$$\nabla\cdot\underline{u} = \frac{\partial u_i}{\partial x_i} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}$$

$$\underline{\underline{d}} = \underline{\nabla}\,\underline{u} = \frac{\partial u_i}{\partial x_j} = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{bmatrix}$$

a. $\nabla\cdot\left(\overbrace{r^n\mathbf{r}}^{u}\right) = (n+3)r^n$

$\boxed{\nabla\cdot u \to \text{ordem } 0}$

$$r = \|\underline{r}\|$$

$$\underline{r}\cdot\underline{r} = \underbrace{\|\underline{r}\|}_{r}\,\underbrace{\|\underline{r}\|}_{r}\cdot\overbrace{\cos\theta}^{1 \text{ al ser } \theta=0}$$

$$\underbrace{x_i x_i = \underline{r}\cdot\underline{r} = r^2}$$

$$\underline{r} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\frac{\partial(\overbrace{r^2}^{r\cdot r})}{\partial x_i} = 2\cdot r\cdot\frac{\partial r}{\partial x_i}$$

$$\frac{\partial(r^2)}{\partial x_i} = \frac{\partial}{\partial x_i}(\underline{r}\cdot\underline{r}) = \frac{\partial}{\partial x_i}(x_j x_j) = \overbrace{\frac{\partial x_j}{\partial x_i}}^{\overbrace{\delta_{ji}}^{x_i}} x_j + x_j\cdot\overbrace{\frac{\partial x_j}{\partial x_i}}^{\overbrace{\delta_{ji}}^{x_i}} = 2x_i$$

$$A_{ji} = \frac{\partial x_j}{\partial x_i} = \delta_{ji} \qquad A_{12} = \frac{\partial x_1}{\partial x_2} = 0$$

$$A_{22} = \frac{\partial x_2}{\partial x_2} = 1$$

$$\cancel{2}\,r\cdot\frac{\partial r}{\partial x_i} = \cancel{2}\,x_i \Rightarrow \boxed{\frac{\partial r}{\partial x_i} = \frac{x_i}{r}}$$

$$\frac{\partial r}{\partial x_i} = \frac{\partial\sqrt{x_i x_i}}{\partial x_i}$$

$$\overbrace{\nabla\cdot}^{\text{div}}(r^n\underline{r}) = \frac{\partial}{\partial x_i}\left(r^n\overbrace{(\underline{r})_i}^{x_i}\right) = \frac{\partial(r^n)}{\partial x_i}\cdot x_i + r^n\cdot\overbrace{\frac{\partial x_i}{\partial x_i}}^{\overbrace{\delta_{ii}}^{3}}$$

$$= n\cdot r^{n-1}\cdot\frac{\partial r}{\partial x_i}\cdot x_i + 3r^n = n\cdot r^{n-1}\,\overbrace{\frac{x_i}{r}\,x_i}^{r^2} + 3r^n$$

$$= n \cdot r^n + 3r^n = (n+3)r^n$$

b. $\nabla \times \left(r^n \mathbf{r}\right) = \underline{0}$ $\rightarrow$ orden 1

$$\hookrightarrow \underbrace{\varepsilon_{ijk}\, x_j \cdot x_k}_{0} \cdot n \cdot r^{n-2}$$

d. $e_{ijk} A_j A_k = \underline{0}$

c. $\Delta\left(r^n\right) = n(n+1)r^{n-2}$ $\rightarrow$ $r = \|\underline{r}\|$

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$$\Delta(r^n) = \frac{\partial (r^n)}{\partial x_i \partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\partial}{\partial x_i} r^n\right) = \frac{\partial}{\partial x_i}\left(\frac{\partial r^n}{\partial r} \cdot \frac{\partial r}{\partial x_i}\right)$$

$$= \frac{\partial}{\partial x_i}\left(n \cdot r^{n-1} \cdot \frac{x_i}{r}\right) = n \cdot \left[\frac{\partial (r^{n-2})}{\partial x_i} \cdot x_i + r^{n-2} \cdot \overbrace{\frac{\partial x_i}{\partial x_i}}^{\delta_{ii}}\right]$$

$$= n(n+1) \cdot r^{n-2}$$