SECTION A [25 *marks*]
This section consists of 3 questions. Answer all the questions given.

1. (a) Given $f(x)=\dfrac{x^2-1}{x-1}$, find $\lim\limits_{x\to 1} f(x)\ -\ \lim\limits_{x\to 0} f(x)$.

[4 marks]

(b) Evaluate $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x^2-1}}{3x+5}$ [5 marks]

2. (a) Given that $y=x\ln x-3x$, find the value of x and y when $\dfrac{dy}{dx}=0$.

[5 marks]

(b) Given $y=\alpha e^{2x}+\beta e^{-2x}$ where α and β are constants. Show that $\dfrac{d^2y}{dx^2}=4y$.

[6 marks]

3. Given $f(x)=x^2-\dfrac{2}{x}$, where $x\neq 0$. Find the coordinate of stationary point and determine whether it is relative maximum or relative minimum.

[5 marks]

SECTION B [75 marks]
This section consists of 7 questions. Answer all questions.

1. Given that $z_1=2+3i$, $z_2=3+2i$ and $z_3=a+bi$ are three complex numbers, where a and b are real.

(a) Find z_1+z_2. Hence, determine the modulus of z_1+z_2.

(b) If $z=\dfrac{z_1z_3}{z_2}$, find z in terms of a and b. Giving your answer in the form $x+yi$, where $x,y\in\mathbb{R}$.

(c) From part (b), given that $z=\dfrac{17}{13}-\dfrac{7}{13}i$,

(i) find the values of a and b,

(ii) find $\arg z$, giving your answer in radians to 3 decimal places.

(iii) Hence, express z in polar form.

[9 Marks]

2. (a) Solve the equation $\log_5 5x + \log_x 5 = \log_4 64$.

(b) Determine the value of x for $3 - x \le x(x-3) < 10 - 3x$. Express your answer in the interval form.

[11 Marks]

3. The function f is defined by $f(x) = \dfrac{x+2}{x-2}$, $x \ne k$ and $g(x) = \dfrac{2x+2}{x-1}$

(a) State the value of k.

(b) Determine whether f is one-to-one function algebraically.

(c) Find $f \circ g$ and hence state the relationship between f and g.

[8 marks]

4. The functions f and g are defined by $f(x) = \ln(3x-1)$, $g(x) = e^x$

(a) Determine the inverse function of f.

(b) State the domain and range of f^{-1}.

(c) Sketch the graphs of f and f^{-1} on the same axes.

(d) Find $g \circ f$.

[10 marks]

5. Given that

$$f(x) = \begin{cases} \dfrac{2 - x^2 - p}{x}, & x < q \\ r\ x + 6\ , & q \le x < 2 \\ x^3 - p, & x \ge 2 \end{cases}$$

Find the values of p, q and r such that $f(2) = p$ and $f(x)$ is continuous everywhere.

[11 marks]

6. (a) A curve has the equation $y^2 + 6x = 3xy + y$. Find the coordinates of the points on the curve and its gradient when $x = 2$. [8 marks]

(c) The parametric equations of a curve are given by $x = \frac{1}{3}\sin^3 2\theta$ and $y = 2\cos^3 2\theta$. Find $\frac{dy}{dx}$ in terms of θ. Hence, find the value of $\frac{dy}{dx}$ when $\theta = \frac{\pi}{6}$.

[8 marks]

7. Water is leaking from the bottom of a conical tank with radius 1.5 meter and height 2 meter at a rate of 0.25 cubic meter per minute. The tank was initially full. If the height of water is 1 meter then, find the rate of change of

(a) the water level

(b) the radius of the water surface [9 marks]

ANSWER

Section A

1. (a) 1 (b) $\frac{1}{3}$

2. (a) $x = e^2$ $y = -e^2$ (b) shown

3. Stationary point (-1, 3), relative minimum.

Section B

1. (a) $5+5i$, $5\sqrt{2}$ (b) $= \frac{12a-5b}{13} + \frac{(5a+12b)}{13}i$

(c) (i) $a = 1, b = -1$ (ii) $\arg z = -0.391\ rad$

(iii) $z = \sqrt{2}\left(\cos(-0.391 rad) + i\sin(-0.391 rad)\right)$

2. (a) $x = 5$ (b) $\left(-\sqrt{10}, -1\right] \cup \left[3, \sqrt{10}\right)$

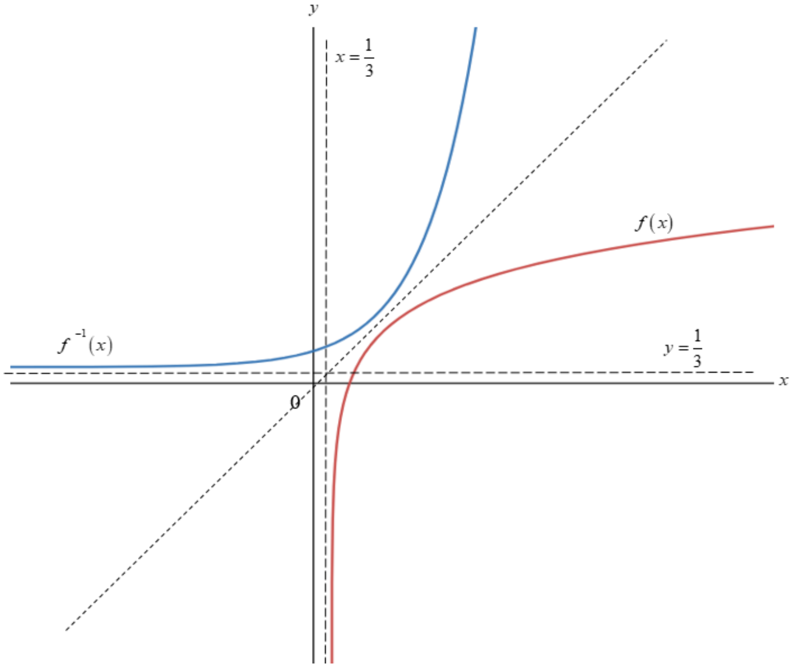
3. (a) $k = 2$

(c) $f\left[g(x)\right] = x, \therefore$ f and g are inverse of each other

4. (a) $f^{-1}(x)=\dfrac{e^{x}+1}{3}$

(b) $D_{f^{-1}}=R_f=(-\infty,\infty),\quad R_{f^{-1}}=D_f=\left(\dfrac{1}{3},\infty\right)$

(c)



(d) $3x-1$

5. $p=4, r=\dfrac{1}{2}, q=0$

6. (a) $2,4$ *and* $2,3$; $gradient = 6,-3$ (b) $\dfrac{dy}{dx}=-6\cot 2\Theta$; $-2\sqrt{3}$

7. (a) -0.1415 m/min (b) -0.1061m/min