SECTION A [25 *marks*]

This section consists of 3 questions. Answer **all** questions.

1 Evaluate the following limits

i. $\lim\limits_{x \to -3} \dfrac{2x^3 + 5x^2 - 2x + 3}{x^2 + x - 6}$

[5 *marks*]

ii. $\lim\limits_{x \to 2^-} \dfrac{x-2}{\left|2x^2 + x - 10\right|}$

[4 *marks*]

2 (a) Find the derivative of $f(x) = \dfrac{1}{2\sqrt{x}}$ by using the first principles.

[5 *marks*]

(b) If $f(x) = \sqrt{x + \sqrt{x}}$, find $f'(x)$ and deduce the value of $f'(1)$.

[4 *marks*]

3 Find the critical numbers for $f(x) = x^3 + 4x^2 - 16x$. Hence, find the x-coordinate of the point of inflection and state the range of values x when $f(x)$ is decreasing.

[7 *marks*]

SECTION B [75 *marks*]

This section consists of 7 questions. Answer **all** questions.

1 Given $z=\dfrac{\sqrt{3}+i}{1+i\sqrt{3}}$. Express z in the form of $a+bi$. Hence, determine modulus and argument of z.

[5 *marks*]

2 (a) Given $\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{2}-\sqrt{3}-\sqrt{5}}$. Rationalise the denominator and simplify.

[5 *marks*]

(b) Solve $\dfrac{1}{|x-3|}>\dfrac{3}{|x+1|}$

[6 *marks*]

3 **(a)** If $g(x)=\ln\left(\dfrac{1-x}{1+x}\right)$, show that $g(x)+g(y)=g\left(\dfrac{x+y}{1+xy}\right)$.

[4 *marks*]

(b) Given that $f(x)=\dfrac{ax+3}{x-b}$. Find the values of a and b if $f(2)=7$ and $f^{-1}(1)=-4$.
Hence, show $f(x)$ is a one to one function.

[6 *marks*]

4 Given $f(x)=e^{x+3}-2$.

(a) Find $f^{-1}(x)$, if it exist.

[3 *marks*]

(b) On the same axis, sketch the graphs of $f(x)$ and $f^{-1}(x)$.
Hence, state the domain and range of $f(x)$ and its inverse.

[5 *marks*]

(c) Show that $f\circ f^{-1}(x)=x$.

. [2 *marks*]

5 (a) Sketch the graph $f(x)=\sqrt{9x^2-1}$ and state the intervals where f is continuous.

$[3\ marks]$

(b) Function g is defined as follows. $g(x)=\begin{cases} mx^2+3 & , \quad x\in\left[-\frac{1}{3},\frac{1}{3}\right] \\ \sqrt{9x^2-1} & , \quad x\notin\left[-\frac{1}{3},\frac{1}{3}\right] \end{cases}$

Given that g is continuous for all values of x. Find the value of the constant m.

$[3\ marks]$

(c) Function h is defined as $h(x)=\dfrac{3x-1}{\sqrt{9x^2-1}}$. State the intervals of continuity of h.

Find all the asymptotes of h. Hence sketch the graph h.

[8 *marks*]

6 **(a)** Given $y=(ax+b)e^{-3x}$ where *a* and *b* are constant. Show that $\dfrac{d^2y}{dx^2}+6\dfrac{dy}{dx}+9y=0$

$[6\ marks]$

(b) The parametric equations of a curve are given by $x=\dfrac{1}{3}\sin^3 2\theta$ and $y=2\cos^3 2\theta$.

Find $\dfrac{dy}{dx}$ in terms of θ. Hence, find the value of the parameter θ if $\dfrac{dy}{dx}=-2\sqrt{3}$

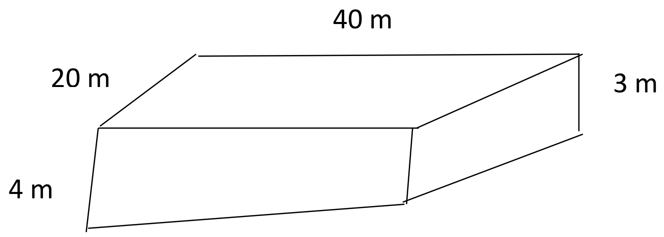
where $0<\theta<\dfrac{\pi}{2}$.

$[6\ marks]$

(c) Given $xe^y+y\ln x=5$, find $\dfrac{dy}{dx}$ in terms of *x* and *y*.

$[4\ marks]$

7 The diagram shows a swimming pool measures 20 m wide, 40 m long, 3 m deep at the shallow end and 4 m deep at the other end.



If the pool is being filled at a rate of $0.8\,\text{m}^3/\text{min}$, what is the rising rate of the water-level when the water-level at the deepest end is $0.5\,\text{m}$? How long does it take before the water-level start to rise at a constant rate?

[9 *marks*]

END OF QUESTION PAPER

Answers

Section A

1 (a) $-\dfrac{22}{5}$ (b) $-\dfrac{1}{9}$

2 (a) $\dfrac{-1}{4x\sqrt{x}}$ (b) $f'(x)=\dfrac{1+2\sqrt{x}}{4\sqrt{x}\sqrt{x+\sqrt{x}}}$, $f'(1)=\dfrac{3\sqrt{2}}{8}$

3 Critical numbers are $x=\dfrac{4}{3}$, $x=-4$; No x-coordinate of the inflection point ;

$f(x)$ is decreasing at $-4<x<\dfrac{4}{3}$

Section B

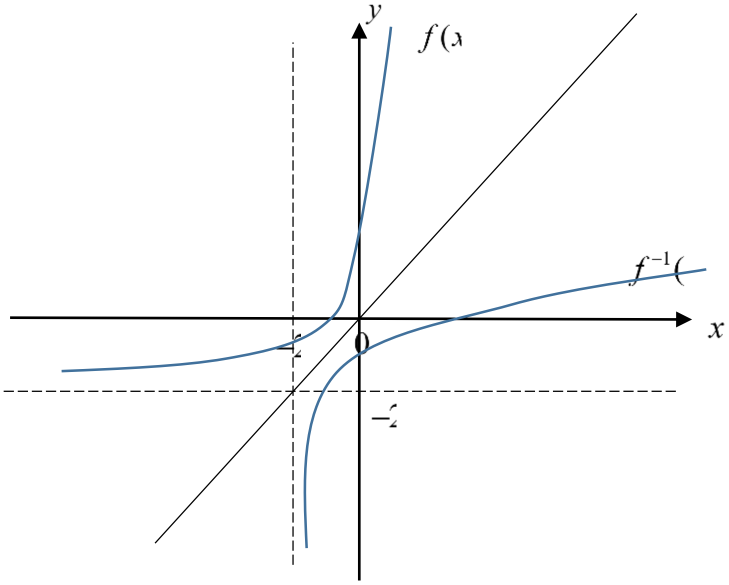
1 $\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i$, $|z|=1$, $\arg(z)=-\dfrac{\pi}{6}$

2 (a) $\dfrac{\sqrt{6}-\sqrt{10}+2}{-4}$ (b) $\{x:2<x<5 \;\; ; x\neq 3\}$

3 (a) DIY (b) $a=2, b=1$; DIY

4 (a) $f^{-1}(x)=\ln(x+2)-3$

(b)

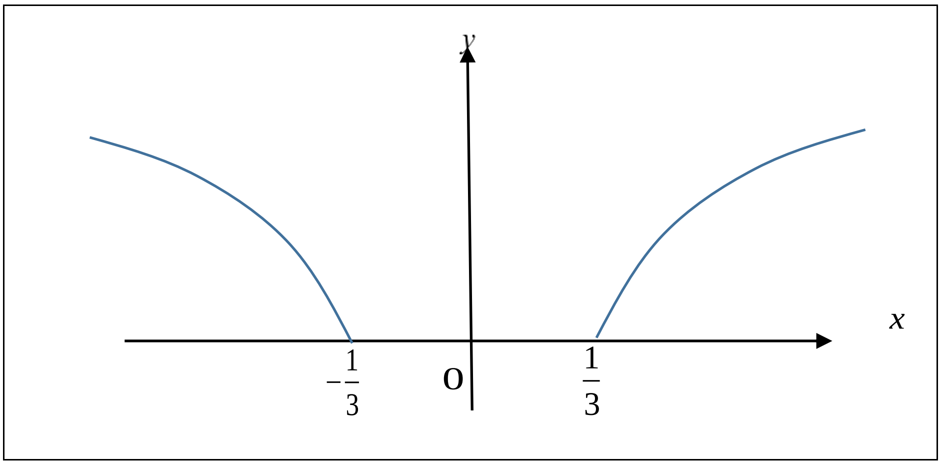


$D_f=(-\infty,\infty)$ $R_f=(-2,\infty)$

$D_{f^{-1}}=(-2,\infty)$ $R_{f^{-1}}=(-\infty,\infty)$

(c) Shown

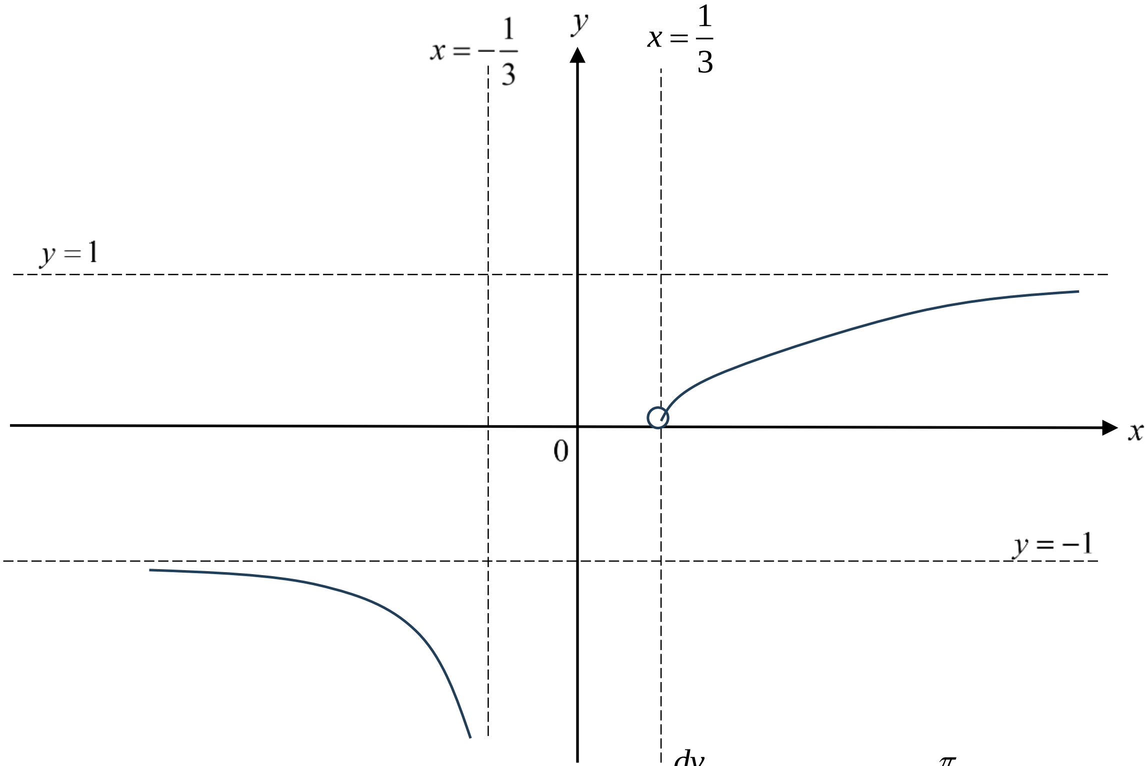
5 (a) f is continuous on the interval $\left(-\infty,-\frac{1}{3}\right]\cup\left[\frac{1}{3},+\infty\right)$



(b) $m=-27$ (c) $\left(-\infty,-\frac{1}{3}\right)\cup\left(\frac{1}{3},+\infty\right)$

Vertical asymptotes $x=-\frac{1}{3}$

Horizontal asymptotes at $y=-1$ and $y=1$



6 (a) Shown (b) $\frac{dy}{dx}=-6\cot 2\theta$; $\theta=\frac{\pi}{6}$

(c) $\frac{dy}{dx}=\frac{-xe^{y}-y}{x\left(xe^{y}+\ln x\right)}$

7 $\frac{dy}{dt}=0.002m/s, T=500$ minutes