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First we want to complete the data. We need the probabilities $p(F|G), p(A|F,G)$ and $p(G)$.

	men		women	
field	$p(F\|G)$	$p(A\|F,G)$	$p(F\|G)$	$p(A\|F,G)$
A	0.62	0.32	0.82	0.06
B	0.63	0.22	0.68	0.01
C	0.37	0.13	0.34	0.32
D	0.33	0.16	0.35	0.20
E	0.28	0.07	0.24	0.21
F	0.06	0.10	0.07	0.19
	0.585		0.415	

Total Causal Effect

1. we can straight forward calculate the conditional probability $p_1(A = \text{true}|do(G))$:

$$p_1(A = \text{true}|do(G)) = p_1(A = \text{true}|G) = \sum_F p(F|G)p(A = \text{true}|F, G) \tag{1}$$

2. Now we can calculate

$$p_1(A = \text{true}|do(G = \text{male})) = 0.32 \cdot 0.62 + 0.22 \cdot 0.63 + ... = 0.46$$
$$p_1(A = \text{true}|do(G = \text{female})) = 0.0.06 \cdot 0.82 + 0.01 \cdot 0.68 + ... = 0.30$$

This indicates that women might be discriminated.

Direct Causal Effect

1. we can calculate $p_2(A = \text{true}|G)$:

$$\begin{aligned}
p_2(A = \text{true}|G) &= p_1(A = \text{true}|G, \text{cut}(G \to A)) \\
&= \sum_F p_1(F|G)p_1(A = \text{true}|F, G, \text{cut}(G \to A)) \\
&= \sum_F \sum_{\tilde{G}} p_1(F|G)p_1(A = \text{true}|F, \tilde{G})p(\tilde{G}) \\
&= \sum_{\tilde{G}} p(\tilde{G}) \sum_F p_1(F|G)p_1(A = \text{true}|F, \tilde{G}) \\
&= p(\tilde{G} = \text{male}) \sum_F p_1(F|G)p_1(A = \text{true}|F, \tilde{G} = \text{male}) \\
&+ p(\tilde{G} = \text{female}) \sum_F p_1(F|G)p_1(A = \text{true}|F, \tilde{G} = \text{female})
\end{aligned}$$

2. Now we can calculate

$$\begin{aligned}
p_2(A = \text{true}|G = \text{male}) &= 0.585 \cdot p_1(A = \text{true}|do(G = \text{male})) + 0.415 \cdot (0.82 \cdot 0.32 + ...) \\
&= 0.585 \cdot 0.46 + 0.415 \cdot 0.54 = 0.49 \\
p_2(A = \text{true}|G = \text{female}) &= 0.415 \cdot p_1(A = \text{true}|do(G = \text{female})) + 0.585 \cdot (0.06 \cdot 0.62 + ...) \\
&= 0.415 \cdot 0.30 + 0.585 \cdot 0.30 = 0.30
\end{aligned}$$

The results yield the same conditional probability for women in both models indicating that there is no discrimination in that respect. But for men the direct causal effect yields a higher conditional probability thus indicating that there might be little discrimination against male applicants.