# 1 Selfstudy for Tutorial 3

*Polking* §**9.2:** 2, 6, 15, 17, 31; §**9.3:** 11, 14, 17, 23; §**9.5:** 7, 10

# 2 Exercises for Tutorial 3

1. Let us look at the harmonic oscillator with mass $m = 1$, spring constant $k > 0$ and damping $b \geq 0$
$$x'' = -kx - bx'.$$
   (a) Rewrite this second order ODE as a first order system.
   (b) Give conditions on $k, b$ such that the system has complex roots, double roots or real distinct roots.
   (c) Determine the general solution in each case.
   (d) Describe the motion of the mass as it is released from rest $x(0) = 1$ and $x'(0) = 0$ for each of the three cases (1b). Sketch the time series $(t, x(t))$ as well as the phase portrait in the $(x, x')$-plane.

2. Below we list four matrices. We have given the corresponding eigenvalues and eigenvectors, to focus on the outcome, but it may be a useful exercise to derive these yourself.
   a For each system $x' = Ax$, write down the general (real!) solution
   b Determine the type of each system, i.e. classify the origin according to the eigenvalues.
   c Sketch the phase portrait.

   (a) $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$, $\lambda_1 = 1$, $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\lambda_2 = 3$, $v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.
   (b) $A = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$, $\lambda_1 = 2$, $v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\lambda_2 = -1$, $v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.
   (c) $A = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$, $\lambda_1 = -5$, $v_1 = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$, $\lambda_2 = 2$, $v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.
   (d) $A = \begin{pmatrix} 2 & 5 \\ -5 & -4 \end{pmatrix}$, $\lambda_1 = -1 + 4i$, $v_1 = \begin{pmatrix} 5 \\ -3 + 4i \end{pmatrix}$, $\lambda_2 = -1 - 4i$, $v_2 = \begin{pmatrix} 5 \\ -3 - 4i \end{pmatrix}$.

3. Solve the initial value problem $y' = A_i y,\ y(0) = y_0,\ i = 1, 2$ for
$$A_1 = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \quad \text{en} \quad A_2 = \begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix}.$$

4. Given the matrix $A = \begin{pmatrix} -2 & 1 & -1 \\ 1 & -3 & 0 \\ 3 & -5 & 0 \end{pmatrix}$.
   find a set of fundamental solutions for the system $x' = Ax$. You will have to resort to generalised eigenvectors, i.e. $Aw = \lambda w + v$.