SET 2

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SESI 2023/2024

CONTENTS

SECTION A
(25 %)

1 a) Find

i. $\lim_{t\to -2}\sqrt{t^3+3t^2}$.

[2 *marks*]

ii. $\lim_{x\to 1^-}\dfrac{x^2}{x^2+2x-3}$.

[3 *marks*]

b) Given $f(x)=x^2-3x+4$. Find $\lim_{x\to 1}\dfrac{f(x)-f(1)}{x-1}$.

[4 *marks*]

CONTENTS

SECTION A
(25 %)

2 a) Find the derivative of $y = \sqrt{x-2}$ from first principle.

[4 *marks*]

b) Differentiate $y = \dfrac{(2x+1)^2}{\sqrt{x^2+x}}$ with respect to *x*. Give the answer in the simplest form.

[5 *marks*]

3 A curve is given by $y = (x+2)^2(2x-3)$.

a) Determine the relative extremum points by using the first derivative test.

[6 *marks*]

b) State the intervals of *x* where the curve increases and decreases.

[1 *mark*]

CONTENTS

SECTION B
(75%)

Q1 Q2 Q3 Q4 Q5 Q6 Q7

1 If z is a complex number satisfying $|z+1| = z + 2(1+i)$, find z.

[5 *marks*]

2 Find the solution for the following inequalities.

(a) $2 \le (x+1)^2 - 2 < x^2 + 3$. [7 *marks*]

(b) $2|x+1| + x > 5x - 6$. [4 *marks*]

3 Given $f(x) = e^{2x-1} + 2$ and $(g \circ f)(x) = 2x - 1$.

Find $g(x)$ and state its domain. Hence, find $(f \circ g)(x)$.

[8 *marks*]

4 Given $f(x) = x^2 + 2x - 3$.

(a) Show that $f(x)$ is not invertible. Restrict the domain of $f(x)$ such that $f^{-1}(x)$ exists (give only one interval of x).

[5 *marks*]

(b) Referring to restricted domain in (a), find $f^{-1}(x)$.

[3 *marks*]

(c) Sketch the graph of $f(x)$ and $f^{-1}(x)$ on the same axes. State the domain of $f^{-1}(x)$ [4 *marks*]

CONTENTS

SECTION B
(75%)

Q1 Q2 Q3 Q4 Q5 Q6 Q7

5 a) State the conditions for a function $g(x)$ to be continuous at $x=c$. [2 *marks*]

b) A function g is defined by $g(x)=\begin{cases}\dfrac{2x-1}{x+1}, & x<-2\\ \dfrac{k}{1-x}, & -2\le x<1 \text{ or } x>1.\end{cases}$

i. If $g(x)$ is continuous at x=-2, find the value of k. [3 *marks*]

ii. Determine the vertical and horizontal asymptotes of $g(x)$. [6 *marks*]

iii. Sketch the graph of $g(x)$. [3 *marks*]

6 a) Given $e^y=\ln\left(\dfrac{e^x+1}{e^x-1}\right)$. Show that $\dfrac{dy}{dx}=-\dfrac{2e^{x-y}}{e^{2x}-1}$.

[6 *marks*]

b) A parametric equation is given by $y=\cos^3\theta,\ x=\ln\left(\sin^2\theta\right)$.

Find $\dfrac{dy}{dx}$, hence show that $\dfrac{d^2y}{dx^2}=-\dfrac{3}{4}\left(\sin\theta\tan\theta\right)\left(3\cos^2\theta-1\right)$.

[10 *marks*]

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SECTION B
(75%)

7 **FIGURE 1** shows a tank in the shape of a cuboid with a rectangular base x m by $4x$ m and no top. The height of the box is h m. It is given $h = \dfrac{75 - x^2}{x}$.

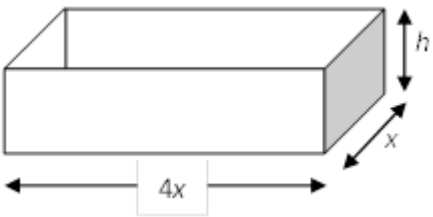

FIGURE 1

a) Find the value x for which the tank has the maximum volume, fully justifying the fact that it gives the maximum volume.

[5 *marks*]

b) Water flows into the tank at the rate of 5 $m^3 min^{-1}$. Find the rate of change of the water level in the tank (use the dimension when the volume is maximum).

[4 *marks*]

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Program WOW

SECTION A

SCORE

/25

SECTION B

SCORE

/75