Intro to vectors

- Linear Combinations are the vector-represented basis for addition, with multiplication through scalars. e.g. $c\underline{u}+d\underline{v}+e\underline{w}$
- "component at a time"; respective procedure of computing.

  e.g. $v_i + w_i = (v_1 + w_1, v_2 + w_2 \dots v_i + w_i)$
- Line → Plane → Space

Important Qns:

Suppose $u, v, w \in$ 3D space:, what is the picture of <u>all</u> combinations for the cases:

cu
$cu + dv$
$cu + dv + ew$?

} This picture depends on the nature of the vectors we are dealing with.

Base case:

if u, v, w are zero vectors, then every combination would also be zero

Recursive case:

if u, v, w are any other values, then ... this picture is painted:

The picture painted by combinations of cu is that of a line filling through (0,0,0)

For $cu + dv$, it is a plane through (0,0,0)

Finally, for $cu + dv + ew$, it fills the entirety of 3D space ~~at~~ (Note: Contingent on third line ∉ plane of u and v) worked Examples

1.1A Given that $cv + dw$ fill a plane in R^3, as a linear combination, it can be described as a plane that passes through $(0,0,0)$. It can be represented as:

$$cv + dw = c\begin{bmatrix}1\\1\\0\end{bmatrix} + d\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}c+0\\c+d\\0+d\end{bmatrix} = \begin{bmatrix}c\\c+d\\d\end{bmatrix}.$$

This ~~[illegible]~~ vector representation fills a plane.

1.1B Given $v = \begin{bmatrix}1\\0\end{bmatrix}$ $w = \begin{bmatrix}0\\1\end{bmatrix}$, describe all points, cv with:

1) whole numbers c
2) non-negative numbers, $c \geq 0$.

they are asking about the graphical representation of these vectors

Then, add all vectors dw and describe all $cv + dw$

Let's enumerate our cases to describe:

1a) $c_1 v = c\begin{bmatrix}1\\0\end{bmatrix}$ (whole no.)

$\downarrow$

1b) $c_1 v + dw$

2a) $c_2 v = c\begin{bmatrix}1\\0\end{bmatrix}$ (non-neg no.)

$\downarrow$

2b) $c_2 v + dw$

1.1c find 2 equations for c and d, so that the linear combination $cv + dw$ equals b:

$$v = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad w = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$