BIOMETRICS 000, 000–000 DOI: 000
000 0000

Supporting Information for "Bayesian Functional Covariance Regression"

John Shamshoian[1], Damla Şentürk[1], Shafali Jeste[2], and Donatello Telesca[1,*]

[1]Department of Biostatistics, University of California, Los Angeles, California 90095, U.S.A.

[2] Department of Psychiatry and Biobehavioral Sciences, University of California, Los Angeles, California 90095, U.S.A.

*email: donatello.telesca@ucla.edu

This paper has been submitted for consideration for publication in *Biometrics*

1. Web Appendix A: Markov-Chain Monte Carlo Sampling Algorithm

In this section we give a detailed Markov-Chain Monte Carlo (MCMC) algorithm to sample from the posterior. Let N be the number of independent functional responses and assume all response functions are observed on a common grid $T = \{t_1, \ldots, t_n\}$. Let B be an $n \times p$ matrix with $B_{ij} = b_j(t_i)$. Let X be an $N \times r(d_1)$ matrix with row i equal to $\tilde{\boldsymbol{X}}(\boldsymbol{x}_i)$. Let Y be an $N \times n$ matrix with $Y_{ij} = y_i(t_j)$, so that each row represents one discretized functional response. Let Γ_j be an $N \times N$ diagonal matrix with rth diagonal element equal to η_{rj} for $j = 1, \ldots, k$.

(1) Update β:

Let $\Omega_r = \tau_{1xr}\tilde{K}_r + \tau_{1tr}\tilde{K}$ if $p_r > 1$. Otherwise set $\Omega_r = \tau_{1tr}K$. Construct $\Omega = \mathrm{blkdiag}(\Omega_1, \ldots, \Omega_R)$.

Let $C = \sigma^{-2}X^\top X \otimes B^\top B + \Omega$

Let $A = \sigma^{-2}\mathrm{vec}[\{B^\top Y^\top - B^\top B(\sum_{j=1}^{k} \Lambda_j X^\top \Gamma_j)\}X]$

Sample $\mathrm{vec}(\beta) \sim N(C^{-1}A, C^{-1})$

(2) Update Λ_j:

Let $\Omega_r = \tau_{2xr}\tilde{K}_r + \tau_{2tr}\tilde{K} + \tau_{rj}^{*}\phi_{rj}$ if $p_r > 1$. Otherwise set $\Omega_r = \tau_{2tr}K + \tau_{rj}^{*}\phi_{rj}$.

Construct $\Omega = \mathrm{blkdiag}(\Omega_1, \ldots, \Omega_R)$

Let $C = \sigma^{-2}X^\top \Gamma_j^2 X \otimes B^\top B + \Omega$

Let $A = \sigma^{-2}\mathrm{vec}[\{B^\top Y^\top - B^\top B(\beta + \sum_{j' \neq j} \Lambda_{j'} X^\top \Gamma_{j'})\}\Gamma_j X]$

Sample $\Lambda_j \sim N(C^{-1}A, C^{-1})$

(3) Update η_{ij}, $j = 1, \ldots, k$:

Let $\boldsymbol{\eta}_i = (\eta_{i1}, \ldots, \eta_{ik})$.

Let $\ddot{X}_i$ be a $p \times k$ matrix with column j equal to $\Lambda_j \tilde{\boldsymbol{X}}(\boldsymbol{x}_i)$.

Let $C = \sigma^{-2}\ddot{X}_i^\top B^\top B \ddot{X}_i + I_k$, where I_k is the $k \times k$ identity matrix.

Let $A = \sigma^{-2}\ddot{X}_i^\top\{B^\top Y_{i\cdot} - B^\top B\beta\tilde{\boldsymbol{X}}(\boldsymbol{x}_i)\}$

Sample $\boldsymbol{\eta}_i \sim N(C^{-1}A, C^{-1})$