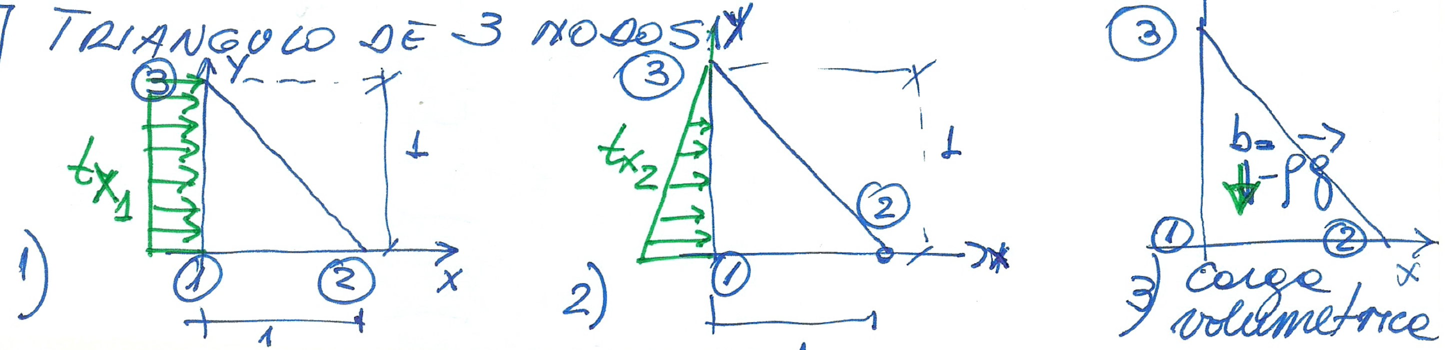
EJERCICIO 1

Calcular analíticamente en los elementos de las figuras el vector de fuerzas nodales equivalentes, para las siguientes cargas: 1) Presión uniforme en la dirección x; $t_x = \mathcal{H}$. 2) Presión triangular en x, $t_x = \mathcal{H}(1-y)$ y 3) carga distribuida uniforme por unidad de volumen, $b_y = -\rho g$.
$\mathcal{H}$: valor constante

a] TRIANGULO DE 3 NODOS.



NOTA: Coinciden las coordenadas naturales con x, y.

$\Rightarrow N_1 = 1 - x - y.$
$N_2 = x.$
$N_3 = y.$
Funciones de forma.

e = espesor.

1) $t = \begin{bmatrix} \mathcal{H} \\ 0 \end{bmatrix}$ en nodos 1-3

$$\Rightarrow f_{1,2} = \oint_l \underbrace{[N^T]}_{\text{Matriz func. de forma}} \underbrace{\underline{t}}_{\text{carga}} \underbrace{e}_{\text{espesor}} \, dl.$$

l (long. lado)

↓ ojo con el espesor.

$$f_b = \int_A \underbrace{[N^T]}_{\text{Funciones de forma}} \underline{b} \, dA$$

$\underset{\text{constante}}{\underline{t_1}} = \begin{bmatrix} \sigma \\ 0 \end{bmatrix}$ $\qquad$ $\underset{\text{triangular.}}{\underline{t_2}} = \begin{bmatrix} \sigma(1-\sigma) \\ 0 \end{bmatrix}$

1) $f_{1,2} = \oint_e [N^T] \underline{t}\, e\, dl.$

<u>Nodo 1</u> $f^1_{t_1x} = \int_{1-3} N_1\Big]_{x=0} \sigma e\, dy = \int_0^1 (1-\cancel{y}) \sigma e \underset{dy.}{\underbrace{dl}} = \int_0^1 (1-y)\sigma e\, dy$

$$\boxed{f^1_{t_1x} = \frac{\sigma e}{2}.}$$

<u>Nodo 3</u> $f^1_{t_3x} = \int_{13} N_3\Big]_{x=0} \sigma e\, dy = \int_0^1 y \sigma e\, dy = \frac{\sigma e}{2}$

Se reparten las cargas los nodos $\Rightarrow$.

$f^{1\,T}_{\underline{t}} = \frac{\sigma e}{2}\left[1, 0 \,|\, 0, 0 \,|\, 1, 0\right]$ $\qquad$ $K.u = \begin{bmatrix} \frac{\sigma e}{2} \\ 0 \\ 0 \\ 0 \\ \frac{\sigma e}{2} \\ 0 \end{bmatrix}$

Carga total: σe

—— x ——

2) $\underline{t} = \begin{bmatrix} \sigma(1-\cancel{y}) \\ 0 \end{bmatrix}$

<u>Nodo 1:</u> $f^2_{t_1x} = \int_{13} N_1\Big|_{x=0} \sigma(1-y) e\, dy = \int_0^1 (1-y)^2 \sigma e\, dy = \frac{\sigma e}{3}$

<u>Nodo 3</u> $f^2_{t_3x} = \int_{13} N_3\Big|_{x=0} \sigma(1-y) e\, dy = \int_0^1 y(1-y)\sigma e\, dy = \frac{\sigma e}{6}.$

$f^{2\,T}_{t} = \frac{\sigma e}{2}\left[\frac{2}{3}, 0 \,\Big|\, 0, 0 \,\Big|\, \frac{1}{3}, 0\right]$ $\quad$ Carga Total $= \frac{\sigma e}{2}\left(\frac{2}{3} + \frac{1}{3}\right) = \frac{\sigma e}{2}$

3) Carga volumétrica: Ej. peso propio

$\underline{b} = \begin{bmatrix} 0 \\ -\rho g \end{bmatrix} \qquad f_b = \int_A N^T b \, dA$ e: espesor

<u>Nodo 1</u> $f_{b1y} = -\rho g e \int_A N_1 \, dA = -\rho g e \int_{x=0}^{1} \int_{y=0}^{1-x} (1-x-y) \, dx\,dy = -\frac{\rho g e}{6}$

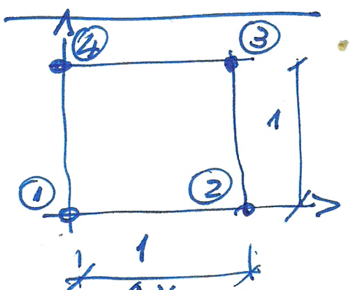
<u>Nodo 2</u> $f_{b2y} = -\rho g e \int_A N_2 \, dA = -\rho g e \int_{x=0}^{1} \int_{y=0}^{1-x} x \, dx\,dy = -\frac{\rho g e}{6}.$

<u>Nodo 3</u> $f_{b3y} = -\rho g e \int_A N_3 \, dA = -\rho g e \int_{x=0}^{1} \int_{y=0}^{1-x} y \, dx\,dy = -\frac{\rho g e}{6}.$

$f_b^T = \frac{-\rho g e}{6} [0,1 \,|\, 0,0 \,\vdots\, 0,1]$ <u>Carga Total</u>: $-\frac{\rho g e}{2}.$

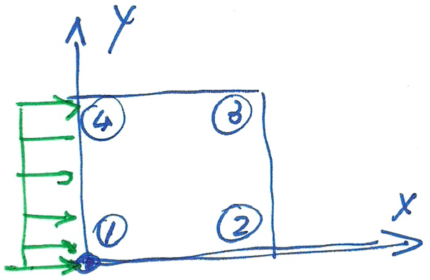
<u>Porque se divide por 2?</u>

b] <u>Rectángulo</u> <u>Bilineal</u>.

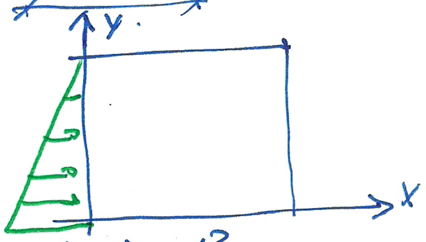


e: espesor

<u>Cargas</u>



1)

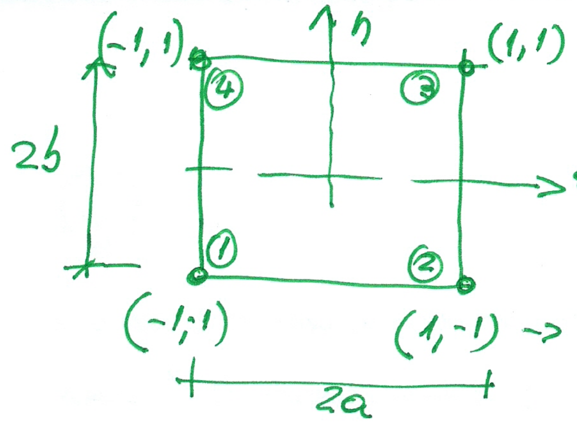


2) $(1-y) = t_x^2$

3) Peso Propio $b = -\rho g$

<u>Coordenadas</u> <u>Naturales</u>

III



$$N_1=\frac{1}{4}\left(1-\frac{\xi}{a}\right)\left(1-\frac{\eta}{b}\right)$$
$$N_2=\frac{1}{4}\left(1+\frac{\xi}{a}\right)\left(1-\frac{\eta}{b}\right)$$
$$N_3=\frac{1}{4}\left(1+\frac{\xi}{a}\right)\left(1+\frac{\eta}{b}\right)$$
$$N_4=\frac{1}{4}\left(1-\frac{\xi}{a}\right)\left(1+\frac{\eta}{b}\right)$$

Si $a=b=1/2$ $\begin{cases}\xi = x-1/2 \\ \eta = y-1/2\end{cases} \Rightarrow \begin{matrix} d\xi = dx \\ d\eta = dy.\end{matrix}$

$$\Rightarrow N_1=\frac{1}{4}(1-2\xi)(1-2\eta)$$
$$N_2=\frac{1}{4}(1+2\xi)(1-2\eta)$$
$$N_3=\frac{1}{4}(1+2\xi)(1+2\eta)$$
$$N_4=\frac{1}{4}(1-2\xi)(1+2\eta)$$

1) $t^1=\begin{bmatrix}\gamma\\0\end{bmatrix}$ en $\frac{\text{lado}}{1-4}$ $\quad f_{t^1}=\oint_l N^T \underline{t}^1 e\, dl.$

Nodo 1 $\Rightarrow f^1_{t_{1x}}=\int_{14} N_1\Big|_{\xi=-1/2}\gamma e\, d\eta=\int_{-1/2}^{1/2}\frac{1}{2}(1-2\eta)\gamma e\, d\eta=\frac{\gamma e}{2}$

Nodo 4 $f^1_{t_{4x}}=\int_{-1/2}^{1/2} N_4\Big|_{\xi=-1/2}\gamma e\, d\eta=\frac{\gamma e}{2}$

$f^T_{t^1}=\left[\frac{\gamma e}{2},0\,\middle|\,0,0\,\middle|\,0,0\,\middle|\,\frac{\gamma e}{2},0\right]$ Carga Total $=\underline{\gamma e}$

2] $t^2=\begin{bmatrix}\gamma(1-y)\\0\end{bmatrix}$ en 1-4 $\quad \underline{t}=\begin{bmatrix}\gamma(1/2-\eta)\\0\end{bmatrix}$

Nodo 1 $f^2_{t_{1x}}=\int_{1-4} N_1\Big|_{\xi=-1/2}\underline{t}^2 e\, dl=\int_{-1/2}^{1/2}\gamma e\frac{1}{2}(1-2\eta)(1/2-\eta)\, d\eta$

$$\boxed{f^2_{t_{1x}}=\frac{\gamma e}{3}}$$

IV

Nodo 4 $\quad f_{t_{4x}}^{2} = \int_{l_{1-4}} N_4\Big|_{\xi=-1/2} t_x^2 \, e \, dl = \int_{-1/2}^{+1/2} \mathcal{H} e \, \frac{1}{2}(1+2\eta)\left(\frac{1}{2}-\eta\right) d\eta.$

$$\boxed{f_{t_{4x}}^{2} = \frac{\mathcal{H}e}{6}.}$$

$$f_{t_x}^{2} = \frac{1}{2}\mathcal{H}e\left[\frac{2}{3}, 0 \,\middle|\, 0, 0 \,\middle|\, 0, 0 \,\middle|\, \frac{1}{3}, 0\right]$$

$$\text{Carga Total} = \frac{\mathcal{H}e}{2}\left(\frac{2}{3}+\frac{1}{3}\right) = \frac{\mathcal{H}e}{2}.$$

3) Carga volumétrica : Peso Propio

$$b = \begin{bmatrix} 0 \\ -\rho g \end{bmatrix} \qquad f_b = \int_A N^T \underline{b} \, dA$$

Nodo 1 $\quad f_{b_{1y}} = -\rho g e \int_A N_1 \, dA = -\rho g e \int_{-1/2}^{1/2}\int_{-1/2}^{1/2} \frac{1}{4}(1-2\xi)(1-2\eta) \, d\xi \, d\eta$

$$f_{b_{1y}} = -\frac{\rho g e}{4} \qquad f_{b_{2y}} = f_{b_{1y}} = f_{b_{3y}} = f_{b_{4y}}$$

$$f_b^T = -\rho g e\left[0, 1/4 \,\middle|\, 0, 1/4 \,\middle|\, 0, 1/4 \,\middle|\, 0, 1/4\right]$$

$$\boxed{\text{Carga Total} = -\rho g e A.}$$

—— x ——