6.004 Tutorial Problems L04a – Boolean Algebra

Problem 1

Write the truth tables for each of the following expressions

- A \overline{A}
- $\mathbf{B} \quad A \cdot B$
- **C** $\overline{A \cdot B}$ Hint: Consider DeMorgan's law!
- $\mathbf{D} \quad A + B$
- E $\overline{A+B}$
- $\mathbf{F} \quad (A + B) \cdot (\overline{A \cdot B})$
- $\mathbf{G} \quad (A \cdot S) + (B \cdot \overline{S})$

Problem 2

Consider the truth table on the right, which defines something called a full adder (more on that in a later lecture!). It has two functions C_{out} and S of three input variables (A, B, and C).

For each function, write it in **normal form**, then find a **minimal sum of products** (minimal SOP) expression.

Α	В	C _{in}	\mathbf{C}_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Normal form for $C_{out}(A, B, C_{in}) =$	
Minimal SOP for C _{out} (A, B, C _{in}) =	
Normal form for S(A, B, C _{in}) =	
Minimal SOP for S(A, B, C _{in}) =	

Problem 3

Consider the 3-input Boolean function $G(A,B,C) = \overline{A} \cdot \overline{C} + A \cdot \overline{B} + \overline{B} \cdot \overline{C}$.

A How many 1's are there in the output column of G's 8-row truth table?

B Give a minimal sum-of-products expression for G.

C Can a sum-of-products expression involving 3 input variables with greater than 4 product terms *always* be simplified to a sum-of-products expression using fewer product terms?

Problem 4

Simplify the following Boolean expressions by finding a *minimal sum-of-products* expression for each one:

A
$$A \cdot C + B + C$$

$$\mathbf{B}(A+B)\cdot C+\overline{C}\cdot A+B\cdot (\overline{A}+C)$$

$$\mathbf{C} A \cdot (B+C) \cdot (B+A \cdot (B+C))$$

D
$$A \cdot (B+C \cdot (D+E \cdot F))$$