Topic 1 - Math Basics

Quantum Machine Learning Course

Outline

- Complex Numbers Vs. Real Numbers.
- Scalars and Vectors.
- Orthogonal Vectors.
- Eigen Values and Eigen Vector
- Matrices and their Operations.
- Tensor.
- Orthogonal and Unitary Matrices.
- Probability.

Complex Numbers and Real Numbers

- Real numbers consist of all
  - rational numbers.
    - Examples: 5 , -3.14 , $^1/_2$
  - Irrational numbers (real numbers cannot be expressed as a ratio of integers)
    - Examples: $\sqrt{2}, \pi$
- In quantum mechanics, real numbers are primarily associated with the physical observables and measurement.
- The outcomes of measurements, which are real values, are crucial in understanding the physical reality of a quantum system
- Imaginary numbers are multiples of the imaginary unit ' i ', where $i^2 = -1$
  - Examples: $2i$, $3i$
- Complex numbers are numbers of the form $\mathrm{a} + bi$, where a and b are real numbers and i is the imaginary unit.
- Complex numbers play fundamental role in representing the state of a quantum system and its evolution over time.
- Examples: $3 + 2i, -1 - 4i$

Operations on Real and Complex Numbers

- Addition, subtraction, multiplication, division.

- Real Numbers: $2 + 3 = 5$, $4 - 2 = 2$, $3 \times 2 = 6$, ${}^{6}/_{2} = 2$.

- Comples Numbers: $(3 + 2i) + (1 - 4i) = 4 - 2i$

$$(3 + 2i) - (1 - 4i) = 2 + 6i$$

$$(3 + 2i) \times (1 - 4i) = 11 + 10i$$

Absolute Value of Complex Numbers

- The absolute value (or modulus) of a complex number is the distance of the number from the origin in the complex plane
- The absolute value (or modulus) of a complex number $a + bi$ is $|a + bi|$= $\sqrt{(a^2 + \ b^2)}$
- Examples: $|3 + 2i| = \sqrt{(3^2 + 2^2)}$ =$\sqrt{13}$

Scalars and Vectors

- What are Scalars?
  - Scalars are quantities that have only magnitude (size) but no direction.
    - Examples: Mass, temperature, time, distance.
  - "scalar" is often used to refer to a complex number that scales a quantum state vector without changing its direction

- What are Vectors?
  - Vectors are quantities that have both magnitude and direction.
    - Examples: Displacement, velocity, acceleration, force.
  - Quantum state is represented by wave function or state vector.

- These vectors can be multiplied by complex scalars during the evolution of the quantum system.

Representation of Vectors in Quantum

- In quantum computing, a vector is denoted by a ket $|v\rangle$ or a bra $\langle w|$.
- Examples: $|v\rangle$ = $\begin{bmatrix} 1 \\ 3 \\ 2 \\ 5 \end{bmatrix}$, or $\langle w| = [1, 3, 2, 5]$.
- The magnitude (or length) of a vector is denoted by $|\ |v\rangle\ |$ or $||\ |v\rangle\ ||$.
- Examples: $|\ |v\rangle\ |$ = 4.

Vector Operations

- Addition: $|a\rangle + |b\rangle = |a+b\rangle = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{bmatrix}$.
- Subtraction: $|a\rangle - |b\rangle = |a+b\rangle = \begin{bmatrix} a_1 - b_1 \\ a_2 - b_2 \\ \vdots \\ a_n - b_n \end{bmatrix}$.
- Scalar multiplication of a vector: $x|a\rangle = \begin{bmatrix} xa_1 \\ xa_2 \\ \vdots \\ xa_n \end{bmatrix}$.

Vector Operations Examples

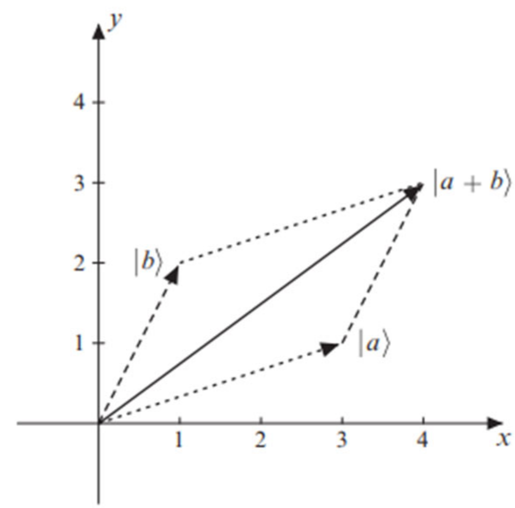
- Example 1: $|a\rangle$ = $\begin{bmatrix}1\\4\\2\\1\end{bmatrix}$, $|b\rangle = \begin{bmatrix}2\\3\\1\\5\end{bmatrix}$, $|a\rangle$ + $|b\rangle$ = $\begin{bmatrix}3\\7\\3\\6\end{bmatrix}$.
- Example 2: $|v\rangle$ = $\begin{bmatrix}1\\3\\4\\2\end{bmatrix}$, $\langle w| = [1, 3, 2, 5]$, $|v\rangle$ + $\langle w|$= $\begin{bmatrix}2\\6\\6\\7\end{bmatrix}$.
- Example 3: $|x\rangle$ = $\begin{bmatrix}4\\5\\2\\6\end{bmatrix}$, $|y\rangle = \begin{bmatrix}2\\3\\1\\5\end{bmatrix}$, $|x\rangle$ - $|y\rangle$ = $\begin{bmatrix}2\\2\\1\\1\end{bmatrix}$.
- Example 4: $2|a\rangle = \begin{bmatrix}2\\8\\4\\2\end{bmatrix}$.
- Example 5: $\langle s| = [2, 1, 1, 3]$, $\langle r| = [1, 2, 4, 3]$, $\langle s| + \langle r| = [3, 3, 5, 6]$.

Orthogonal Vectors

- Two vectors $|a\rangle$ and $|b\rangle$ are perpendicular if and only if $\left||a\rangle\right|^2 + \left||b\rangle\right|^2 = \left||a\rangle + |b\rangle\right|^2$.

- Orthogonal vectors are vectors that are perpendicular to each other.

- In a vector space, two vectors are orthogonal if their dot product is zero.
  - This means the dot or inner product of the two vector is zero
    - *ie.,* $V_i \cdot V_j = 0$ for all $i \neq j$



Orthogonal Vectors

- Example 1: 2D Space: Consider two vectors in a 2D space, $|v1\rangle$ = [3, 1] and $|v2\rangle$ = [-1, 3].

- To determine if they are orthogonal, we calculate their dot product:
  - $|v1\rangle \cdot |v2\rangle$ = (3 * -1) + (1 * 3) = 0.

- Example 2: Consider two vectors in 3D space, $|v1\rangle$ = [1, 0, -2] and $|v2\rangle$ = [2, -1, 1].

- The dot product of $|v1\rangle \cdot |v2\rangle$ = (1×2) + (0×-1) + (-2×1) = $2 + 0 - 2$ = 0.

- Therefore, $|v1\rangle$ and $|v2\rangle$ are orthogonal vectors.

Importance of Orthogonal Vectors in Quantum Computing (Cont)

- Orthogonal vectors are fundamental in quantum computing for several reasons:

- Basis States: Orthogonal vectors serve as basis states for qubits in quantum computing.

- The basis states $|0\rangle$ and $|1\rangle$ are orthogonal vectors that represent the two possible states of a qubit.

- In quantum mechanics, quantum states are often represented as vectors in a Hilbert space.
  - The basis states of this space are orthogonal, forming a set of mutually perpendicular vectors.

  - The orthogonality of basis states simplifies calculations and provides a convenient representation of quantum states

Importance of Orthogonal Vectors in Quantum Computing

- Quantum gates: Used to perform operations on qubits in quantum circuits.
- Orthogonal vectors play a crucial role in defining and implementing these gates.
- Measurement: The final step in quantum computations and involves collapsing the quantum state to one of the basis states.
- Orthogonal vectors are used to determine the probabilities of obtaining specific measurement outcomes.

Orthogonal and Orthonormal Vectors

Orthonormal vectors

- A set of vectors *V* is orthonormal if it is orthogonal and normalized.
- This means every vector in *V* has a unit magnitude of 1 and the set of vectors are mutually orthogonal
- Quantum states can be expressed as linear combinations of orthonormal basis vectors, simplifying the mathematical representation of the states

Orthonormal Vectors - Example

if $V = \{u_1, u_2, u_3\}$

where $u_1 = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix}$ $u_2 = \begin{bmatrix} 1/2 \\ 1/\sqrt{2} \\ 1/2 \end{bmatrix}$ $u_3 = \begin{bmatrix} 1/2 \\ -1/\sqrt{2} \\ 1/2 \end{bmatrix}$

The set of vectors u_1, u_2, u_3 are orthonormal

if $u_1 \cdot u_2 = 0$, $u_1 \cdot u_3 = 0$ and $u_2 \cdot u_3 = 0$

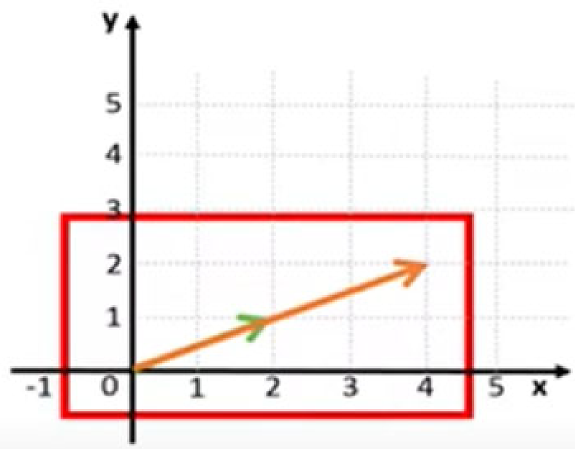
and

$|u_1|^2 = 1, |u_2|^2 = 1$ $|u_3|^2 = 1$

Eigen Vector - Example

$$v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$



$$Av = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

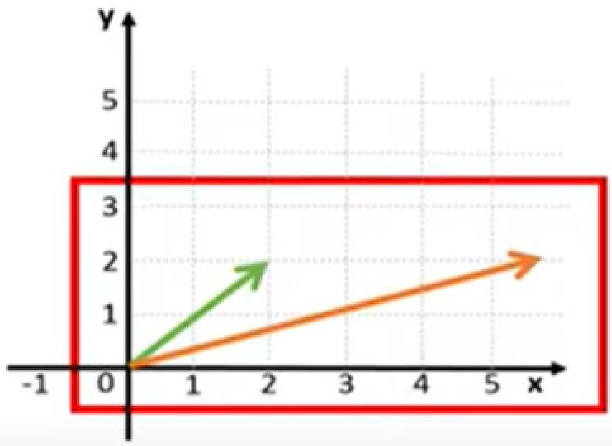
- New vector has the same direction as the original vector.

- New vector is only scaled and not rotated

Not An Eigen Vector - Example

$$v = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$



$$Av = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

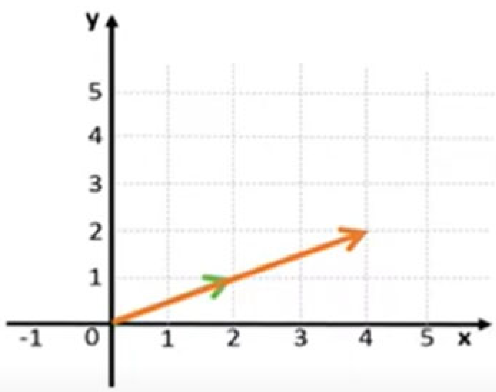
Direction of the new vector (orange color) compared to the original vector (green color) is changed

Eigen Value - Example

- Eigenvalue tells us how much the eigenvector changes in size when multiplied with the matrix

$$v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

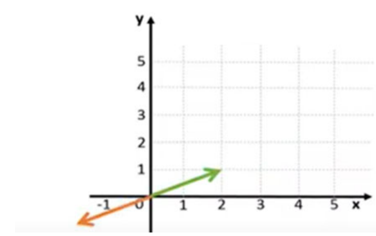
$$A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$



$$\boxed{\lambda = 2}$$

$$Av = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \qquad Av = \boxed{\lambda} v$$

- If we multiply matrix A by the eigen vector, we get a new vector
- This new vector is in the same direction as v, multiplied with some scalar, which is our eigen value.
- Scaling factor, eigen value can be negative or positive



Eigen Value and Vector – Definition and Application

- Eigenvalues and eigenvectors provide a powerful and elegant mathematical framework for representing and understanding quantum states, observables, and the dynamics of quantum systems.

- The eigenvector of a linear operator A (For example - Matrix) on a vector space V is a vector $|v\rangle$ such that

$$A\,|v\rangle = \lambda\,|v\rangle$$

λ is the eigenvalue, and $|v\rangle$ is the corresponding eigenvector

Basis Vector and Qubit

- Usually in quantum computing, a vector can be represented in terms of an underlying basis vectors eg $|0\rangle$ and $|1\rangle$.
  - Based on the assumption - any vector can be expressed as a linear sum of its basis vectors.

- If the quantum state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ of a qubit is represented as $\boxed{\begin{bmatrix} \alpha \\ \beta \end{bmatrix}}$,
  - then $|0\rangle$ and $|1\rangle$ are the basis vectors of this column vector representation of the qubit.

Matrices

- A matrix is a rectangular array of numbers or symbols arranged in rows and columns.
- It is often denoted using uppercase letters, such as $A, B, or\ C$.
- Each entry in a matrix is called a matrix element.
- The position of an element is specified by its row and column indices:
- $A_{3\times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}.$
- Example of a 3×3 matrix: $A_{3\times 3} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 1 & 6 \\ 1 & 4 & 7 \end{bmatrix}.$

Matrix Operations

Adding

To add two matrices: add the numbers in the matching positions:

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} \overset{3+4=7}{=} \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$$

These are the calculations:

3+4=7	8+0=8
4+1=5	6−9=−3

Subtracting

To subtract two matrices: subtract the numbers in the matching positions:

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} \overset{3-4=-1}{=} \begin{bmatrix} -1 & 8 \\ 3 & 15 \end{bmatrix}$$

These are the calculations:

3−4=−1	8−0=8
4−1=3	6−(−9)=15

Multiply by a Constant

We can multiply a matrix by a **constant** *(the value 2 in this case)*:

$$2 \times \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} \overset{2\times4=8}{=} \begin{bmatrix} 8 & 0 \\ 2 & -18 \end{bmatrix}$$

These are the calculations:

2×4=8	2×0=0
2×1=2	2×−9=−18

We call the constant a **scalar**, so officially this is called "scalar multiplication".

Matrix Addition and Subtraction

- Matrix Addition

$$\begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ -2 & 4 \end{bmatrix}$$

- Matrix Subtraction

$$\begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 6 & 4 \end{bmatrix}$$

Matrix Multiplication

Multiplying a Matrix by Another Matrix

But to multiply a matrix **by another matrix** we need to do the "dot product" of rows and columns ... what does that mean? Let us see with an example:

To work out the answer for the **1st row** and **1st column**:

"Dot Product"

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & \\ & \end{bmatrix}$$

The "Dot Product" is where we **multiply matching members**, then sum up:

$$(1, 2, 3) \bullet (7, 9, 11) = 1\times7 + 2\times9 + 3\times11$$
$$= 58$$

We match the 1st members (1 and 7), multiply them, likewise for the 2nd members (2 and 9) and the 3rd members (3 and 11), and finally sum them up.

Matrix Multiplication

Want to see another example? Here it is for the 1st row and **2nd column**:

$$\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}\times\begin{bmatrix}7 & 8\\ 9 & 10\\ 11 & 12\end{bmatrix}=\begin{bmatrix}58 & 64\\ & \end{bmatrix}$$

$$(1, 2, 3) \bullet (8, 10, 12) = 1\times8 + 2\times10 + 3\times12$$
$$= 64$$

We can do the same thing for the **2nd row** and **1st column**:

$$(4, 5, 6) \bullet (7, 9, 11) = 4\times7 + 5\times9 + 6\times11$$
$$= 139$$

And for the **2nd row** and **2nd column**:

$$(4, 5, 6) \bullet (8, 10, 12) = 4\times8 + 5\times10 + 6\times12$$
$$= 154$$

And we get:

$$\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}\times\begin{bmatrix}7 & 8\\ 9 & 10\\ 11 & 12\end{bmatrix}=\begin{bmatrix}58 & 64\\ 139 & 154\end{bmatrix}$$

Matrix Multiplication

- Matrix Multiplication (Scalar * Matrix)

$$2 \times 3 = 6$$

$$2 \times \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 8 & 2 \end{bmatrix}$$

- Dot product (Matrix * Matrix)

$$\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \bullet \begin{bmatrix} 8 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} (3 \times 8) + (2 \times 5) & (3 \times 6) + (2 \times 7) \\ (4 \times 8) + (1 \times 5) & (4 \times 6) + (1 \times 7) \end{bmatrix}$$

$$\begin{bmatrix} (24) + (10) & (18) + (14) \\ (32) + (5) & (24) + (7) \end{bmatrix} = \begin{bmatrix} 34 & 32 \\ 37 & 31 \end{bmatrix}$$

Transpose of a Matrix

- Transposing: To transpose a matrix, swap the rows and columns.

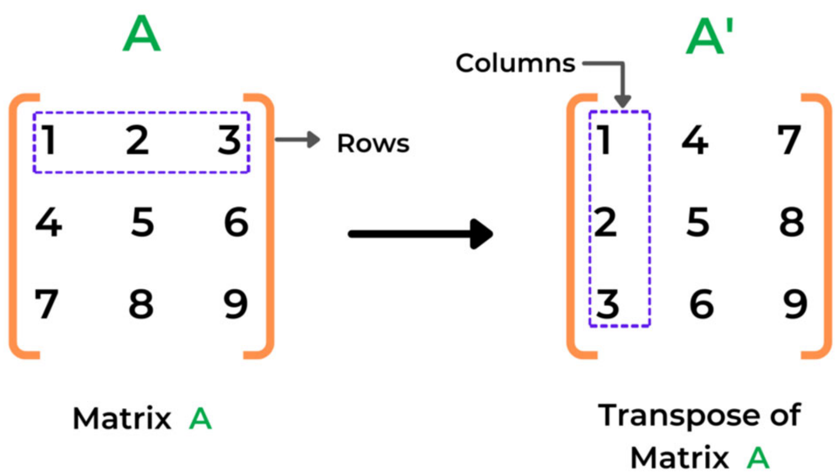
- We put a T in the top right-hand corner to mean transpose:

- $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, A^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}.$

- Example: $A = \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix}, A^T = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}.$

Transpose of a Matrix

$$A = \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix} \qquad A^T = \begin{bmatrix} A_{1,1} & A_{2,1} \\ A_{1,2} & A_{2,2} \end{bmatrix}$$



Trace of a Matrix

$$A = \begin{bmatrix} -2 & 3 & 4 \\ 1 & 7 & 9 \\ 8 & 5 & 6 \end{bmatrix}$$

$$Tr(A) = -2 + 7 + 6 = 11$$

Identity of a Matrix

- The identity matrix is a special matrix that preserves the dimensions of any matrix it multiplies.
- Denoted as I, it has ones on the diagonal and zeros elsewhere.
- Example: $I_{3\times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A \times I = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

Orthogonal Matrices

- An orthogonal matrix A is a square matrix whose columns and rows are orthogonal unit vectors.
- $A^T A = I$.
- Example A= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
- Consider that x and y are vectors and A is an orthogonal matrix:
- Orthogonal matrices preserve vector lengths: $||Ax||$= $||x||$.
- Orthogonal matrices preserve angles between vectors: dot product $(x, y) = (Ax, Ay)$.

Orthogonal Matrices

- Example: $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.
- A represents a rotation in a 2D plane by an angle θ.
- A is orthogonal matrix because its columns and rows are orthogonal unit vectors.
- $A^T A = I$, confirming its orthogonality.

Unitary Matrices

- In complex linear algebra, a unitary matrix is the counterpart of an orthogonal matrix.

- A unitary matrix A is a square matrix whose $A^*A = I$, where A^* represents the conjugate transpose of A.

- To find the conjugate transpose A^* of A:

  - First, we find A^T, then we find the complex conjugate of each element in A^T.

  - The complex conjugate of a number in the form of $a + bi$ is $a - bi$.

  - The complex conjugate of a number n is n.

Unitary Matrices

- Example: Example: $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}i & \frac{-1}{\sqrt{2}}i \end{bmatrix}$, find A^*.

- To find A^*, we need to find A^T, then the complex conjugate of each element in A^T.

- A^T= $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}i \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}}i \end{bmatrix}$

- Then A^*= $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}}i \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}i \end{bmatrix}$.

- A is a unitary matrix because $A^*A = I$.

- A represents the Hadamard transform, an important operation in quantum computing and signal processing.

Orthogonal and Unitary Matrices Applications

- Computer graphics: Transformations, rotations, and reflections.
- Signal processing: Fourier analysis and wavelet transforms.
- Quantum mechanics: Unitary evolution of quantum states.
- Error correction: Encoding and decoding operations in coding theory.

Tensor Product

A = $\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ B = $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$$A \otimes B = \begin{bmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{bmatrix}$$

Math Basics – Vectors, Matrix and Tensor

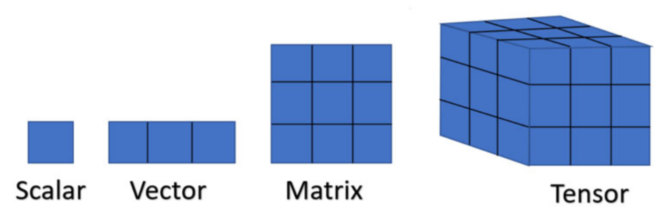
- Vectors are one dimensional matrix with only one row or only one column. Both $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are vectors

- A Matrix an m x n array of numbers, where m is the number of rows and n is the number of columns

$$\begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix} \qquad \boldsymbol{A} \in \mathbb{R}^{m \times n}$$

- A Tensor is a multidimensional array

$$\begin{bmatrix} [1 & 2] & [3 & 4] \\ [5 & 6] & [7 & 8] \\ [9 & 0] & [1 & 2] \end{bmatrix}$$



Probability (Cont.)

- Probability is a measure of uncertainty.
- It quantifies the likelihood of an event occurring.
- In quantum computing, probability is used to describe the behavior and outcomes of quantum states and measurements.
- Superposition is a fundamental concept in quantum mechanics: It allows a quantum system to exist in multiple states simultaneously.
- The probabilities associated with each state determine the likelihood of measuring a particular outcome.

Probability (Cont.)

- Consider a classical coin flip, where we have a fair coin that can result in either heads (h) or tails (t).
- The probability of getting h is 0.5, and the probability of getting t is also 0.5.
- In quantum computing, we can have a quantum coin that can be in a superposition of both heads ($|h\rangle$) and tails ($|t\rangle$).
- The probability amplitudes describe the chances of measuring a particular outcome.
- For a fair quantum coin, the probability amplitudes of $|h\rangle$ and $|t\rangle$ would be $\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$, respectively.

Probability (Cont.)

- A qubit is the basic unit of information in quantum computing.
- It can be in a superposition of two states: $|0\rangle$ and $|1\rangle$.
- The probabilities of measuring $|0\rangle$ and $|1\rangle$ are given by their probability amplitudes.
- If a qubit is in the state $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$, the probabilities of measuring $|0\rangle$ and $|1\rangle$ are 0.5 each.
- Upon measurement, the qubit will collapse into either $|0\rangle$ or $|1\rangle$, with equal probabilities.

Probability Applications in Quantum Mechanics

- Quantum state often represents a superposition of **different possible states** until a measurement is made, collapsing the system into **one of the possible outcomes.**

- Quantum state is represented by **wave function or state vector**.

Probability Applications in Quantum Computing

- Superposition – Phenomenon of Qubit that can exist in multiple state… Probability is used to find a qubit in a particular state

- Measurement – When a quantum system is measured, it "collapses" to one of the basis states… Outcome of the measurement is probabilistic…

Summary

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Next Week Topics

- Topic 2 - Introduction to Quantum Computing – Gates; Operators and Circuits
  - Read Text Book (QML- An Applied Approach - Ganguly) – Ch. 1
  - Read Ref. Book 1 (Intro to QC –Bernhardt) – pp. 118-140

Appendix

Linear Operator

- A linear operator A maps a vector $|v\rangle$ in vector space V to a vector $|w\rangle$ in the vector space W and is linear in its inputs.

if $|v\rangle = \sum_i \boldsymbol{a}_i |v_i\rangle$, then for a linear operator;

$A\sum_i \boldsymbol{a}_i |v_i\rangle = \sum_i \boldsymbol{a}_i A |v_i\rangle$

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Eigen Values and Eigen Vectors

- The eigenvalues can be interpreted as the possible values of observable A. Since A is an observable, operator A is Hermitian

- The eigenvalues of an operator can be found by solving for its characteristic equation, $\det|A - \lambda I| = 0$

- The characteristic function corresponding to the characteristic equation of the linear operator is defined as $c(\lambda) = \det | A - \lambda I |$

- Comples Numbers: $(3 + 2i) + (1 - 4i) = 4 - 2i$, $(3 + 2i) - (1 - 4i) = 2 + 6i$, $(3 + 2i) * (1 - 4i) = 11 - 10i$, $(3 + 2i) / (1 - 4i) = (-1/17) + (14/17)i$.