Problem set 1.1

X 1a. $\begin{bmatrix}1\\2\\3\end{bmatrix}$ and $\begin{bmatrix}3\\6\\9\end{bmatrix}$ in linear comb $= c\begin{bmatrix}1\\2\\3\end{bmatrix} + d\begin{bmatrix}3\\6\\9\end{bmatrix}$

In this case, it fills ~~the~~ a plane in xyz space.

✓ 1b. for $\begin{bmatrix}1\\0\\0\end{bmatrix}$ and $\begin{bmatrix}0\\2\\3\end{bmatrix} = c\begin{bmatrix}1\\0\\0\end{bmatrix} + d\begin{bmatrix}0\\2\\3\end{bmatrix} = \begin{bmatrix}c\\2d\\3d\end{bmatrix}$

Fills plane. in $\mathbb{R}^3$

✓ 1c. $\begin{bmatrix}2\\0\\0\end{bmatrix}, \begin{bmatrix}0\\2\\2\end{bmatrix}, \begin{bmatrix}2\\2\\2\end{bmatrix}$

Fills all of $\mathbb{R}^3$

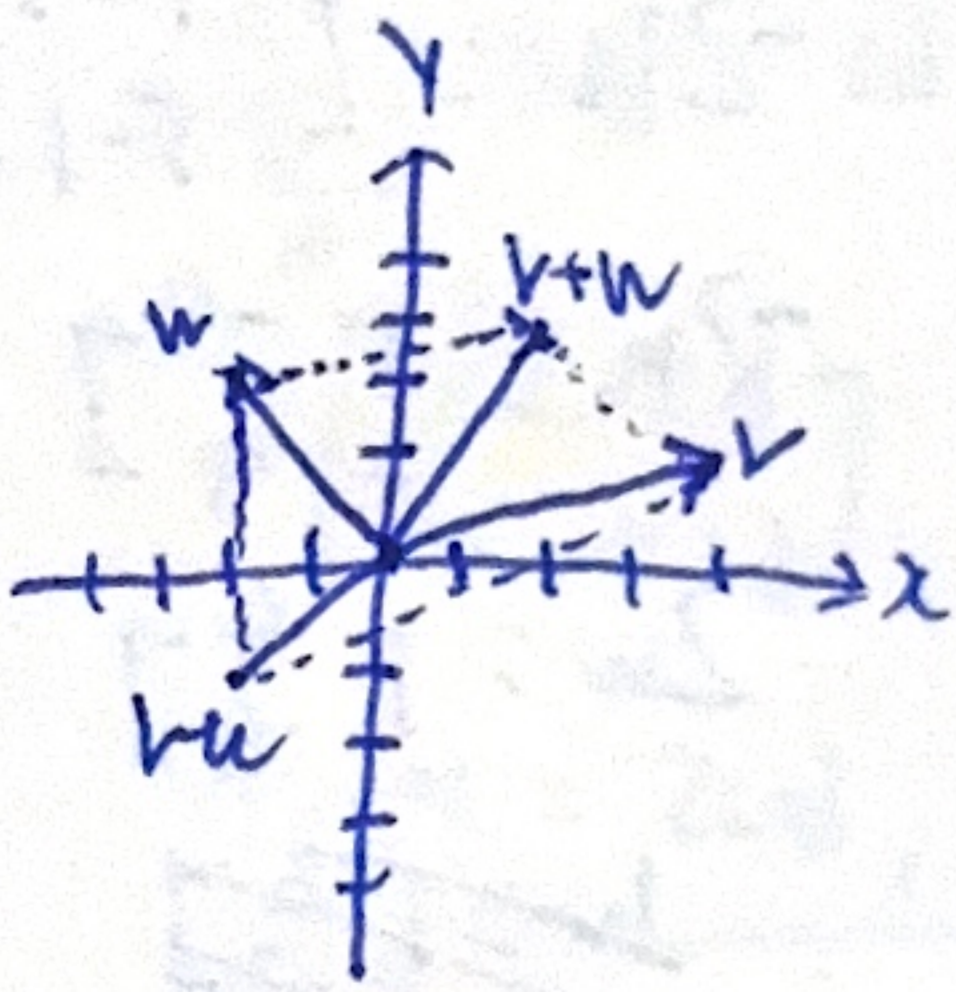
✓ 2. $v = \begin{bmatrix}4\\1\end{bmatrix}$

$w = \begin{bmatrix}-2\\2\end{bmatrix}$

$v+w = \begin{bmatrix}4\\1\end{bmatrix} + \begin{bmatrix}-2\\2\end{bmatrix}$

$= \begin{bmatrix}4-2\\1+2\end{bmatrix}$

$= \begin{bmatrix}2\\3\end{bmatrix}$



$v-w = \begin{bmatrix}4\\1\end{bmatrix} - \begin{bmatrix}-2\\2\end{bmatrix}$

$= \begin{bmatrix}-4+2\\1-2\end{bmatrix}$

$= \begin{bmatrix}-2\\-1\end{bmatrix}$

3. ① $v + w = \begin{bmatrix}5\\1\end{bmatrix}$ ② $v - w = \begin{bmatrix}1\\5\end{bmatrix}$

③ $v = \begin{bmatrix}5\\1\end{bmatrix} - w$

From ③, into ②

$$\begin{bmatrix}5\\1\end{bmatrix} - w - w = \begin{bmatrix}1\\5\end{bmatrix}$$

$$\begin{bmatrix}5\\1\end{bmatrix} - 2w = \begin{bmatrix}1\\5\end{bmatrix}$$

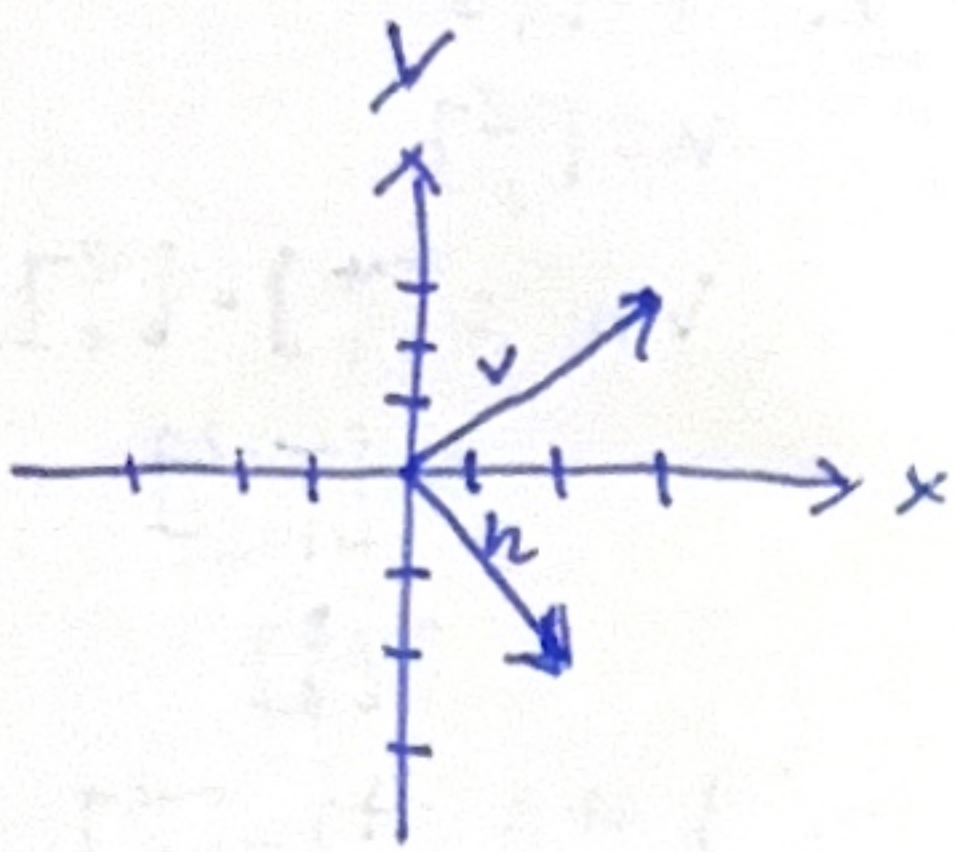
$$-2w = \begin{bmatrix}1\\5\end{bmatrix} - \begin{bmatrix}5\\1\end{bmatrix}$$

$$-2w = \begin{bmatrix}1-5\\5-1\end{bmatrix}$$

$$-2w = \begin{bmatrix}-4\\4\end{bmatrix}$$

$$2w = \begin{bmatrix}4\\-4\end{bmatrix}$$

$$\cancel{w = \begin{bmatrix}2\\-2\end{bmatrix}}$$



④ $w = \begin{bmatrix}2\\-2\end{bmatrix}$

From ④, into ①

$$v + \begin{bmatrix}2\\-2\end{bmatrix} = \begin{bmatrix}5\\1\end{bmatrix}$$

$$v = \begin{bmatrix}5\\1\end{bmatrix} - \begin{bmatrix}2\\-2\end{bmatrix} = \begin{bmatrix}3\\3\end{bmatrix}$$

$$= \begin{bmatrix}5-2\\1+2\end{bmatrix}$$

4. $v = \begin{bmatrix}2\\1\end{bmatrix}$ $w = \begin{bmatrix}1\\2\end{bmatrix}$

Find components of

$3v + w$

$$3\begin{bmatrix}2\\1\end{bmatrix} + \begin{bmatrix}1\\2\end{bmatrix}$$

$cv_1 = 3(2) = 6$

$cv_2 = 3(1) = 3$

$dw_1 = 1$

$dw_2 = 2$

$$= \begin{bmatrix}6+1\\3+2\end{bmatrix}$$

$$= \begin{bmatrix}7\\5\end{bmatrix}$$

Components = 7, 5

$cv + dw$

$$c\begin{bmatrix}2\\1\end{bmatrix} + d\begin{bmatrix}1\\2\end{bmatrix}$$

$cv_1 = 2c$

$cv_2 = c$

$dw_1 = d$

$dw_2 = 2d$

$$= \begin{bmatrix}2c+d\\c+2d\end{bmatrix}$$

Components = $2c+d$, $c+2d$

5. ~~x~~ Compute; assuming: $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $v = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}$ $w = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$

$u + v + w$

$$= \begin{bmatrix} 1 - 3 + 2 \\ 2 + 1 - 3 \\ 3 - 2 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$2u + v + w$

$$\begin{bmatrix} 2 - 3 + 2 \\ 4 + 1 - 3 \\ 6 - 2 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

How do we know that u, v, w lie in a plane?

Using $w = cu + dv$ test:

$$\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = c \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + d \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} c - 3d \\ 2c + d \\ 3c - 2d \end{bmatrix}$$

$2 = c - 3d$

$-3 = 2c + d$

$1 = 3c - 2d$

$c = 2 + 3d$

From 4 into 3,

$1 = 3(2 + 3d) - 2d$

$1 = 6 + 9d - 2d$

$1 = 6 + 7d$

$1 - 6 = 7d$

$-5 = 7d$

$\Rightarrow$ 5 $d = -\frac{5}{7}$

From 5 into 2, $-3 = 2c + \cancel{(}{-}\frac{5}{7})$

$-3 = 2c - \frac{5}{7}$

$-3 + \frac{5}{7} = 2c$

6 $c = \dfrac{\cancel{-3}(\frac{5}{7} - 3)}{2}$

The fact that I can even find a c and d, as evidenced in 5 and 6, is proof that this particular linear combination of u, v, w lie in a plane.

If I cannot find a c and d for it, then it means that ~~this~~ linear combination does not lie on a plane.

Correction: Actually, the test for this is to find a linear combination that $= (0,0,0)$.

In this case, $u + v + w = (0,0,0)$, which I did not do.

6. $v = (1, -2, 1)$ $w = (0, 1, -1)$

$$\begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix} = c\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + d\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -c \\ -2c+d \\ c-d \end{bmatrix}$$

① $3 = -c$

② $3 = -2c + d$

③ $-6 = c - d$

From ①:

④ $c = -3$

From ③:

$-6 = -3 - d$

$-6 + 3 = -d$

$-3 = -d$

⑤ $d = 3$

why is $(3, 3, 6)$ impossible?

$$\begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix} = c\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + d\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$= \begin{matrix} -c \\ -2c+d \\ c-d \end{matrix}$$

① $3 = -c$

② $3 = -2c + d$

③ $6 = c - d$

From ①:

④ $c = -3$

From ③:

$6 = -3 - d$

$6 + 3 = -d$

$9 = -d$

$d = -9$

Hmm.. isn't it possible? Because I can find a c and d, as evidenced in ① and ③.

Correction: ~~[illegible]~~ similarly I did not do a simple check of component summing = 0 check. why..?

7. $c,d = 0,0,$ $\boxed{\begin{matrix}0,0 & 0,1 & 0,2\\ 1,0 & 1,1 & 1,2\\ 2,0 & 2,1 & 2,2\end{matrix}}$

$\cancel{\#1: \begin{bmatrix}0\\0\end{bmatrix}}$ $n_{c,d}$, where $c \in [0,1,2]$, $d \in [0,1,2]$

$1_{0,0} = \begin{bmatrix}0\\0\end{bmatrix}$

$2_{0,1} = \begin{bmatrix}0\\1\end{bmatrix}$

$\cancel{3_{1,0} = \begin{bmatrix}2\\1\end{bmatrix}}$

$\cancel{4_{1,1} = \begin{bmatrix}2\\1+1\end{bmatrix} = \begin{bmatrix}2\\2\end{bmatrix}}$

~~5~~

$\cancel{3_{0,2}}$

$\cancel{3_0}$

$3_{0,2} = \begin{bmatrix}\cancel{2+0}\\\cancel{1+2}\end{bmatrix}$
$= \begin{bmatrix}0\\2\end{bmatrix}$

$\cancel{4_{1}}$

4_1

$4_{1,0} = \begin{bmatrix}2\\1\end{bmatrix}$

$5_{1,1} = \begin{bmatrix}2\\1+1\end{bmatrix} = \begin{bmatrix}2\\2\end{bmatrix}$

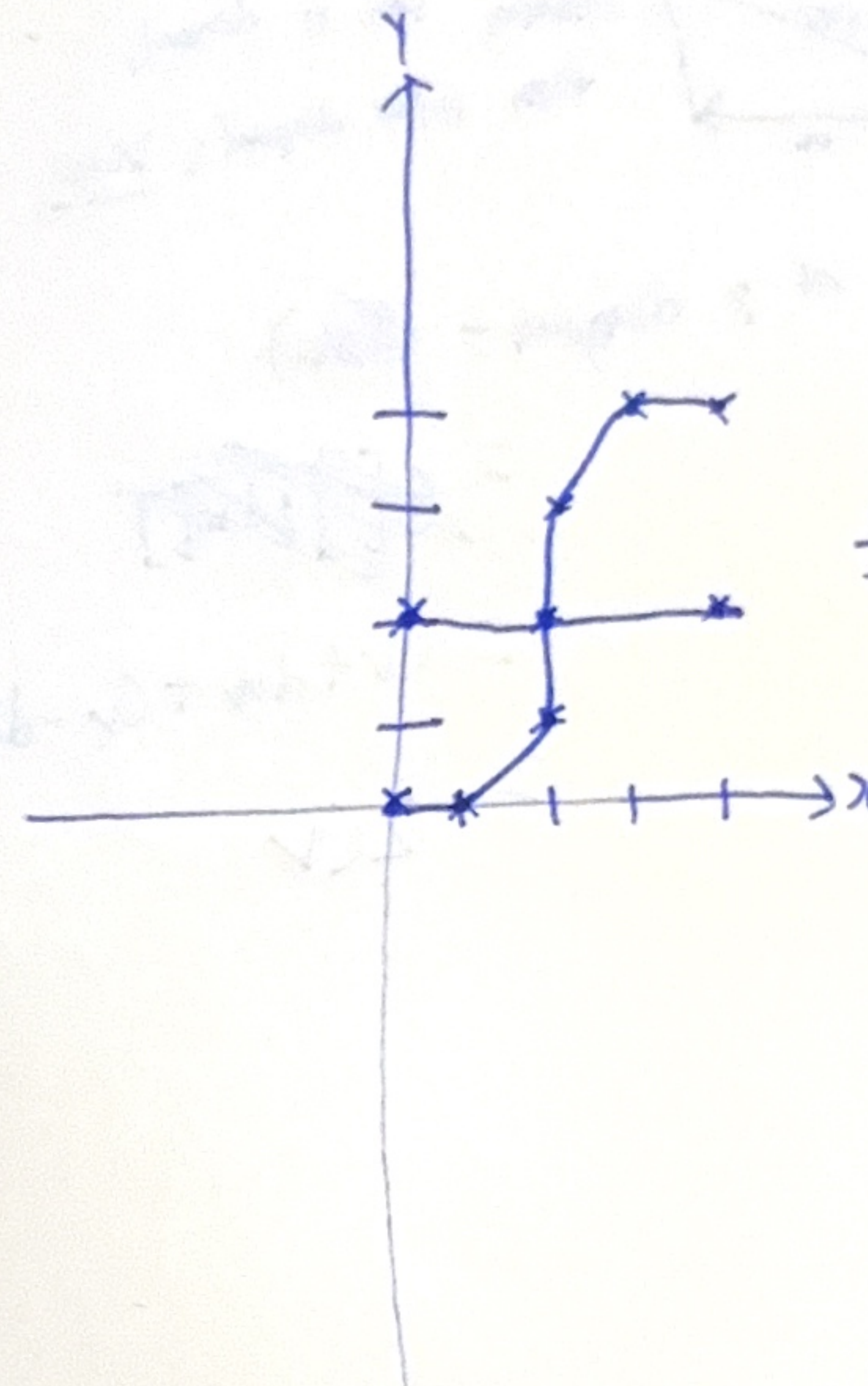
$6_{1,2} = \begin{bmatrix}2\\1+2\end{bmatrix} = \begin{bmatrix}2\\3\end{bmatrix}$

$7_{2,0} = \begin{bmatrix}4\\2\end{bmatrix}$

$8_{2,1} = \begin{bmatrix}4+0\\2+1\end{bmatrix} = \begin{bmatrix}4\\3\end{bmatrix}$

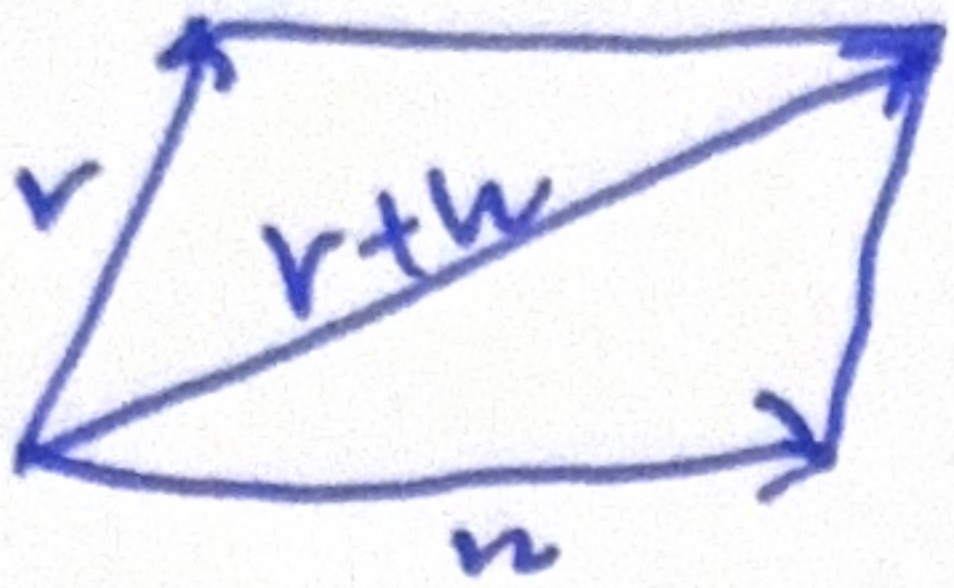
$9_{2,2} = \begin{bmatrix}4\\2+2\end{bmatrix} = \begin{bmatrix}4\\4\end{bmatrix}$

1. (0,0)
2. (0,1)
3. ~~(2,3)~~ (0,2)
4. (2,1)
5. (2,2)
6. (2,3)
7. (4,2)
8. (4,3)
9. (4,4)



f?!!

8. ✓



Assuming $v+w$ is diagonal, then other diagonal is $\underline{v-w}$.

Sum of 2 diagonals =

=

$= cv + dw + cv - dw$

$= 2cv$

9. Given that 3 corners of a parallelogram are

$(1,1)$ $(4,2)$ $(1,3)$, what are all

3 of the possible 4^{th} corner?