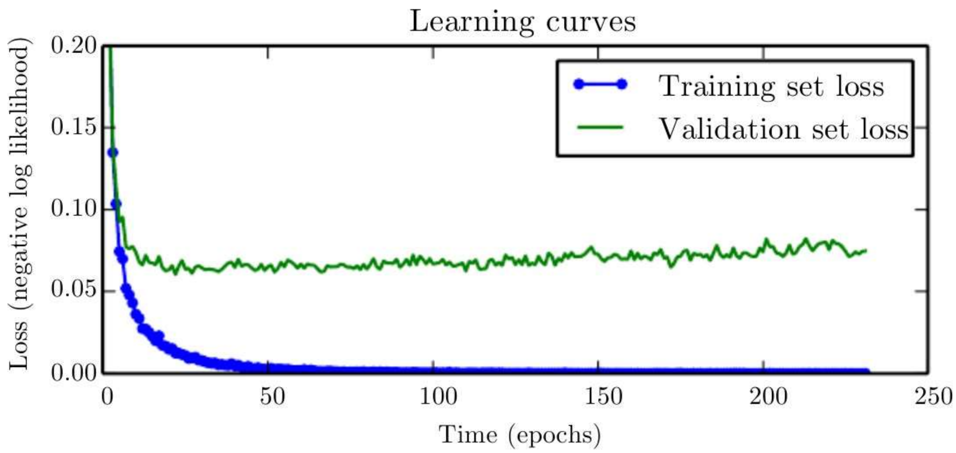
Early Stopping



- While training error decreases over time, validation error starts increasing again
- Thus: train for some time and **return parameters with lowest validation error**

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Early Stopping

Early Stopping:

- Most commonly used form of regularization in deep learning
- Effective, simple and computationally efficient form of regularization
- Training time can be viewed as hyperparameter ⇒ model selection problem
- Efficient as a single training run tests all hyperparameters (unlike weight decay)
- Only cost: periodically evaluate validation error on validation set
- Validation set can be small, and evaluation less frequently

Remark: If little training data is available, one can perform a second training phase where the model is retrained from scratch on all training data using the same number of training iterations determined by the early stopping procedure

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Ensemble Methods

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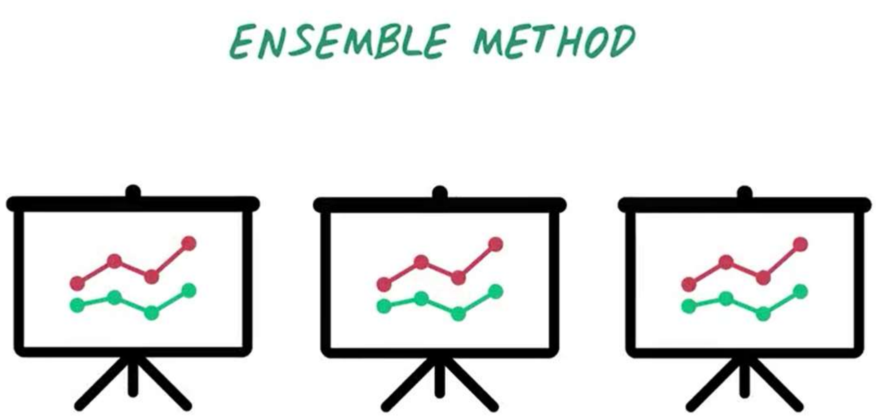

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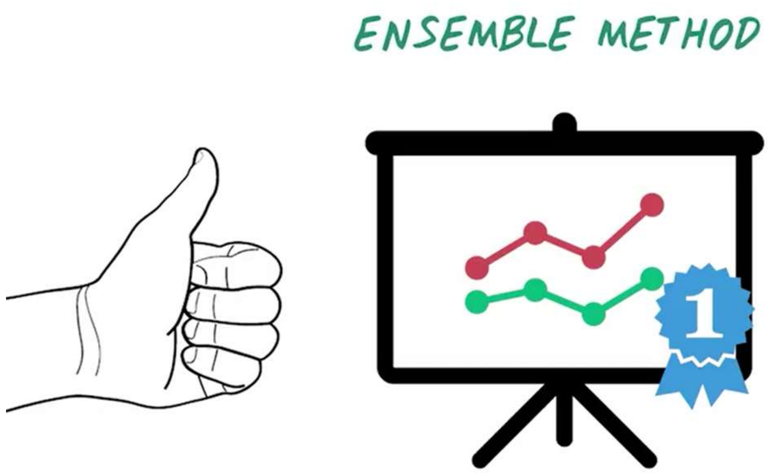

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Traditionally ensemble method

- Refer to combining models using different algorithms.
- Ensemble model often outperform other classification method.

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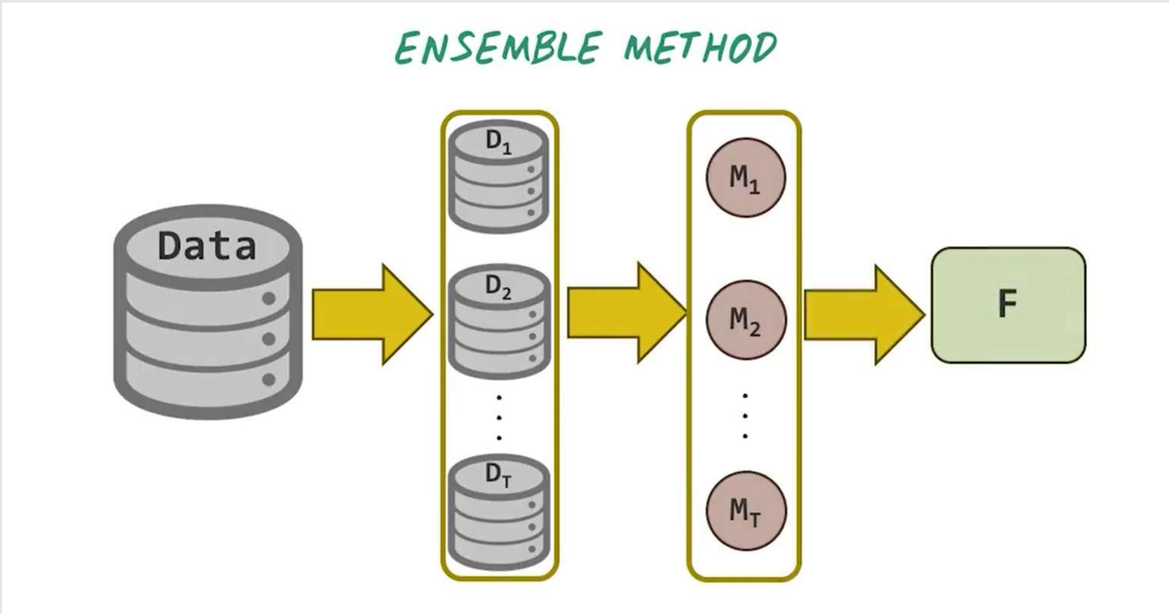

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BAIS Vs Variance

- Bias refers to the prediction error due to the wrong modeling assumption.
- The variance on the other hand, refers to the error from the sensitivity to small fluctuation in the training data set.
- Ideally, we want a model to have both low variance and low bias.

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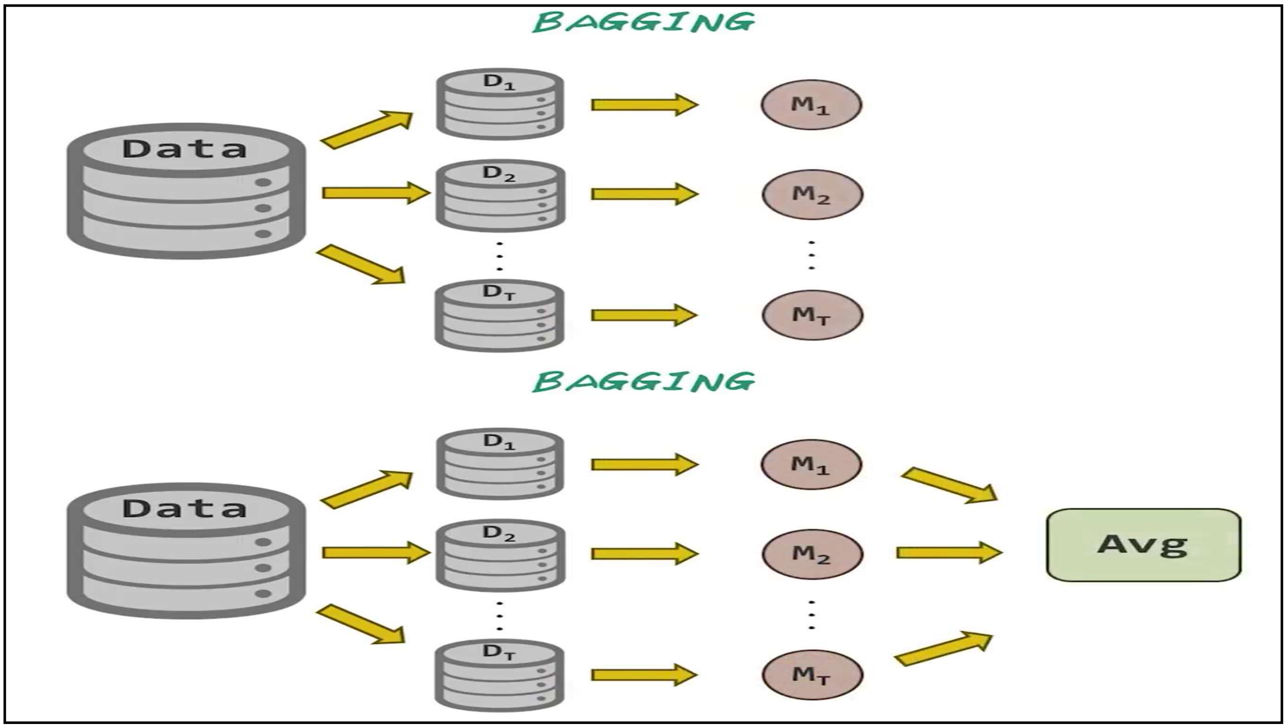
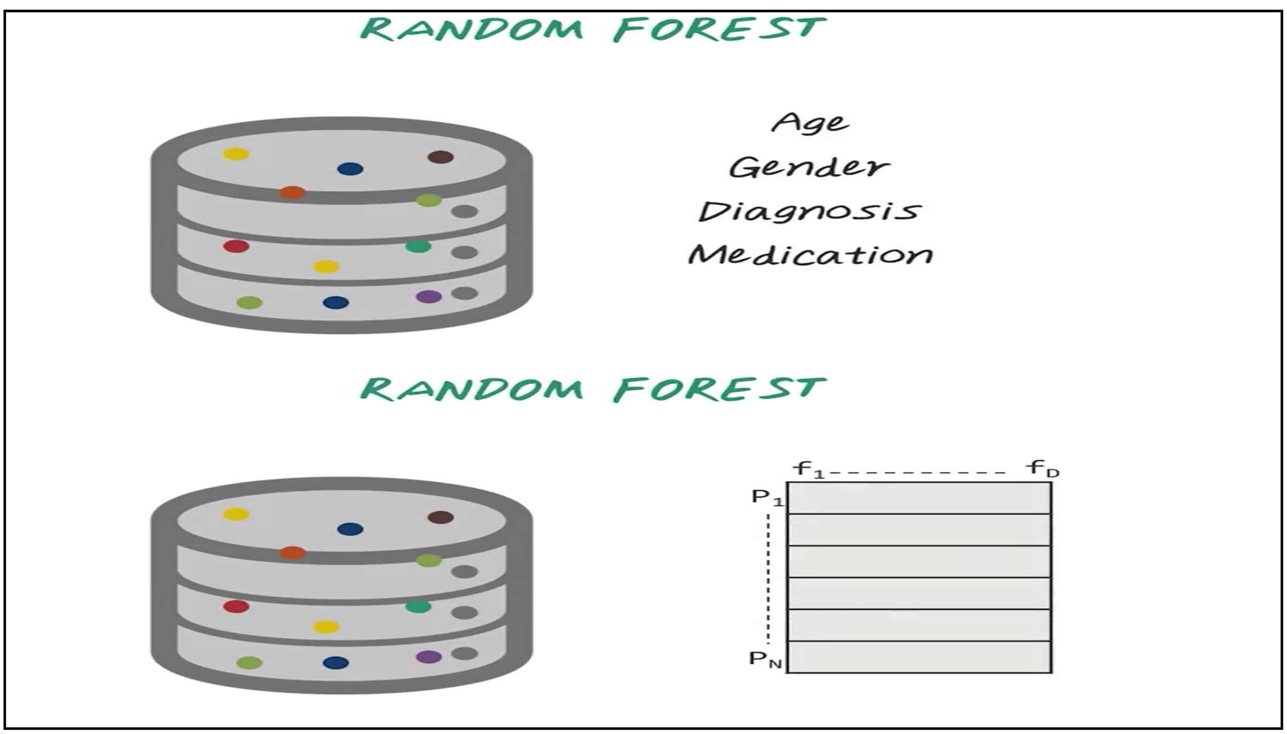

BAGGING
Data
D1
D2
DT
M1
M2
MT
BAGGING
Data
D1
D2
DT
M1
M2
MT
Avg

31


RANDOM FOREST
Age
Gender
Diagnosis
Medication
RANDOM FOREST
f1
fD
P1
PN

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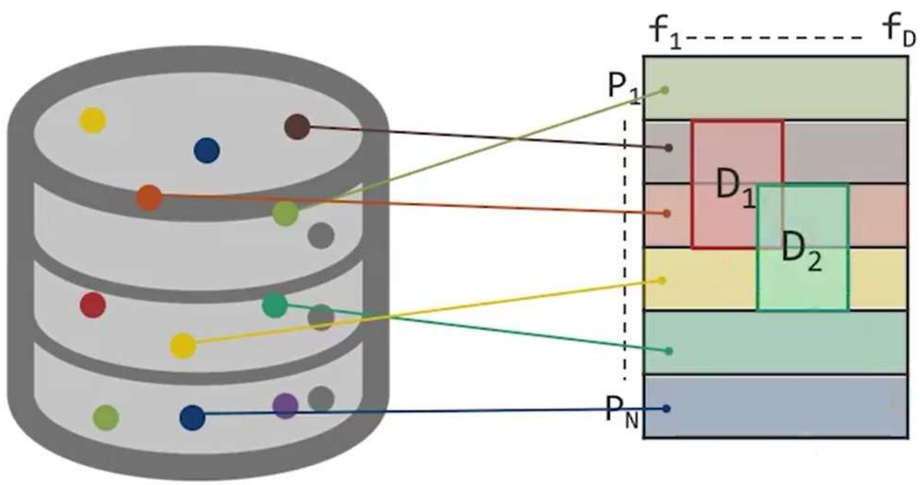
RANDOM FOREST



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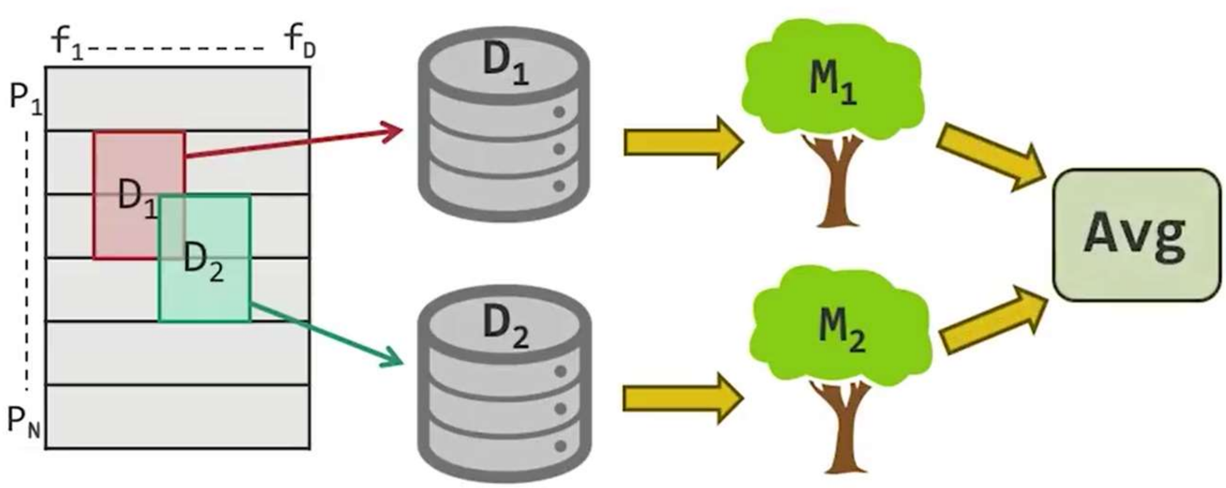
RANDOM FOREST



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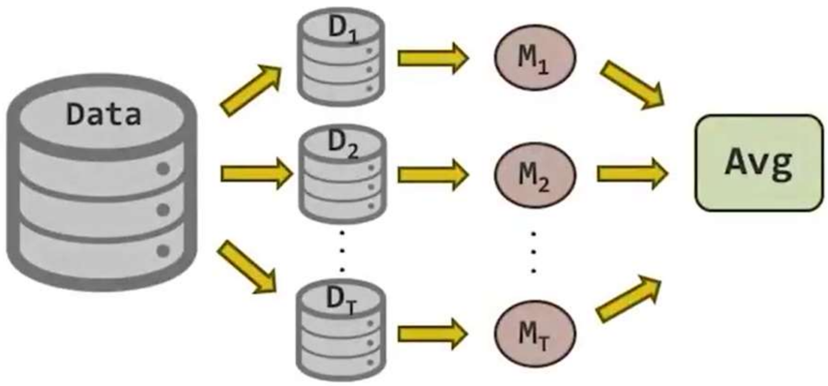
WHY BAGGING WORKS

Reduce Variance Without Increasing Bias

$$Var(\bar{X}) = \frac{Var(X)}{T} \quad \text{(when X are independent)}$$



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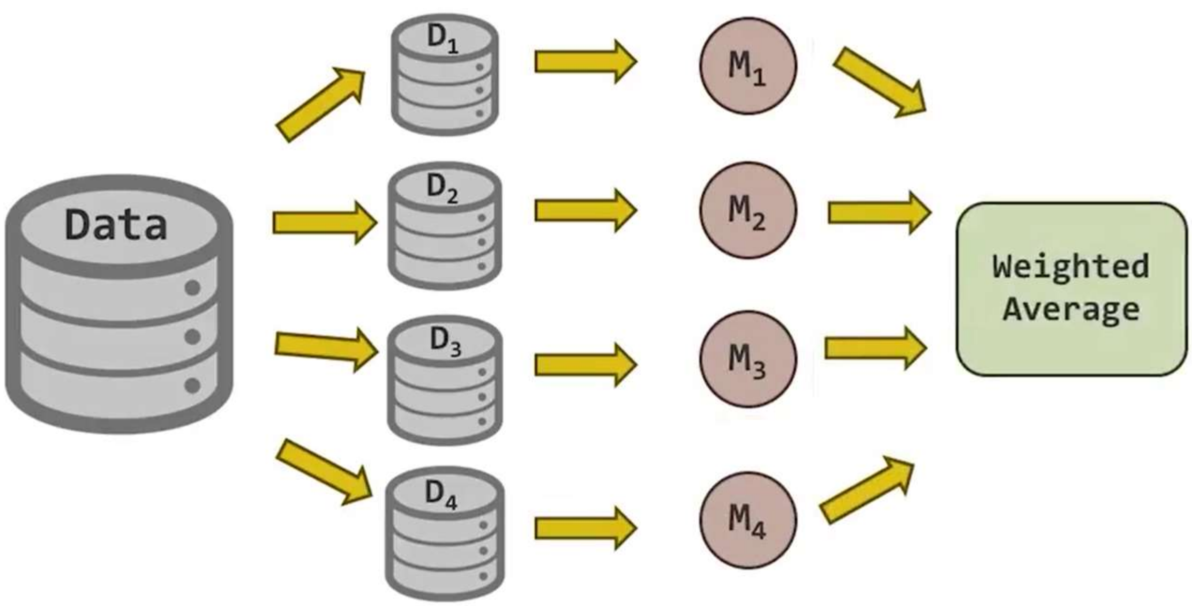
BOOSTING



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BAGGING VS. BOOSTING QUIZ

	BAGGING	BOOSTING
COMBINING METHOD	✓ Simple average ○ Weighted average	○ Simple average ✓ Weighted average
PARALLEL COMPUTING	○ Hard ✓ Easy	✓ Hard ○ Easy
SENSITIVE TO NOISE	✓ Less ○ More	○ Less ✓ More
ACCURACY	✓ Good in all cases ○ Better in most cases	○ Good in all cases ○ Better in most cases

BAGGING VS. BOOSTING QUIZ

	BAGGING	BOOSTING
COMBINING METHOD	✓ Simple average ○ Weighted average	○ Simple average ✓ Weighted average
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