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The Two Body Problem and the Effects of Mass and Eccentricity

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ABSTRACT

The Two-Body Problem was first introduced by Isaac Newton in his book Philosophiæ Naturalis Principia Mathematica (1687), He described the motion of two objects in motion while they were under the influence of Newtonian Gravity and provided a way to solve one body and two body problems. However, three body and n-body problems are still unsolved to this day. In this paper, it will be explained how to reduce a two body problem into a one body problem. It will then be explained how to solve those equations with varying mass ratios and eccentricities and the effect of mass and eccentricity on orbits will be shown through graphs. We will also explain the importance of this work and potential future developments.

Keywords: Two-Body Problem; Newtonian Gravity; orbits; eccentricity

INTRODUCTION

The two body problem originally seen in Newton's work named Newton's Philosophiae Naturalis Principia Mathematica, written in 1687. He proposed notions of celestial bodies and how those bodies gravity interacted with each other. He solved one body and second body problems but failed to solve 3rd body problems as well as nth body problems. When three bodies orbit each other, the resulting dynamical system is chaotic for most initial conditions. Generally these problems only apply in astrology and closed systems, it might not seem important at first glance but it helps us understand natural phenomenon. If we only consider the Moon and Earth, they would constitute a two body problem as well as the Earth and Sun. The two body us to calculate and predict their orbits with reasonable accuracy, which is crucial for tasks like satellite tracking, planetary motion studies, and understanding the solar system. The scientific importance of this problem is it allows scientists a base line in which to observe and predict systems.Newton (1687)

REDUCING THE TWO-BODY PROBLEM TO A ONE-BODY PROBLEM

The approach how to reduce the Two-Body Problem to a one-body problem is simply turning it into two one body problems by having a central point and concatenating the masses. We have to set the relative positional vectors to one single vector, allowing us to analyze their positions relative to each other. $\overrightarrow{r_1}$ + F = $\overrightarrow{r}$ afterwards we need to calculate the center of mass, which we denote with $\overrightarrow{R} = \frac{\overrightarrow{r_1}m_1+\overrightarrow{r_2}m_2}{m_1+m_2}$ it allows us to separate into the motion of the center of mass and the motion of the bodies relative to the center of mass. We also need to calculate the reduced mass as one averaged mass.μ=$\frac{m_1 m_2}{m_1+m_2}$ then calculate the force acting on this reduced mass $\overrightarrow{F}$= -$\frac{Gm_1m_2}{r^2}\hat{r}$ then we combine them into one function $\mu\frac{d^2\overrightarrow{r}}{dt^2}$=$\overrightarrow{F}$ allowing us to use the same methods we would apply to a single body moving under a central force using simple integration by parts.Banjara (2013)

SOLVING THE TWO BODY PROBLEM

The initial conditions chosen in order to solve the Two Body Problem were for the initial x position to be -1, the initial y position to be 0, the x component of the initial velocity was 0, and the y component of the initial velocity

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was $\sqrt{1+e}$ We tested the eccentricities 0, 0.25, 0.5, and 0.75 and the mass ratios 1:1, 1:2, 1:4, and 1:16. The period of time a body in orbit would take was calculated using the formula $\frac{2\pi}{(1-e)^{3/2}}$ to create the time interval of the orbit. This information was then all plugged into the SciPy function solve_ivp in order to find the solutions of the Two Body System of Equations. The method used by solve_ivp was Runge-Kutta of the fifth order. Then in order to find the x position for the orbit of 1 mass we did $\frac{m_2}{\text{total mass}} \cdot x$ where x in this case is the x component found by solving the two body system of equations. To find the y position, the same thing was done $\frac{m_2}{\text{total mass}} \cdot$ but the y solution was used. We then did the same thing for the other mass in orbit but in this case we used $\frac{m_1}{\text{total mass}} \cdot x$ and $\frac{m_1}{\text{total mass}} \cdot y$. These formulas were derived from the formula for Newtonian Gravity $F = G\frac{m_1 m_2}{r^2}$ Finally, the x and y positions for both orbiting masses was plotted to obtain the following figures. As can be seen the greater the difference in masses, the greater the difference in circumferences between the orbits of two masses. Moreover, the higher the eccentricity, the more elliptical the shape of the orbit.

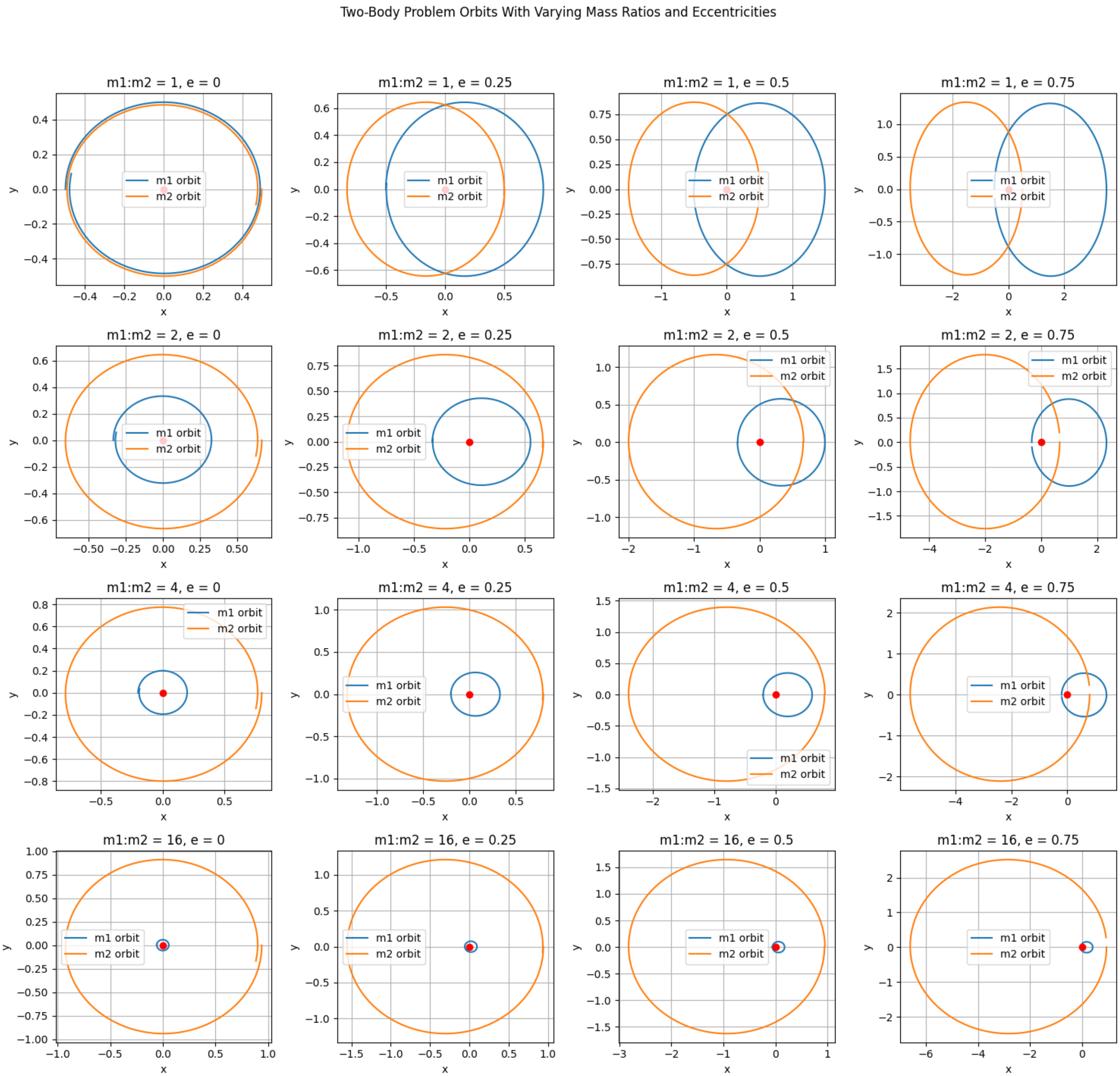

Figure 1. The orbits of two bodies which were solved using the above method.

SUMMARY / CONCLUSION

An isolated dynamical system with two independently moving objects exerting forces on each other is termed a *Two-Body Problem*. We can *reduce* the mass of the isolated system of two interacting objects to that of an equivalent system consisting of a single point object moving in a fixed force field by substituting both respective centers of mass into the equations of motion of our two objects. Thus we find that both equations yield $\mu \frac{d^2 r}{dt^2} = \mathbf{f}$, where $\mu = \frac{m_1 m_2}{m_1 + m_2}$. In the reduced problem, the force $\mathbf{f}$ is the same acting on both objects as in the *Two-Body Problem*, but the mass, μ, is *reduced*. The solution to the *Two-Body Problem* was done in Jupyter notebooks using the SciPy solve.ivp and *Runge-Kutta* method of the fifth order. Demonstrated in Figure 1, the greater the difference in masses, the greater the difference in circumferences between the orbits of two masses. Moreover, the higher the eccentricity, the more elliptical the shape of the orbit. There are many future practical applications of the *Two-Body Problem* method in industry, such as spacecraft navigation, particle interaction modeling in nuclear engineering, nanopartical interactions, and many others. In the future, we could expand on these analytical methods to try and target the Three Body Problem and/or the Nth Body Problem.Fitzpatrick (2005)

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