

# Лабораторная работа № 1

Решить следующие смешанные задачи методом разделения переменных. Произвести визуализацию с использованием пакета Wolfram Mathematica. Первая задача посвящена колебанию тонкой струны, вторая – прямоугольного контура. Сумма, при необходимости, должна состоять не менее чем из 100 слагаемых.

## Вариант 1.

1.

$$\begin{cases} u_{tt} - u_{xx} = x + t, & 0 < x < 1, t > 0 \\ u|_{t=0} = x^2 \\ u_t|_{t=0} = 2x \\ u_x + u|_{x=0} = 1 \\ u|_{x=1} = 0 \end{cases}.$$

2.

$$\begin{cases} u_{tt} = \Delta u \\ u|_{x=0} = u_x|_{x=1} = 0 \\ u|_{y=0} = u|_{y=9} = 0 \\ u|_{t=0} = 1 \\ u_t|_{t=0} = 1 \end{cases}.$$

## Вариант 2.

1.

$$\begin{cases} u_{tt} - u_{xx} = x, & 0 < x < 2, t > 0 \\ u|_{t=0} = 0 \\ u_t|_{t=0} = x \\ u|_{x=0} = 2 + t \\ u_x - 2u|_{x=2} = t \end{cases}.$$

2.

$$\begin{cases} u_{tt} = \Delta u \\ u_x|_{x=0} = u|_{x=2} = 0 \\ u|_{y=0} = u_y|_{y=5} = 0 \\ u|_{t=0} = x + y \\ u_t|_{t=0} = x - y \end{cases}.$$

## Вариант 3.

1.

$$\begin{cases} u_{tt} - u_{xx} = e^t \cos(x), & 0 < x < 3, t > 0 \\ u|_{t=0} = \cos(x) \\ u_t|_{t=0} = 2x \\ u_x + 3u|_{x=0} = 2t \\ u|_{x=3} = t - 1 \end{cases}.$$

2.

$$\begin{cases} u_{tt} = \Delta u \\ u_x|_{x=0} = u_x|_{x=3} = 0 \\ u|_{y=0} = u|_{y=7} = 0 \\ u|_{t=0} = x \\ u_t|_{t=0} = y \end{cases}.$$

#### Вариант 4.

1.

$$\begin{cases} u_{tt} - u_{xx} = 4t \sin(x), & 0 < x < 4, t > 0 \\ u|_{t=0} = 0 \\ u_t|_{t=0} = 0 \\ u|_{x=0} = 3 \\ u_x - 4u|_{x=4} = t^2 + t \end{cases}.$$

2.

$$\begin{cases} u_{tt} = \Delta u \\ u_x|_{x=0} = u|_{x=4} = 0 \\ u_y|_{y=0} = u_y|_{y=6} = 0 \\ u|_{t=0} = xy \\ u_t|_{t=0} = 1 \end{cases}.$$

#### Вариант 5.

1.

$$\begin{cases} u_{tt} - u_{xx} = x(2+t), & 0 < x < 5, t > 0 \\ u|_{t=0} = x \\ u_t|_{t=0} = 0 \\ u_x + 5u|_{x=0} = 1 - t \\ u|_{x=5} = t \end{cases}.$$

2.

$$\begin{cases} u_{tt} = \Delta u \\ u|_{x=0} = u_x|_{x=5} = 0 \\ u_y|_{y=0} = u|_{y=9} = 0 \\ u|_{t=0} = e^{x+y} \\ u_t|_{t=0} = e^{x-y} \end{cases}.$$

#### Вариант 6.

1.

$$\begin{cases} u_{tt} - u_{xx} = te^{-x}, & 0 < x < 6, t > 0 \\ u|_{t=0} = 0 \\ u_t|_{t=0} = x^2 \\ u|_{x=0} = t \\ u_x - 6u|_{x=6} = 2t - 1 \end{cases}.$$

2.

$$\begin{cases} u_{tt} = \Delta u \\ u_x|_{x=0} = u_x|_{x=6} = 0 \\ u|_{y=0} = u|_{y=8} = 0 \\ u|_{t=0} = \cos\left(\frac{\pi}{6}x\right) \sin\left(\frac{3\pi}{8}y\right) \\ u_t|_{t=0} = \cos\left(\frac{5\pi}{6}x\right) \sin\left(\frac{7\pi}{8}y\right) \end{cases}.$$

### Вариант 7.

1.

$$\begin{cases} u_{tt} - u_{xx} = e^{t+x}, & 0 < x < 7, t > 0 \\ u|_{t=0} = 1 \\ u_t|_{t=0} = 0 \\ u_x + 7u|_{x=0} = 1 \\ u|_{x=7} = e^t \end{cases}.$$

2.

$$\begin{cases} u_{tt} = \Delta u \\ u_x|_{x=0} = u|_{x=4} = 0 \\ u|_{y=0} = u_y|_{y=9} = 0 \\ u|_{t=0} = \cos\left(\frac{\pi}{8}x\right) \sin\left(\frac{3\pi}{18}y\right) \\ u_t|_{t=0} = 1 \end{cases}.$$

### Вариант 8.

1.

$$\begin{cases} u_{tt} - u_{xx} = t^3, & 0 < x < 8, t > 0 \\ u|_{t=0} = 0 \\ u_t|_{t=0} = \cos(x) \\ u|_{x=0} = t^2 \\ u_x - 8u|_{x=8} = 1 \end{cases}.$$

2.

$$\begin{cases} u_{tt} = \Delta u \\ u|_{x=0} = u_x|_{x=3} = 0 \\ u|_{y=0} = u|_{y=7} = 0 \\ u|_{t=0} = 1 \\ u_t|_{t=0} = \sin\left(\frac{5\pi}{6}x\right) \sin\left(\frac{3\pi}{7}y\right) \end{cases}.$$

### Вариант 9.

1.

$$\begin{cases} u_{tt} - u_{xx} = 2t - x^2, & 0 < x < 9, t > 0 \\ u|_{t=0} = \sin(x) \\ u_t|_{t=0} = \cos(x) \\ u_x + 9u|_{x=0} = e^t \\ u|_{x=9} = 0 \end{cases}.$$

2.

$$\begin{cases} u_{tt} = \Delta u \\ u|_{x=0} = u|_{x=2} = 0 \\ u|_{y=0} = u|_{y=5} = 0 \\ u|_{t=0} = \sin\left(\frac{\pi}{2}x\right) \sin\left(\frac{7\pi}{5}y\right) \\ u_t|_{t=0} = \sin\left(\frac{5\pi}{2}x\right) \sin\left(\frac{3\pi}{5}y\right) \end{cases}.$$

**Вариант 10.**

1.

$$\begin{cases} u_{tt} - u_{xx} = \cos(t), \quad 0 < x < 10, \quad t > 0 \\ u|_{t=0} = 0 \\ u_t|_{t=0} = \sin(x) \\ u|_{x=0} = 1 \\ u_x - 10u|_{x=10} = 0 \end{cases}.$$

2.

$$\begin{cases} u_{tt} = \Delta u \\ u|_{x=0} = u|_{x=5} = 0 \\ u_y|_{y=0} = u|_{y=8} = 0 \\ u|_{t=0} = 1 \\ u_t|_{t=0} = y \end{cases}.$$

**Вариант 11.**

1.

$$\begin{cases} u_{tt} - u_{xx} = t^2 - x, \quad 0 < x < 11, \quad t > 0 \\ u|_{t=0} = 2 \\ u_t|_{t=0} = 0 \\ u_x + 11u|_{x=0} = t \\ u|_{x=11} = -1 \end{cases}.$$

2.

$$\begin{cases} u_{tt} = \Delta u \\ u|_{x=0} = u_x|_{x=6} = 0 \\ u_y|_{y=0} = u|_{y=7} = 0 \\ u|_{t=0} = x \\ u_t|_{t=0} = 1 \end{cases}.$$

**Вариант 12.**

1.

$$\begin{cases} u_{tt} - u_{xx} = \sin(t), \quad 0 < x < 12, \quad t > 0 \\ u|_{t=0} = x \\ u_t|_{t=0} = 1 \\ u|_{x=0} = 2t + 1 \\ u_x - 12u|_{x=12} = 0 \end{cases}.$$

2.

$$\begin{cases} u_{tt} = \Delta u \\ u|_{x=0} = u|_{x=4} = 0 \\ u_y|_{y=0} = u|_{y=5} = 0 \\ u|_{t=0} = e^y \sin\left(\frac{\pi}{4}x\right) \\ u_t|_{t=0} = y \sin\left(\frac{3\pi}{4}x\right) \end{cases}.$$

### Вариант 13.

1.

$$\begin{cases} u_{tt} - u_{xx} = 2, & 0 < x < 13, t > 0 \\ u|_{t=0} = x^2 + 1 \\ u_t|_{t=0} = x \\ u_x + 13u|_{x=0} = 1 + t \\ u|_{x=13} = 1 - 2t \end{cases}.$$

2.

$$\begin{cases} u_{tt} = \Delta u \\ u|_{x=0} = u_x|_{x=5} = 0 \\ u|_{y=0} = u|_{y=8} = 0 \\ u|_{t=0} = e^x \sin\left(\frac{\pi}{8}y\right) \\ u_t|_{t=0} = x \sin\left(\frac{5\pi}{8}y\right) \end{cases}.$$

### Вариант 14.

1.

$$\begin{cases} u_{tt} - u_{xx} = x - t, & 0 < x < 14, t > 0 \\ u|_{t=0} = 1 - x \\ u_t|_{t=0} = 2x \\ u|_{x=0} = 0 \\ u_x - 14u|_{x=14} = t^2 \end{cases}.$$

2.

$$\begin{cases} u_{tt} = \Delta u \\ u|_{x=0} = u_x|_{x=7} = 0 \\ u|_{y=0} = u|_{y=9} = 0 \\ u|_{t=0} = xe^y \\ u_t|_{t=0} = ye^x \end{cases}.$$

### Вариант 15.

1.

$$\begin{cases} u_{tt} - u_{xx} = x, & 0 < x < 15, t > 0 \\ u|_{t=0} = x^2 - x \\ u_t|_{t=0} = 0 \\ u_x + 15u|_{x=0} = 2t \\ u|_{x=15} = 0 \end{cases}.$$

2.

$$\begin{cases} u_{tt} = \Delta u \\ u|_{x=0} = u_x|_{x=5} = 0 \\ u|_{y=0} = u_y|_{y=7} = 0 \\ u|_{t=0} = xy \\ u_t|_{t=0} = x + y \end{cases} .$$