Exercise 1

Let's first make up some arbitrarily chosen probabilities. We chose:

$$\left.\begin{array}{ll} p(C=0) = 0{,}7 & , \; p(C=1) = 0{,}3 \\ p(A=0\mid C=0) = 0{,}4 \, , & p(A=1\mid C=0) = 0{,}6 \\ p(B=0\mid C=0) = 0{,}7 \, , & p(B=1\mid C=0) = 0{,}3 \\ p(A=0\mid C=1) = 0{,}5 \, , & p(A=1\mid C=1) = 0{,}5 \\ p(B=0\mid C=1) = 0{,}2 \, , & p(B=1\mid C=1) = 0{,}8 \end{array}\right\}$$

each line sums up to 1 as required by normalization

Based on these probabilities, we can now create 2 tables for the conditional probability $p(A,B\mid C)$ for $C=0$ and $C=1$. Assuming $A \perp B \mid C$, we can use

$$p(A,B\mid C) = p(A\mid C)\cdot p(B\mid C)$$

to obtain

$C=0$

A	B	$p(A\mid C)$	$p(B\mid C)$	$p(A,B\mid C)$
0	0	0,4	0,7	0,28
0	1	0,4	0,3	0,12
1	0	0,6	0,7	0,42
1	1	0,6	0,3	0,18

$C=1$

A	B	$p(A\mid C)$	$p(B\mid C)$	$p(A,B\mid C)$
0	0	0,5	0,2	0,1
0	1	0,5	0,8	0,4
1	0	0,5	0,2	0,1
1	1	0,5	0,8	0,4

Now, we can marginalize over C to compute

$$p(A) = p(A\mid C=0)\cdot p(C=0) + p(A\mid C=1)\cdot p(C=1)$$ (analogously for $p(B)$)

and

$$p(A,B) = p(A,B\mid C=0)\cdot p(C=0) + p(A,B\mid C=1)\cdot p(C=1)$$

$\Rightarrow$

A	B	$p(A)$	$p(B)$	$p(A,B)$		$p(A)\cdot p(B)$
0	0	0,43	0,55	0,226	$\neq$	0,2365
0	1	0,43	0,45	0,204	$\neq$	0,1935
1	0	0,57	0,55	0,324	$\neq$	0,3135
1	1	0,57	0,45	0,246	$\neq$	0,2565

As one can see, $p(A,B) \neq p(A)\cdot p(B)$ in all 4 cases and thus A and B are not independent ($A \not\perp B$) although $A \perp B \mid C$ holds

$\Rightarrow A \perp B \mid C$ does not imply $A \perp C$!