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UCL PHD UPGRADE REPORT

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## Qubit Coherence and Control

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*Author:*

David F. WISE

*Supervisor:*

Prof. John MORTON

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## ABSTRACT

Scientific documents often use  $\text{\LaTeX}$  for typesetting. While numerous packages and templates exist, it makes sense to create a new one. Just because.

# CONTENTS

1	INTRODUCTION	3
2	THEORY	7
2.1	Electron Spin Resonance . . . . .	7
2.1.1	Basic Theory . . . . .	7
2.1.2	Pulsed Electron Spin Resonance and Qubits . . . . .	8



# LIST OF FIGURES

2.1	Free electron level splitting . . . . .	7
2.2	Hahn echo sequence . . . . .	8



# 1 INTRODUCTION

*At midnight all the agents and  
superhuman crew come out and round up  
everyone that knows more than they do*

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Quantum computing has been an active field of research since the concept was first suggested by Richard Feynman in the early 1980s [6]. Originally proposed as an efficient method for simulating chemical processes (something that traditional computers find extremely taxing) the discovery of several algorithms offering significant speed increases over classical computers has further fuelled research [17, 18]. Strong initial scepticism abounded regarding the potential for quantum computers to exist in the real world, primarily due to the concerns that error correction with such a complex device would be impossible [13]. This pessimism gave way with the identification of an error threshold for quantum computation. Given an error rate below a critical threshold it was possible to perform an arbitrarily long computation with negligible possibility of significant error [1, 16]. Following these discoveries serious attention has been given to the development of error correcting codes that might be implemented to allow a physical quantum computer to be *fault tolerant*.

Gottesman identified the class of error correcting codes known as *stabilizer codes*, where codes are defined by the group of logical operators that leave the code unchanged [7]. This class of quantum error correcting codes have become the dominant form in theoretical research. Of these one in particular has become the focus of much of the ongoing research of quantum computing, *the surface code*, developed by Bravyi and Kitaev [4]. Although other quantum error correcting codes (such as colour codes [19]) exist, the surface code has become the focus for experimental implementations. This is due mainly to the relatively high error threshold that it is able to tolerate,  $\sim 0.5\%$ , and the simple architecture of a planar grid of qubits.

In recent years there has been rapid progress in the development of physical qubits. Groups in both the academic and private sectors have shown small numbers of qubits functioning with error rates above the fault tolerant threshold [3, 14]. The successful recent approaches have tended to focus on superconducting circuits to produce their qubits. Whilst these have proved excellent for the small numbers of qubits currently in use, it is likely that they will present significant additional challenges when scaling to numbers sufficient for useful, fault-tolerant quantum computation. With the numbers required likely to be close to  $100 \times 10^6$  and the current size of these qubits close to  $1\text{mm}^2$ , it will likely be impossible to use these qubits in their current form in a fault-tolerant quantum computer.

Although there are many alternative systems that could provide a qubit, this paper will focus on the use of the spins of nuclei and electrons bound to donors in semiconductors, particularly sili-

con. A seminal paper by Kane [9] stimulated much of the research interest in this field. He proposed using the spin of phosphorus nuclei in silicon as qubits with the ability to mediate interactions between neighbouring donor nuclei using the interaction between the electrons bound to each. These types of qubit are attractive due to their exceptionally long coherence times, the time that the qubit reliably stores quantum information for. A long coherence time relative to qubit gate times is essential, as this determines the error rate of the qubits. Coherence times as long as several seconds have been reported for donor spin qubits in silicon, whilst gate times can be as low as several nanoseconds [20]. Despite these advantages, development of these types of qubits for quantum computers has lagged behind the superconducting and ion trap versions [2]. This is due to the difficulty of isolating and addressing single donors in a silicon lattice. Kane's proposal required sub nano-metre precision in qubit placement to facilitate inter-qubit interactions. Even if this precision were achieved there remains the issue of how the requisite control circuitry could be integrated into such a dense design. This has led to the development of more modern proposals to both overcome these difficulties and also to implement surface code based error correction.

One such proposal is from O'Gormann et al [11]. This proposal takes a similar approach to Kane, with qubits being donor spins in a silicon lattice. Where it differs significantly is in its use of two lattices of qubits. One of these is for the storage of data, whilst the other performs measurements on these data qubits. This measurement stage is placed above the data stage and held close, within 40nm, and moved in a repeating cycle over the data qubits. This allows each measurement qubit to perform  $\hat{X}$  or  $\hat{Z}$  measurement on groups of four data qubits, the stabilizer measurements that make up the fundamental units of surface code. This architecture allows for data qubits to be placed much farther apart - since no direct interaction between them is required. Several key research questions remain:

1. What donor species should be used for both types of qubit?
2. How are the measurement qubits to be read out?
3. How are the qubits to be controlled?

Whilst donors in silicon make excellent choices for the data qubits due to the properties stated above, a different qubit species is required for the measurement qubits to avoid an unwanted exchange interaction between the two lattices. Optical qubit readout is suggested in the proposal by O'Gormann et al and is a well studied means of reading the state of spin qubits, particularly in nitrogen-vacancy (NV) centres in diamond [10]. A concern with optical readout is the impact that illumination can have on the coherence times of donors in silicon. Silicon has a band gap energy equivalent to approximately 1058nm or photon energy of 1.17eV. Illumination at shorter wavelengths than this will create free electrons in the silicon conduction band. These can scatter off the electrons bound to donors, causing them to relax and shortening the  $T_1$  time of the qubits as a whole [15]. NV centres are read out at between 500nm and 600nm, illumination that would reduce data qubit coherence times and increase the qubit error rate. Alternative optically addressed spin qubits at higher wavelengths exist, such as the di-vacancy centre in silicon carbide [5], but a vital question is what wavelengths can be used for read-out without compromising data qubit coherence times. This report addresses this question by examining the effect of various laser



wavelengths close to the silicon band gap on the coherence times of electrons bound to phosphorus donors in silicon.

Another key question presented by the O’Gormann proposal is how to control the data qubits. The frequencies traditionally used for electron spin resonance are between 8GHz and 10GHz. Electromagnetic radiation at these frequencies require large coaxial cables and cavities for transmission. This makes it almost impossible to individually address qubits using microwaves. Instead the solution proposed by Kane is to use global microwave pulses, addressing all qubits at once. To selectively control qubits the DC Stark shift can be employed - DC electric fields can be used to shift the electron spin transition frequency meaning that a global microwave field will not effect them. Unlike RF radiation, DC signals are easily multiplexed and the commercial electronics industry has achieved fabrication precision well within that required by the O’Gormann proposal. The stark shift of donors in silicon has been well studied but has yet to be quantified in certain systems [12]. Among these is the hyperfine interaction between the donor electron and the silicon-29 nuclei present in natural silicon. The nuclear bath has been identified as a potential quantum register and the ability to tune the interaction between data qubits and memory could be of use.

$$\hat{H} = \mu_b B g_e S_z + \mu_n g_n B I_z + A \hat{S} \cdot \hat{I} \quad (1.1)$$



# 2 THEORY

## 2.1 ELECTRON SPIN RESONANCE

### 2.1.1 BASIC THEORY

Electron spin resonance (EPR) functions by detecting the energy difference between the spin states of an electron. Normally degenerate, the presence of a magnetic field separates the two spin states parallel to it in energy. The spin state parallel to the magnetic field has a lower energy whilst the anti-parallel state has a higher energy. These are described as the up,  $|\uparrow\rangle$ , and down,  $|\downarrow\rangle$ , spin states. For an electron in free space the energy difference is given by:

$$\Delta E = g_e \mu_b B, \quad (2.1)$$

where  $g_e$  is the free electron g-factor,  $\mu_b$  is the Bohr magneton and  $B$  is the magnetic field strength. This results in an energy splitting as seen in figure 2.1.

In practice this energy splitting can be detected using continuous wave EPR. Transitions between the spin states will be driven by incident electromagnetic radiation of photon energy equal to the energy gap (i.e.  $h\nu = g_e \mu_b B$ ). The presence of an EPR transition in a sample can be detected either by applying a constant magnetic field and sweeping EM frequency incident on that sample or vice versa. In practice, it is the latter that is used for experimental simplicity. Measuring reflection of radiation from the sample whilst sweeping magnetic field will reveal a drop in reflection at the transition field - when photons are absorbed by spins moving from the lower to higher energy state.

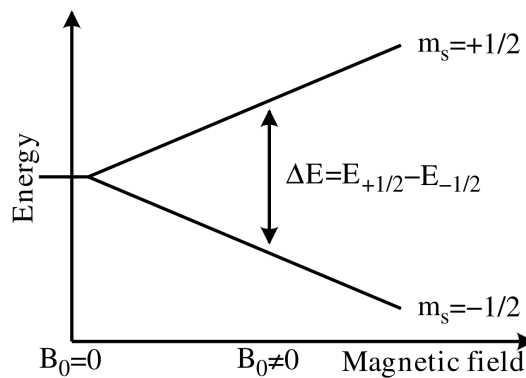


Figure 2.1: Spin state energy splitting for a free electron in a magnetic field.

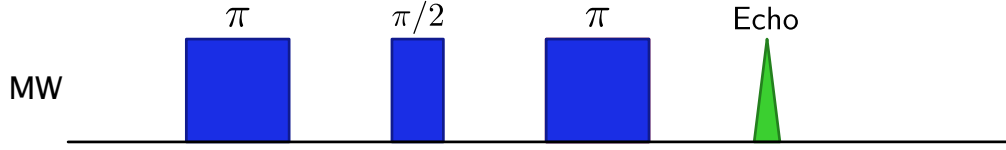


Figure 2.2: Cartoon showing a Hahn echo pulse sequence. A  $\frac{\pi}{2}$  pulse causes the spins to precess in the  $x-y$  plane. Loss of phase coherence is reversed via a  $\pi$  pulse following time interval  $T$  and a signal is detected following another interval of  $T$ .

### 2.1.2 PULSED ELECTRON SPIN RESONANCE AND QUBITS

The description above details continuous wave (CW) ESR, a technique that has proven invaluable for studying the electronic structure of materials. Another ESR has proven more popular for the manipulation of spins for use of qubits: pulsed ESR. This uses short bursts of microwave radiation, on resonance with the spin transition, to control spin states and allows the spin to function as a qubit. Pulses are described in terms of a rotation angle, with a  $\pi$  pulse taking the ensemble of spins from the down to up state or vice versa. A  $\pi/2$  pulse takes the spins into the plane normal to the magnetic field, termed the  $x-y$  plane. This causes the spins to precess in the magnetic field at the Lamor frequency given by:

$$\omega = \frac{eg}{2m} B_0 \quad (2.2)$$

#### HAHN ECHO AND DETECTION

In pulsed ESR the spins are detected via the electromagnetic radiation they emit when precessing in a magnetic field. This radiation is of the same frequency as the resonant control radiation (easily shown using equations 2.1 and 2.2). This emitted radiation can be demodulated with the control radiation giving a DC signal. In a perfectly homogeneous magnetic field all spins would precess at the same rate giving a constant DC signal but in reality all spins will precess at slightly different rates due to small, static differences in the magnetic field each experiences. So, following a  $\pi/2$  pulse, the signal from the spins will rapidly decay as the ensemble of spins lose phase coherence. A technique, known as a spin or Hahn echo, to reverse this loss of phase was developed by Erwin Hahn in 1950 [8]. This follows a  $\pi/2$  pulse with a  $\pi$  pulse after a set time interval,  $T$ . Static magnetic field differences now act to reverse the loss of phase coherence. This results in a brief re-phasing of the spins following another interval  $T$ , detected as a rise and fall of a DC signal or and 'Echo'. A cartoon of this sequence is shown in figure 2.2.

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