

Soalan Pra PSPM SM015 Sesi 2023/2024

(Set 1)

Part A

1. Evaluate the following limits (if exist):

(a) $\lim_{x \rightarrow 5^+} \frac{25 - 4x^2}{|5 - 2x|}$ [3 marks]

(b) $\lim_{x \rightarrow -1} \frac{\sqrt{3-x} - 2}{x+1}$ [3 marks]

(c) $\lim_{x \rightarrow \infty} \left(\frac{8x^2 - 5x + 1}{x^2 + 3} \right)^{\frac{1}{3}}$ [3 marks]

2. a) Find the derivative of $\sqrt{5x+4}$ by using the first principle. [5 marks]

b) Find $\frac{dy}{dx}$ for $y = \ln \frac{\sin(x)}{x-1}$. [4 marks]

3. A curve is given by the function $f(x) = \sqrt[3]{x-1}$. Determine the nature of the critical points whether maximum, minimum or point of inflection.

[7 marks]

Part B

1. Given $z_1 = 1 - 3i$ and $z_2 = 2 + 5i$.

a) Express $\frac{1}{z_1} + \frac{1}{z_2}$ in the form of $a + bi$

[3 marks]

b) Find argument of \bar{z}_2 in radian

[2 marks]

2. Solve the following inequalities.

a) $4x + \frac{9}{x} \geq 12$

[3 marks]

b) $\left| \frac{x+5}{2x-4} \right| \geq 1$

[7 marks]

3. Given $f(x) = x^2 - 4x + 6$ and $(g \circ f)(x) = \frac{4}{(x-2)^2+1}$

a) Find $g(x)$.

[5 Marks]

b) Hence, find $g^{-1}(x)$.

[3 Marks]

4. Given that $f(x) = e^{4x} + 2$ and $g(x) = \frac{1}{4} \ln(x-2)$.

i) Show using algebraic method, that $f(x)$ is one-to-one function.

[3 marks]

ii) Find $(f \circ g)(x)$ and $(g \circ f)(x)$. Hence, state the relationship between $f(x)$ and $g(x)$.

[6 marks]

iii) Given that $f(x) = \frac{4}{3x}$ and $\frac{f(x)}{g(x)} = \frac{x+3}{2x^2+x}$. Find $g(x)$.

[4 marks]

5. a) Given that $f(x)$ and $g(x)$ are non zero functions where $\lim_{x \rightarrow 2} \frac{2f(x) + 4g(x)}{5x+2} = 1$ and

$$\lim_{x \rightarrow 2} (3f(x) + 8g(x) - 2x^2) = 4.$$

i) Find the value of $\lim_{x \rightarrow 2} f(x)$ and $\lim_{x \rightarrow 2} g(x)$.

[4 marks]

ii) Given that function $g(x) = \begin{cases} kx+3, & x < 3 \\ qx, & x \geq 3 \end{cases}$

Find the value of k and hence, the value of q if $g(x)$ is continuous at $x = 3$.

[4 marks]

b) Find the vertical and horizontal asymptote for $f(x) = \frac{\sqrt{x^2-9}}{x-3}$

[8 marks]

6. a) Given the parametric equations for the curve $x = -9 \sin^3 t$ and

$$y = 3 - 3 \cos^3 t.$$

i) Show that $\frac{dy}{dx} = -\frac{1}{3} \cot t$. [5 Marks]

ii) Hence, solve $\frac{dy}{dx} = 0$ for $0 \leq t \leq \pi$. [3 Marks]

b) Given $y = e^{-2x} \ln x$, show that $x^2 \left[\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} \right] + e^{-2x} (2x + 1) = 0$.

[6 Marks]

7. a) The function $f(x) = x^3 - 3x^2 - 9x - 4$ is defined on the interval $[-2, 6]$. Find the critical points of $f(x)$ on this interval and determine whether each critical point is local minimum or maximum. [5 marks]

b) The total cost, $RM C$, of manufacturing n boxes of cookies in time t hour given by $C = 2n^2 + 3n + 60$, where $n = t^2 + 12t$. Calculate the rate of change of the total cost with respect to time when $t = 2$. [4 marks]