1 Selfstudy for Tutorial 1

Polking **§2.2:** 3, 8, 15, 17; **§2.4:** 3, 9, 14, 18; **§2.6:** 9, 11, 19

2 Exercises for Tutorial 1

1. Consider the differential equation

$$ty' = 2y - t.$$

   (a) Determine the general solution for this equation. Sketch some of the solutions of this family of curves.

   (b) Try to find a solution satisfying the initial value $y(0) = 2$. If not possible, explain why.

2. (a) Determine two solutions of the initial value problem

$$x' = tx^{1/3}, \qquad x(0) = 0.$$

   (b) Show this initial value problem has infinitely many solutions.
   Hint: Glue the two solutions together at some particular point and show this solution is differentiable at that point.

3. Consider Bernoulli's equation [1]

$$x' = a(t)x + f(t)x^n, \qquad n \neq 0, 1. \tag{1}$$

   (a) Use the substitution of variables $z = x^\alpha$ to a obtain a differential equation for z.

   (b) For which value of α do you arrive at a linear inhomogeneous ODE?

   (c) Solve this new ODE for $n = 2,\ f(t) = t,\ a(t) = 1$.

   (d) Using the inverse transformation $z \to x$, write down all solutions of (1).

   (e) Make a sketch of the direction field for $n = 2, f = t, a = 1$ and some solution curves.

4. We may find the equation $P(x, y) + Q(x, y)\frac{dy}{dx} = 0$ is not exact, but there could be a function μ such that the equation

$$\mu(x, y)P(x, y) + \mu(x, y)Q(x, y)\frac{dy}{dx} = 0, \qquad \mu(x, y) \neq 0$$

   is exact though obtaining μ is not always possible. We refer to such a μ as an *integrating factor*, and then we can solve the new equation.

   (a) Write down the condition for μP and μQ such that $\mu P + \mu Q y' = 0$ is exact.

   (b) Assume the differential equation $(xy - 1) + (x^2 - xy)\frac{dy}{dx} = 0$ has an integrating factor depending on x only, i.e. $\mu = \mu(x)$. Determine this integrating factor and find the general solution for this particular ODE.

[1] See also *Polking*, Exercise 2.4.22

5. (If time permits. Practicing substitution of variables) Consider the following general differential equation

$$P(x, y) + Q(x, y)\frac{dy}{dx} = 0.$$

A function is homogeneous of degree n if $G(\alpha x, \alpha y) = \alpha^n G(x, y)$.[2]

(a) Check that the coefficients P and Q in the following equation are homogeneous

$$y(ax + by) - ax^2 y' = 0. \tag{2}$$

(b) Apply the substitution of variables $y = xv(x)$ to equation (2) and simplify the ODE for v.

(c) Solve the differential equation (2); first for v and next for y.

[2]For more details see *Polking*, §2.6, p73.