Boolean Algebra

Useful Boolean Algebra Properties

 Using the axioms, we can derive several useful properties to manipulate and simplify Boolean expressions:

commutative	a·b = b·a	a+b = b+a
associative	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$	a+(b+c) = (a+b)+c
distributive	$a \cdot (b+c) = a \cdot b + a \cdot c$	$a+b\cdot c = (a+b)\cdot (a+c)$
complements	$a \cdot \overline{a} = 0$	$a+\overline{a}=1$
absorption	$a \cdot (a+b) = a$	$a+a\cdot b = a$
reduction	$a \cdot b + a \cdot \overline{b} = a$	$(a+b)\cdot(a+\overline{b}) = a$
DeMorgan's Law	$\overline{a \cdot b} = \overline{a} + \overline{b}$	$\overline{a+b} = \overline{a} \cdot \overline{b}$

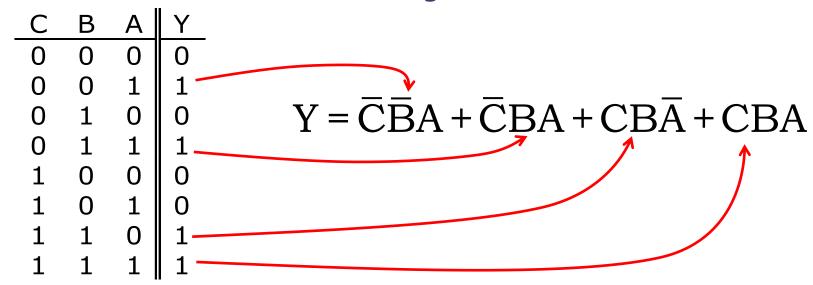
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- Many of these properties are easy to remember because they match the ones for integer algebra, but be aware of the differences
 - e.g., distributive property for Boolean "+" a+b·c = (a+b)·(a+c) does not hold for integer "+"!
- To familiarize yourself with the properties, we recommend that you simply prove them
 - Example: DeMorgan's Law

а	b	a·b	ā+b̄
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Equivalence and Normal Form

 Given a truth table, it is easy to derive an equivalent Boolean expression: write a sum of product terms where each term covers a single 1 in the truth table



- This representation is called the function's normal form
 - It is unique, but there may be simpler expressions
- Corollary: Boolean expressions can represent any combinational function