PART A

(25 marks)

1. Evaluate the following limits (if exists).

(a) $\lim\limits_{x \to 4} \dfrac{x^4 - 16}{x - 2}$ [3 marks]

(b) $\lim\limits_{x \to 4} \dfrac{2 - \sqrt{x}}{4 - x}$. [4 marks]

(c) $\lim\limits_{x \to -\infty} \dfrac{3}{4x^2}$ [2 marks]

2. **(a)** By using the definition $f'(a) = \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$, if f is a function with $f'(4) = 2$, find $\lim\limits_{x \to 4} \dfrac{f(x) - f(4)}{\sqrt{x} - 2}$. [5 marks]

(b) Differentiate each of the following function with respect to *x*.

i) $f(x) = (\ln x)^3$ [2 marks]

ii) $f(x) = 3e^{3x}$ [2 marks]

iii) $f(x) = 4\sin^3 x$ [2 marks]

3. Given $y = 3x + \dfrac{12}{x},\ x > 0$. Find the coordinates of the stationary point. Hence determine the nature of the point. [5 marks]

PART B

(75 marks)

1. (a) Find integer values of *m* and *n* for which $m - n\log_3 2 = 10\log_9 6$. [4 marks]

(b) The complex numbers z_1 and z_2 are given by $z_1 = p + 2i$ and $z_2 = 1 - 2i$ where p is an integer

i) Find $\dfrac{z_1}{z_2}$ in the form of $a + bi$ where a and b are real numbers. [3 marks]

ii) Hence, find the possible values of p when $\left|\dfrac{z_1}{z_2}\right| = 13.$ [4 marks]

2. Solve

a) $-2\left|2x+3\right| + 14 \geq -16$ [3 marks]

b) $\left|\dfrac{x-1}{3x+1}\right| \geq 1$ [7 marks]

3. Given the functions $f(x) = 2 - x^2$ and $g(x) = x + 2$, find $f \circ g$ and $g \circ f$. Hence, determine the value of x such that $(f \circ g)(x) = (g \circ f)(x)$. [6 marks]

4. A function f is defined as $f(x) = 3 + \sqrt{x-2}$

i) Show that the function $f^{-1}(x)$ exists and hence, find $f^{-1}(x)$ [5 marks]

ii) State the domain and range of $f^{-1}(x)$ [2 marks]

iii) On the same axes, sketch the graphs of *f (x)* and $f^{-1}(x)$. [5 marks]

State the relationship between the two graphs.

5. a) Find the horizontal asymptote of $f(x) = \dfrac{2x}{(x+1)(x-2)}$. [4 marks]

b) The function f is defined by $f(x) = \begin{cases} \dfrac{x-1}{x+2}, & 0 \le x < 2 \\ ax^2 - 1, & x \ge 2 \end{cases}$. Find the value of *a* if $\lim_{x \to 2} f(x)$ exist. Hence, determine whether *f* is continuous at x = 2. [7 marks]

6. (a) A curve has an equation $x^2 + y^2 - 2y = 4$

i. Find $\dfrac{dy}{dx}$ in the terms of *x* and *y*.

ii. Determine the gradient of the curve at point (1,3)

iii. Express $\dfrac{d^2y}{dx^2}$ in terms of *y*.

[11 marks]

(b) The curve defined by the parametric equations, $x = t^2 - 3$ and $y = t^3 + t$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ [5 marks]

7. A closed rectangular box has a base with its length twice its width, and the total surface area of the box is 300 cm^2. If the width of the base of the box is $\boldsymbol{x}$ cm, and the volume of the box is $\boldsymbol{V}$ cm^3. Show that $V = 100x - \frac{4}{3}x^3$. Find the height of the box when its volume is maximum. Hence, find the maximum volume of the box. [9 *marks*]

END OF QUESTIONS PAPER

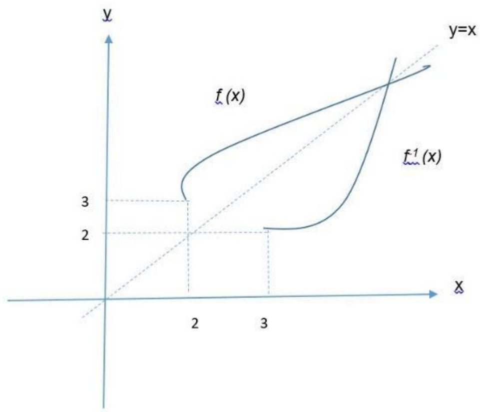
ANSWERS

PART A

1. (a) 120 (b) $\frac{1}{4}$ (c) 0
2. (a) 8 (bi) $\frac{3}{x}(\ln x)^2$ (bii) $9e^{3x}$ (biii) $12\sin^2 x\cos x$
3. (2 , 12) , minimum point

PART B

1. (a) m = 5, n = -5 (bi) $\frac{p-4}{5}+\frac{2}{5}(p+1)i$ (bii) $p = \pm 29$
2. (a) $-9 \le x \le 6$ (b) $\left[-1,-\frac{1}{3}\right) \cup \left(-\frac{1}{3},0\right]$
3. $x = -\frac{3}{2}$
4. (ai) $f^{-1}(x) = (x-3)^2 + 2$ (aii) $D_{f^{-1}} = [3,\infty)$, $\mathrm{R}_{f^{-1}} = [2,\infty)$
   (aiii)



$f^{-1}(x)$ is a reflection of graph f (x) about the line y = x.

5. (a) y = 0 (b) $a=\frac{5}{16}$, f (x) is continuous at x = 2

6. (ai) $\frac{x}{1-y}$ @ $\frac{-x}{y-1}$ (aii) $-\frac{1}{2}$ (aiii) $\frac{5}{(1-y)^3}$

(b) $\frac{dy}{dx}=\frac{3t^2+1}{2t}$, $\frac{d^2y}{dx^2}=\frac{3t^2-1}{4t^3}$

7. $\frac{1000}{3}cm^3$

TARGET A	SM015	KMS

PART A

(25 marks)

NO	ANSWER SCHEMES	REMARKS
1a	$\lim\limits_{x\to 4}\dfrac{x^4-16}{x-2}=120$	K1KIJ1
1b	$\lim\limits_{x\to 4}\dfrac{2-\sqrt{x}}{4-x} \quad =\lim\limits_{x\to 4}\dfrac{2-\sqrt{x}}{4-x}\times\dfrac{2+\sqrt{x}}{2+\sqrt{x}}$ $=\lim\limits_{x\to 4}\dfrac{4-x}{(4-x)\left(2+\sqrt{x}\right)} =\lim\limits_{x\to 4}\dfrac{1}{\left(2+\sqrt{x}\right)}$ $=\dfrac{1}{2+2}=\dfrac{1}{4}$	K1 K1K1 J1
1c	$\lim\limits_{x\to -\infty}\dfrac{3}{4x^2}=\dfrac{3}{\infty}=0$	K1J1
	TOTAL	**9 marks**
2a	Given $f'(a)=\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$, $f'(4)=2$ $\lim\limits_{x\to 4}\dfrac{f(x)-f(4)}{\sqrt{x}-2} \quad =\lim\limits_{x\to 4}\dfrac{f(x)-f(4)}{\sqrt{x}-2}\times\dfrac{\sqrt{x}+2}{\sqrt{x}+2}$ $=\lim\limits_{x\to 4}\dfrac{f(x)-f(4)}{x-4}\times\left(\sqrt{x}+2\right)=\left[f'(4)\right]\times\lim\limits_{x\to 4}\left(\sqrt{x}+2\right)$ $=(2)\times(2+2)=8$	K1 K1K1 K1J1
2bi	$f(x)=(\ln x)^3$ $f'(x)=3(\ln x)^2\dfrac{d}{dx}(\ln x)$ $=3(\ln x)^2\dfrac{1}{x}$ @ $\dfrac{3}{x}(\ln x)^2$	K1 J1
2bii	$f(x)=3e^{3x}$ $f'(x)=9e^{3x}$	K1 J1

2biii	$f(x)=4\sin^3 x$ $f'(x)=4\left[3\sin^2 x\right]\dfrac{d}{dx}(\sin x)$ $=12\sin^2 x\cos x$	**K1** **J1**
	TOTAL	**11 marks**
3	$y=3x+\dfrac{12}{x},\ x>0$ $\dfrac{dy}{dx}=3-\dfrac{12}{x^2}$ $\dfrac{dy}{dx}=0\ \rightarrow\ 3-\dfrac{12}{x^2}=0$ $x^2=\dfrac{12}{3}$ $x=\sqrt{4}=2\ ,x>0$. When x = 2 , $y=3(2)+\dfrac{12}{2}=12$ Coordinates of the stationary point = (2, 12)	**B1** (differentiate) **K1** ($\dfrac{dy}{dx}$ equal to 0) **K1** (find y)
	$\dfrac{d^2y}{dx^2}=-12(-2)\left(x^{-3}\right)=24x^{-3}$ $when\ x=2,\ \dfrac{d^2y}{dx^2}=\dfrac{24}{(2)^3}=3>0\,(\text{min})$ (2, 12) is a minimum point.	**K1** **J1**
	TOTAL	**5 marks**

TARGET A	SM015	KMS

PART B

(75 marks)

NO	ANSWER SCHEMES	REMARKS
1	**(a)** $m-n\log_3 2=10\log_9 6$ $=10\left(\dfrac{\log_3 6}{\log_3 3^2}\right)$ $=10\left(\dfrac{\log_3(3\bullet 2)}{2}\right)$ $=5\left[\log_3(3\bullet 2)\right]$ $=5\left[\log_3 3+\log_3 2\right]$ $=5\log_3 3+5\log_3 2$ $=5+5\log_3 2$ $\therefore m=5, n=-5$	**B1** (change base) **K1** (law of log) **K1** (simplify) **J1**
	(b)i. $\dfrac{p+2i}{1-2i}=\left(\dfrac{p+2i}{1-2i}\right)\left(\dfrac{1+2i}{1+2i}\right)$ $=\dfrac{p+2pi+2i+4i^2}{1-4i^2}$ $=\dfrac{p-4+(2p+2)i}{5}$ $=\dfrac{p-4}{5}+\dfrac{2}{5}(p+1)i$	**K1** (multiply with conjugate) **K1** (expand) **J1** (must in a +bi)

	bii) $\left\|\frac{z_1}{z_2}\right\|=13$ $\sqrt{\left(\frac{p-4}{5}\right)^2+\left[\frac{2}{5}(p+1)\right]^2}=13$ $\left(\frac{p-4}{5}\right)^2+\left[\frac{2}{5}(p+1)\right]^2=169$ $(p-4)^2+4(p+1)^2=169(25)$ $p^2-8p+16+4p^2+8p+4=4225$ $5p^2=4205$ $p=\pm 29$	K1K1 Sub in correct formula K1 Attempt to solve J1 both
	TOTAL	**11 marks**
2a)	$-2\|2x+3\|+14\geq -16$ $\|2x+3\|\leq 15$ $-15\leq 2x+3\leq 15$ $-9\leq x\leq 6$ *or* $[-9,6]$	**K1** **K1** **J1**
2b)	$\left\|\frac{x-1}{3x+1}\right\|\geq 1$ $\frac{x-1}{3x+1}\geq 1$ *or* $\frac{x-1}{3x+1}\leq -1$ $\frac{x-1-(3x+1)}{3x+1}\geq 0$ $\quad$ $\frac{x-1+(3x+1)}{3x+1}\leq 0$ $\frac{-2x-2}{3x+1}\geq 0$ $\quad$ $\frac{4x}{3x+1}\leq 0$	**K1** **K1** **K1K1**

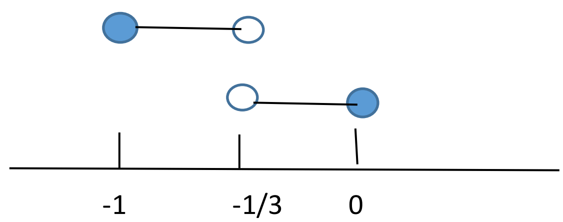
	-∞	-1	-1/3	∞
-2x-2	+	-	-	
3x+1	-	-	+	
		+		

	- ∞	-1/3	0	∞
4x	-	-	+	
3x+1	-	+	+	
		-		

J1

$\left[-1,-\frac{1}{3}\right)$ $\quad$ $\left(-\frac{1}{3},0\right]$

K1

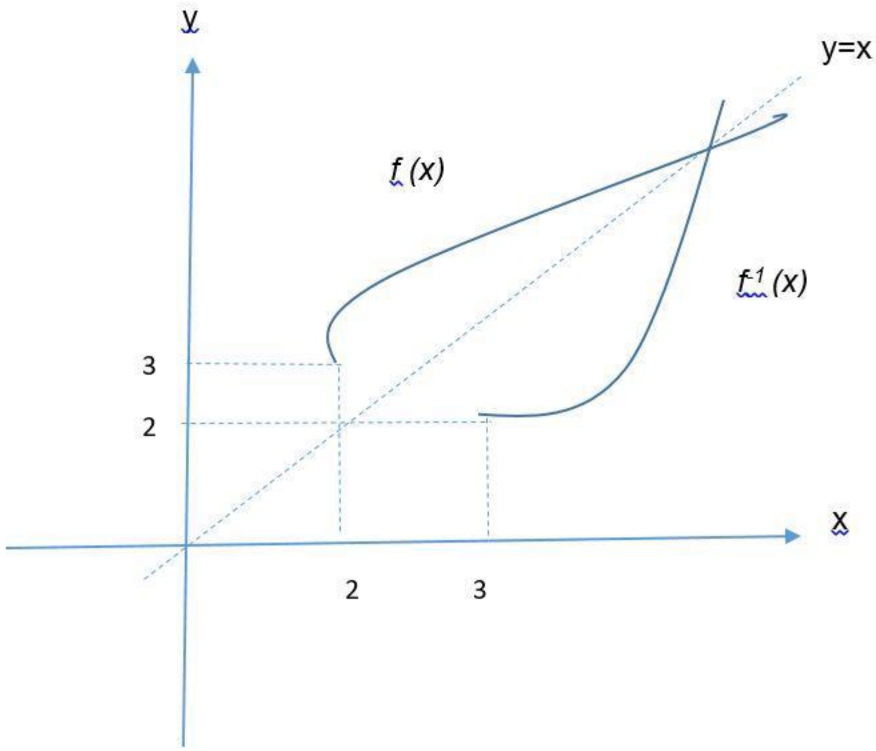


J1

$$\left[-1,-\frac{1}{3}\right)\cup\left(-\frac{1}{3},0\right]$$

	TOTAL	**10 marks**
3	$f\circ g=f\left[g(x)\right]=f(x+2)=2-\left(x+2\right)^2=-x^2-4x-2$	**K1J1**
	$g\circ f=g\left[f(x)\right]=g(2-x^2)=2-x^2+2=4-x^2$	**K1J1**
	$(f\circ g)(x)=\left(g\circ f\right)(x)$	**K1**
	$-x^2-4x-2=4-x^2$	**J1**
	$\therefore x=-\frac{3}{2}$	
	TOTAL	**6 marks**
4a)i.	For $x_1,x_2\in D_f$	**K1**
		K1

	$If \quad f(x_1) = f(x_2)$ $3+\sqrt{x_1-2} = 3+\sqrt{x_2-2}$ $\sqrt{x_1-2} = \sqrt{x_2-2}$ $\left(\sqrt{x_1-2}\right)^2 = \left(\sqrt{x_2-2}\right)^2$ $x_1 - 2 = x_2 - 2$ $x_1 = x_2$	**J1**
	Thus, f(x) is a one-to-one function. Therefore $f^{-1}(x)$ exists.	**K1**
ii.	$f \circ f^{-1}(x) = x$ $3+\sqrt{f^{-1}(x)-2} = x$ $\sqrt{f^{-1}(x)-2} = x-3$ $\therefore\ f^{-1}(x) = (x-3)^2 + 2$ *OR* $\therefore\ f^{-1}(x) = x^2 - 6x + 11$	**J1** **B1** **B1**
	$D_{f^{-1}} = [3, \infty)$ $R_{f^{-1}} = [2, \infty)$	
iii.		**R1** (shape f(x)) **R1** (shape inverse) **R1** (complete – label, line y=x)

	 **Graf** $f^{-1}(x)=(x-3)^2+2$ Graf $f(x)=3+\sqrt{x-2}$ $\therefore f^{-1}(x)$ is a reflection of graph f (x) about the line y=x	**J2**
	TOTAL	**12 marks**
5(a)	$f(x)=\dfrac{2x}{(x+1)(x-2)}=\dfrac{2x}{x^2-x-2}$ $\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty}\dfrac{\frac{2x}{x^2}}{\frac{x^2}{x^2}-\frac{x}{x^2}-\frac{2}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{\frac{2}{x}}{1-\frac{1}{x}-\frac{2}{x^2}}=\dfrac{0}{1-0-0}=0$ $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\dfrac{\frac{2x}{x^2}}{\frac{x^2}{x^2}-\frac{x}{x^2}-\frac{2}{x^2}}=\lim\limits_{x\to-\infty}\dfrac{\frac{2}{x}}{1-\frac{1}{x}-\frac{2}{x^2}}=\dfrac{0}{1-0-0}=0$	K1 K1 Divide by highest degree of denominator K1

	$\therefore$ Horizontal Asymptote, y = 0	**J1**
(b)	$$\lim_{x\to 2^-}\frac{x-1}{x+2}=\lim_{x\to 2^+}(ax^2-1)$$	K1K1
	$$\frac{2-1}{2+2}=a(2)^2-1$$	K1 Attempt to solve
	$$\frac{1}{4}=4a-1$$	
	$$\therefore a=\frac{5}{16}$$	J1
	Hence,	
	$$f(2)=\frac{5}{16}(2)^2-1=\frac{1}{4}$$	K1 Find f(2)
	$$\lim_{x\to 2}f(x)=\frac{1}{4}$$	
	$$\lim_{x\to 2}f(x)=f(2)=\frac{1}{4}$$	K1
	$\therefore f(x)$ is continuous at x = 2	J1
	TOTAL	**11 marks**

NO	ANSWER SCHEMES	REMARKS
6a(i)	$$2x+2y\frac{dy}{dx}-2\frac{dy}{dx}=0$$	
	$$2y\frac{dy}{dx}-2\frac{dy}{dx}=-2x$$	**K1**
	$$\frac{dy}{dx}=\frac{-x}{y-1}\ @\ \frac{x}{1-y}$$	**J1**

6a(ii)	*At point* $(1,3)$ @ $x=1, y=3$ $\frac{dy}{dx}=\frac{x}{1-y}=\frac{1}{1-3}=-\frac{1}{2}$	**K1J1**
6a(iii)	$\frac{dy}{dx}=\frac{x}{1-y}$ $u=x \qquad v=1-y$ $u'=1 \qquad v'=-\frac{dy}{dx}=-\frac{x}{1-y}$ $\frac{d^2y}{dx^2}=\frac{vu'-uv'}{v^2}=\frac{(1-y)(1)-(x)\left(-\frac{x}{1-y}\right)}{(1-y)^2}$ $\frac{d^2y}{dx^2}=\frac{(1-y)^2+(x)^2}{(1-y)^3}$	**K1K1** **K1** **J1**
	$x^2+y^2-2y=4 \quad @ \quad x^2=4-y^2+2y$ $\frac{d^2y}{dx^2}=\frac{(1-y)^2+(x)^2}{(1-y)^3}$ $=\frac{(1-2y+y^2)+(4-y^2+2y)}{(1-y)^3}$ $=\frac{5}{(1-y)^3}$	**K1K1** **J1**

b	$x = t^2 - 3 \qquad y = t^3 + t$ $\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = 3t^2 + 1$ $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{3t^2+1}{2t}$	K1 J1
	$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\left(\frac{dt}{dx}\right)$ $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{3t^2+1}{2t}\right)\left(\frac{1}{2t}\right)$ $\frac{d^2y}{dx^2} = \left[\frac{(2t)(6t)-(2)(3t^2+1)}{(2t)^2}\right]\left(\frac{1}{2t}\right)$ $\frac{d^2y}{dx^2} = \left[\frac{6t^2-2}{8t^3}\right] = \left[\frac{3t^2-1}{4t^3}\right]$	K1 K1 J1
	TOTAL	**16 marks**
7	Let the height of the box = $\boldsymbol{h}$ cm Total surface area of the box = 300 cm^2 $2(2x^2) + 2hx + 2(2xh) = 300$ $4x^2 + 6hx = 300$ $2x^2 + 3hx = 150$ $h = \frac{150 - 2x^2}{3x}$	K1

	Volume of the box , $V = 2x^2h$	J1
	$V = 2x^2\left(\dfrac{150-2x^2}{3x}\right)$ $= 100x - \dfrac{4}{3}x^3$	K1
	$\dfrac{dV}{dx} = 100 - \dfrac{4}{3}\left(3x^2\right)$ $= 100 - 4x^2$	K1 J1
	When $\dfrac{dV}{dx} = 0,\ \ 100 - 4x^2 = 0$ $x^2 = 25$ $x = 5$	K1
	$\dfrac{d^2V}{dx^2} = -8x$	J1
	When $x = 5,\ \ \dfrac{d^2V}{dx^2} = -40(<0)$ (maximum value)	K1 J1
	the height of the box , $h = \dfrac{150-2(5)^2}{3(5)} = \dfrac{20}{3}\,cm$	

	the maximum volume of the box, $V = 100(5) - \frac{4}{3}(5)^3 = \frac{1000}{3} cm^3$	
	TOTAL	**9 marks**
		75 MARKS