part 2:

$\xi = \begin{pmatrix} v \\ w \end{pmatrix} \quad \hat{\xi} = \begin{pmatrix} \hat{w} & v \\ 0^T & 0 \end{pmatrix}$

$\therefore \hat{\xi}^2 = \begin{pmatrix} \hat{w} & v \\ 0^T & 0 \end{pmatrix}\begin{pmatrix} \hat{w} & v \\ 0^T & 0 \end{pmatrix} = \begin{pmatrix} \hat{w}^2 & \hat{w}v \\ 0^T & 0 \end{pmatrix}$

$\hat{\xi}^3 = \hat{\xi}^2\hat{\xi} = \begin{pmatrix} \hat{w}^2 & \hat{w}v \\ 0^T & 0 \end{pmatrix}\begin{pmatrix} \hat{w} & v \\ 0^T & 0 \end{pmatrix} = \begin{pmatrix} \hat{w}^3 & \hat{w}^2 v \\ 0^T & 0 \end{pmatrix}$

$\therefore \hat{\xi}^n = \begin{pmatrix} \hat{w}^n & \hat{w}^{n-1}v \\ 0^T & 0 \end{pmatrix}$

$\therefore \exp(\hat{\xi}) = \sum_{n=0}^{\infty} \frac{(\hat{\xi})^n}{n!} = I + \hat{\xi} + \frac{\hat{\xi}^2}{2!} + \cdots = \begin{pmatrix} I & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \hat{w} & v \\ 0^T & 0 \end{pmatrix} + \frac{1}{2!}\begin{pmatrix} \hat{w}^2 & \hat{w}v \\ 0^T & 0 \end{pmatrix} + \cdots$

$= \begin{pmatrix} I + \hat{w} + \frac{1}{2!}\hat{w} + \cdots & v + \frac{1}{2!}\hat{w}v + \frac{1}{3!}\hat{w}^2 v + \cdots \\ 0^T & 1 \end{pmatrix}$

$= \begin{pmatrix} \sum_{n=0}^{\infty} \frac{1}{n!}(\hat{w})^n & \sum_{n=0}^{\infty} \frac{1}{(n+1)!}(\hat{w})^n v \\ 0^T & 1 \end{pmatrix}$

for $\sum_{n=0}^{\infty} \frac{1}{(n+1)!}(\hat{w})^n v$: let $\hat{w} = \|\hat{w}\| \cdot \frac{\hat{w}}{\|\hat{w}\|} = \theta \cdot \hat{a}$ $\begin{cases} \theta = \|\hat{w}\| \\ \hat{a} = \frac{\hat{w}}{\|\hat{w}\|} \end{cases}$ $\quad \hat{a}^2 = aa^T - I$ $\quad \hat{a}^3 = -\hat{a}$

$\therefore = \sum_{n=0}^{\infty} \frac{(\theta\hat{a})^n}{(n+1)!} = I + \frac{1}{2!}\theta \cdot \hat{a} + \frac{1}{3!}\theta^2\hat{a}^2 + \cdots$

$= I + \frac{1}{\theta}\left(\frac{\theta^2}{2!}\hat{a} + \frac{\theta^4}{4!}\hat{a}^3 + \cdots\right) + \frac{1}{\theta}\left(\frac{\theta^3}{3!}\hat{a}^2 + \frac{\theta^5}{5!}\hat{a}^4 + \cdots\right)$

$= I + \frac{1}{\theta}\left(\frac{\theta^2}{2!} - \frac{\theta^4}{4!} + \cdots\right)\hat{a} + \frac{1}{\theta}\left(\frac{\theta^3}{3!} - \frac{\theta^5}{5!} \cdots\right)\hat{a}^2$

recall: $\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$

$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$

$= I + \frac{1}{\theta}(1-\cos\theta)\hat{a} + \frac{1}{\theta}(\theta - \sin\theta)\hat{a}^2$

$= I + \frac{1}{\theta}(1-\cos\theta) \cdot \frac{\hat{w}}{\theta} + \frac{1}{\theta}(\theta - \sin\theta) \cdot \frac{(\hat{w})^2}{\theta^2}$

$= I + \frac{1-\cos\theta}{\theta^2}\hat{w} + \frac{\theta - \sin\theta}{\theta^3}\hat{w}^2 = J$