

SECTION A*(25 marks)*

This section consists of **3** questions. Answer **all** questions.

- 1.** Evaluate the following limits (if exist):

(a) $\lim_{x \rightarrow 3^-} \frac{x^2 - 9}{|x - 3|}$. **(3 marks)**

(b) $\lim_{x \rightarrow 5} \frac{2x - 10}{\sqrt{2x - 1} - 3}$. **(3 marks)**

(c) $\lim_{x \rightarrow +\infty} \frac{2\sqrt{x} - 4}{x - 4}$. **(3 marks)**

- 2.** (a) Evaluate $\frac{dy}{dx}$ when $x = 0$ for each of the following

(i) $y = \ln(x^2 + \sqrt{x^2 + 1})$. **(3 marks)**

(ii) $y = \frac{e^x(2x^2 + 1)}{\sqrt{x + 1}}$. **(4 marks)**

(b) Given $y = 3^{x^2}$, find $\frac{dy}{dx}$. **(3 Marks)**

- 3.** The function $f(x) = x^3 - 3x^2 - 9x - 4$ is defined on the interval $[-2, 6]$. Find the critical points of $f(x)$ on the interval and determine whether each critical point is a minimum or maximum. **(6 marks)**

SECTION B*(75 marks)*

This section consists of 7 questions. Answer **all** questions.

1. Given $z_1 = 3 + 2i$ and $z_2 = 1 - 3i$. Without using calculator, find $z_3 = \frac{z_2}{\bar{z}_1}$ and hence,

evaluate $|z_3|$. **(5 marks)**

2. (a) Solve the equation $(\log x)^2 = \log x^3$. **(5 marks)**

(b) Find the interval notation of the inequality $2 + \left| \frac{5x+2}{x-3} \right| \geq 5$. **(8 marks)**

3. Given the functions f and g as follows

$$f(x) = x^2 + 3x + 1,$$

$$g(x) = x - 2.$$

(a) Find $f \circ g$ and $g \circ f$. **(4 marks)**

(b) State domain and range of $g \circ f$. **(3 marks)**

(c) Determine the value of x such that $f \circ g(x) = g \circ [g \circ f(x)]$. **(3 marks)**

4. The functions f and g are defined as $f(x) = \frac{x+1}{x-5}, x \neq 5$ and $g(x) = 4 - x$.

(a) Find $f^{-1}(x)$ and $g^{-1}(x)$. **(5 marks)**

(b) Evaluate $(f \circ g^{-1})(2)$. **(3 marks)**

5. (a) Given $f(x) = \begin{cases} 7 - 2x, & x \leq p \\ \frac{x^2 + (q - 2)x - 2q}{x - 2}, & p < x \leq 5 \\ 10 - (x - 7)^2, & x > 5 \end{cases}$

with $\lim_{x \rightarrow p^+} f(x) = 3$ and the function f is continuous for all real values of x .

Determine the values of p and q . (7 marks)

(b) A function f is defined by $f(x) = \begin{cases} \frac{2(1-x)}{x-2}, & x < \frac{3}{2} \\ 2, & x \geq \frac{3}{2} \end{cases}$

(i) Use the definition to show that f is continuous at $x = \frac{3}{2}$. (1 mark)

(ii) Sketch the graph of f . (6 marks)

6. (a) Find $\frac{dy}{dx}$ in terms of x and y if $x^2 \sin y + 2x = y$. (7 marks)

(b) Differentiate $\cos^3(\ln(2x - 1))$ with respect to x . (4 marks)

(c) Given $y = 5 \sin(3x) + \sqrt{x}$. Find the value of $\frac{dy}{dx}$ when $x = \frac{\pi}{2}$. (4 marks)

7. (a) Find the stationary points of the curve has an equation $y = \frac{1}{3}x^3 + x^2 - 8x$. (4 marks)

(b) Air is pumped into a spherical balloon at a rate $54 \text{ cm}^3 \text{ s}^{-1}$. Find the rate at which the radius is increasing when the volume of balloon is $36\pi \text{ cm}^3$. (6 marks)

END OF QUESTIONS PAPER

ANSWER:

PART A

1. (a) -6 (b) 6 (c) 0

2. (a) (i) 0 (ii) $\frac{1}{2}$

(b) $\frac{dy}{dx} = 3^{x^2} (\ln 3)(2x)$

3. $(-1, 1)$ maximum point, $(3, -31)$ minimum point

PART B

1. $z_3 = \frac{9}{13} - \frac{7}{13}i$, $|z_3| = 0.8771$

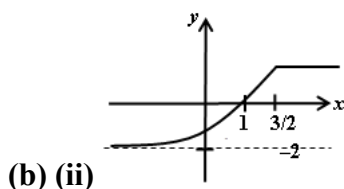
2. (a) $x=1, 1000$ (b) $\left(-\infty, -\frac{11}{2}\right] \cup \left[\frac{7}{8}, 3\right) \cup (3, \infty)$

3. (a) $f \circ g = x^2 - x - 1$, $g \circ f = x^2 + 3x - 1$ (b) $D_{g \circ f} = (-\infty, \infty)$, $R_{g \circ f} = \left[-\frac{13}{4}, \infty\right)$

(c) $x = \frac{1}{2}$

4. (a) $f^{-1}(x) = \frac{5x+1}{x-1}$, $g^{-1}(x) = 4-x$ (b) -1

5. (a) $p=2, q=1$



6. (a) $\frac{dy}{dx} = \frac{2(1+x \sin y)}{1-x^2 \cos y}$ (b) $\frac{dy}{dx} = -\frac{6}{2x-1} \cos^2(\ln(2x-1)) \sin(\ln(2x-1))$

(c) 0.399

7. (a) $\left(2, -\frac{28}{3}\right)$ and $\left(-4, \frac{80}{3}\right)$. (b) $\frac{dr}{dt} = 0.4775 \text{ cms}^{-1}$