6.004 Tutorial Problems
L04a – Boolean Algebra

Problem 1

Write the truth table for each of the following expressions

A $\overline{A}$

A	Out
0	1
1	0

B $A \cdot B$

A	B	Out
0	0	0
0	1	0
1	0	0
1	1	1

C $\overline{A \cdot B}$

A	B	Out
0	0	1
0	1	1
1	0	1
1	1	0

D $A + B$

A	**B**	**Out**
0	0	0
0	1	1
1	0	1
1	1	1

E $\overline{A + B}$

A	B	Out
0	0	1
0	1	0
1	0	0
1	1	0

F $(A + B) \cdot (\overline{A \cdot B})$

A	B	Out
0	0	0
0	1	1
1	0	1
1	1	0

G $(A \cdot S) + (B \cdot \overline{S})$

A	B	S	Out
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Problem 2

Consider the truth table on the right, which defines a full adder, with two functions C_{out} and S of three input variables (A, B, and C).

For each function, write it in normal form, then find a minimal sum of products (minimal SOP) expression.

A	B	C_{in}	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Normal form for $C_{out}(A, B, C_{in})$ = A'BC + AB'C + ABC' + ABC

Minimal SOP for $C_{out}(A, B, C_{in})$ = BC + AC + AB

Normal form for $S(A, B, C_{in})$ = A'B'C + A'BC' + AB'C' + ABC

Minimal SOP for $S(A, B, C_{in})$ = A'B'C + A'BC' + AB'C' + ABC

Problem 3

Consider the 3-input Boolean function G(A,B,C) = $\overline{A} \cdot \overline{C} + A \cdot \overline{B} + \overline{B} \cdot \overline{C}$.

A How many 1's are there in the output column of G's 8-row truth table?

Four.

A	B	C	G
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

B Give a minimal sum-of-products expression for G.

AB' + A'C'

C Can a sum-of-products expression involving 3 input variables with greater than 4 product terms *always* be simplified to a sum-of-products expression using fewer product terms?

Yes. A 3 input system has 8 possible input states, so if there are say 5 inputs that give a truthy output, there are 3 inputs that generate a falsey one, so create a normal form expression for the inverse of the system and then flip it using DeMorgan's law.

Problem 4

Simplify the following Boolean expressions by finding a *minimal sum-of-products expression* for each one:

A $A \cdot C + B + C$

C + B

B $(A + B) \cdot C + \overline{C} \cdot A + B \cdot (\overline{A} + C)$

A + BC + A'B

C $A \cdot (B + C) \cdot (B + A \cdot (B + C))$

AB + AC

D $A \cdot (B + C \cdot (D + E \cdot F))$

AB + ACD + ACEF