

# Steady-State Tram Heating Model for Electric Energy Consumption Estimation

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**T**his paper describes a model that estimates the electric energy consumption of a tram fleet for heating. A general thermodynamic system is derived for a single tram vehicle. This system is simplified under certain assumptions and parameters are chosen appropriately for the specific *Cobra* tram type. Subsequently, the instantaneous and the total model results are validated demonstrating reasonable accuracy compared to experimental data. The model is developed for the *TempTrim* project, associated with the *Energy Now! 2.0* challenge at the *Energy Science Center at ETH Zurich* with support of *Verkehrsbetriebe Zürich*.

## I. INTRODUCTION

*Verkehrsbetriebe Zürich* (VBZ), as a public transport operator, observed that the energy demand for heating is equal to the energy required for driving during cold days [1]. Major energy savings could be achieved from lowering the temperature by a few degrees. A measurement campaign was deployed by VBZ to measure their potential savings [1]. During the *Energy Now!* challenge in 2022, the idea came up to generalise the findings for public access.

The *TempTrim* project associated with the *Energy Now! 2.0* challenge, organized by the *Energy Science Center at ETH Zurich* takes up this idea and realises a tool for public access. This tool enables users to input certain tram and operational specifications and estimates the energy consumption for heating under various conditions. The tool is based on a thermodynamic model, freely accessible and can be found on <https://temptrim.streamlit.app>. The code is open-source available and can be found together with additional information in the *GitHub* repository on [https://github.com/TempTrim/temp\\_trim](https://github.com/TempTrim/temp_trim).

First, a model is defined that comprises a tram fleet consisting of different vehicle types throughout a selected period of time under consideration of environmental aspects (section II). Second, a general thermodynamic system of a single tram is specified (section III). Various assumptions are taken to reduce the complexity of the model. In a third

step, remaining parameters are selected according to specific data from VBZ (section IV). Additionally, the required electricity consumption is calculated based on the thermodynamically derived heat demand (section V). The resulting model is validated (section VI) and the results discussed (section VII). All symbols used are summarised and described in table 1 (continues on page 2).

Table 1: List of symbols

Symb.	Description	Unit
$A$	Area <ul style="list-style-type: none"> <li><math>abs, n</math>: effective absorption area normal to sun (windows)</li> <li><math>conv</math>: effective convection area</li> <li><math>door, tot</math>: total door area</li> <li><math>door, open</math>: average open door area</li> <li><math>front, n</math>: front area normal to sun</li> <li><math>side, n</math>: side area normal to sun</li> <li><math>left/right/back, n</math>: left / right / back area normal to sun</li> </ul>	$m^2$
$c_{p,air}$	Specific heat capacity of air	$\frac{J}{kgK}$
$COP$	Coeff. of performance (heat pump)	—
$f$	Fraction coefficients <ul style="list-style-type: none"> <li><math>door, open</math>: time fraction where doors are open</li> <li><math>sun</math>: time fraction where tram is in sun (during sunshine-time)</li> <li><math>win</math>: area fraction of windows in tram mantle</li> </ul>	—
$h_{tram}$	Tram height	$m$
$\dot{H}$	Enthalpy flow <ul style="list-style-type: none"> <li><math>vent, in/out</math>: via ventilation</li> <li><math>door, in/out</math>: through open doors</li> <li><math>\Delta_{air}</math>: net enthalpy via ventilation and through open doors</li> </ul>	$W$
$i$	Index for heat pumps	—

Symb.	Description	Unit
$I_{sun}$	Solar irradiation intensity	$\frac{W}{m^2}$
$k$	Heat transfer coefficient <ul style="list-style-type: none"> <li>• <i>chas</i>: combined conduction and convection through/at chassis</li> <li>• <i>cond</i>: conduction through chassis</li> <li>• <i>conv</i>: convection at chassis</li> <li>• <i>rail</i>: conduction to rail</li> </ul>	$\frac{W}{m^2K}$
$l_{tram}$	Tram length	$m$
$\dot{m}$	Mass flow (air) <ul style="list-style-type: none"> <li>• <i>door, in/out</i>: through doors</li> <li>• <i>vent, in/out</i>: via ventilation</li> </ul>	$\frac{kg}{s}$
$n$	Number of specific day in year	–
$n_{pass}$	Passenger number per tram	–
$p_{\infty}$	Pressure (environment)	$hPa$
$\dot{P}_{el}$	Electric power <ul style="list-style-type: none"> <li>• <i>res</i>: sum of resistive heaters</li> <li>• <i>HP, i</i>: heat pump (with index <i>i</i>)</li> </ul>	$W$
$\dot{Q}$	Heat flow (generation/losses) <ul style="list-style-type: none"> <li>• <i>aux</i>: auxiliary tram devices</li> <li>• <i>conv</i>: convection on surface</li> <li>• <i>cond</i>: conduction through chassis</li> <li>• <i>cond, rail</i>: conduction to rails</li> <li>• <i>heat</i>: sum of tram heaters</li> <li>• <i>HP, i</i>: heat pump (with index <i>i</i>)</li> <li>• <i>pass</i>: sum of passengers</li> <li>• <i>pers</i>: single person</li> <li>• <i>rad, sun</i>: solar radiation</li> <li>• <i>rad, tram</i>: radiative losses</li> <li>• <i>res</i>: sum of resistive heaters</li> </ul>	$W$
$t$	Time	$s$
$T$	Temperature <ul style="list-style-type: none"> <li>• <i>surf</i>: tram interior</li> <li>• <i>tram</i>: tram surface</li> <li>• <math>\infty</math>: environment</li> </ul>	$^{\circ}C$
$v_{out/in}$	Specific volume of air	$\frac{m^3}{kg}$
$v_{air}$	Air speed through open door	$\frac{m}{s}$
$\dot{V}$	Volume flow <ul style="list-style-type: none"> <li>• <i>door</i>: through doors</li> <li>• <i>vent</i>: via ventilation</li> </ul>	$\frac{m^3}{s}$
$w_{tram}$	Tram width	$m$
$\dot{W}_{flow}$	Flow work ( <i>in/out</i> )	$W$
$\underline{x}$	Position	$m$

Symb.	Description	Unit
$\delta$	Earth declination angle	$rad$
$\eta$	Efficiency (resistive heater)	–
$\omega$	Hour angle	$rad$
$\phi$	Angle <ul style="list-style-type: none"> <li>• <i>A</i>: azimuth</li> <li>• <i>L</i>: longitude</li> <li>• <i>Z</i>: zenith</li> </ul>	$rad$
$\rho_{air}$	Gravimetric density of air	$\frac{kg}{m^3}$

## II. MODELLING

In the following, a model of the tram operation and the calculation of the electricity consumption required for tram heating is specified. This model is capable to comprise all tram vehicles of an operator during a specified time period up to one year. Cooling demand of the tram is neglected.

As trams are usually longer operated than parked, it has been decided to neglect transient effects at the start and end of the operation. In addition, transients caused by environmental and operational changes during one day are neglected. To account for different environmental changes over a day – in particular variations of temperatures and solar irradiation – a steady-state system is modelled for each tram vehicle type with an hourly resolution. It is assumed that in general, the variations between consecutive days are negligible compared to the hourly variations over a day and between the months. Hence, each month is modelled by one exemplary day with monthly averaged values for the hourly temperature and solar irradiation.

The thermodynamic system describes a generic tram and is solved for its heat demand. Subsequently, for each tram vehicle type the hourly electricity consumption is derived based on the heat demand and the heating devices available. In a last step, the total electricity demand is calculated by summation over all trams. For all trams of the same vehicle type, the electricity demand is derived by multiplying the heat demand per tram by the amount of operated vehicles of that specific type. The electric energy consumption of a single tram of a certain type is derived by the summation over the hourly energy consumptions. The model is implemented in *Python*.

### III. THERMODYNAMIC SYSTEM

The general form of the thermodynamic system model describing a single tram of a certain vehicle type is visualized in figure 1. It considers the solar radiation  $\dot{Q}_{rad,sun}$ , heat generation in the tram by the heaters  $\dot{Q}_{heat}$ , passengers  $\dot{Q}_{pass}$  and auxiliary devices  $\dot{Q}_{aux}$  as well as losses by radiation  $\dot{Q}_{rad,tram}$ , convection  $\dot{Q}_{conv}$  and conduction via the wheels  $\dot{Q}_{cond,rail}$ . Furthermore, mass flow based enthalpy flows by incoming and outgoing air via the ventilation system ( $\dot{H}_{vent,in/out}$ ) and through open doors ( $\dot{H}_{door,in/out}$ ) are considered. Those flows over the system boundary are accompanied by flow work ( $\dot{W}_{flow,in/out}$ ). The environmental temperature and pressure are denoted by  $T_\infty$  and  $p_\infty$ , respectively, while  $T_{tram}$  describes the temperature within and  $T_{surf}$  on the surface of the tram.

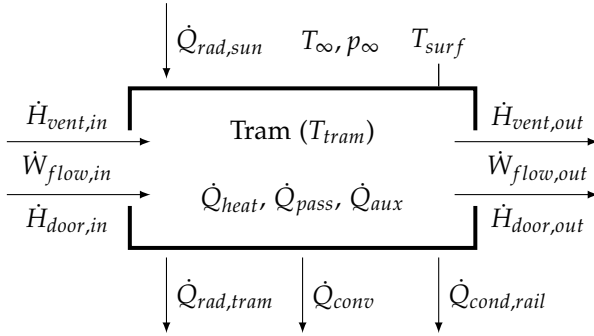


Figure 1: General thermodynamic system of a tram

The general system depicted in figure 1 is simplified to reduce its complexity and the number of necessary parameters. In order to do so, the following assumptions are made:

1. No transient phenomena occur and a steady-state system can be considered, i.e.,  $\frac{d(\dots)}{dt} = 0$ .
2. The tram, its surface and the environment temperatures are spatially uniform, such that  $\frac{dT_{tram}}{dx} = \frac{dT_{surf}}{dx} = \frac{dT_\infty}{dx} = 0$ .
3. The conductive heat transfer via the wheels to the rails is given by

$$\dot{Q}_{cond,rail} = A_{cond} k_{rail} (T_{surf} - T_\infty), \quad (1)$$

where the area  $A_{cond}$  is the contact area of the tram wheels with the rails and  $k_{rail}$  is the conduction coefficient. As the contact area is comparably small considering the dimensions of a tram, conductive losses can be neglected, i.e.,  $\dot{Q}_{cond,rail} = 0$ .

4. The radiative emissions from the tram are small compared to the heat losses by convection and the enthalpy flows by ventilation and through the doors. As a consequence, radiative tram emissions are neglected, i.e.,  $\dot{Q}_{rad,tram} = 0$ .
5. The influence of humidity and deviations in the molar composition as well as the density of the air is negligible. Hence, the specific heat capacity for the air flow  $c_{p,air}$  and its density  $\rho_{air}$  is constant. The tabulated values  $\rho_{air} = 1.275 \frac{kg}{m^3}$  and  $c_{p,air} = 1.004 \frac{kJ}{kgK}$  for  $T = 20^\circ C$  and  $p_\infty = 1050 hPa$  are used. [2].
6. The amount of air entering the tram via the ventilation system leaves the tram through the same system and air entering the tram when the doors are open leaves the tram again through the doors. It follows for the mass flows that

$$\dot{m}_{vent} \equiv \dot{m}_{vent,in} = \dot{m}_{vent,out} \quad (2)$$

and

$$\dot{m}_{door} \equiv \dot{m}_{door,in} = \dot{m}_{door,out}. \quad (3)$$

7. All air flows enter with the environment temperature  $T_\infty$  and leave with the tram temperature  $T_{tram}$ .
8. The system is isobaric. Hence, the net flow work is given by

$$\Delta \dot{W}_{flow} = (\dot{m}_{vent} + \dot{m}_{door}) p_\infty (v_{out} - v_{in}). \quad (4)$$

The difference in specific volumes  $v_{out} - v_{in}$  is assumed to be negligible, which yields  $\dot{W}_{flow,in} = \dot{W}_{flow,out}$  and therefore is the net flow work negligible, i.e.,  $\Delta \dot{W}_{flow} = 0$ .

9. The solar radiative heating through the windows dominates and the solar radiation on the chassis can be neglected. Furthermore, the window reflectivity is neglected and all irradiation is assumed to pass and directly heat the tram interior.
10. There are sufficiently many trams in operation and they are operated equally in all directions. As a consequence, to derive the effective tram area  $A_{abs,n}$  normal to the sun irradiation, the average area for the azimuth angle interval  $\phi_A \in [0, 2\pi]$  can be calculated. According to [3], the earth declination  $\delta$  on the day  $n \in [1, 365]$  in a non-leap year is

$$\delta = \frac{23.45^\circ \pi}{180^\circ} \sin \left( \frac{2\pi}{365} (284 + n) \right) \quad (5)$$

with the hour angle

$$\omega = \frac{15^\circ \pi}{180^\circ} (t - 12) \quad (6)$$

at time  $t \in [0, 24[$  (measured in hours without summer time shift), the instantaneous zenith angle  $\phi_Z$  is derived by

$$\phi_Z = \arccos(\cos \phi_L \cos \delta \cos \omega + \sin \phi_L \sin \delta) \quad (7)$$

taking the latitude  $\phi_L$  into account. To be precise, the solar time has to be considered for  $\omega$ , which is neglected here, as hourly averages are calculated.

For the radiative absorption from the sun, an average effective area normal to the sun irradiation is calculated by

$$A_{abs,n} = f_{win} (\bar{A}_{front,n} + \bar{A}_{left,n} + \bar{A}_{right,n} + \bar{A}_{back,n}), \quad (8)$$

where  $f_{win}$  is defined to be the fraction of the tram front, back and side mantle that consists of window areas. Defining the azimuth angle  $\phi_A = 0$  for the sun irradiation being perpendicular to the tram direction of travel, where due to symmetry considerations only a  $\phi_A \in [0, \frac{\pi}{2}]$  needs to be considered, the average area can be calculated by

$$A_{abs,n} = f_{win} (\bar{A}_{side,n} + \bar{A}_{front,n}), \quad (9)$$

which is derived by

$$A_{abs,n} = \frac{2 f_{win}}{\pi} \int_0^{\frac{\pi}{2}} (A_{side,n} + A_{front,n}) d\phi_A. \quad (10)$$

Inserting the equations for the side

$$A_{side,n} = l_{tram} h_{tram} \sin \phi_Z \cos \phi_A \quad (11)$$

and the front area

$$A_{front,n} = w_{tram} h_{tram} \sin \phi_Z \sin \phi_A \quad (12)$$

into equation (10), the resulting average normal absorption area is

$$A_{abs,n} = \frac{2 f_{win} \sin \phi_Z}{\pi} h_{tram} (l_{tram} + w_{tram}). \quad (13)$$

11. The tram has a constant conductive heat transfer coefficient  $k_{cond}$  through and a convective coefficient  $k_{conv}$  over its entire effective convection surface  $A_{conv}$ , that is considered to be a cuboid, i.e.,

$$A_{conv} = 2(h_{tram} l_{tram} + h_{tram} w_{tram} + l_{tram} w_{tram}). \quad (14)$$

12. Due to shading, for example by buildings, the tram is not always exposed to sun irradiation during the day. Independent of the solar azimuth and altitude angle, an unshaded coefficient  $0 < f_{sun} < 1$  is assumed to indicate the fraction of the time during which the tram is exposed to sun radiation. Its value is assumed to be constant trough-out the year.

13. The average heat generation of a single person  $\dot{Q}_{pers}$  is constant.

14. The average heat generation of the auxiliary devices  $\dot{Q}_{aux}$  is constant.

15. Heating from the traction system is neglected.

Applying these assumptions yields a simplified thermodynamic system, depicted in figure 2. This model was derived independently, comparison to the literature shows similar approaches [4], [5].

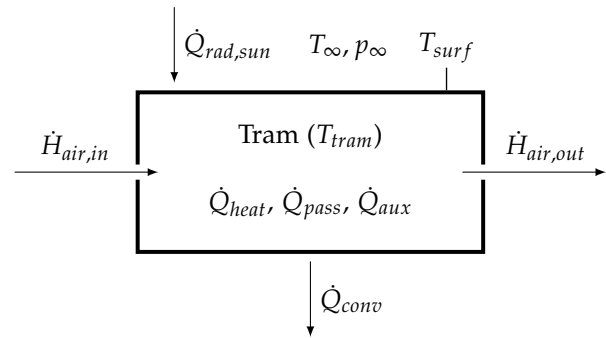


Figure 2: Simplified thermodynamic system of a tram

The overall steady-state power balance of this simplified system model results in

$$0 = \dot{Q}_{heat} + \dot{Q}_{pass} + \dot{Q}_{aux} + \dot{Q}_{rad,sun} + \Delta \dot{H}_{air} - \dot{Q}_{conv}. \quad (15)$$

The individual terms are derived in the following. The passenger heat production is given by

$$\dot{Q}_{pass} = n_{pass} \dot{Q}_{pers}, \quad (16)$$

where  $n_{pass}$  is the average passenger number in a tram. The radiative heating of the tram by the sun is estimated to cumulate into

$$\dot{Q}_{rad,sun} = f_{sun} I_{sun} A_{abs,n}, \quad (17)$$

where  $I_{sun}$  is the average sun intensity (areal power density) on an area normal to the sun. For the ventilation and door opening air flow, the net enthalpy flow is given by

$$\Delta \dot{H}_{air} = (\dot{V}_{vent} + \dot{V}_{door}) \rho_{air} c_{p,air} (T_{\infty} - T_{tram}) \quad (18)$$

for the volume flows  $\dot{V}_{vent}$  and  $\dot{V}_{door}$  with

$$\dot{m} = \rho_{air} \dot{V}. \quad (19)$$

The air volume exchanged via ventilation is calculated by

$$\dot{V}_{door} = \frac{1}{2} f_{door,open} A_{door,open} v_{door,air}, \quad (20)$$

where  $A_{door,open}$  is the average effective open door area,  $v_{door,air}$  is the average air speed measured at the open door and  $f_{door,open}$  the fraction of the time during which the doors are open. The prefactor  $\frac{1}{2}$  is necessary, as fresh air enters over half of the door area and air flows out of the tram over the other half. The net convective heat losses result in

$$\dot{Q}_{conv} = A_{conv} k_{conv} (T_{surf} - T_{\infty}). \quad (21)$$

As the surface temperature is linearly dependent on the tram inner and environmental temperature, it can be derived by the conductive heat transfer through the chassis

$$\dot{Q}_{cond} = A_{conv} k_{cond} (T_{tram} - T_{surf}), \quad (22)$$

where the same effective area  $A_{conv}$  as for convective heat transfer is assumed, and  $k_{cond}$  is the conductive heat transfer coefficient through the chassis. The steady-state energy conservation for the chassis yields

$$\dot{Q}_{cond} = \dot{Q}_{conv}. \quad (23)$$

Implementing equations (22) and (21) and solving for  $T_{surf}$  results in

$$T_{surf} = \frac{k_{conv} T_{\infty} + k_{cond} T_{tram}}{k_{conv} + k_{cond}}. \quad (24)$$

Inserting  $T_{surf}$  into equation (21) and defining the combined conductive and convective heat transfer coefficient

$$k_{chas} \equiv \frac{k_{conv} k_{cond}}{k_{conv} + k_{cond}} \quad (25)$$

for the chassis yields

$$\dot{Q}_{conv} = A_{conv} k_{chas} (T_{tram} - T_{\infty}). \quad (26)$$

#### IV. PARAMETERS

The remaining parameters are determined for the VBZ *Cobra* tram. Moreover, measurements from the literature are used and further quantities derived. The results as well as the mathematical derivations are described in the following. For the *Cobra* tram, the parameters are summarized in table 2.

- Typical values for the convective heat transfer coefficient  $k_{chas}$  for the tram mantle range from 2.5 to  $3.2 \frac{W}{m^2 K}$  [4]. For the *Cobra* tram,  $k_{chas} \approx 2.8 \frac{W}{m^2 K}$  is assumed [5].
- For the *Cobra* tram, the outer dimensions are  $l_{tram} = 35.9m$ ,  $w_{tram} = 2.4m$  and  $h_{tram} = 3.6m$  [6].
- In winter, the *Cobra* tram ventilation system is assumed to operate at 90% of its maximum throughput with one third of the air being supplied from the environment, such that  $\dot{V}_{vent} \approx 1345 \frac{m^3}{h} \approx 0.37 \frac{m^3}{s}$  [5].
- For the air exchange at the doors, VBZ conducted measurements for the *Cobra* tram. It has a total door area of  $A_{door,tot} = 18.2m^2$ , an average open door area of  $A_{door} = 12.1m^2$  (as not all doors are opened at every stop), an average air speed of  $v_{air} = 0.56 \frac{m}{s}$  for the open doors and an average hourly open door time of  $10.5 \frac{min}{h}$  [5]. A open door time fraction  $f_{door,open} = 0.175$  can be derived. According to equation (20), the resulting average volume air flow through the open doors is  $\dot{V}_{door} \approx 2134 \frac{m^3}{h} \approx 0.59 \frac{m^3}{s}$ . As the calculation of the heat losses via the open doors is subject to high uncertainties, it was assumed based on the experiments that the actual open door losses are 100 kWh higher than estimated [5]. Accordingly, the door volume flow has been increased to  $\dot{V}_{door} \approx 3215 \frac{m^3}{h} \approx 0.89 \frac{m^3}{s}$ .
- VBZ measured an average passenger number per *Cobra* tram of  $n_{pass} \approx 27$  [5].
- A sitting person generates an average heat power of  $\dot{Q}_{pers} = 116W$  [7].
- The total resistive heating power of a *Cobra* tram is  $\dot{Q}_{res}^{max} = 102.3kW$  [5].
- The auxiliary device heat generation for a *Cobra* tram stems from the lighting with a power of  $\dot{Q}_{aux} = 1.5kW$  [5].

- For solar irradiation it is assumed that the tram is 80% of the sunshine-time in the sun, i.e.,  $f_{sun} = 0.8$  and that half of it's mantle consist of windows areas, i.e.,  $f_{win} = 0.5$ .

As climate data, hourly monthly-averaged solar irradiation and temperature data is required. This data is taken from the EU Photovoltaic Geographical Information System (PVGIS 5.2) [8].

**Table 2:** VBZ Cobra tram system parameters

Param.	Type	Unit	VBZ
$T_{tram}$	user-defined	$^{\circ}\text{C}$	
$T_{\infty}$	climate	$^{\circ}\text{C}$	
$I_{sun}$	climate	$\frac{\text{W}}{\text{m}^2}$	
$\phi_H$	location	$\text{rad}$	
$l_{tram}$	tram specification	$m$	25.9
$w_{tram}$	tram specification	$m$	2.4
$h_{tram}$	tram specification	$m$	3.6
$k_{chas}$	tram specification	$\frac{\text{W}}{\text{m}^2\text{K}}$	2.8
$\dot{Q}_{aux}$	tram specification	$\text{kW}$	1.5
$\dot{Q}_{res,i}^{max}$	tram specification	$\text{kW}$	102.3
$\dot{Q}_{HP,i}^{max}$	tram specification	$\text{kW}$	—
$COP_{HP,i}$	tram specification	—	—
$\dot{V}_{vent}$	tram specification	$\frac{\text{m}^3}{h}$	1345
$\dot{V}_{door}$	tram specification / statistics (location)	$\frac{\text{m}^3}{h}$	3215
$n_{pass}$	statistics (location)	—	27

## V. POWER CONSUMPTION

To estimate the electric power demand of a single tram, its instantaneous heat demand  $\dot{Q}_{heat}$  is calculated according to equation (15). Subsequently, the heater specification of the tram is used to map the heat to the electricity demand.

To avoid a non-linear optimization problem, it is assumed that for all resistance heaters  $\eta_{res} = 1$  and all coefficients of performance (COPs) of the heat pumps are constant and equal to the respective COP at maximum thermal power. As a consequence, a linear optimization problem needs to be solved to derive the heating power of every heater. Solving the linear optimization problem shows that first,

the heat pumps should be used ordered with respect to a decreasing COP and then, the remaining heat demand needs to be supplied by the resistive heaters.

The electric power can be calculated based on the thermal power for each heater separately. For the total resistive heating,

$$P_{el,res} = \dot{Q}_{res} \quad (27)$$

and for each heat pump,

$$P_{el,HP,i} = \frac{\dot{Q}_{HP,i}}{COP_{HP,i}}. \quad (28)$$

The total instantaneous electric power is the sum of the consumption of all heaters:

$$P_{el} = P_{el,res} + \sum P_{el,HP,i}. \quad (29)$$

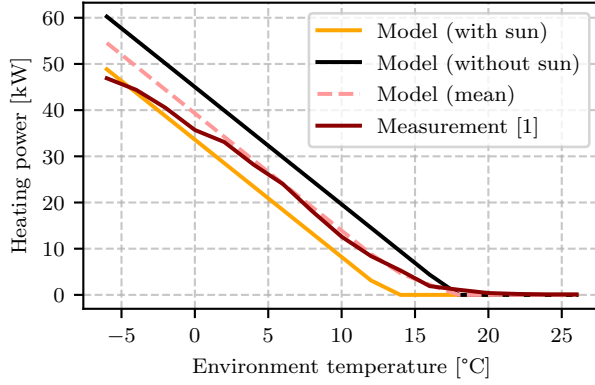
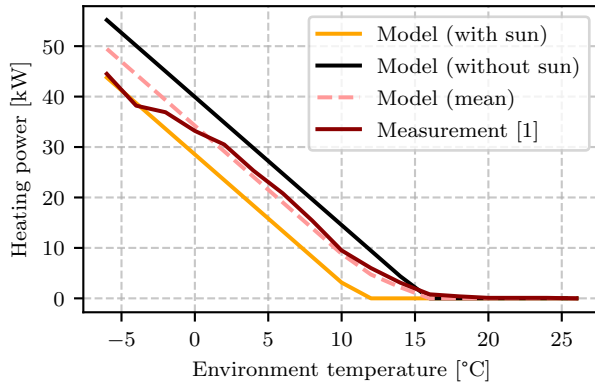
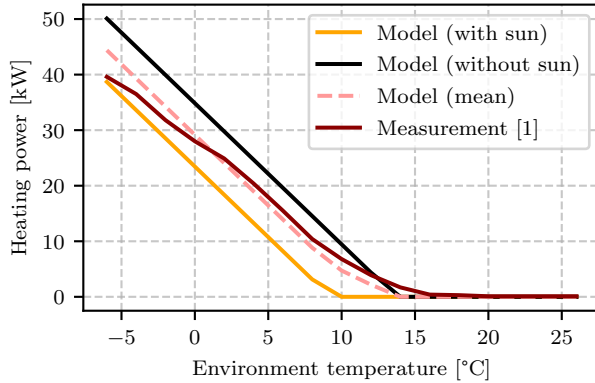
This instantaneous electric power demand is then summed over the whole operation duration and all trams. This results in the total electric energy demand.

## VI. VALIDATION

The instantaneous model as well as the by VBZ provided or above derived parameters were validated against additional experimental data. VBZ measured and provided data on the average instantaneous electric power demand in dependence of the average daily temperature [1]. No data is stored on the solar irradiation during the measurements.

To validate the thermodynamic system, the instantaneous model has been evaluated for the temperature range  $T_{\infty} \in [-6^{\circ}\text{C}, 26^{\circ}\text{C}]$  taken from the experimental dataset. The evaluation was pursued for two extreme cases: First, no solar irradiation, and second, the maximum solar irradiation. In Zurich, the maximum average solar irradiation of  $I_{sun} = 731.9 \frac{\text{W}}{\text{m}^2}$  occurs in July at 11 am [8]. According to the hourly average temperatures for every month, the minimum and maximum average temperatures in Zurich are  $0.29^{\circ}\text{C}$  and  $25.43^{\circ}\text{C}$ , respectively [8]. Hence, the temperature interval  $T_{\infty} \in [0^{\circ}\text{C}, 26^{\circ}\text{C}]$  is in particular important for the validation.

For three different temperature set-points  $T_{tram} \in \{20^{\circ}\text{C}, 18^{\circ}\text{C}, 16^{\circ}\text{C}\}$ , the model evaluation (with and without solar irradiation) is compared to the experimental values. The resulting heating powers in dependence of the environment temperatures are depicted in figures 3 to 5. In the diagrams, also the mean model value of both cases (with and without sun) is depicted.

Figure 3: Model validation for  $T_{tram} = 20^\circ\text{C}$ Figure 4: Model validation for  $T_{tram} = 18^\circ\text{C}$ Figure 5: Model validation for  $T_{tram} = 16^\circ\text{C}$ 

The average coefficient of variation between the individual model mean values and measurement data points has been calculated over the temperature range  $T_\infty \in [0^\circ\text{C}, 26^\circ\text{C}]$ . The results are compared for  $T_{tram} \in \{20^\circ\text{C}, 18^\circ\text{C}, 16^\circ\text{C}\}$  in table 3. It can be seen that the average coefficients of variation are in the range of 2.2% to 4.8%.

It shall be mentioned that the measurement data is derived as average heat demand in dependence

of the average daily temperature, while the instantaneous model uses the exact instantaneous model input. Hence, the comparability is restricted. Given the level of generalization of the model and based on the data available, it is assumed that the linear modelling approach and the resulting accuracy is acceptable.

Table 3: Coefficients of variation for model validation

Temperature $T_{tram}$	$20^\circ\text{C}$	$18^\circ\text{C}$	$16^\circ\text{C}$
Avg. coeff. of variation	0.0220	0.0479	0.0349

For temperatures  $T_\infty < 0^\circ\text{C}$ , larger deviations of the instantaneous model results from the experimental data can be observed in figures 3 to 5. Possible reasons for these effects are not discussed here as the temperature interval lies outside the relevant range given by the climate data.

## VII. RESULTS & DISCUSSION

The tram heating model based on a steady-state thermodynamic system is successfully derived, simplified, parametrized, validated and implemented in *Python*. The instantaneous hourly-averaged model results show reasonable accuracy with daily averaged experimental data.

To evaluate the total model results, the total electricity consumption and savings with respect to different temperature set-points  $T_{tram}$  are analysed. The results are compared to the experimental data provided in [1] as well as a measurement-based model derived in [5]. To ensure comparability of the inputs, all three sources assume an average daily operation of 18 hours (5:30 am to 11:30 pm for this paper's model) and 70 *Cobra* trams operated (80% of all 88 vehicles).

The savings with respect to electricity consumption over a whole year are compared in table 4. The savings are considered with a reduction of the temperature set-point from  $20^\circ\text{C}$  to  $18^\circ\text{C}$  as well as from  $20^\circ\text{C}$  to  $16^\circ\text{C}$ .

Table 4: Electricity savings from models and measurements

$T_{tram}$ reduction	$20^\circ\text{C} \rightarrow 18^\circ\text{C}$	$20^\circ\text{C} \rightarrow 16^\circ\text{C}$
This paper's model	1.27 GWh	2.39 GWh
Measurement in [1]	1.0 GWh	2.3 GWh
Model/meas. in [5]	1.32 GWh	2.64 GWh

The model results derived in this paper deviate from the measured values in [1] by 27% for the temperature reduction from 20°C to 18°C and by 4% for 16°C instead of 20°C. Comparing this model's savings with [5] shows deviations of 4% and 10% for temperature reductions from 20°C to 18°C and 16°C, respectively.

As all three sources derive their results differently, deviations were expectable. The data provided in [1] only captures a single year and is therefore subject to higher environmental uncertainties, while this paper's model uses averaged climate data for several years. Hence, the different climate data sources could be an explanation for the deviating results to some extent. Furthermore, in [1] daily average temperatures were measured, which could also be a reason for the deviations from the model with hourly resolution. This could explain why the mean instantaneous values are much less deviating between the different data sources, as they do not consider climate data.

Climate data is used for Zurich, however, different climate data sources are exploited. The model in [5] is, for example, based on individual measurements and *MeteoSwiss* data. The model discussed here uses climate data from the EU PVGIS 5.2 database [8]. This paper's modelling approach is in addition not completely independent of the model in [5], as certain parameter assumptions were taken from the master's thesis. Furthermore, in [5], only trams operated on certain routes are analysed, while it is assumed that with this paper's approach the average over all routes in a region is represented.

As for the instantaneous model validation, it shall be underlined that the comparability of the different models and measurements is not fully given. However, as gathering or measuring additional data for validation exceeds this paper's scope, the different results were still used for a basic comparison. The deviations between the sources and this paper are significant, however, the results are in the same order of magnitude. It is assumed that for the *TempTrim* project requirements, this paper's model is accurate enough to be used.

For a further validation of this paper's model, additional measurements would be required. As the parameters vary for each tram, the effort for further measurements for the *Cobra* tram does not seem to

be sensible for an overall model improvement. On the contrary, deriving parameters for other tram types and comparing their measurements with the model results would be required to further validate the general model.

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