6.004 Tutorial Problems
L06 – Boolean Algebra and Logic Synthesis

Note: A small subset of essential problems are marked with a red star (★). We especially encourage you to try these out before recitation.

Problem 1. ★

Consider the truth table on the right, which defines two functions F and G of three input variables (A, B, and C).

For each function, write it in **normal form**, then find a **minimal sum of products** (minimal SOP) expression.

A	B	C	F	G
0	0	0	1	1
0	0	1	1	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	1
1	0	1	0	0
1	1	0	0	1
1	1	1	1	0

Normal form for F(A,B,C) = ____

Minimal sum of products for F(A,B,C) = ____

Normal form for G(A,B,C) = ____

Minimal sum of products for G(A,B,C) = ____

Problem 2. ★

Consider the 3-input Boolean function G(A,B,C) = $\bar{A} \cdot \bar{C} + A \cdot \bar{B} + \bar{B} \cdot \bar{C}$

1. How many 1's are there in the output column of G's 8-row truth table?

2. Give a minimal sum-of-products expression for G.

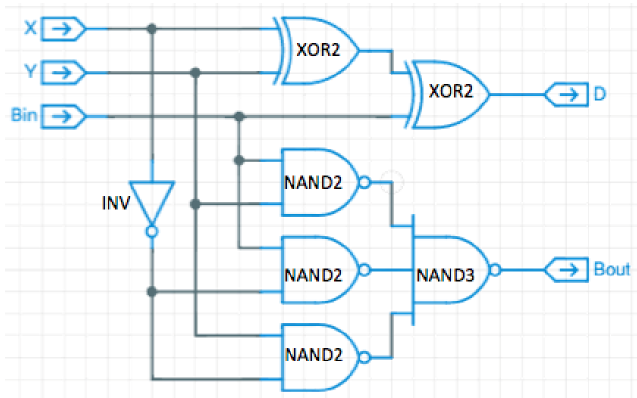
3. There's good news and bad news: the bad news is that the stockroom only has G gates. The good news is that it has as many as you need. Using only combinational circuits built from G gates, one can implement (choose the best response):

   (A) Any Boolean function (G is functionally complete)
   (B) Only functions with 3 inputs or less
   (C) Only functions with the same truth table as G

4. Can a sum-of-products expression involving 3 input variables with greater than 4 product terms *always* be simplified to a sum-of-products expression using fewer product terms?

Problem 3. ★

Consider the logic diagram shown below, which includes XOR2, NAND2, NAND3, and INV (inverter) gates.



Gate	t_{PD}
INV	1.0ns
NAND2	1.5ns
NAND3	1.8ns
XOR2	2.5ns

1. Using the t_{PD} information for the gate components shown in the table above, compute the t_{PD} for the circuit.

t_{PD} = __________ns

2. Find minimal sum-of-products expressions for both outputs, **D** and **Bout**.

   NOTE: The gates implement the following functions:
   - $NAND2(a,b) = \overline{a \cdot b}$
   - $NAND3(a,b,c) = \overline{a \cdot b \cdot c}$
   - $XOR2(a,b) = a \cdot \overline{b} + \overline{a} \cdot b$

Minimal sum of products for D(X,Y,Bin) = ____

Minimal sum of products for Bout(X,Y,Bin) = ____

Problem 4.

Simplify the following Boolean expressions by finding a *minimal sum-of-products expression* for each one:

1. $\overline{ac + b + c}$
2. $(a + b)c + \bar{c}a + b(\bar{a} + c)$
3. $a\overline{(b + c)}(b + a(b + c))$
4. $a(b + c(d + ef))$

Problem 5.

There are some Boolean expressions for which no assignment of values to variables can produce True (e.g., $a\bar{a}$). Those Boolean expressions are said to be *non-satisfiable*. Are the following Boolean expressions satisfiable? If the expression is satisfiable, give an assignment to variables that makes the expression evaluate to True. If the expression is non-satisfiable, prove it.

1. $(a + b)c + \bar{c}a + b(\bar{a} + c)$
2. $(x + y)(x + \bar{y})(z + \bar{y})(y + \bar{x})$
3. $(x + y + z)(x + y + \bar{z})(x + \bar{y} + z)(\bar{x} + y + z) \cdot$
   $(x + \bar{y} + \bar{z})(\bar{x} + y + \bar{z})(\bar{x} + \bar{y} + z)(\bar{x} + \bar{y} + \bar{z})$
4. $\overline{xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}}$

Problem 6.

(A) Simplify the following Boolean expressions by finding a minimal sum-of-products expression for each one. (*Note:* These expressions can be reduced into a minimal SOP by repeatedly applying the Boolean algebra properties we saw in lecture.)

1. $\overline{(a + b \cdot \bar{c})} \cdot d + c$

2. $a \cdot \overline{(b + c)}(c + a)$

(B) There are Boolean expressions for which no assignment of values to variables can produce True (e.g., $a \cdot \bar{a}$). These Boolean expressions are said to be *non-satisfiable*.

Are the following Boolean expressions satisfiable? If the expression is satisfiable, give an assignment to variables that makes the expression evaluate to True. If the expression is non-satisfiable, explain why.

1. $(\bar{x} + y\bar{z}) \cdot (\bar{y}x + z) \cdot (\bar{z}y + x)$

2. $(\bar{x} + y\bar{z}) \cdot (\bar{y}x + z) \cdot (\bar{z}y + x) + (\bar{x} + yz) \cdot (\bar{y}x + z) \cdot (\bar{z}y + x)$