CSE 150. Assignment 1 *Winter 2015*

Out: *Tue Jan 13*
Due: *Tue Jan 20* (in class)
Supplementary reading: RN, Ch 13; KN, Ch 1.

1.1 Kullback-Leibler distance

Often it is useful to measure the difference between two probability distributions over the same random variable. For example, as shorthand let

$$\begin{aligned} p_i &= P(X\!=\!x_i|E), \\ q_i &= P(X\!=\!x_i|E') \end{aligned}$$

denote the conditional distributions over the random variable X for different pieces of evidence $E \neq E'$. The Kullback-Leibler (KL) distance between these distributions is defined as:

$$\mathrm{KL}(p,q) = \sum_i p_i \log(p_i/q_i).$$

(a) By sketching graphs of $\log x$ and $x-1$, verify the inequality

$$\log x \leq x - 1,$$

with equality if and only if $x\!=\!1$. Confirm this result by differentiation of $\log x - (x\!-\!1)$. (Note: all logarithms in this problem are *natural* logarithms.)

(b) Use the previous result to prove that $\mathrm{KL}(p,q) \geq 0$, with equality if and only if the two distributions p_i and q_i are equal.

(c) Provide a counterexample to show that the KL distance is not a symmetric function of its arguments:

$$\mathrm{KL}(p,q) \;\neq\; \mathrm{KL}(q,p).$$

Despite this asymmetry, it is still common to refer to $\mathrm{KL}(p,q)$ as a measure of distance between probability distributions.

1.2 Conditional independence

Show that the following three statements about random variables X, Y, and Z are equivalent:

$$\begin{aligned} P(X,Y|Z) &= P(X|Z)P(Y|Z) \\ P(X|Y,Z) &= P(X|Z) \\ P(Y|X,Z) &= P(Y|Z) \end{aligned}$$

In other words, show that each one of these statements implies the other two. You should become fluent with all these ways of expressing that X is conditionally independent of Y given Z.

1.3 Creative writing

Attach events to the binary random variables X, Y, and Z that are consistent with the following patterns of commonsense reasoning. You may use different events for the different parts of the problem.

(a) Explaining away:

$$\begin{aligned} P(X\!=\!1|Y\!=\!1) &> P(X\!=\!1), \\ P(X\!=\!1|Y\!=\!1, Z\!=\!1) &< P(X\!=\!1|Y\!=\!1) \end{aligned}$$

(b) Accumulating evidence:

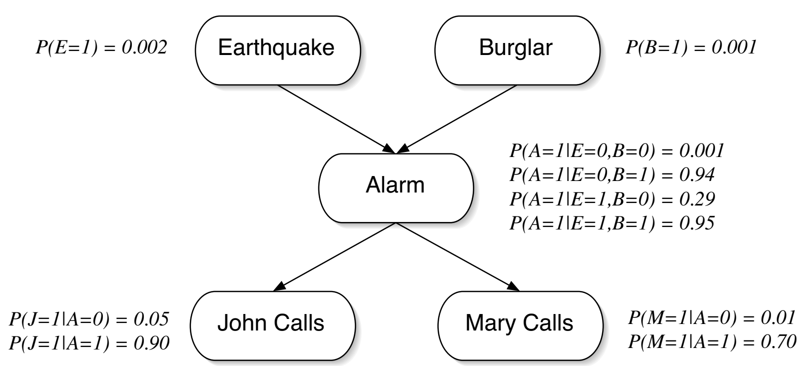
$$P(X\!=\!1) > P(X\!=\!1|Y\!=\!1) > P(X\!=\!1|Y\!=\!1, Z\!=\!1)$$

(c) Conditional independence:

$$\begin{aligned} P(X\!=\!1, Y\!=\!1) &> P(X\!=\!1)P(Y\!=\!1) \\ P(X\!=\!1, Y\!=\!1|Z\!=\!1) &= P(X\!=\!1|Z\!=\!1)P(Y\!=\!1|Z\!=\!1) \end{aligned}$$

1.4 Probabilistic inference

Recall the probabilistic model that we described in class for the binary random variables $\{E = \mathrm{Earthquake}, B = \mathrm{Burglary}, A = \mathrm{Alarm}, J = \mathrm{JohnCalls}, M = \mathrm{MaryCalls}\}$. We also expressed this model as a belief network, with the directed acyclic graph (DAG) and conditional probability tables (CPTs) shown below:



Compute numeric values for the following probabilities, exploiting relations of conditional independence as much as possible to simplify your calculations. You may re-use numerical results from lecture, but otherwise *show your work*. Be careful not to drop significant digits in your answer.

(a) $P(E\!=\!1|A\!=\!1)$ (c) $P(A\!=\!1|J\!=\!0)$ (e) $P(A\!=\!1|M\!=\!1)$
(b) $P(E\!=\!1|A\!=\!1, B\!=\!1)$ (d) $P(A\!=\!1|J\!=\!0, M\!=\!0)$ (f) $P(A\!=\!1|M\!=\!1, E\!=\!0)$

Consider your results in (b) versus (a), (d) versus (c), and (f) versus (e). Do they seem consistent with commonsense patterns of reasoning?