logistic-regression

May 8, 2018

1 1 Loading the Dataset

```
In [4]: import numpy as np
        from sklearn.datasets import load_digits
        digits = load_digits()
        print(digits.keys())
        data = digits["data"]
        images = digits["images"]
        target = digits["target"]
        target_names = digits["target_names"]
        print(data.shape)
```

```
dict_keys(['data', 'target', 'target_names', 'images', 'DESCR'])
(1797, 64)
```

Extract instances showing "3" or "8", append a column of "1s" and create a vector of ground-truth labels where 1 corresponds to 3 and -1 to 8.

```
In [5]: X = data[np.logical_or(target == 3, target == 8)]
        y = target[np.logical_or(target == 3, target == 8)]

        X = np.concatenate((X, np.ones([len(y),1])), axis = 1)

        y[y==3] = 1
        y[y==8] = -1
```

2 1.1 Classification with sklearn

```
In [6]: from sklearn.linear_model import LogisticRegression
        from sklearn.model_selection import cross_val_score, train_test_split

In [7]: num_splits = 10
        lambdas = [0.001,0.01,0.1,1,10,100,1000]
        scores = np.zeros([len(lambdas), 2]) # save mean score and std for each lambda

        for i, C in enumerate(lambdas):
```

```
            logistic = LogisticRegression(C = C)
            curr_scores = cross_val_score(logistic, X, y, cv = num_splits)
            scores[i,0] = np.average(curr_scores)
            scores[i,1] = np.std(curr_scores)
```

```
In [8]: import pandas as pd
        pd.options.display.float_format = '{:,.4f}'.format
```

```
In [9]: display(pd.DataFrame(
                data = scores,
                index = lambdas,
                columns = ['mean','std'])
                .rename_axis('C', axis = 'columns'))
```

C	mean	std
0.0010	0.9688	0.0531
0.0100	0.9691	0.0464
0.1000	0.9860	0.0288
1.0000	0.9860	0.0288
10.0000	0.9803	0.0283
100.0000	0.9775	0.0304
1,000.0000	0.9747	0.0341

The accuracy of the prediction varies only very slowly when modifying the regularization parameter. The best accuracy is obtained for $\lambda = 0.1, 1, 10$, thus a very large range. In the following we will proceed using $\lambda = 1$.

3 1.2 Optimization Methods

```
In [10]: def sigmoid(z):
             return 1 / (1 + np.exp(-z))
```

```
In [11]: def predict(beta, X):
             # if X*beta > 0 --> y=1, if X*beta < 0 --> y=-1
             y = (X.dot(beta) >= 0) * 2 - 1
             return y
```

```
In [12]: def zero_one_loss(y_prediction, y_truth):
             return np.sum(np.not_equal(y_prediction, y_truth))
```

```
In [13]: def gradient(beta, X, y, lambda_ = 1):
             # distinguish the cases of one single / several training instances
             if np.isscalar(y):
                 grad = beta / lambda_ - sigmoid(-X.dot(beta) * y) * y * X
             else:
                 grad = beta / lambda_ - np.average((sigmoid(-X.dot(beta) * y) * y)[:,None] * X
             return grad
```

```
In [14]: def GD(X, y, beta, tau_0, gamma, m):
             for _ in range(m):
                 tau = tau_0 / (1 + gamma * _)
                 beta = beta - tau * gradient(beta, X, y)
             return beta
```

To sample single random instances, we do sampling without replacement since we want to avoid computing the gradient from the same instance repetitively. Instead, we want to use as much different samples as possible to compute the gradient. Only in the case of mini-batches, we do sampling with replacement since the training set is not large enough to sample 150 times a mini-batch witout replacing "old" samples.

```
In [15]: from sklearn.utils import shuffle
         def SGD(X, y, beta, tau_0, gamma, m):
             X, y = shuffle(X,y)
             for i in range(m):
                 tau = tau_0 / (1 + gamma * i)
                 beta = beta - tau * gradient(beta, X[i,:], y[i])
             return beta

In [16]: def SG_minibatch(X, y, beta, tau_0, gamma, m, B = 1):
             # here we do not replace the samples since there would not be enough samples
             # to perform m=150 optimization steps for larger B
             for i in range(m):
                 X, y = shuffle(X,y)
                 tau = tau_0 / (1 + gamma * i)
                 beta = beta - tau * gradient(beta, X[:B,:], y[:B])
             return beta

In [17]: def SG_momentum(X, y, beta, tau_0, gamma, mu, m):
             X, y = shuffle(X,y)
             # initialize g
             g = np.zeros(len(beta))
             for i in range(m):
                 tau = tau_0 / (1 + gamma * i)
                 g = mu * g + (1 - mu) * gradient(beta, X[i,:], y[i])
                 beta = beta - tau * g
             return beta

In [18]: def ADAM(X, y, beta, m, tau=10**-4, mu1=0.9, mu2=0.999, eps=10**-8):
             X, y = shuffle(X,y)
             # initialize g, q
             g = np.zeros(len(beta))
             q = np.zeros(len(beta))
             for i in range(m):
                 grad = gradient(beta, X[i,:], y[i])
                 g = mu1 * g + (1 - mu1) * grad
                 q = mu2 * q + (1 - mu2) * np.square(grad)
                 beta = beta - tau / (np.sqrt(q) + eps) * g
             return beta
```

In [19]:
```
# so wie im Skript
def Newton_Raphson(X, y, beta, m, lambda_=0.001):# hier muss lambda kleiner gewählt w
    N,D = X.shape
    for _ in range(m):
        z = X.dot(beta)
        y_tilde = y / sigmoid(-y * z)
        W = np.diag(lambda_ / N * sigmoid(z) * sigmoid(-z))
        inv = np.linalg.inv(1 / lambda_ * np.identity(D) + np.dot(X.T, np.dot(W, X)))
        beta = beta + np.dot(inv, np.dot(X.T, np.dot(W, y_tilde)) - beta / lambda_)
    return beta
```

In [20]:
```
# so wie auf dem Zettel
def Newton_Raphson1(X, y, beta, m, lambda_=0.001):
    N,D = X.shape
    for _ in range(m):
        z = X.dot(beta)
        y_tilde = y / sigmoid(-y * z)
        W = lambda_ / N * np.diag(sigmoid(z) * sigmoid(-z))
        inv = np.linalg.inv(np.identity(D) + np.dot(X.T, np.dot(W, X)))
        beta = np.dot(inv, np.dot(X.T, W)).dot(z + y_tilde)
    return beta
```

In [21]:
```
def dual_coordinate_ascent(X, y, m, lambda_=1):
    # initialization:
    N = X.shape[0]
    alpha = np.random.uniform(size=N)
    beta = lambda_ * np.average((alpha * y)[:,None] * X, axis=0)
    X, y = shuffle(X, y)
    for i in range(m):
        f_prime = y[i] * np.dot(X[i,:], beta) + np.log(alpha[i] / (1 - alpha[i]))
        f_2prime = lambda_ / N * np.dot(X[i,:], X[i,:].T) + 1 / (alpha[i] * (1 - alpha
        alpha_old = alpha[i].copy()
        alpha[i] = max(0, min(1, alpha[i] - f_prime / f_2prime))
        beta += lambda_ / N * y[i] * X[i,:] * (alpha[i] - alpha_old)
    return beta
```

In [22]:
```
def SAGD(X, y, beta_init, tau_0, gamma, m, lambda_ = 0.001):
    #init
    try:
        X, y = shuffle(X, y)
        N    = X.shape[0]
        beta = beta_init
        g_stored = np.zeros(N)
        g_vec    = np.zeros(N)
        for i in range(N):
            #print(sigmoid(-y[i] * np.dot(X[i,:],beta)))
            g_stored[i] = -y[i] * X[i, :] * sigmoid(-y[i] * np.dot(X[i,:],beta))
        g = np.average(g_stored)
```

```
            #iterate
            for i in range(m):
                g_vec[i] = -y[i] * X[i,:] * sigmoid(-y[i] * np.dot(X[i,:],beta))
                g     = g + 1. / N * (g_vec[i] - g_stored[i])
                tau_t = tau_0 / (1 + gamma * i)
                g_stored[i] = g_vec[i]
                beta  = beta * (1 - tau_t / lambda_) - tau_t * g
            return beta
        except:
            raise NotImplementedError('Not yet implemented')
```

dual coordinate ascent

```
In [23]: X_train, X_test, y_train, y_test = train_test_split (X, y, test_size = 0.3 ,random_sta
         beta = dual_coordinate_ascent(X_train, y_train, 150)

         errors = zero_one_loss(predict(beta, X_test), y_test)
         print(errors / len(y_test))
```

```
0.0277777777778
```

SAGD

```
In [24]: X_train, X_test, y_train, y_test = train_test_split (X, y, test_size = 0.3 ,random_sta
         beta = np.zeros(65)
         tau_0 = 0.001
         gamma = 0.0001
         beta = SAGD(X_train, y_train, beta, tau_0, gamma, 150)

         errors = zero_one_loss(predict(beta, X_test), y_test)
         print(errors / len(y_test))
```

```
        ---------------------------------------------------------------------------

        ValueError                                Traceback (most recent call last)

        <ipython-input-22-028ad3c55fc7> in SAGD(X, y, beta_init, tau_0, gamma, m, lambda_)
         10             #print(sigmoid(-y[i] * np.dot(X[i,:],beta)))
    ---> 11             g_stored[i] = -y[i] * X[i, :] * sigmoid(-y[i] * np.dot(X[i,:],beta))
         12         g = np.average(g_stored)


        ValueError: setting an array element with a sequence.


    During handling of the above exception, another exception occurred:
```

```
NotImplementedError                       Traceback (most recent call last)

<ipython-input-24-f91d9cc6115d> in <module>()
  3 tau_0 = 0.001
  4 gamma = 0.0001
----> 5 beta = SAGD(X_train, y_train, beta, tau_0, gamma, 150)
  6
  7 errors = zero_one_loss(predict(beta, X_test), y_test)


<ipython-input-22-028ad3c55fc7> in SAGD(X, y, beta_init, tau_0, gamma, m, lambda_)
 20         return beta
 21     except:
---> 22         raise NotImplementedError('Not yet implemented')


NotImplementedError: Not yet implemented
```

GD

```
In [25]: X_train, X_test, y_train, y_test = train_test_split (X, y, test_size = 0.3 ,random_sta
         beta = np.zeros(65)
         beta = GD(X_train, y_train, beta, 0.001, 0.0001, 10)

         errors = zero_one_loss(predict(beta, X_test), y_test)
         print(errors / len(y_test))
```

```
0.0185185185185
```

Newton Raphson

```
In [26]: X_train, X_test, y_train, y_test = train_test_split (X, y, test_size = 0.3 ,random_sta
         beta = np.zeros(65)
         beta = Newton_Raphson(X_train, y_train, beta, 150)

         errors = zero_one_loss(predict(beta, X_test), y_test)
         print(errors / len(y_test))
```

```
0.0185185185185
```

```
In [27]: X_test.shape
```

```
Out[27]: (108, 65)
```

SGD w/o replacement

```
In [28]: X_train, X_test, y_train, y_test = train_test_split (X, y, test_size = 0.3 ,random_sta
         beta = np.zeros(65)
         tau_0 = 0.001
         gamma = 0.0001
         beta = SGD(X_train, y_train, beta, tau_0, gamma, 150)

         errors = zero_one_loss(predict(beta, X_test), y_test)
         print(errors / len(y_test))
```

```
0.0185185185185
```

SGD with minibatch

```
In [29]: X_train, X_test, y_train, y_test = train_test_split (X, y, test_size = 0.3 ,random_sta
         beta = np.zeros(65)
         tau_0 = 0.001
         gamma = 0.0001
         beta = SG_minibatch(X_train, y_train, beta, tau_0, gamma, 150, B = 20)

         errors = zero_one_loss(predict(beta, X_test), y_test)
         print(errors / len(y_test))
```

```
0.0185185185185
```

ADAM

```
In [30]: X_train, X_test, y_train, y_test = train_test_split (X, y, test_size = 0.3 ,random_sta
         beta = np.zeros(65)
         tau_0 = 0.001
         gamma = 0.0001
         mu = 0.1
         beta = ADAM(X_train, y_train, beta, 150)

         errors = zero_one_loss(predict(beta, X_test), y_test)
         print(errors / len(y_test))
```

```
0.0185185185185
```

4 1.3 Comparison

```
In [31]: from sklearn.cross_validation import KFold
         from operator import itemgetter
         import itertools
```

```
/usr/local/lib/python3.6/site-packages/sklearn/cross_validation.py:41: DeprecationWarning: This
  "This module will be removed in 0.20.", DeprecationWarning)
```

```
In [32]: optimizers = ['GD', 'SGD', 'SGD_mini', 'SGD_mom', 'ADAM', 'DualCA', 'NR']
         #tbd: 'SAGD'

         class LogisticRegression:
             def __init__(self, X, y, type_, beta_0):
                 self.X = X
                 self.y = y
                 self.type = type_
                 self.beta = beta_0

             def compare_parameters(self, tau_0_, mu_, gamma_, iter_ = 150, num_results_shown =
                 if num_results_shown is None:
                     num_results_shown = len(tau_0_)*len(mu_)*len(gamma_)
                 if self.type in ['NR', 'GD']:
                     iter_ = 10
                 X_tr, X_test, y_tr, y_test = train_test_split (self.X, self.y, test_size = 0.3
                 cross_val_results = []
                 beta = self.beta
                 for (tau_0, mu, gamma) in itertools.product(tau_0_, mu_, gamma_):
                     kf = KFold(y_tr.shape[0], n_folds=10)
                     errors = 0
                     for train_idx, val_idx in kf:
                         if self.type == 'SGD':
                             beta = SGD(X_tr[train_idx], y_tr[train_idx], beta, tau_0, gamma, i
                         elif self.type == 'GD':
                             beta = GD(X_tr[train_idx], y_tr[train_idx], beta, tau_0, gamma, it
                         elif self.type == 'SGD_mini':
                             #print('SGD minibatch running with B = ', B)
                             beta = SG_minibatch(X_tr[train_idx], y_tr[train_idx], beta, tau_0
                         elif self.type == 'SGD_mom':
                             beta = SG_momentum(X_tr[train_idx], y_tr[train_idx], beta, tau_0,
                         elif self.type == 'ADAM':
                             #ADAM(X, y, beta, m, tau=10**-4, mu1=0.9, mu2=0.999, eps=10**-8):
                             beta = ADAM(X_tr[train_idx], y_tr[train_idx], beta, iter_)
                         elif self.type == 'DualCA':
                             beta = dual_coordinate_ascent(X_tr[train_idx], y_tr[train_idx], it
                         elif self.type == 'NR':
                             beta = Newton_Raphson(X_tr[train_idx], y_tr[train_idx], beta, iter
                         elif self.type == 'SAGD':
                             #beta = self.type(X_tr[train_idx], y_tr[train_idx], beta, tau_0, g
                             raise NotImplementedError('tbd')

                         errors += zero_one_loss(predict(beta, X_tr[val_idx]), y_tr[val_idx])
```

```
                    cross_val_results.append((tau_0, mu, gamma, errors))

                sort = np.array(sorted(cross_val_results, key = itemgetter(3)))
                print('\nUsing ' , self.type, ' optimizer.')
                print('Listing results starting with lowest error score: \n')
                display(pd.DataFrame(
                    data = sort[:num_results_shown - 1, :3],
                    index = sort[:num_results_shown - 1, -1],
                    columns = ['tau_0','mu', 'gamma'])
                    .rename_axis('errors', axis = 'columns'))

                return sort[0]
                #print(sorted(cross_val_results, key = itemgetter(3)))
```

```
In [33]: tau, mu, gamma = [0.001, 0.01, 0.1], [0.1, 0.2, 0.5], [0.0001, 0.001, 0.01]
         LR = LogisticRegression(X, y, 'SGD', np.zeros(65))
         LR.compare_parameters(tau, mu, gamma)
```

```
Using  SGD  optimizer.
Listing results starting with lowest error score:
```

errors	tau_0	mu	gamma
0.0000	0.0010	0.1000	0.0100
0.0000	0.0010	0.5000	0.0100
1.0000	0.0010	0.2000	0.0100
2.0000	0.0010	0.2000	0.0010
3.0000	0.0010	0.5000	0.0010
4.0000	0.0010	0.1000	0.0001
4.0000	0.0010	0.1000	0.0010
4.0000	0.0010	0.2000	0.0001
5.0000	0.0010	0.5000	0.0001
11.0000	0.0100	0.5000	0.0100
16.0000	0.0100	0.2000	0.0100
18.0000	0.0100	0.2000	0.0010
21.0000	0.0100	0.1000	0.0100
27.0000	0.0100	0.5000	0.0001
28.0000	0.0100	0.1000	0.0010
38.0000	0.0100	0.5000	0.0010
40.0000	0.1000	0.5000	0.0100
47.0000	0.0100	0.2000	0.0001
62.0000	0.0100	0.1000	0.0001
65.0000	0.1000	0.2000	0.0100
66.0000	0.1000	0.5000	0.0001
73.0000	0.1000	0.1000	0.0100
78.0000	0.1000	0.2000	0.0010

```
78.0000 0.1000 0.5000 0.0010
87.0000 0.1000 0.2000 0.0001
96.0000 0.1000 0.1000 0.0010
```

```
Out[33]: array([ 0.001,  0.1  ,  0.01 ,  0.   ])
```

listing the n best results of each optimizer

```
In [34]: def list_n_best(X, y, n, opti_list, tau, mu, gamma):
             overview = np.empty([7], dtype=[('opt', 'U10'), ('tau', 'f4'), ('mu', 'f4'), ('gam
             for i, opt in enumerate(opti_list):
                 LR = LogisticRegression(X, y, opt, np.zeros(65))
                 best = LR.compare_parameters(tau, mu, gamma, num_results_shown=n)
                 overview['opt'][i] = opt
                 overview['tau'][i] = best[0]
                 overview['mu'][i] = best[1]
                 overview['gamma'][i] = best[2]
             print("\n \n Overall best parameters: \n")
             display(pd.DataFrame(
                     data = overview, index=overview['opt'],
                     columns = ['tau','mu', 'gamma']).rename_axis('optimizer', axis = 'columns
             return overview

In [35]: params = list_n_best(X, y, 10, optimizers, tau, mu, gamma)
```

```
Using  GD  optimizer.
Listing results starting with lowest error score:
```

```
errors  tau_0     mu  gamma
3.0000 0.0010 0.1000 0.0100
3.0000 0.0010 0.2000 0.0001
3.0000 0.0010 0.2000 0.0010
3.0000 0.0010 0.2000 0.0100
3.0000 0.0010 0.5000 0.0001
3.0000 0.0010 0.5000 0.0010
3.0000 0.0010 0.5000 0.0100
3.0000 0.0100 0.1000 0.0001
3.0000 0.0100 0.1000 0.0010
```

```
Using  SGD  optimizer.
Listing results starting with lowest error score:
```

```
errors  tau_0     mu  gamma
0.0000 0.0010 0.5000 0.0001
1.0000 0.0010 0.2000 0.0010
1.0000 0.0010 0.5000 0.0100
2.0000 0.0010 0.1000 0.0010
2.0000 0.0010 0.1000 0.0100
2.0000 0.0010 0.2000 0.0100
5.0000 0.0010 0.2000 0.0001
6.0000 0.0010 0.1000 0.0001
6.0000 0.0010 0.5000 0.0010


Using  SGD_mini  optimizer.
Listing results starting with lowest error score:


errors  tau_0     mu  gamma
1.0000 0.0010 0.2000 0.0001
2.0000 0.0010 0.1000 0.0010
2.0000 0.0010 0.5000 0.0010
2.0000 0.0010 0.5000 0.0100
3.0000 0.0010 0.1000 0.0001
3.0000 0.0010 0.1000 0.0100
3.0000 0.0010 0.2000 0.0010
3.0000 0.0010 0.2000 0.0100
3.0000 0.0010 0.5000 0.0001


Using  SGD_mom  optimizer.
Listing results starting with lowest error score:


errors  tau_0     mu  gamma
1.0000 0.0010 0.1000 0.0100
1.0000 0.0010 0.2000 0.0010
1.0000 0.0010 0.2000 0.0100
1.0000 0.0010 0.5000 0.0100
2.0000 0.0010 0.2000 0.0001
2.0000 0.0010 0.5000 0.0010
3.0000 0.0010 0.1000 0.0010
4.0000 0.0010 0.5000 0.0001
4.0000 0.0100 0.2000 0.0100


Using  ADAM  optimizer.
```

```
Listing results starting with lowest error score:


errors  tau_0     mu  gamma
1.0000 0.0100 0.1000 0.0100
1.0000 0.1000 0.5000 0.0001
2.0000 0.0010 0.2000 0.0001
2.0000 0.0010 0.2000 0.0010
2.0000 0.0100 0.2000 0.0010
2.0000 0.0100 0.2000 0.0100
2.0000 0.0100 0.5000 0.0001
2.0000 0.1000 0.1000 0.0001
2.0000 0.1000 0.2000 0.0010


Using  DualCA  optimizer.
Listing results starting with lowest error score:


errors   tau_0     mu  gamma
14.0000 0.0010 0.1000 0.0001
16.0000 0.0100 0.2000 0.0100
16.0000 0.0100 0.5000 0.0100
17.0000 0.1000 0.5000 0.0001
18.0000 0.1000 0.2000 0.0001
19.0000 0.0100 0.1000 0.0001
19.0000 0.0100 0.5000 0.0001
19.0000 0.1000 0.2000 0.0100
21.0000 0.0010 0.1000 0.0100


Using  NR  optimizer.
Listing results starting with lowest error score:


errors   tau_0     mu  gamma
27.0000 0.0010 0.1000 0.0001
27.0000 0.0010 0.1000 0.0010
27.0000 0.0010 0.1000 0.0100
27.0000 0.0010 0.2000 0.0001
27.0000 0.0010 0.2000 0.0010
27.0000 0.0010 0.2000 0.0100
27.0000 0.0010 0.5000 0.0001
27.0000 0.0010 0.5000 0.0010
```

```
27.0000 0.0010 0.5000 0.0100



 Overall best parameters:



optimizer    tau     mu  gamma
GD        0.0010 0.1000 0.0100
SGD       0.0010 0.5000 0.0001
SGD_mini  0.0010 0.2000 0.0001
SGD_mom   0.0010 0.1000 0.0100
ADAM      0.0100 0.1000 0.0100
DualCA    0.0010 0.1000 0.0001
NR        0.0010 0.1000 0.0001
```

speed test

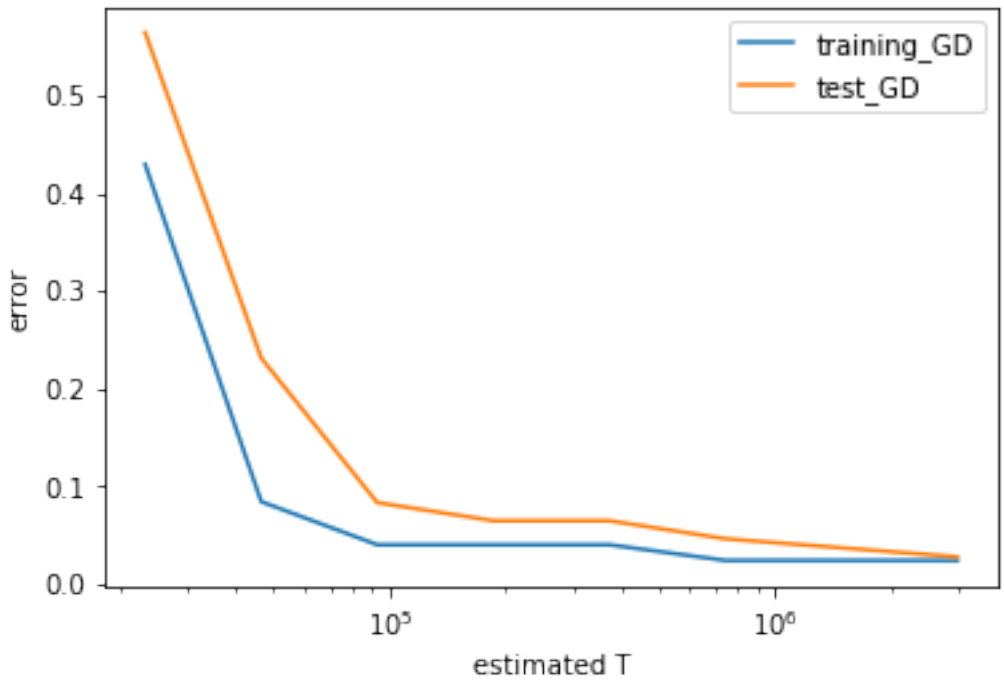
```
In [105]: import matplotlib.pyplot as plt
          %matplotlib inline

          data = np.empty(7, dtype=[  ('opt', 'U10'),
                                      ('tau', 'f4'),
                                      ('mu', 'f4'),
                                      ('gamma', 'f4'),
                                      ('compC', 'i8'),
                                      ('iterdata', '(8, 4)f4')]) #use for iterations-training
          #do iterations from 1 = 2^0 to 256=2^8 in steps of x2 as standard, ergo 9 steps. for
          # iterations from 1 to 17 in steps of +2

          for i, struct in enumerate(data):
                  struct['opt'] = params['opt'][i]
                  struct['tau'] = params['tau'][i]
                  struct['mu'] = params['mu'][i]
                  struct['gamma'] = params['gamma'][i]
                  if 0:#struct['opt'] in ['NR', 'GD']:
                      struct['iterdata'] = [[1.,0.,0.,0.],
                                            [5.,0.,0.,0.],
                                            [9.,0.,0.,0.],
                                            [13.,0.,0.,0.],
                                            [17.,0.,0.,0.],
                                            [21.,0.,0.,0.],
                                            [25.,0.,0.,0.],
                                            [29.,0.,0.,0.]]
                  else:
                      struct['iterdata'] = [[1.,0.,0.,0.],
```

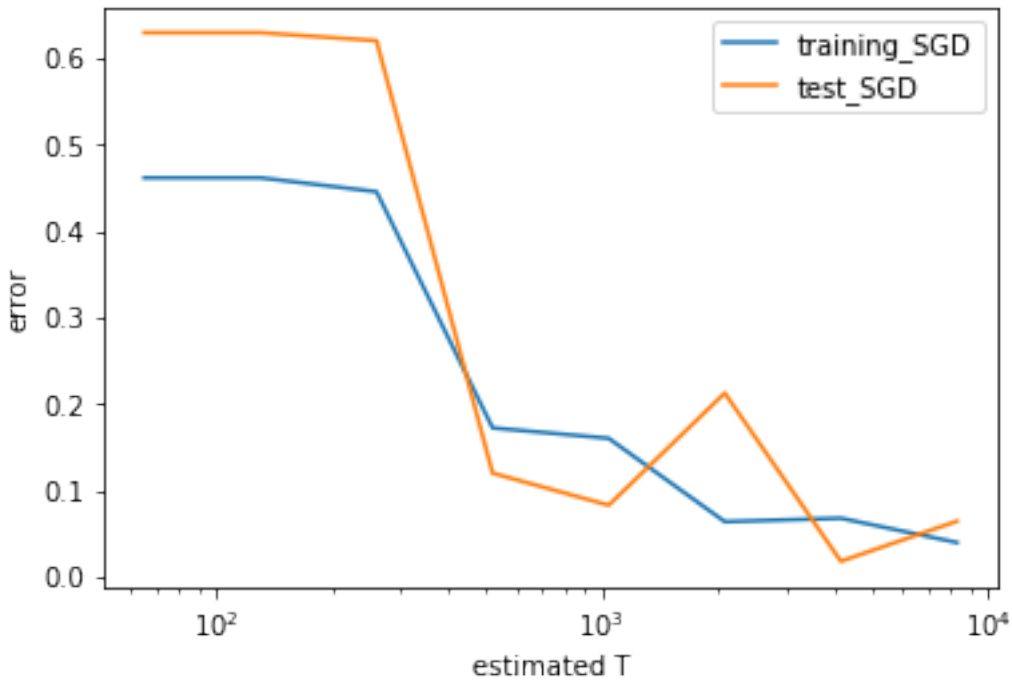
```
                                    [2.,0.,0.,0.],
                                    [4.,0.,0.,0.],
                                    [8.,0.,0.,0.],
                                    [16.,0.,0.,0.],
                                    [32.,0.,0.,0.],
                                    [64.,0.,0.,0.],
                                    [128.,0.,0.,0.]]
```

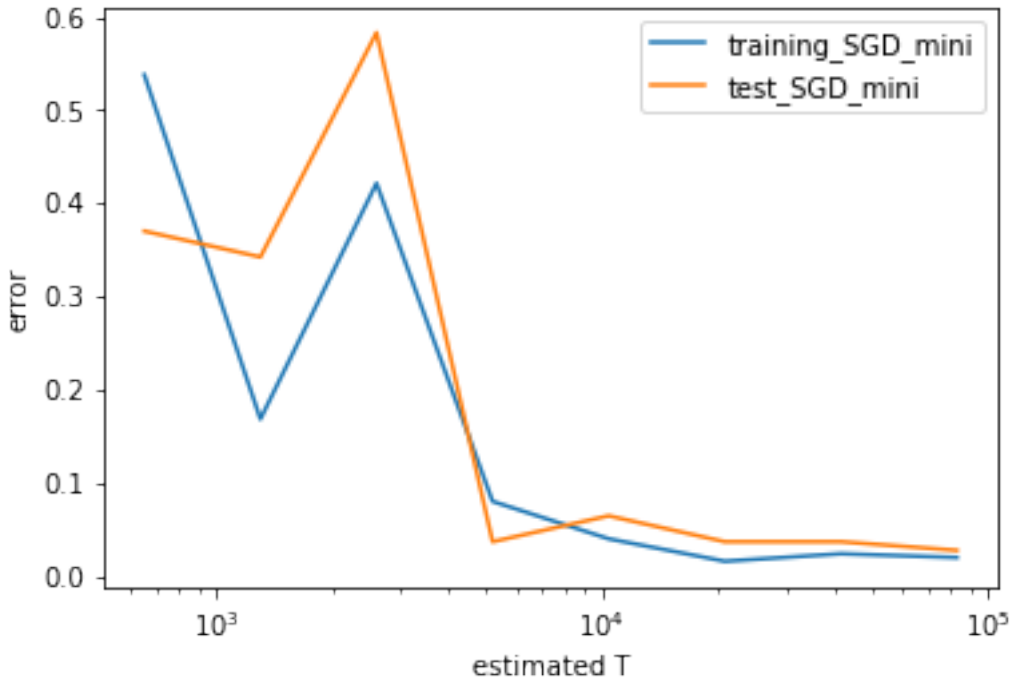
```
#insert computational complexiies
D = 65
N = 357
B = 10
data['compC'] = D
data['compC'][data['opt'] == 'GD'] = N*D
data['compC'][data['opt'] == 'SGD_mini'] = B*D
data['compC'][data['opt'] == 'NR'] = N*D**2
```

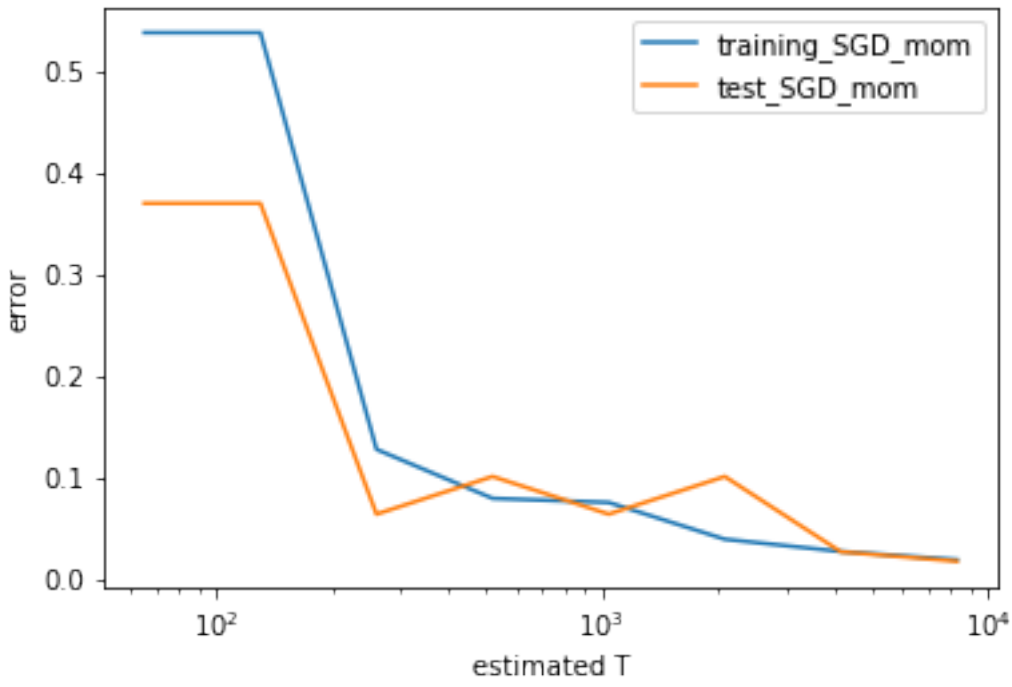
```
X_tr, X_test, y_tr, y_test = train_test_split (X, y, test_size = 0.3 ,random_state =
for i, struct in enumerate(data):
        beta = np.zeros(65)
        tau_0 = struct['tau']
        mu = struct['mu']
        gamma = struct['gamma']
        for j, iterdata in enumerate(struct['iterdata']):
            iter_ = int(iterdata[0])
            errors = 0
            if struct['opt'] == 'SGD':
                beta = SGD(X_tr, y_tr, beta, tau_0, gamma, iter_)
            elif struct['opt'] == 'GD':
                beta = GD(X_tr, y_tr, beta, tau_0, gamma, iter_)
            elif struct['opt'] == 'SGD_mini':
                #print('SGD minibatch running with B = ', B)
                beta = SG_minibatch(X_tr, y_tr, beta, tau_0, gamma, iter_, B = 10)
            elif struct['opt'] == 'SGD_mom':
                beta = SG_momentum(X_tr, y_tr, beta, tau_0, gamma, mu, iter_)
            elif struct['opt'] == 'ADAM':
                #ADAM(X, y, beta, m, tau=10**-4, mu1=0.9, mu2=0.999, eps=10**-8):
                beta = ADAM(X_tr, y_tr, beta, iter_)
            elif struct['opt'] == 'DualCA':
                beta = dual_coordinate_ascent(X_tr, y_tr, iter_)
            elif struct['opt'] == 'NR':
                beta = Newton_Raphson(X_tr, y_tr, beta, iter_)
            elif struct['opt'] == 'SAGD':
                #beta = self.type(X_tr, y_tr, beta, tau_0, gamma, mu, iter_)
                raise NotImplementedError('tbd')
```

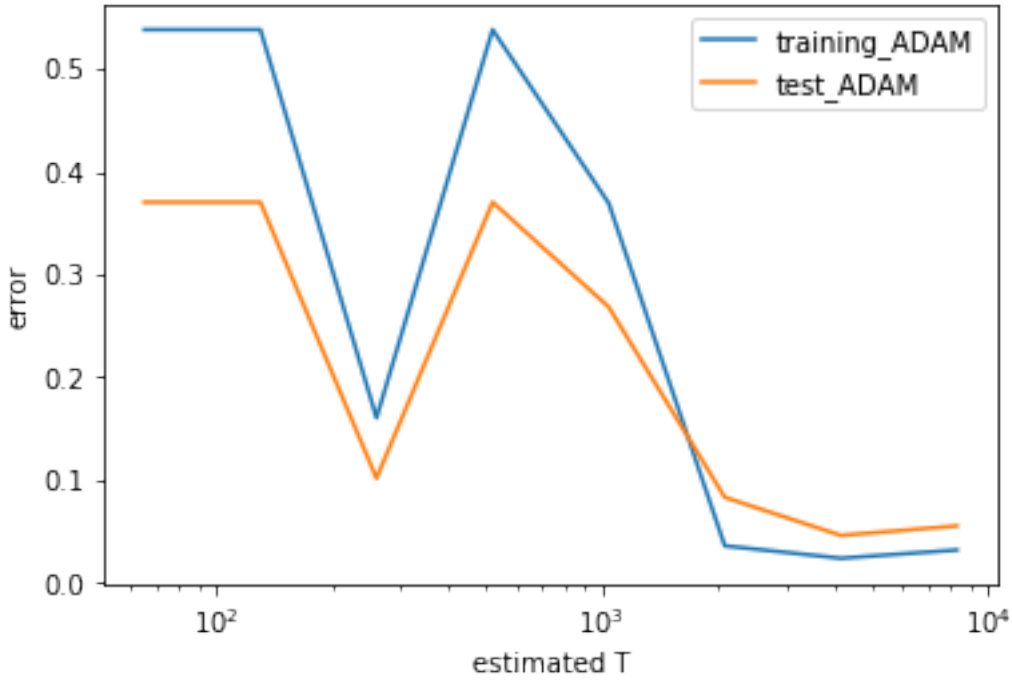
```
        iterdata[1] = zero_one_loss(predict(beta, X_tr), y_tr)/len(y_tr)
        iterdata[2] = zero_one_loss(predict(beta, X_test), y_test)/len(y_test)
        iterdata[3] = iterdata[0]*struct['compC']
    #print(struct['iterdata'][:,3])
    #print(struct['iterdata'])
    plt.figure()
    plt.semilogx(struct['iterdata'][:,3],struct['iterdata'][:,1],label='training_
    plt.semilogx(struct['iterdata'][:,3],struct['iterdata'][:,2],label='test_'+st
    plt.ylabel('error')
    plt.xlabel('estimated T')
    plt.legend()
    #plt.plot(iterdata[3],iterdata[2],label='test')
```

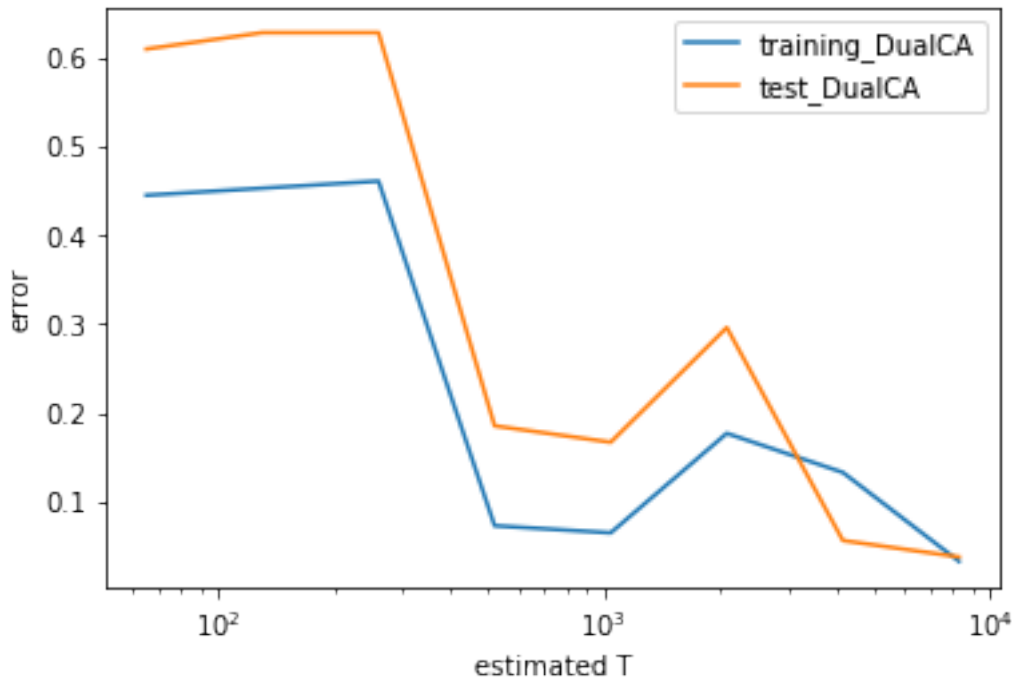


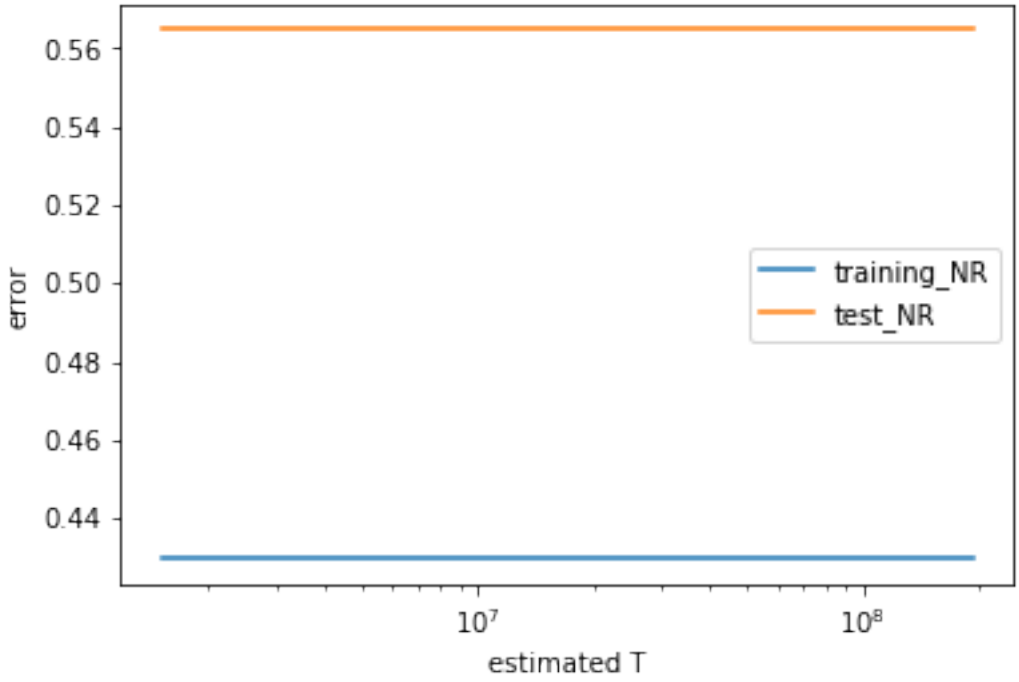

training_SGD
test_SGD
error
estimated T
0.0
0.1
0.2
0.3
0.4
0.5
0.6
10^2
10^3
10^4


training_SGD_mini
test_SGD_mini
error
estimated T
0.0
0.1
0.2
0.3
0.4
0.5
0.6
10^3
10^4
10^5


training_SGD_mom
test_SGD_mom
error
0.0
0.1
0.2
0.3
0.4
0.5
10^2
10^3
10^4
estimated T


training_ADAM
test_ADAM
error
0.0
0.1
0.2
0.3
0.4
0.5
10^2
10^3
10^4
estimated T


training_DualCA
test_DualCA
error
0.6
0.5
0.4
0.3
0.2
0.1
10^2
10^3
10^4
estimated T


training_NR
test_NR
error
0.56
0.54
0.52
0.50
0.48
0.46
0.44
10^7
10^8
estimated T