Exercise 8

Deadline: 18.07.2018

This exercise deals with Hidden Markov Models and Causal Analysis.

Regulations

Please create a Jupyter notebook `robot.ipynb` (along with export `robot.html`) for task 1 and a PDF file `berkeley.pdf` for task 2. Zip all files into a single archive with naming convention (sorted alphabetically by last names)

`lastname1-firstname1_lastname2-firstname2_exercise08.zip`

or (if you work in a team of three)

`lastname1-firstname1_lastname2-firstname2_lastname3-firstname3_exercise08.zip`

and upload it to Moodle before the given deadline. We will give zero points if your zip-file does not conform to the naming convention.

1 Robot on a Circle (20 Points)

Consider the following tracking problem. A Robot is constrained to a circular corridor with S discrete positions.

Transition

At each timestep, a command is sent to the robot to either move clockwise or counter-clockwise. The transition probabilities for the commands are supposed to be independent

$$A = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}.$$

However, the probability that the robot receives the signal correctly is only $\epsilon < 1$. When the command is incorrectly received, the robot will move in the opposite direction. Adjust your model accordingly.

Measurement

You do not know the robot's command and can only get a faulty measurement of its position. Your sensor fails completely with a probability of ω (model this as an additional state). It gives you a completely random position (uniformly distributed) with probability τ. In all other cases, your measurement will yield the correct position (probability $1 - \omega - \tau$).

Tasks:

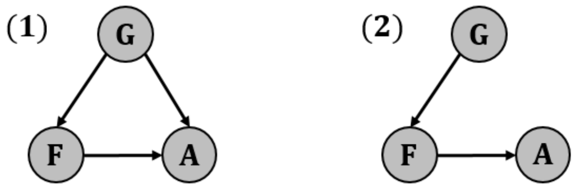
- Describe this process with a hidden Markov Model (draw the nodes and include their possible states).
- Compute all transition matrices for $S = 50$, $\epsilon = 0.4$, $\omega = 0.3$ and $\tau = 0.1$.
- Create your own implementation of the Baum-Welch algorithm.
- Use the sequence of measurements given in file `train_walk.npy` on Moodle to learn the values of the transition matrices using the Baum-Welch algorithm.
- Use the Viterbi algorithm from the previous homework to compute the globally most likely path, given the measurements in file `test_walk.npy` in Moodle. Compare this to the locally most likely position in each timestep.

2 Berkeley Admission (10 Bonus Points)

We consider again the Berkeley admission example from the lecture. Recall that the University of Berkeley was sued because it apparently preferred men over woman. This exercise constructs a counter-argument against this claim of gender discrimination. Applications vs. admissions data for six fields of study are given in the following table:

field	men		woman	
	applied	admitted	applied	admitted
A	825	512	108	89
B	560	353	25	17
C	325	120	593	202
D	417	138	375	131
E	191	53	393	94
F	272	16	341	24

The training data can be used to compute the joint probability $p(G, F, A)$ (with G = gender $\in$ {male, female}, F = field of study $\in \{A, ..., F\}$, and A = admission $\in$ {true, false}). We can express the two competing claims in terms of causal models that treat F as a mediating variable between G and A, and either contain or do not contain a direct causal influence from G to A.



Total Causal Effect

The total causal effect from gender to admission in model (1) is defined by the interventional distributions

$$p_1(A = \text{true}|\, do(G = \text{male})) \quad \text{and} \quad p_1(A = \text{true}|\, do(G = \text{female}))$$

Since gender is a root variable in the present application, these distributions coincide with the conditional distributions here:

$$p_1(A = \text{true}|\, do(G)) = p_1(A = \text{true}|\, G)$$

Tasks:

1. Derive the formula to compute these conditional probabilities from the factorization according to model (1).
2. Use the given data to calculate the conditional probability of A = true for male and female applicants and prove that men indeed had a higher chance of being accepted, indicating a total causal effect in favor of men.

Direct Causal Effect

The University of Berkeley argued that only a *direct* causal effect from gender to admission would constitute a case of discrimination. To this end, we compare the conditional probabilities derived from models (1) and (2). If both conditionals are (nearly) the same, there is no discrimination. If the elimination of the direct link increases the acceptance chances for woman or men, a case of discrimination in the respective direction has been found.

When the factorization for model (1) is known, we can derive the factorization of model (2) by means of the "cut" operator. The critical factor to be considered is the conditional probability of A:

$$\begin{aligned} p_2(A \mid F) =& p_2(A \mid \mathrm{PA}_2(A)) = p_1(A \mid \mathrm{PA}_1(A), \mathit{cut}(G \to A)) = p_1(A \mid G, F, \mathit{cut}(G \to A)) \\ =& \sum_{\widetilde{G}} p_1(A \mid \widetilde{G}, F) p_1(\widetilde{G}) \end{aligned}$$

where $\mathrm{PA}(A)$ indicates the parents of A, and $p_1(\widetilde{G})$ is the marginal distribution of G in model (1). In words, we introduce an independent copy $\widetilde{G}$ of variable G into the model and reconnect A's incoming arc from G to $\widetilde{G}$. The variable $\widetilde{G}$ is distributed according to the marginal distribution of G and immediately marginalized out, because we are not interested in it.

Task:

1. Derive the complete formula for computing $p_2(A \mid G)$ by applying the cut-operator to model (1).
2. Use the given data to calculate the conditionals $p_2(A = \text{true} | \, G)$ according to model (2). Check if there is discrimination by comparing these results with $p_1(A = \text{true} | \, G)$.