

**SECTION A** (25 marks)

This section consists of **3** questions. Answer **all** questions.

1. Evaluate the following limits, if exists:

(a)  $\lim_{x \rightarrow -\infty} \frac{3x}{\sqrt{x^2 - 36}}$ . [4 marks]

(b)  $\lim_{x \rightarrow 3} \frac{\sqrt{x^2 + 7} - 4}{x^2 - 3x}$ . [5 marks]

2. (a) Find the derivative of  $f(x) = \sqrt{8x - 3}$  by using the first principle of differentiation. [4 marks]

(b) Find  $\frac{dy}{dx}$  for the following:

(i)  $y = \sin(\ln 3x)$  [3 marks]

(ii)  $y = \frac{x}{x^3 + 1} + e^{2x} \tan x$ . [4 marks]

3. Find the inflection point of the function  $f(x) = x^3 + 3x^2 - 24x + 20$ . [5 marks]

**SECTION B** (75 marks)

This section consists of **7** questions. Answer **all** questions.

1. Given  $z_1 = 3 + 4i$  and  $z_2 = 5 - 2i$ . Find  $z_3$  in  $a + bi$  form if  $\frac{1}{z_3} = \frac{1}{z_1} + \frac{1}{z_1 z_2}$ .

Hence, express  $\overline{z_3}$  in polar form.

[9 marks]

2. (a) Solve the equality  $|3x + 1| = |2x - 7|$ .

[4 marks]

- (b) Obtain solution set for the following inequalities

(i)  $|2x + 7| + 1 \geq 6x$ .

[4 marks]

(ii)  $\left| \frac{2x + 1}{x - 3} \right| \geq 2$ .

[4 marks]

3. (a) A function  $f(x)$  is defined as

$$f(x) = \frac{3 - \ln(4 - x)}{2}, \quad x \in \mathbb{R} \text{ and } x < 4.$$

- (i) Determine the inverse function,  $f^{-1}(x)$ .

[3 marks]

- (ii) Determine whether  $f^{-1}(x)$  is one to one.

[3 marks]

- (b) Given the function  $g(x) = e^{x+1}$ . Find the function  $h(x)$  if  $(h \circ g)(x) = x + 1$ .

[6 marks]

4. A function  $f$  is defined as

$$f(x) = -(x - 2)^2 + 1, \quad x \geq 2$$

Determine the inverse function of  $f$ .

Hence, sketch the graph of  $f$  and  $f^{-1}$  on the same axes.

[6 marks]

5. (a) Find the vertical and horizontal asymptotes of

$$f(x) = \frac{2x^2 + 1}{4x^2 - 25}$$

[6 marks]

(b) Given

$$f(x) = \begin{cases} \frac{x^2 - 16}{x^2 - 4x}, & x \neq 4 \\ p & x = 4 \end{cases}$$

where  $p$  is a constant.

(i) Determine  $\lim_{x \rightarrow 4} f(x)$  if exists. [3 marks]

(ii) Find the value of  $p$  if  $f(x)$  is continuous at  $x = 4$ . [2 marks]

6. (a) The parametric equations of a curve is given by  $x = -9\sin^3 t$  and  $y = 3 - 3\cos^3 t$ .

(i) Show that  $\frac{dy}{dx} = -\frac{1}{3}\cot t$ . Hence, solve  $\frac{dy}{dx} = 0$  for  $0 \leq t \leq \pi$ . [8 marks]

(ii) Show that  $\frac{d^2y}{dx^2} = -\frac{1}{81\sin^4 t \cdot \cos t}$ . [4 marks]

(b) Given  $e^{-2y} + 2xy + \ln(1 - 2x) = 2$ . Calculate the value of  $\frac{dy}{dx}$  when  $x = 2$  and  $y = 0$ . [4 marks]

7. (a) Given a water tank which has the shape of an inverted circular cone water tank with a base radius of 3 meters and a height of 5 meters. If water is being pumped into the tank at the rate of  $2 \text{ m}^3/\text{minute}$ , find the rate of change in water level rising when the water is 3 meters deep.

[5 marks]

(b) Air is pumped into a spherical balloon at the rate of  $12\pi \text{ cm}^3 \text{ s}^{-1}$ . Find the rate of increase of the surface area when its radius is 6 cm.

[4 marks]

END OF QUESTION

**Final Answer:**

**SECTION A**

1. (a) -3 (b)  $\frac{1}{4}$
2. (a)  $\frac{4}{\sqrt{8x-3}}$  (b)(i)  $\frac{\cos(\ln 3x)}{x}$  (b)(ii)  $\frac{1-2x^3}{(x^3+1)^2} + e^{2x}(\sec^2 x + 2 \tan x)$
3. Point of inflection = (-1, 46)

**SECTION B**

1.  $z_3 = \frac{11}{4} + \frac{13}{4}i$  ;  $\bar{z}_3 = 4.26(\cos(-0.8685) + i \sin(-0.8685))$
2. (a)  $x = \frac{6}{5}$ ,  $x = -8$   
(b) (i)  $\{x : x \leq 2\}$  (ii)  $\left\{x : x \geq \frac{5}{4}, x \neq 3\right\}$
3. (a)(i)  $f^{-1}(x) = 4 - e^{3-2x}$  (ii)  $f^{-1}(x)$  is one to one  
(b)  $h(x) = \ln x$
4.  $f^{-1}(x) = 2 + \sqrt{1-x}$ ,  $x \leq 1$ , D.I.Y
5. (a) V.A:  $x = \frac{5}{2}$ ,  $x = -\frac{5}{2}$ , H.A:  $y = \frac{1}{2}$  (b) (i)  $\lim_{x \rightarrow 4} f(x) = 2$  (ii)  $p = 2$
6. (a) (i)  $\frac{\pi}{2}$  (ii) D.I.Y (b)  $\frac{dy}{dx} = -\frac{1}{3}$
7. (a)  $\frac{50}{81\pi} \text{ cm s}^{-1}$  (b)  $4\pi \text{ cm}^2 \text{ s}^{-1}$