Contents

1 Introduction

In aquaponic systems, it is commonly reported that certain plant nutrients are lacking. Furthermore, it is oftentimes observed that some compounds reach a concentration plateau even though they are continuously supplied to the system. Thus, there has to be a constraint that is controlling the concentration of these compounds in the water. If microbial uptake is neglected, two potential constraints are

- the chemical satiation concentration
- nutrient depletion via water exchange

. It was found that the satiation concentrations of most of the plant nutrients are about three orders of magnitude higher than the concentrations observed in aquaponic systems. Thus, the water exchange might be what the right thing to look for in some cases.

2 Equations

2.1 General considerations

In general, the mass balance of an aquaculture system can be described considering the mass inputs m_{in} and outputs m_{out}.

$$m_{acc} = m_{in} - m_{out} \tag{1}$$

Depending on the values of M_{in} and m_{out} the resulting scenarios in terms of the mass m_{acc} of a compound in the system are

- $m_{in} > m_{out}$: Accumulation
- $m_{in} = m_{out}$: Balance
- $m_{in} < m_{out}$: Depletion

The accumulation rate over time can be described as

$$\Delta m_{acc} = \frac{dV_{tot}c}{dt} = V_{tot}\frac{dc}{dt} \tag{2}$$

, due to the constant volume V_{tot} of the system. We assume that a substance (for instance a nutrient) is added continuously to the system at a constant mass. Thus, we assume a constant percentage water exchange of the total system volume, a constant mass input of feed and other substances. We can then write

$$V_{tot}\frac{dc}{dt} = Q_{in}c_{in} - Q_{out}c_{out} \tag{3}$$

, with Q_{in} and Q_{out} describing the exchange volume per day and thus a volume flow per time unit and c_{in} and c_{out} as the concentration of the substance in the liquid entering or leaving the system, respectively.

2.2 Scenario 1: Accumulation

Considering a system with a water discharge of 0%, which would be the case if only evaporation water was replaced, Equation 3 would be simplified to a constant mass input:

$$V_{tot}\frac{dc}{dt} = Q_{in}c_{in} \tag{4}$$

The constant input would eventually lead to a linear increase in the concentration of the targeted substance.

2.3 Scenario 2: Balance

The starting point of the inflow-outflow mass balance is Equation 3. Because we are replacing the amount of water that we remove with the same amount of exchange water, we can write

$$Q_{in} = Q_{out} = Q \tag{5}$$

. We can then substitute equation 5 into equation 3, divide by Q and obtain

$$\frac{V_{tot}}{Q}\frac{dc}{dt} = c_{in} - c_{out} \tag{6}$$

. Eventually, this equation can be integrated and rearranged with the following steps (see also Howe et al., Principles of Water Treatment).

$$\frac{Q}{V_{tot}}\int_0^t dt = \int_0^c \frac{dc}{c_{in} - c_{out}} \tag{7}$$

$$\frac{1}{-1}\ln|c_{in} - c_{out}|_0^c = \frac{Q}{V}t \tag{8}$$

$$-\ln|c_{in} - c_{out}| - (-\ln|c_{in} - c_{out}|) = \frac{Q}{V}t \tag{9}$$

$$-\ln(c_{in} - c_{out}) + \ln(c_{in}) = -\frac{Q}{V}t \tag{10}$$

$$\ln(\frac{c_{in} - c_{out}}{c_{in}}) = -\frac{Q}{V}t \tag{11}$$

$$c_{in} - c_{out} = c_{in} \cdot \exp^{-\frac{Q}{V}t} \tag{12}$$

$$1 - \frac{c_{out}}{c_{in}} = \exp^{-\frac{Q}{V}t} \tag{13}$$

$$\frac{c_{out}}{c_{in}} = 1 - \exp^{-\frac{Q}{V}t} \tag{14}$$

The expression $\frac{Q}{V}$ in the equation is denoting for the **hydraulic retention time (HRT)**

$$\frac{Q}{V} = \tau \tag{15}$$

, so that equation 15 can be substituted into 14, resulting in

$$\frac{c_{out}}{c_{in}} = 1 - \exp^{-\frac{t}{\tau}} \tag{16}$$

. Equation 16 can now be rearranged and gives

$$c_{out} = c_{in}(1 - \exp^{-\frac{t}{\tau}}) \tag{17}$$

. This model is used in chemical engineering and is generally known as the **Continuous Flow Stirred Tank Reactor (CFSTR)** model. As shown in Figure 1, the model approaches 1 for $\lim_{t\to\infty} f(t)$.

The input routes of nutrients into aquaculture systems are

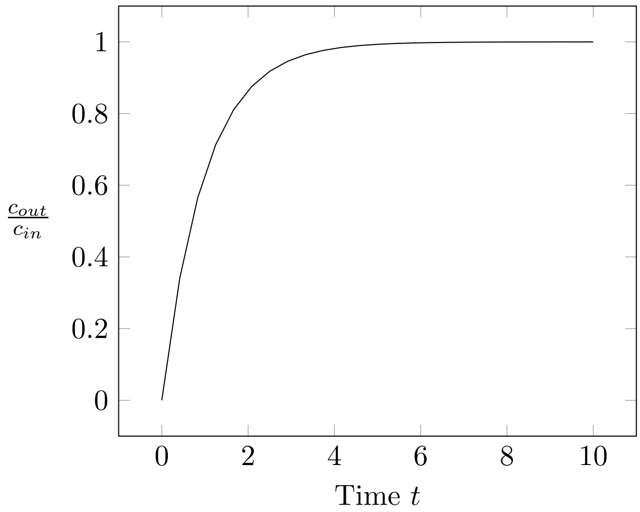

Figure 1: Asymptotic behaviour of the CFSTR model for $\lim_{t\to\infty} f(t)$.

1. Source water
2. Fish feed
3. "Auxiliary substances" for water management

. The input concentration c_{in} can therefore be rewritten as

$$c_{in} = \frac{m_{in}}{V_{in}} = \frac{m_w + m_f + m_a}{V_{\text{in}}} \tag{18}$$

, with m_w, m_f and m_a denoting for the nutrient mass supplied by water, feed and auxiliary substances, respectively.

It is usually stated that the main proportion of nutrients enters the system via the fish feed. However, when having a closer look at the actual contribution of the feed and water to the total daily input (see **??**), it becomes obvious that this claim is actually lacking any basis. The variables controlling the proportion of the nutrient that is provided by either feed or source water are the stocking density (and thus the biomass), the feeding rate, the concentration and mass fraction of the nutrient in the water and the feed, respectively, and the water exchange rate.

3 Assumptions

Let it be assumed that the water in the system contains a concentration $c_{NUT,\mathrm{H_2O}}$ of a nutrient. On a daily base, the amount $m_{NUT,feed}$ is introduced to the system via feed. The additional nutrients are homogeneously dissolved in the system water, thus adding

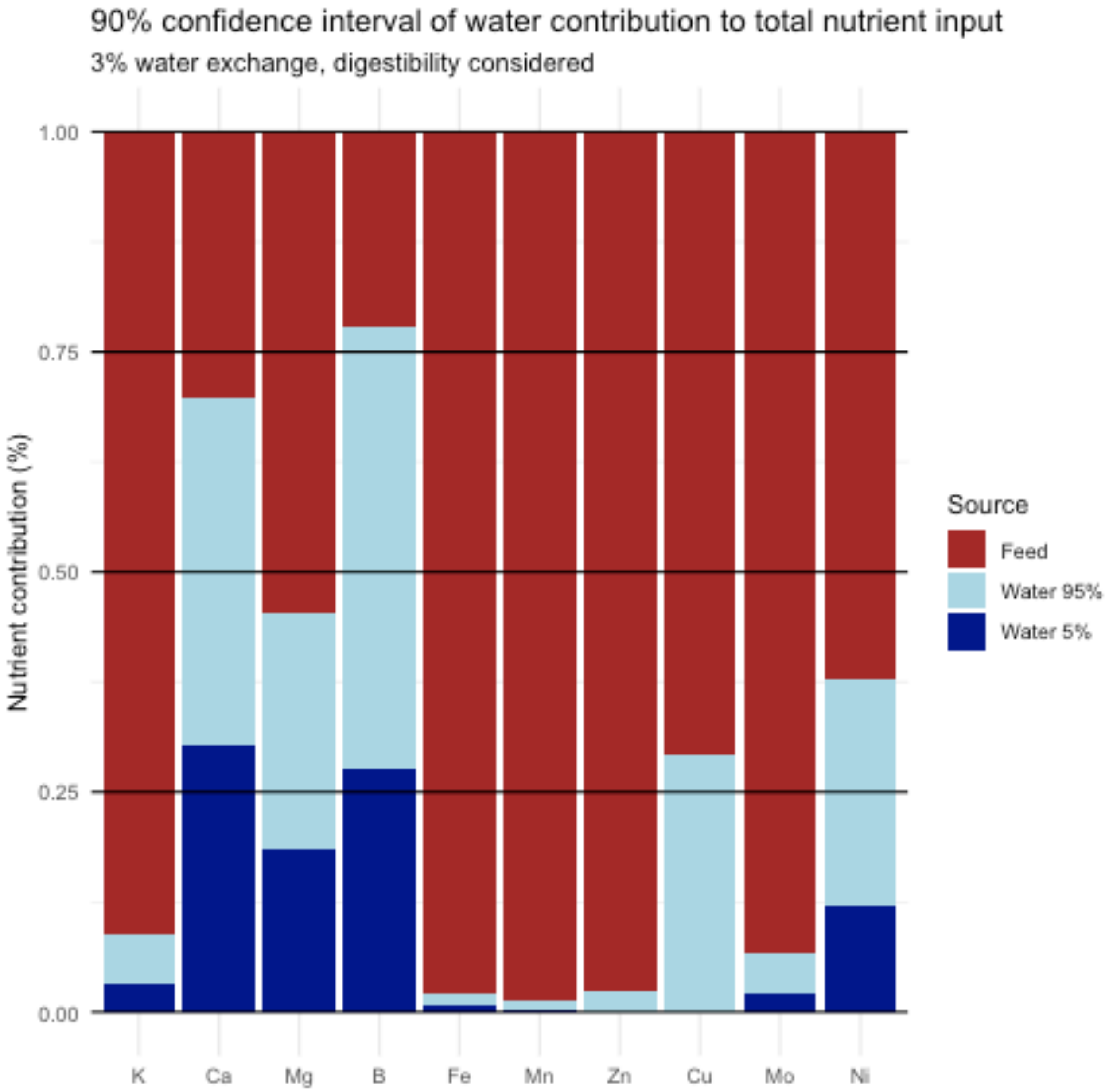

Figure 2: Percentage of plant nutrients originating from fish feed and source water under the assumption that 3% of the system water evaporated and was replaced by tap water (no nutrient outflow). Light blue color indicates the 90% confidence interval of the corresponding concentrations in tap water. Tap water data obtained from 26 municipal tap water suppliers from 12 countries around the world. Average nutrient composition of 16 commercial fish feeds was used, regardless the fish species. Nutrient digestibility and retention in fish was considered, assuming an apparent digestibility of 50% for each compound.

Table 1: Model inputs and descriptions.

Abbreviation	Unit	Description
V_{tot}	L	Total volume of system
V_r	L	Rearing volume of system
V_{ex}	$\mathrm{L\,d^{-1}}$	Volume of daily exchanged water
ρ_{fish}	$\mathrm{kg\,m^{-3}}$	Stocking density of fish
m_{fish}	kg	Total biomass of fish in the system
m_{feed}	$\mathrm{kg\,d^{-1}}$	Total mass of feed fed daily
m_{NUT}	$\mathrm{mg\,d^{-1}}$	Daily mass input of nutrient
r_{ex}	$\%\,\mathrm{d^{-1}}$	Daily water exchange rate
r_{feed}	$\%\,\mathrm{d^{-1}}$	Daily feeding rate
$c_{NUT,\mathrm{H_2O}}$	$\mathrm{mg\,L^{-1}}$	Concentration of compound in system water
c_{in}	$\mathrm{mg\,L^{-1}}$	Concentration of compound in source water
ω	$\mathrm{mg\,kg^{-1}}$	Mass fraction of compound in feed

$\frac{m_{NUT,feed}}{V_{tot}} = c_{feed}$ to the system. The resulting concentration is $c + c_{feed} = c_{tot}$. For the water exchange, a proportion of the system water that is given by $r_{ex}V_{tot} = V_{ex}$ is removed, resulting in the mass $m_{ex} = c_{tot}V_{ex}$ leaving the system. The removed water is then replaced by tap water with the nutrient concentration $c_{NUT,\mathrm{H_2O}}$.
The initial assumptions used for the creation of **??** and for further calculations are shown in 2. These values are partly taken from literature.
The described behaviour can be modelled using a **Continuous Flow Stirred Tank Reactor (CFSTR)** model with stepwise addition of a conservative tracer.

Table 2: Initial assumptions.

Abbreviation	Assumptions
V_{tot}	15 000 L
V_r	10 000 L
V_{ex}	$4500\,\mathrm{L\,d^{-1}}$
ρ_{fish}	$25\,\mathrm{kg\,m^{-3}}$
m_{fish}	250 kg
m_{feed}	$5\,\mathrm{kg\,d^{-1}}$
r_{ex}	$3\,\%\,\mathrm{d^{-1}}$
r_{feed}	$2\,\%\,\mathrm{d^{-1}}$