SECTION A
(25 MARKS)

ANSWER ALL QUESTIONS BELOW

1. Evaluate the following limits:

   (a) $\lim_{x \to -3} \frac{x+3}{x^3+27}$ [4 *marks*]

   (b) $\lim_{x \to \infty} \sqrt{\frac{5x+7}{6x-5}}$ [3 *marks*]

   (c) $\lim_{x \to 2^-} \frac{x+1}{x^2-4}$ [2 *marks*]

2. Find the first derivatives of :

   (a) (i) $y = 8^{7x+6}$ [2 *marks*]

   (ii) $f(x) = \ln\left(\frac{x^2-1}{\sqrt{x-1}}\right)$ [3 *marks*]

   (b) Find the value of $\frac{dy}{dx}$ for $y = \sin 2x - x\cos x$, when $x = \frac{\pi}{3}$ [4 *marks*]

3. Given a curve $f(x) = 27 + 9x - 3x^2 - x^3$.

   (a) Find the coordinates of the stationary points of the curve. [5 *marks*]

   (b) Determine whether the stationary points are maximum or minimum.

   [2 *marks*]

SECTION B
(75 MARKS)
ANSWER ALL QUESTIONS BELOW

1. Given that $z_1 = 3-3i$ and $z_2 = 3+2i$, where $i=\sqrt{-1}$.

   Express $\dfrac{\left(\overline{z_1}z_2\right)}{13}+\overline{\left(\dfrac{i^3}{-z_2}\right)}$ in the form of $a+bi$ where $a,b\in\mathbb{R}$ [6 *marks*]

2. (a) Solve the equation $x^{\ln x-2}=e^3$ [6 *marks*]

   (b) Obtain the solution set for $x-1\le x^2+3x\le x+3$ [6 *marks*]

3. Given $f(x)=\dfrac{x+a}{x-a}$ and $g(x)=bx+3$, where a and b are constants.

   Find the possible values of a and b if $fg(1)=2$ and $gf(1)=-1$. [6 *marks*]

4. Given the function $g(x)=\dfrac{1}{2x-5}$.

   (a) Find the domain and range for $g(x)$. [2 *marks*]

   (b) Show that $g(x)$ is a one to one function. Hence find $g^{-1}(x)$. [5 *marks*]

   (c) On the same axis, sketch the graph of $g(x)$ and $g^{-1}(x)$. [3 *marks*]

   (d) Show that $g\circ g^{-1}(x)=x$. [2 *marks*]

5. Functions $f(x)=\begin{cases} x+2, & x\ge 2\\ x^2-4x+a, & x<2\end{cases}$ and $g(x)=\begin{cases} \dfrac{b}{x}, & x\ge 2\\ 4-x, & x<2\end{cases}$ are continuous at $x=2$

   (a) (i) Find the values of a and b [4 *marks*]

   (ii) Show that the function $f+g$ is continuous at $x=2$ [4 *marks*]

(b) Find the horizontal asymptote(s) for the function $\frac{\sqrt{x^2+9}}{x}$, if they exists

[6 *marks*]

6. (a) A curve is defined by the parametric equations $x = 3t - \frac{1}{t}$ and $y = t + \frac{3}{t}$, where $t \neq 0$.

(i) Show that $\frac{dy}{dx} = \frac{t^2-3}{3t^2+1}$. [4 *marks*]

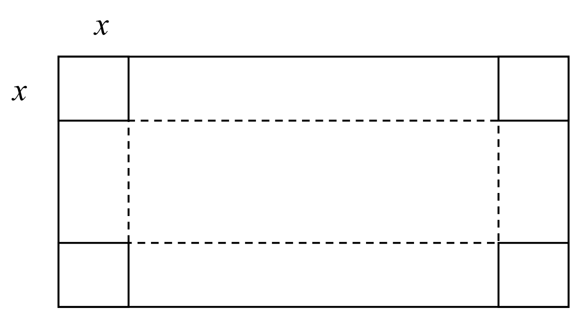
(ii) Find $\frac{d^2y}{dx^2}$. If $t \in \mathbb{Z}$, evaluate $\frac{d^2y}{dx^2}$ when $y = \frac{7}{2}$

[9 *marks*]

(b) Show that $\frac{dy}{dx}$ can be expressed as $\frac{dy}{dx} = \frac{1}{3} - \frac{10}{3\left(3t^2+1\right)}$.

[2 *marks*]

7. An open rectangular box is to be made from a piece of paper of width 8 cm and length 15 cm. A square of width x cm is cut from each corner of the paper and the sides are folded up as shown in the figure below. Find the width, length and height of the box that gives the maximum volume.



[10 *marks*]

END OF QUESTION PAPER

ANSWERS

SECTION A

1. (a) $\frac{1}{27}$ (b) $\frac{\sqrt{30}}{6}$ (c) $-\infty$

2. (a)(i) $\frac{dy}{dx} = 7(\ln 8)8^{7x+6}$ (ii) $f'(x) = \frac{2x}{x^2-1} - \frac{1}{2(x-1)}$ (b) -0.5931

3. (a) $(-3,0)$, $(1,32)$ (b) $(-3,0)$ min, $(1,32)$ max

SECTION B

1. (a) $\frac{5}{13} + \frac{12}{13}i$

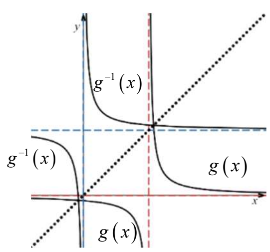
2. (a) $x = e^3$, $x = e^{-1}$ (b) $\{x : -3 \le x \le 1\}$

3. $a = \frac{1}{3}$, $a = 1, b = -2$, $b = 0$

4. (a) $D_g = \left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$ $R_g = (-\infty, 0) \cup (0, \infty)$

 (c) DIY, $g^{-1}(x) = \frac{1}{2x} + \frac{5}{2}$

 (d)



 (e) DIY

5. a) (i) $a = 8$, $b = 4$

 (ii) DIY

 b) HA $y = 1, y = -1$

6. (i)DIY (ii) $\frac{20t^3}{(3t^2+1)^3}$, $\frac{160}{2197}$

7. width $= \frac{14}{3}$, length $= \frac{35}{3}$, height $= \frac{5}{3}$