Example 1.8 Using unit vectors

Given: $\vec{D} = (6.00\hat{i} + 3.00\hat{j} - 1.00\hat{k})\,m$

$\vec{E} = (4.00\hat{i} - 5.00\hat{j} + 8.00\hat{k})\,m$

Find $2\vec{D} - \vec{E}$; or $\vec{F}$.

$\vec{F} = 2(6.00\hat{i} + 3.00\hat{j} - 1.00\hat{k})\,m - (4.00\hat{i} - 5.00\hat{j} + 8.00\hat{k})\,m$

$= (12.00\hat{i} + 6.00\hat{j} - 2.00\hat{k})\,m - (4.00\hat{i} - 5.00\hat{j} + 8.00\hat{k})\,m$

$= [(12.00 - 4.00)\hat{i} + (6.00 + 5.00)\hat{j} + (-2.00 - 8.00)\hat{k}]\,m$

$= \cancel{(8.00\hat{i} + 1.00\hat{j} + 6.00\hat{k})\,m}$

$= (8.00\hat{i} + 11.00\hat{j} - 10.00\hat{k})\,m$

$|F| = \sqrt{F_x^2 + F_y^2 + F_z^2}$

$= \sqrt{(8.00m)^2 + (11.00m)^2 + (-10.00m)^2}$

$\approx 16.88m$

Test your understanding section 1.9

Arrange the following vectors in order of their magnitude:

$\vec{A} = (3\hat{i} + 5\hat{j} - 2\hat{k})\,m$

$\vec{B} = (-3\hat{i} + 5\hat{j} - 2\hat{k})\,m$

$\vec{C} = (3\hat{i} - 5\hat{j} - 2\hat{k})\,m$

$\vec{D} = (3\hat{i} + 5\hat{j} + 2\hat{k})\,m$

→ because calculating the magnitude for all of them results in the same result; or in other words, the displacement for all 4 vectors are the same; there is no need to arrange them at all.

Example 1.9 Calculating a scalar product

Find the scalar product $\vec{A} \cdot \vec{B}$ of the two vectors in Fig. 1.28, given that $|A| = 4.00$, $|B| = 5.00$

<u>via magnitude and angles</u>

Angle between $\vec{A}$ and $\vec{B}$: $130.0^\circ - 53.0^\circ = 77.0^\circ$
ϕ

$$\vec{A} \cdot \vec{B} = AB\cos\phi = (4.00)(5.00)\cos 77.0^\circ = 4.50$$

<u>via component of vectors</u>

$$A_x = (4.00)\cos(53.0^\circ)$$

$$A_y = (4.00)\sin(53.0^\circ)$$

$$B_x = (5.00)\cos(130.0^\circ)$$

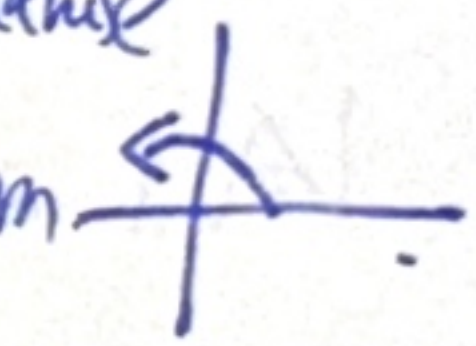
$$B_y = (5.00)\sin(130.0^\circ)$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\approx \underline{\underline{4.50}}$$

Notes to Self

1) It seems that the convention for evaluating components is derived with respect to angles from the positive x-axis, so generally anti-clockwise in the Cartesian coordinate system.



2) Dot product general formula is defined as:

$$\vec{A} \cdot \vec{B} = \sum_{i=1}^{n} A_i B_i$$

The provided example is within the realms of the 2D ~~plane~~, where $n=2$.

However, the inclusion of the z axis introduces the notion of 3D, where $n=3$. While not needed here, I suspect that beyond completeness, the author also wants to get the reader/learner acquainted w/ dealing in 3D, as Physics problems w/ reality is in the $n=3$ case...

Example 1.10 Find an angle w/ scalar products.

Find the angle between vectors:

$\vec{A} = 2.00\hat{i} + 3.00\hat{j} + 1.00\hat{k}$

$\vec{B} = -4.00\hat{i} + 2.00\hat{j} - 1.00\hat{k}$

$|A| = \sqrt{(2.00)^2 + (3.00)^2 + (1.00)^2}$

$\cancel{|B|} = \sqrt{4.00 + 9.00 + 1.00}$

$= \sqrt{14.00}$

$|B| = \sqrt{(-4.00)^2 + (2.00)^2 + (-1.00)^2}$

$= 16.00 + 4.00 + 1.00$

$= \sqrt{21.00}$

$\vec{A} \cdot \vec{B} = (2.00 + (-4.00))\hat{i} + (3.00 + 2.00)\hat{j} + (1.00 - 1.00)\hat{k}$

$= (2.00 - 4.00)\hat{i} + (3.00 + 2.00)\hat{j} + (1.00 - 1.00)\hat{k}$

$= -2.00\hat{i} + 5.00\hat{j} + 0.00\hat{k}$

$|AB| = \sqrt{(-2.00)^2 + (5.00)^2 + (0.00)^2}$

$= \sqrt{4.00 + 25.00 + 0.00}$

$= \sqrt{29.00}$

$\cos\phi = \dfrac{|A|}{|AB|} = \dfrac{\sqrt{14.00}}{\sqrt{29.00}}$

$\phi = \cos^{-1}\left(\dfrac{\sqrt{14.00}}{\sqrt{29.00}}\right)$

$\approx 45.99°$

Again (Example 1.10)

$$\cos\phi = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

$$\vec{A}\cdot\vec{B} = (2.00)(4.00) + (3.00)(2.00) + (1.00)(-1.00)$$

$$= -3.00$$

$$A = \sqrt{14.00}$$

$$B = \sqrt{21.00}$$

$$\cos\phi = \cancel{\cos^{-1}\left(\frac{\sqrt{14.00}}{\sqrt{21.00}}\right)} \frac{-3.00}{\sqrt{14.00}\sqrt{21.00}}$$

$$= -0.175$$

$$\phi = \cos^{-1}(\cancel{-0.175}\ -0.175)$$

$$\approx 100.08^\circ$$

Formula to note: $\vec{A}\cdot\vec{B} = AB\cos\phi$.