Homework assignment 1 – Mathematical Statistics 2020 (Probability Theory and estimation)
Hand in ***your own solutions at the start of the tutorial*** *on September 11.*
Suppose that $X, X_1, \ldots, X_n$ are i.i.d. (independent and identically distributed) with probability density function

$$f(x) = \begin{cases} \frac{4\theta^4}{x^5}, & \text{if } x \geq \theta \\ 0, & \text{if } x < \theta \end{cases}$$

a) Determine the first three moments $E(X)$, $E\left(X^2\right)$, $E\left(X^3\right)$ and $var(X)$.

b) Express $E(X-\mu)^3$ in the first three moments in order to determine the skewness coefficient γ_1: Is the skewness visible in the graph of f (to the left or to the right?)?

c) Derive the density function of $Y = \sqrt{X}$ (define in parts!) and compute $E(Y)$.

d) Recall that $\overline{X} = \frac{X_1+\cdots+X_n}{n}$. Show that $\frac{3}{4}\overline{X}$ is an unbiased and consistent estimator of θ.

e) Consider all estimators $T = a \cdot \overline{X}$, with real constant $a \in \mathbb{R}$. For which value of a is the mean squared error for θ minimized?

To simplify computations we will consider a small sample of only $n = 2$ observations.

f) Determine the density function of $M = \min(X_1, X_2)$ and express $E(M)$ and $var(M)$ in θ.

g) Which of the estimators of θ, $\frac{3}{4}\overline{X}$ in part d. or M in part f., has the smaller mean squared error?

Grading:	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**Total**
	4×½	½+1	1+½	½+1	1½	1	1	**10**