Boolean Algebra

Useful Boolean Algebra Properties

- Using the axioms, we can derive several useful properties to manipulate and simplify Boolean expressions:

commutative	$a \cdot b = b \cdot a$	$a+b = b+a$
associative	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$	$a+(b+c) = (a+b)+c$
distributive	$a \cdot (b+c) = a \cdot b + a \cdot c$	$a+b \cdot c = (a+b) \cdot (a+c)$
complements	$a \cdot \overline{a} = 0$	$a+\overline{a} = 1$
absorption	$a \cdot (a+b) = a$	$a+a \cdot b = a$
reduction	$a \cdot b + a \cdot \overline{b} = a$	$(a+b) \cdot (a+\overline{b}) = a$
DeMorgan's Law	$\overline{a \cdot b} = \overline{a}+\overline{b}$	$\overline{a+b} = \overline{a} \cdot \overline{b}$

Useful Boolean Algebra Properties

- Many of these properties are easy to remember because they match the ones for integer algebra, but be aware of the differences
  - e.g., distributive property for Boolean "+" $a+b\cdot c = (a+b)\cdot(a+c)$ does not hold for integer "+"!

- To familiarize yourself with the properties, we recommend that you simply prove them
  - *Example: DeMorgan's Law*

a	b	$\overline{a\cdot b}$	$\overline{a}+\overline{b}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Equivalence and Normal Form

- Given a truth table, it is easy to derive an equivalent Boolean expression: write a *sum of product* terms where each term covers a single 1 in the truth table

C	B	A	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$Y = \bar{C}\bar{B}A + \bar{C}BA + CB\bar{A} + CBA$$

- This representation is called the function's normal form
  - It is unique, but there may be simpler expressions
- Corollary: Boolean expressions can represent any combinational function